Use of simple finite elements for mechanical systems impact analysis based on stereomechanics, stress wave propagation, and energy method approaches[†]

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Abstract

This paper examines the effectiveness of analyzing impact events in mechanical systems for design purposes using simple or low ordered finite elements. Traditional impact dynamics analyses of mechanical systems namely stereomechanics, energy method, stress-wave propagation and contact mechanics approaches are limited to very simplified geometries and provide basic analyses in making predictions and understanding the dominant features of the impact in a mechanical system. In engineering practice, impacted systems present a complexity of geometry, stiffness, mass distributions, contact areas and impact angles that are impossible to analyze and design with the traditional impact dynamics methods. In real cases, the effective tool is the finite element (FE) method. The high-end FEA codes though may be not available for typical engineer/designer. This paper provides information on whether impact events of mechanical systems can be successfully modeled using simple or low-order finite elements. FEA models using simple elements are benchmarked against theoretical impact problems and published experimental impact results. As a case study, an FE model using simple plastic beam elements is further tested to predict stresses and deflections in an experimental structural impact.

Keywords: Impact dynamics; Stereomechanics; Stress wave; Energy method; FEA impact simulation; FEA impact benchmarks

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1. Introduction

The nature of impact involves a minimum of a two body mechanical system. Impact phenomena is quite varied depending on the velocity of the impactor and the characteristics of the target or struck system. Zukus [1, 2] presents that for high velocity impacts, the response of the impacted system to the impact is very local and is highly dependent on the constitution of the material type impacted. For low velocity impacts, both the geometry and material respond to the impactor. Shivaswamy [3] showed through his experimental work that the nature of impacts involves very short contact times combined with large contact forces. In addition, the stiffness of the impacted system greatly influences the contact time and force. It was shown that contact force, contact time and system stiffness are inversely related. As system stiffness increases, contact forces increase, but contact time decreases.

In an attempt to classify at what defines an impact condition, Blake [4], Faupel [5] and Juvinall [6] present loading classifications. For a load whose rise time is greater than three times the fundamental period of free vibration of the mechanical system, the loading is considered static and conventional deflection and stress analysis techniques using classical material properties apply. For a loading rise time of less than three, but greater than one and one-half times the fundamental period, is classified as rapid loading. Vibration methods for stress and deflection analysis should be incorporated for rapid loading conditions. However, in practice for rapid loading conditions, classical methods of static stress and deflection analysis are used with multiplying the stresses and deflections by a factor of two. Craig [7] demonstrates how the factor of two rule is derived from classical vibration theory. If the time of load application is less than 0.5 times the fundamental period of the mechanical system, the loading is defined as impact. The static methods of stress, strain and deflection analyses are meaningless under impact conditions. This is due to propagation, reflection and interference of elastic/plastic waves traveling within the engineering solid. Accurate calculation of stresses and strains need to be based on wave analysis methods, which are exceedingly complex for practical use and thus used to solve a limited number of simple cases [6]. Other methods such as contact mechanics, energy methods and the FEA method are required to be implemented in order to estimate the effects of impact conditions on mechanical systems that exhibit any degree of complexity.

Faik [8] divides the modeling of impact systems into four

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analysis methods. Depending on the parameters desired from the analysis such as: velocities, stresses, deflections, plastic deformation or energy absorption along with the types of simplifying assumptions about the impact event, each method has its advantages and disadvantages. These four basic methods are: 1) Stereomechanics, 2) Stress Wave Propagation, 3) Contact Mechanics and 4) Plastic Deformation Methods.

Stereomechanics is the application of classical Newtonian mechanics to impacting bodies to predict pre and post-impact velocities. This method uses the conservation of energy and momentum laws along with the impulse-momentum law [9]. The momentum law is a vector equation. For collisions where energy is dissipated, the method uses the coefficient of restitution. The coefficient of restitution is used to relate the effect between the pre and post-impact velocities as the result of energy dissipation. The advantage of stereomechanics is that it is algebraic and thus, easy to apply and accessible to practicing engineers. The disadvantage is the lack of analytical tools to define the coefficient of restitution [10], thus collisions involving energy dissipation are typically analyzed experimentally as evident by vehicle crash testing. Another disadvantage is that neither the contact duration nor the contact force is predicted by stereomechanics approach.

