Use of radiation pressure for measurement of high-power laser emission

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We demonstrate a paradigm in absolute laser radiometry where a laser beam's power can be measured from its radiation pressure. Using an off-the-shelf high-accuracy mass scale, a 530 W Yb-doped fiber laser, and a 92 kW CO_2 laser, we present preliminary results of absolute optical power measurements with inaccuracies of better than 7-13 %. We find negligible contribution from radiometric (thermal) forces. We also identify this scale's dynamic-force noise floor for a 0.1 Hz modulation frequency as 4 $\mu N/Hz^{1/2}$, or as optical power sensitivity, 600 $W/Hz^{1/2}$.

In the 50 years since the invention of the laser, fundamentally accurate measurements of laser output power have been calorimetric in nature. That is, a sensor head maximally absorbs the laser light and the laser energy or power is measured from the change in temperature at the target. This technique affords impressive 0.01 – 1 % absolute accuracies [1, 2] but becomes increasingly difficult as laser powers increase above roughly 10 kW. We describe here a different paradigm in which the laser power is measured not by absorbing it, but rather by reflecting it and measuring the total radiation force experienced by the mirror. We show that for power levels above a few hundred watts, this measurement can be carried out with a portable, "off-the-shelf" mass-weighing scale. This presents the possibility of low-cost, accurate, power-scalable, and fast characterization of high laser powers.

In the most general sense, accurate optical power measurements require a comparison between optical power and a separate fundamental quantity. In traditional laser calorimeters this quantity is electrical energy. An optically-absorbing thermal mass is injected with laser light, and that optical energy is measured by comparing the calorimeter's temperature increase to that induced by an equivalent injection of electrical energy. With this kind of thermal design, the size and response period of a calorimeter will scale linearly with its optical energy capacity. As an example, calorimeters capable of measuring up to hundreds of kilojoules of optical energy have volumes on the order of 1 m^3 and response periods of tens of minutes and even longer recovery (cooling) periods. An improvement to this is a flowing-water power meter where laser light is injected into an absorbing cavity and the heat is removed by rapidly flowing water. The optical power is measured from the water's flow rate and temperature change [3]. This gives an improved response time (tens of seconds) that is less dependent on optical power capacity, but the size (cubic meters) still scales linearly with power.

Measuring optical power by use of radiation force, on the other hand, is dramatically simpler because it eliminates the requirement to absorb the laser power, making the basic design at least an order of magnitude smaller (determined only by the laser beam diameter, rather than the optical power). Absolute optical power can be measured fundamentally by comparison to a reference force or equivalent mass. Such reflective power measurements, are non-perturbative, in the sense that laser power can be assessed in real time while the fully-reflected light is still available for use. The slow process of heating and cooling massive calorimeters is no longer required, enabling fast, power-independent response periods with no recovery period.

The concept of radiation pressure has long been understood and demonstrated [4-6] – photons carry momentum, so a laser beam reflecting from a mirror imparts a radiation force F due to the change in photon momentum, which is proportional to the optical power P,

$$F = \left(2P/c\right) r\cos(\theta), \qquad (1)$$

where $r = R + (1-R)\alpha/2$ accounts for the fact that an absorbed photon imparts all its momentum, and a reflected photon imparts twice its momentum. R is the mirror reflectivity, α indicates the fraction of non-reflected light absorbed by the mirror, θ is the angle of incidence, and c is the speed of light. Equation (1) describes a maximum power-to-force conversion factor of $2/c = 6.67\times10^{-9}$ N/W for normal incidence on a perfectly reflecting mirror.

The idea of radiation pressure as a means of measuring optical energy has been proposed before [7-11]. However, all of these previous designs were based on a torsion balance, which has several limitations. These include a slow time response due to long oscillation periods (tens to hundreds of seconds [12]), difficulty in scaling to larger beam diameters, operation in a vacuum environment, and inability for fundamental calibration to a force standard. Today, with industrial, defense, and research laser power levels from kW to MW, and robust commercial scales with precisions of 1 nN (0.1 μg), a radiation-pressure approach to absolute power measurement in high-power lasers becomes practical.

Our goal in this work is to demonstrate that radiation pressure can be measured for kilowatt-level laser powers with existing scale technology. Our experimental setup is shown in Figure 1. The scale was an off-the-shelf direct-loading force restoration balance [13] with a 100-nN (10-μg) resolution. Our 530 W Yb-doped fiber laser source, centered at 1071 nm with a 5 nm full-width-at-half-maximum bandwidth, is capable of providing 3.6 μN of radiation force. This is at the lower end of our scale's operation range, but provides sufficient signal-to-noise ratio to demonstrate feasibility. The weakly focused light impinges on a first-surface dielectric mirror (25 mm diameter) mounted on the shaft of the scale. The angle of incidence θ of the laser beam with respect to the mirror normal was adjustable between 7° and 45°.We measured the mirror reflectance to be R = 0.997 over this angular range at the operating wavelength.

