ARTICLE NO. 80-4010

C80-019

Use of Multiblade Coordinates for Helicopter Flap-Lag Stability with Dynamic Inflow

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Rotor flap-lag stability in forward flight is studied with and without dynamic inflow feedback via a multiblade coordinate transformation (MCT). The algebra of MCT is found to be so involved that it requires checking the final equations by independent means. Accordingly, an assessment of three derivation methods is given. Numerical results are presented for three- and four-bladed rotors up to an advance ratio of 0.5. While the constant-coefficient approximation under trimmed conditions is satisfactory for low-frequency modes, it is not satisfactory for high-frequency modes or for untrimmed conditions. The advantages of multiblade coordinates are pronounced when the blades are coupled by dynamic inflow.

Nomenclature

a	= slope of lift curve, rad^{-1} = 2π
C_{d_0}	= blade profile drag coefficient
C_L	= harmonic perturbation of roll moment
C_M	= harmonic perturbation of pitch moment coefficient
C_T	= harmonic perturbation of thrust coefficient (in figures also refers to steady value of thrust coefficient)
$\bar{f}$	= helicopter flat plate drag area/πR^2
F_{β_K}	= dimensionless force per unit length, perpendicular to blade and also to direction of rotation
k_m, k_I	= dimensionless apparent mass and inertia of an impermeable disk
m	= number of degrees of freedom per blade
M_k	= unsteady moment component from the kth blade at the rotor hub
n	= number of degrees of freedom of system
N	= number of blades
$N(\psi)$	= inflow coupling matrix, Eq. (6)
p	= dimensionless rotating flapping frequency = $\sqrt{1+\omega_\beta^2}$
P_k	= flapping stiffness of kth blade
r	= radial distance
R	= rotor radius
T_k	= unsteady thrust component from the kth blade at the rotor hub
$\bar{v}$	= dynamic inflow parameter, Eq. (3)
$\{X\}$	= vector of state variables, Eq. (5)
$\{U\}$	= vector of inflow parameters, Eq. (5)
Z_k	= stiffness parameter (equal to zero for zero elastic coupling or $\theta_k = 0$)
$\beta_k(\zeta_k)$	= perturbation flapping (lead-lag) angle of the kth blade
$\bar{\beta}_k$	= equilibrium flapping angle
$\beta(\zeta)$	= flapping (lead-lag) coordinate
β_{pc}	= precone angle
γ	= blade lock number
η	= real portion of lead-lag eigenvalue or lead-lag damping
θ_k	= pitch angle of the kth blade, $\bar{\theta}_k + \theta_\beta(\beta_k - \beta_{pc}) + \theta_\zeta \zeta_k$
$\bar{\theta}_k$	= equilibrium pitch angle of the kth blade, $\theta_0 + \theta_I \cos\psi_k + \theta_{II} \sin\psi_k + \theta_\beta(\bar{\beta}_k - \beta_{pc}) + \theta_\zeta \zeta_k$
$\theta_\beta, \theta_\zeta$	= pitch-flap and pitch-lag coupling ratios
λ	= steady inflow ratio (= ¼ ϕ)
μ	= rotor advance ratio
ν	= inflow perturbation
ν_0, ν_I, ν_{II}	= uniform, longitudinal, and lateral components of induced flow
$\bar{\nu}$	= induced inflow due to steady rotor thrust
ρ	= air density
σ	= rotor solidity
ψ	= azimuth position, dimensionless time
ψ_k	= azimuth position of the kth blade
$\omega_\beta, \omega_\zeta$	= dimensionless rotating flapping frequency
Ω	= rotor angular speed
{ }	= vector
[ ]	= matrix
$(\dot{\ })$	= $\dfrac{d}{d\psi}$

Received Jan. 29, 1979; revision received June 1, 1979. Reprints of this article may be ordered from AIAA Special Publications, 1290 Avenue of the Americas, New York, N.Y. 10019. Order by Article No. at top of page. Member price $2.00 each, nonmember, $3.00 each. **Remittance must accompany order.**

Index categories: Helicopters; Aerodynamics; Nonsteady Aerodynamics.

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Introduction

IN multiblade coordinate transformation (MCT) individual blade deflections are represented by a finite Fourier series in the azimuth angle. The coefficients of this series are nonrotating blade coordinates which describe overall rotor motions. MCT provides a natural reference frame for rotor equations because perturbations due to the dynamic wake, fuselage, or active controls couple with the rotor motion in the form of nonrotating feedback effects. An N-bladed rotor, each blade having m degrees of freedom, is described exactly by Nm multiblade coordinates; each deflection is expressed via N multiblade coordinates: collective, differential (only for N even), and first-, second-, and higher-order longitudinal and lateral cyclic components. For rotors with an even number of blades, the differential component remains in the rotating system, but is reactionless for $N>2$ and does not