Stress wave theory addresses the phenomenon when an impactor strikes elastic solid. At impact, a strain wave is initiated at the contact region and transverses or radiates throughout the solid at a velocity of $(E/\rho)^{1/2}$, the speed of sound in a solid where E is the Young's modulus and ρ is the mass density of the struck material. As time progresses, these original waves contact the boundary surfaces of the solid and reflect inward. This generates standing or interfering strain waves, which produce the larger strains and stresses associated with impact condition than that of ordinary static loading. In theory, the advantage of stress wave method is an accurate stress analysis on the impacted elastic solid. Also, the variation of local strain/stress levels in the solid can be identified as a function of time and space. On the other hand, stress wave propagation is highly mathematical and requires a large amount of simplification of the impacted mechanical system. This limits it to mainly one-dimensional problems such as a rigid body impactor impacting the end of an elastic cantilever rod.

The contact mechanics approach to impact of a mechanical system is through the examination of the contact stress in the contained area of deformation between the colliding bodies. Contact mechanics uses force-deformation equations to estimate local stresses along with elastic/plastic deformations and contact duration times. The method originated with Hertz theory of elastic contact for two spheres in contact [1, 11]. Several investigators including Lankarani, have extended the Hertz theory to include the effects of this contained plastic deformation [12]. The force-deformation equations have also included viscous terms to model energy dissipation at the contact area as discussed by Wu [13]. Contact mechanics is a very powerful method of introducing the forces developed in im-

pacts to be included in multibody dynamics analysis. The disadvantages are the analytical selection of a force-deformation equation and establishing the parameters, which are required, to define the force-deformation equation. Shivaswamy [3] does provide an experimental method to determine parameters, which govern contact force-deformation equations. However, this method does not yield global stresses or displacement results. The classical system solved by contact mechanics is the mid-span impact of a simple beam with a sphere. This problem includes the effect of the indention of the sphere in the beam. Timoshenko [14] was the first to examine this impact scenario. No closed solutions exist for this problem; it has been solved by a numerical method [15].

For impacts where plastic strains occur outside of the contact area, some form of plastic analysis is required. The perfect elastic-plastic analysis exhibits the most practicality in predicting large plastic strains due to impact loading. Bohnenblust [16], Conroy [17, 18] and Symonds [19] developed methods to analyzed beams undergoing plastic deformations using rigid perfect-plastic and elastic-perfect plastic material constitutes.

Faik [8] ignores the use of energy methods to understand impact problems. Blake [4], Faupel [5], and Junvinall [6] all present treatise on the energy method, sometimes referred to as the equivalent load method. The basis for the method is that mechanical energy is conserved in the elastic compression of the impacted system bodies. The kinetic energy of the impactor body is converted into potential energy, which is stored within the struck body or structure. The maximum deflections and stresses of the impacted system occur when the velocity of the impactor is become zero, as all of the impactor's energy has been transferred to target body. Equating the impactor's energies to the system's strain energy yields the maximum deflection of the system. From the known deflections, the stresses of the mechanical system can be approximated as the result of the impact, thus aiding in design of the system. The advantage of the energy method is its practicality. The disadvantage is that the basic energy method ignores the inertial effects of the target and energy conversion. Cox [20] improves on the basic energy method by including inertial effects of the target.

As discussed, these classical methods are quite limited to simple impacts of mechanical systems. The use of the FEA method is the analysis tool of choice for examining an impacted system. FEA has the capability of solving complex systems for which the classical methods cannot. However, incompetently using FEA without knowledge of its inner workings and proper modeling techniques can be disastrous. Zukus [2] evaluating young engineers found it took several months of familiarity of the FEA code before reasonable solutions to practical engineering problems could be achieved. Element types [1], aspect ratios [21], element arrangement [22], mesh type and quality [23], and time step profoundly impact the accuracy of an impacted system when using the FEA method to solve it. This paper extends the use of classi-

cal impact analysis approaches for simple systems, namely the stereomechanics, stress wave propagation, and energy method, to more general mechanical systems by the incorporation of simple or low ordered finite elements.