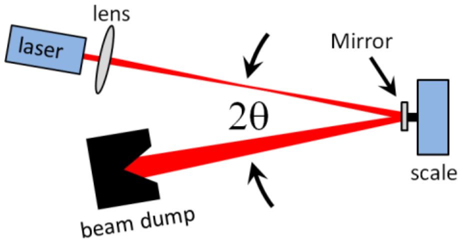

Figure 1. Layout of radiation pressure measurement. In this work, the laser beam propagates entirely in a horizontal plane, requiring the scale to operate in a "vertical" orientation with the mirror surface in a vertical plane.

The direct-loading balance design is integral to the practical operation of this radiation-pressure power meter. The direct-load mechanism [13] does not require gravity for its mechanical operation, so, by removing the spring that compensates for the weight of the balance pan, we were able to operate the balance in an orientation where the force is measured horizontally. This significant feature allows the laser beams to travel and be measured in a horizontal direction (parallel to the plane of the floor). This avoids vertically propagating light, which is a practical laser safety issue, particularly important at the highest laser powers.

To isolate the radiation force from noise effects due to temperature drift and ambient air current, the amplitude of the laser was current-modulated (100 % depth) with a 0.1 Hz square wave. The scale's internal electronics include a digitizer circuit with an effective response period of ~ 2 s. The modulated laser beam irradiated the mirror for 5-10 minutes for signal averaging, while the digitized scale reading was acquired at a sample rate of 18 Hz and converted from mass to force by use of the gravitational constant g = 9.80 m/s². Typical raw output is shown in Figure 2, displaying a drift due to thermal effects and random air currents. We extracted (in software) the amplitude of the modulated radiation force *Fopt* from the drifting background by use of a quadrature heterodyne approach:

$$F_{opt} = \frac{g\pi}{2}\sqrt{\left\langle m(t)\sin(\omega t)\right\rangle^2 + \left\langle m(t)\cos(\omega t)\right\rangle^2} \qquad (2)$$

where $m(t)$ is the time-dependent mass reported by the scale, ω is the angular modulation frequency of the laser, and brackets indicate a time average over the laser injection interval. The force on the scale was measured as a function of laser power from 0 to ~530 W. Results are shown as solid circles in Figure 3, where the horizontal axis is effective optical power $PR\cos(\theta)$, and the solid line indicates the theoretical prediction of Equation (1). As an estimate of the noise, with no laser power incident on the scale, we measure 0.1 μN of force. Within this noise, we see a linear response to laser power with a best-fit slope of 6.23×10-9 N/W (~7 % below the theoretical 2/c value).

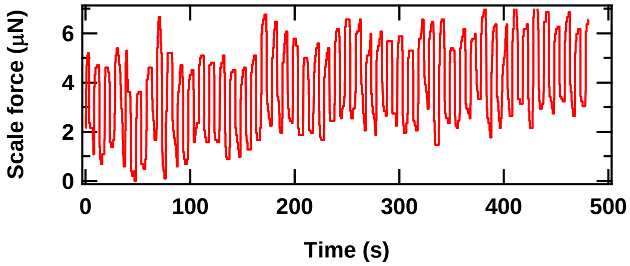

Figure 2. Scale reading (in force units) for 530 W incident power modulated at 0.1 Hz. The average y-axis value is arbitrary since it includes the initial (non-zero) mass reading of the scale.

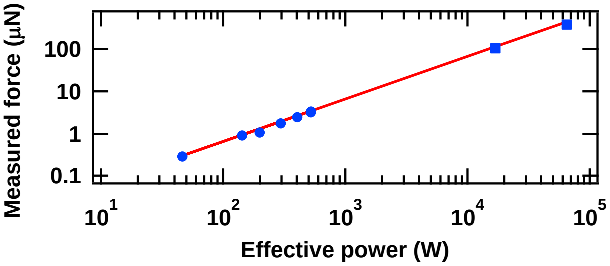

Figure 3. Measured radiation force versus effective laser power (includes mirror reflectivity and angle of incidence). Circles are 1071 nm laser, squares are 10.6 μm laser. Solid line indicates the theoretical force-to-power slope.

To determine the upper power range, we operationally tested our meter with a 100 kW cw CO2 laser. In order to accommodate the larger 10 cm beam diameter without drastically increasing the mirror mass, we used a 20-cm diameter silicon wafer coated with gold and a surface dielectric layer, providing R=0.998 at 10.6 μm and 45 ° incidence angle. With the increased mirror area the balance was particularly susceptible to slight air currents and so it was protected by a windowless housing with cylindrical baffles surrounding the input and output light paths. We measured the resultant force on the scale for a 3-s exposure of unmodulated 24 kW and 92 kW laser powers at a nominal 45 ° angle of incidence. The latter is shown in Figure 4. We experienced no damage during the exposure, but did see a delayed drift in the scale background force reading, which might be due to a temperature rise in the scale mechanical system due to residual heating effects from the light. The measured force was 13 % lower than expected, and the results are shown as squares in Figure 3.