Simple finite elements are element types that possess the following characteristics: 1) low-order DOF vs high-order DOF, 2) simpler spatial representation verses complex spatial representation or 3) low-order integration verses high order integration.

For low order DOF element simplification, the use of 2D planar and 3D solid elements without mid-side nodes will be shown to produce engineering acceptable prediction (typically within 5%) to closed solution to classical impact problems. An example of a low order simplification is a quadrilateral planar element which would exhibit a total of 8 DOF (4 nodes, 2 DOF each node) per element. An example of a higher order DOF element would be a quadrilateral planar element with mid-nodes which would exhibit a total 16 DOF (8 nodes, 2 DOF each node) per element. Order size is important at computational computer time increases linearly with of the number of DOF in the model. Thus the 16-DOF element would require a twofold computation effort and computer memory size per element than that of the simple 8-DOF element which is important for non-linear FEA methods used for impact analysis on a PC basis.

For simpler spatial representation, the use of axisymmetric elements which produces a 2D computation field can represent a 3D geometry. Using more spatial complex elements such 4, 6 and 8-node solid elements increases computational effort due to increased DOF in the solid elements. The axisymmetric triangular element would exhibit a total of 6 DOF per element while a axisymmetric quadrilateral 2D element would exhibit a total of the 8-DOF. This compares to a total of 12-DOF, 18-DOF or 24-DOF for the more complex 4, 6 and 8-node solid elements. The more complex solid elements when used in a model would increase computation time in the range of 3 fold over that of the 2D representation.

Order of element integration impacts computation time to the third power of the order. Elements like 4-node tetrahedrons or meshes will irregular shaped elements produce elements with high distortions requiring high ordered integration for accurate results. For uniform non-distorted shaped elements, 2nd Order integration can be used with satisfactory results reducing computation time. For moderately distorted elements, 3rd ordered should be used and for extremely distorted elements, 4th ordered should be used. Thus, simple uniformed shaped elements help reduce computation time in impact analysis using FEA.

In this paper, FEA analysis of closed-form solution impact problems are examined using simple finite elements as defined above to determine their effectiveness in analyzing impacts.

2. Current classical methods of approach to impact analyses

2.1 Stereomechanics approach to impact analysis

Stereomechanics is the classical theory of impact. The main focus of this theory is to determine the post velocities of impacted bodies given initial velocities and contact angles. The theory is based on the impulse-momentum equation for rigid bodies. The impulse-momentum for a mass m traveling with linear velocity v is given by the vector equation [9]:

$$m\underline{v}_f - m\underline{v}_i = \underline{J} = \int_{t_1}^{t_2} \underline{F} dt .\tag{1}$$

Eq. (1), the change in linear momentum of a mass due to an impact with no other external forces acting on the body is equal to the impulse J acting upon it. The impulse J is defined by the integral expression on the RHS of Eq. (1). Because stereomechanics does not provide a means to determine the contact force F nor the duration of contact, $t_2 - t_1$, the use of Eq. (1) in impact analysis is limited.

Two limiting cases exist in this simplest of impact models. The first case is the impact of perfect elastic bodies. The second case is the impact of perfect plastic bodies. For the first case, both conservation laws of momentum and kinetic energy apply and these two equations are used to determine the vector quantity post impact velocities. As for the second case, the conservation of momentum and the first law apply, but the conservation of kinetic energy does not. However, due to the bodies being perfect plastic, the bodies stick together and thus their post impact velocities are common.

The theory is extended to help accommodate that most impacts fall between the two extreme cases of being perfect elastic or perfect plastic. For these impact phenomena, a portion of the impact kinetic energy is transformed to heat and non-conservative work via material damping and plastic deformations. The energy loss accounted for by the coefficient of restitution (COR) symbolized by e. The COR relates the pre-impact (subscripted with i) and post-impact velocities by (subscripted with f) of body 1 and body 2 by the following equation:

$$v_{1f} - v_{2f} = -e(v_{1i} - v_{2i}) .\tag{2}$$

With the use of the COR and the momentum conservation law, the post-impact of the bodies from a central (direct) impact is determined by [15]:

$$v_{1f} = v_{1i} - \frac{(1+e)m_2(v_{1i} - v_{2i})}{m_1 + m_2}\tag{3}$$

$$v_{2f} = v_{2i} + \frac{(1+e)m_1(v_{1i} - v_{2i})}{m_1 + m_2} .\tag{4}$$

The coefficient of restitution is a dimensionless parameter

ranging from 0 to 1 with $e = 0$ representing a perfect plastic and $e = 1$ representing rigid body or perfectly-elastic impact. The COR is not a material property, but various with the material types impacted, impact velocities and surface geometry such as sphere-sphere contact or sphere-plate contact [4]. Experimental testing has shown that impact velocity has a trend in decreasing the COR. This would be expected as higher velocity impacts would produce more damping and plastic energy dissipation.

The major advantage of using COR to define the impact energy loss is the use of simple algebraic equations to determine post-impact velocities. The major disadvantage of this method is that the COR cannot be effectively determined analytically for any other type of geometry much more sophisticated than sphere-sphere contact [11] and thus, the COR has to be experimentally determined. For the use of momentum-impulse method as an engineering analysis, accurate prior knowledge of the COR for the particular impact circumstances is required.

2.2 Stress-wave impact approach to impact analysis

The theory of elasticity provides the basis for wave analysis in solids. The stress wave approach for impact analysis for a mechanical system is very complicated and in practicality only yields solutions in 1D solids, such as the classical problem of an end impact load applied to a prismatic bar [6]. This approach is also limited to small elastic strains. The wave equations are derived from the combination of: 1) the strain-displacement relations (kinematics), 2) strain compatibility conditions and 3) equations of motion (equilibrium) applied to an infinitesimal element.

An example of stress wave analysis applied to a 1D elastic problem follows. Fig. 1(a) diagrams an impact of rigid body mass M with a velocity v_0 into an elastic rod with properties of density ρ, Young's modulus E and Fig. 1 also shows the elastic stress wave propagating and reflecting in the bar at various time frames. At impact, shown in Fig. 1(b), a stress wave is generated in the solid and this wave travels at the speed of sound $c = (E/\rho)^{1/2}$ towards the opposite side the rod. Also at impact, the differential element of rod mass of $\rho A dx$ experiences a change in velocity of v_0 due to the impulse generated upon it by the impacting rigid body over a differential of time dt. Equating this impulse-momentum relation gives:

$$v_0 \rho A dx = E \frac{d}{dx} u A dt = \sigma_0 A dt. \quad (5)$$

Re-arranging Eq. (5) for the initial impact stress σ_0 at the rod end and noting that dx/dt is the wave front velocity c, σ_0 at impact can be established as:
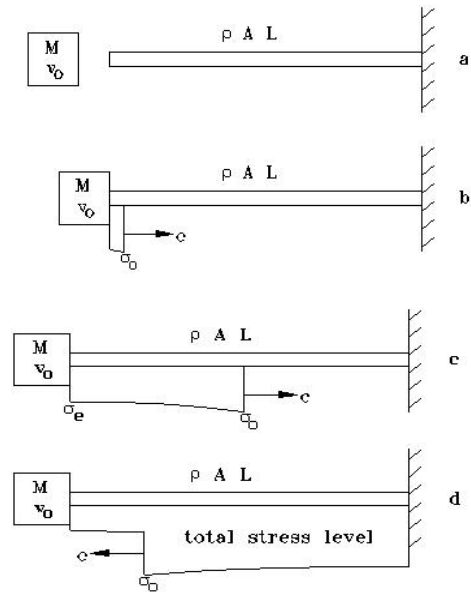
$$\sigma_0 = v_0 \sqrt{E\rho}. \quad (6)$$

Fig. 1. Rod end impact diagramming stress wave in struck elastic object.