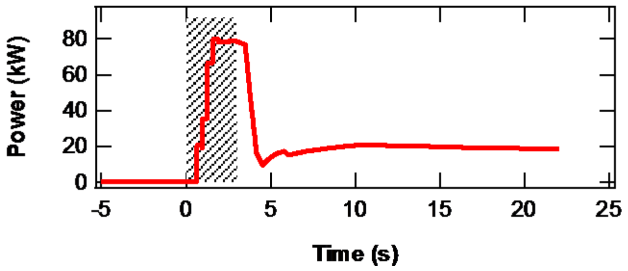

Figure 4. CO_2 laser power vs time (for ~ 92 kW injected power) measured using radiation pressure. Red curve is scale output (corrected for mirror reflectivity and angle of incidence), shaded region indicates injection duration.

We evaluate the significance of the discrepancies in the measured force-to-power ratio by considering the measurement uncertainty. For the 1071 nm fiber laser measurements, the incident laser power was measured indirectly and is known to only ~2 % uncertainty. An advantage of a true mass-reading scale (in contrast to a torsion balance) is that the force reading can be calibrated with a simple mass artifact. Our scale was calibrated with a known mass of 20 g (0.196 N) and its accuracy near the operation range was tested by comparison with a set of calibrated masses down to 1 mg (10 μN), see Figure 5. But the 530 W fiber laser's radiation force ranged from 0.3 to 3 μN, below our verification range. Therefore, we have no direct measurement of the scale's error in this range, but estimate it to be at least 2 % based on the increased disagreement seen in the comparison over the 10-50 μN range (Figure 5). We expect that operating the scale in the non-standard "horizontal force" orientation should have a negligible effect on its calibration due to supporting flexures operating independent of gravity. The uncertainty on the measured mirror reflectivity is only a few tenths of a percent. We estimate the total measurement uncertainty to be on the order of several percent, which is commensurate with the measured 7 % disagreement with theory. More work is required to determine the measurement uncertainty down to the one-percent level.

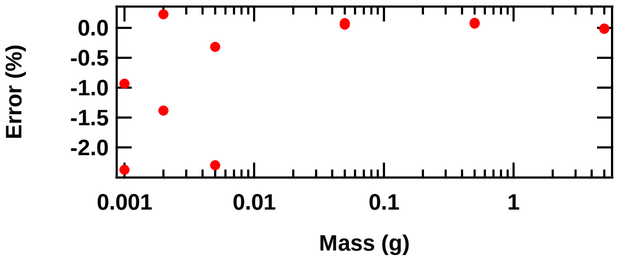

Figure 5. Scale verification showing error as percent disagreement of scale readings with calibrated masses. Repeated measurements indicate measurement variability.

For the CO2 laser, the true power was known only to 6 % uncertainty, and the incidence angle known only to 5 ° (8 % amplitude uncertainty) yielding a total uncertainty of 10 %, which roughly agrees with the 13 % discrepancy measured.

Although radiation force is commonly used as a tool in applications ranging from remote force application [14] to laser cooling [15] to "tractor beams" [16], its accurate measurement is often obscured by "radiometric" forces wherein air flows from the cold side of the mirror to the hot, creating a secondary force that does not follow from Equation (1) [17]. In the past, these radiometric effects have been reduced by operating the measurement in vacuum. However, this is not practical for our application as a robust power meter, and in fact, we find that our measurement conditions render such radiometric effects negligible. We demonstrate the dominance of true radiation forces below, but hypothesize several reasons for the lack of significant radiometric forces. First, our high-reflectance mirrors allow only a very small fraction (0.002-0.003) of the incident light to be available for conversion to heat in the mirror itself. Furthermore, the glass-substrate dielectric mirror (1071 nm) transmits the majority of the unreflected light with little absorption. And in both the 25 mm dielectric mirror and the 200 mm Si substrate mirror, the beam underfills the mirror, meaning any differential (front-to-back) heating that does occur will be relatively far from the mirror edge where this effect takes place, reducing the effect. Nor does the curious rise in measured force for the CO_2 laser (beginning just before 5 s in Figure 4) seem to be a radiometric, effect since the more than 15 s duration of the effect should be more than sufficient to establish equilibrium between mirror front and back, eliminating the molecular flow.