2.3 Energy method approach to impact analysis

The energy method is an approximation technique for the analysis of impact problems. This method has a great deal of design practicality as it can be readily used in the design office to analyze mechanical components with impact loading [6]. This method is also referred to in the literature as the equivalent load method. The basis for the method is that mechanical energy is conserved. The potential and kinetic energies of the impactor are converted into strain energy, which is stored within the struck system or structure. The maximum deflections and stresses of the impacted system occur when the velocity of the impactor becomes zero when all of the impactor's energy has transferred to the target. Equating the impactor's energy to the system's strain energy yields the maximum deflection of the system. From the known deflections, the stresses of the mechanical system can be approximated as the result of the impact, thus aiding in design of the system.

Assumptions of this method are: 1) stresses are instantaneous distributed through the target object, 2) the impact stresses and deflections of the system are identical to that produced by a static load application multiplied by a Dynamic Load Factor (DLF), 3) non-conservative energy losses are not considered and 4) inertia of the target is ignored. Fig. 2 diagrams an ideal case of a mass impacting a mechanical system. The spring analogy of the mechanical system represents the fact that all mechanical systems will possess some degree of elasticity. Equating the impactor kinetic energy to the elastic energy of the system, the system's maximum deflection w_m due to the impact may be estimated as:

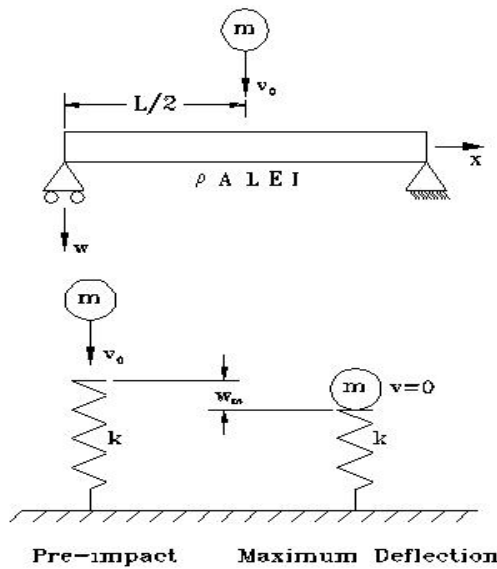$$\frac{1}{2} m v_0^2 = \frac{1}{2} k w_m^2. \quad (7)$$

Fig. 2. Energy method transforms impacted systems to an equivalent static load condition.

$$w_m = \sqrt{\frac{m}{k}} v_0 \ .$$

An improvement can be made to Eq. (7) by accounting for the change of potential energy of the impactor as it deflects w_m with the beam. Equating the energies:

$$\frac{1}{2} m v_0^2 + mg w_m = \frac{1}{2} k w_m^2 \tag{8}$$

$$w_m = \frac{mg}{k}(1 + \sqrt{1 + \frac{k v_0^2}{mg^2}})$$

where g is the acceleration of gravity. Noting that $\frac{mg}{k}$ is the static deflection w_{st} of the impactor's mass upon the system, the dynamic load factor for the impact is:

$$\frac{w_m}{w_{st}} = DLF = 1 + \sqrt{1 + \frac{v_0^2}{g w_{st}}} \ . \tag{9}$$

Eq. (9) indicates that the dynamic deflection w_m of the mechanical system can be determined by knowing the deflection w_{st} of the system under static conditions and then multiplying the static condition by the *DLF* .

3. Simple FEM approach to impact analyses

3.1 FEA emulation of stereomechanics

The advantage of the FEA method over stereomechanics approach in analyzing impacts is that the prior knowledge COR would not be a prerequisite to conduct the analysis. This is illustrated by the examination of a two spheres colliding by the FEA method. The two limiting cases, a perfect elastic or

Fig. 3. Sphere pre-impact and post-impact results predicted by stereomechanics approach (e =0.64 for elastic bodies system).
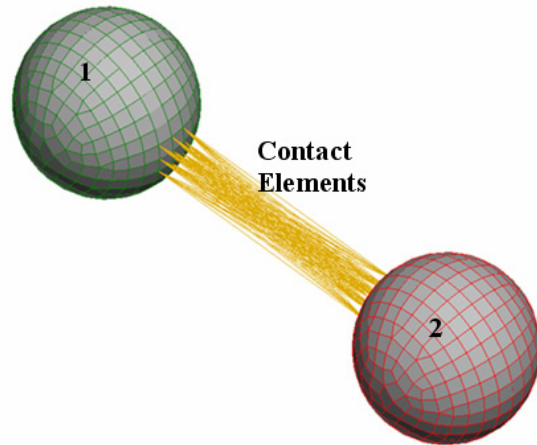

Fig. 4. FEA model of rigid body system impact.

rigid bodies impact (e =1) and an elastic bodies impact (0 < e < 1) impact were examined.

A simulation of a 487.7 cm/s impact between two 2.54 cm-radius rigid body spheres were virtually impacted using a PC based FEA package. The following parameters were used for the analysis the impact event of the two-body mechanical system shown in Fig. 3:

Material: Rigid body or perfectly-elastic
ρ = 8.15E-03 kg/cm^3 (density of steel)
m_1 = m_2 = 100.7 g
COR = 1
v_{1i} = 0 cm/s, v_{2i} = -487.7 cm/s.

From stereomechanics, Eqs. (3) and (4) predict the post-impact velocities for a rigid body collision (COR = 1) of the spheres to be: v_{1f} = -487.7 cm/s, v_{2f} = 0 cm/s. The spheres would simply trade velocities at impact.

Fig. 4 shows the FEA model for the rigid body collision. The model was developed from ALGOR, an FEA package capable of non-linear and event simulation analyses [24]. The body designated as body 1 is on the left of the figure. Each sphere was modeled with (5048) 3-D kinematic elements, a very simple FEA element. A high number of elements were required to obtain results of that predicted by stereomechanics. The contact area was modeled with 81 contact elements with a total contact area of 0.206 cm^2.

Fig. 5 shows the FEA model for the elastic system collision. The body designated as body 1 is one at the bottom of the
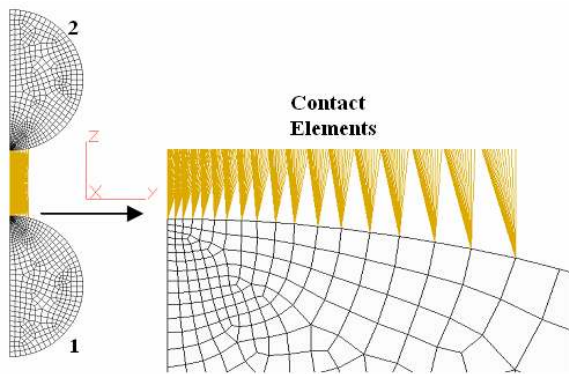
Fig. 5. FEA model of elastic system impact.

Fig. 6. FEA Model of rod end to determine if stress wave can be captured using simple FEA elements.

Fig. 7. Complex stiffness beam-impactor system.

figure. Each sphere was meshed with 773 quad 2D planar elastic elements, again simple or low ordered FEA elements. At the contact region, a fine meshing was required to produce results predicted by stereomechanics theory, and thus a half model was developed for computation efficiency. The contact area was modeled with 338 contact elements with a contact modulus of 206.8 GPa for modeling a steel to steel contact. The contact area was conservatively covered by contact elements, as the exact size of the contact area was unknown before emulation. A technique was found not to make the emulated contact too stiff due to the conservative coverage as the contact distance was set at 0.025 cm. In doing this, only contact elements involved in the actual deformation zone participated whereas contact elements with distances of greater than 0.025 cm did not participate, controlling the impact stiffness.