We demonstrate empirically the lack of radiometric forces through two experiments. First, since photon momentum is a vector quantity and the radiometric heating effect is scalar (independent of the light's angle of incidence), the radiation force imparted to a mirror can be identified by its cosine dependence on the incident angle. We launched ~ 525 W of optical power (1071 nm) onto the dielectric mirror at incidence angles from ~7° to 45°. The measured force (Figure 6) agrees well with the expected cosine behavior, indicating that the measured force is due to radiation pressure and not a heating effect.

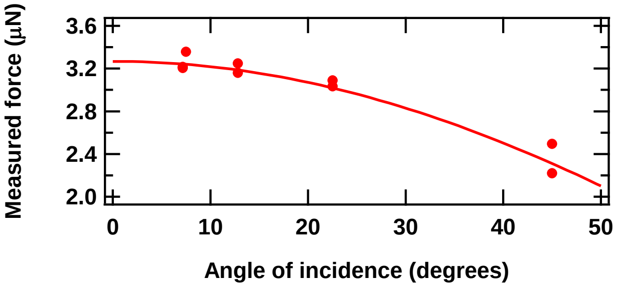

Figure 6. Measured force amplitude (solid circles) for various angles of incidence demonstrating expected cosine dependence (solid line).

As a second means of verifying radiation pressure as the source of the measured force, we drastically reduce the incident optical power but increase the absorbed optical power. This will reduce the radiation pressure signal but increase any thermal effects. We replace the mirror of Figure 1 with an ~ 80 % absorbing disc (BK-7 glass flat coated with a carbon nanotube absorbing layer). We direct 4.6 W of modulated laser light (1071 nm) onto the absorber. The 3.7 W of absorbed power is equivalent to the amount of light available for absorption if 1.2 kW were incident on our dielectric mirror. If the modulation seen in Figure 2 is due to an absorption-based mechanical effect, we should see more than twice the effect here. If it is truly due to radiation force, we should see no scale signal. In fact, we see no signal at the 0.1 Hz modulation frequency (Figure 7). These results demonstrate that the modulation seen in Figures 2 and 3 is indeed due to radiation force and not radiometric effects, and sets an upper limit to the size of such effects at 17 dB below the radiation force signal (calculated from the signal-to-noise ratio of Figure 7 plus 3.8 dB to scale the signal level to the effective 1.2 kW).

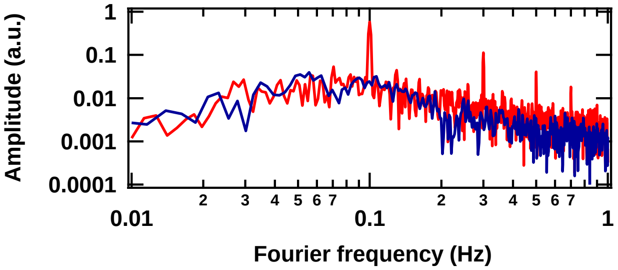

Figure 7. Force spectrum from scale with 0.1 Hz modulated laser power, 533 W incident on mirror (red) and 4.6 W incident on absorber (blue). With no signal, we treat the absorber curve as our noise floor.

Using the signal and noise spectra of Figure 7 and knowing the peak at 0.1 Hz corresponds to a 3.3 μN force, we identify this scale's dynamic force noise floor as $4\mu N/Hz^{1/2}$ at 0.1 Hz modulation frequency, or, in terms of optical power, $600\ W/Hz^{1/2}$. This picture of the noise floor allows investigation of other applications. The nominal 1/*f* frequency dependence of the noise floor (above ~ 0.1 Hz) makes it attractive to consider a reverse calibration (calibrate scale response by using optical power). Generally milligram-level scale calibration is a difficult process involving a static "weighing" of a known mass at zero frequency where the noise floor is a maximum. This "DC" measurement requires that calibrations be done under strict environmentally stable conditions. However, if the reference "mass" were instead applied as modulated laser power, the signal-to-noise ratio could be instantly improved by a factor proportional to the ratio of modulation frequency to measurement bandwidth. This could provide rapid and high-accuracy measurements of scale linearity by use of the laser as a force dither of a well-known amplitude much smaller than available calibrated masses.

Targeting industrial use, high-power laser welding applications can require laser pulse-energy characterization accuracies as low as 1 %. An optical energy meter capable of accurately measuring individual laser pulses (e.g., 10 J/pulse and 10 Hz repetition rate) would be very useful. Again, assuming a simple 1/f noise extrapolation and appropriate filtering, the scale demonstrated here would need an approximately 13 dB reduction in noise level to perform a 1 % measurement of a 10 J pulse. However, commercial scale technology is not optimized for AC measurements and improvements should come relatively easily.

We have shown that optical radiation-force measurements using a commercial scale are feasible for measurement of laser power. Future investigations will more carefully determine the ultimate accuracy of this technique and what modifications may be needed to enable other applications.

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