3.2 FEA emulation of stress-wave approach

To demonstrate the FEA method versus stress wave theory, the impact system illustrated in Fig. 1 was examined by the FEA method and compared to the stress wave solution. The event parameters analyzed are defined as:

ρ = 8.15E-03 kg/cm^3
A = (5.07 cm^2) area of 2.54ϕ cm rod
E = 206.8 GPa
L = 63.5 cm
M = 453.6 g
v_0 = 474.2 cm/s

Fig. 6 diagrams the FEA model developed to analyze the end impact scenario. A 3D model of the rod was constructed using (3200) 2-D axisymmetrical elastic elements possessing 7105 DOF with the density and Young's modulus for steel. In Fig. 6, this rod is the right hand structure and is shown truncated in length. The impactor was modeled as an elastic body with properties of steel. These element types are considered simple elements due to the low DOF and simple spatial representation. The contact interface was represented with 81 contact elements possessing a contact modulus of 206.8 GPa for
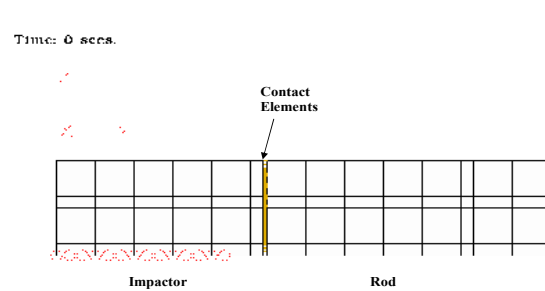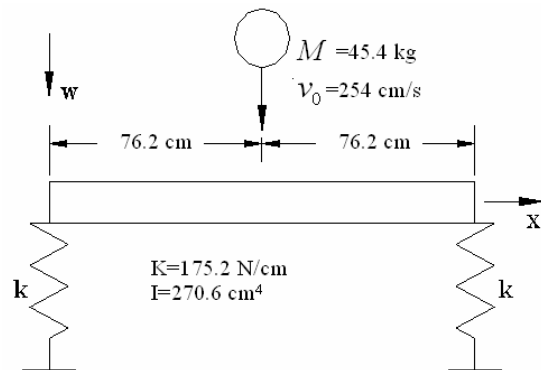
steel and a 0.025 cm contact enforcement distance. The contact area was 0.065 cm^2 per contact element. The kinematic elements on the left side of Fig. 6 were assigned an initial velocity of 474.2 cm/s in the y-direction. To examine stress wave events, it was found that very fine time increments were required of the FEA. A time step of five micro-seconds was required to capture this impact event. Five micro-seconds represented a small portion of the axial fundamental frequency of the rod.

3.3 FEA emulation of energy method in impact analysis

Comparing the FEA method to the energy method, a beam-impactor system was modeled by a simple FEA mass and general stiffness elements. Fig. 7 diagrams a steel beam over a spring suspension subjected to an impact of a 45.4 kg impactor at a velocity of 254 cm/s. For design analysis considerations, the maximum deflection of the beam, spring and system is desired.

Fig. 8 is the FEA model of the impact problem of the complex stiffness beam, shown in Fig. 7. The objective of this model was to determine if the impact event could be emulated using general stiffness FEA elements. The beam and spring stiffness were modeled with one stiffness element each with an area of 6.45 cm^2 and a length of 30.48 cm. With the area and length fixed, the required elastic modulus to provide the stiffness of the beam and spring members calculated at 3,588 MPa and 16.6 MPa. Kinematic element was attached to the
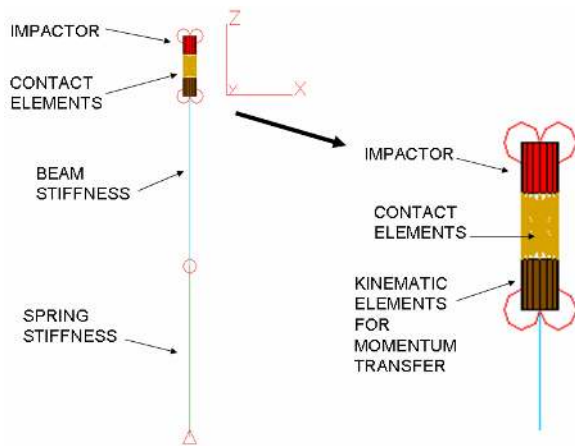
Fig. 8. FEA model of beam-impactor system.

end of the general stiffness element to provide a momentum transfer. Without this element, it was found that an end contact to a beam or truss element was not possible. Contact elements communicated the momentum of the impactor to the kinematic elements. The contact element modulus was 66.7 GPa and exhibited a total contact area of 6.54 cm^2.

4. Results and discussion

4.1 Comparison of stereomechanics approach with FEA approach to impact analysis

Fig. 9 shows velocity traces for the two bodies from the results of the FEA analysis with the rigid body or perfectly-elastic elements. Stereomechanics theory predicts that body 1 would acquire a post-impact velocity of -487.7 cm/s and body 2 would come to a standstill. As seen in these velocity traces, the FEA model was well within engineering accuracy. Body 2 does exhibit a small residual velocity. The model would likely be improved by increasing the contact modulus to provide a better representation of a rigid body system. Using the same impact parameters as the rigid body system, the stereomechanics Eq. (2) and (3) predict the post-impact velocities for an elastic system collision of steel spheres to be; v_{1f} = -399.8 cm/s along with v_{2t} =-87.9 cm/s. A COR of 0.64 was used for the stereomechanics computations taken from experimental steel sphere impacts [15], illustrating the disadvantage of stereomechanics.

Fig. 10 diagrams the velocity traces for the two bodies from the results of the FEA analysis of the elastic system. Body 1 exhibited a post velocity of -384.8 cm/s while body 2 had a post impact velocity of -104.9 cm/s. The difference between the FEA analysis and the experiment using the COR of 0.64 was likely in the type of steels and thus differences in yield strength. The exact steel classification from the impact test from reference [15] could not be determined. The material in this FEA analysis exhibited yield strength of 248.2 MPa and an ultimate of 400 MPa. From the FEA analysis, the COR computed to be at 0.58.
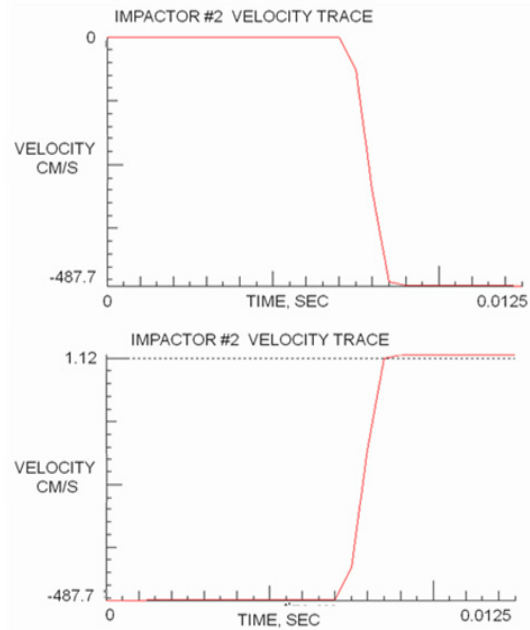

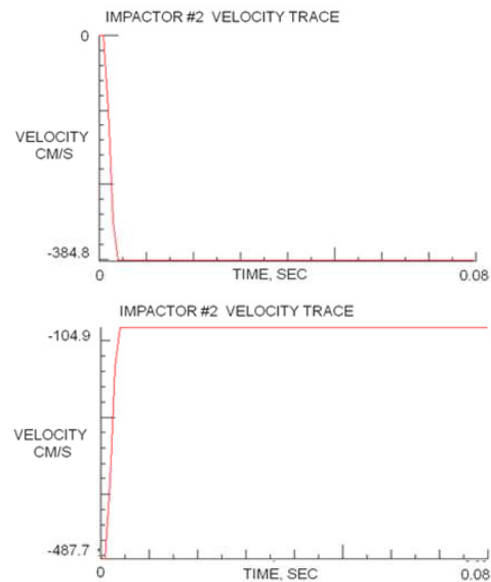Fig. 9. FEA prediction on rigid body system impact.

Fig. 10. FEA prediction on elastic body system impact.

4.2 Comparison of stress-wave propagation approach with FEA approach to impact analyses

Using Eq. (6) and the above system parameters, the local stress predicted at the end of the rod immediately after impact is σ_0 = 194.6 MPa while the theoretical velocity of the stress wave in steel is calculated to be 5,038 m/s.

Fig. 11 presents the results of the FEA analysis for end impact of the rod. The initial local end stress in the rod immediately after impact exhibited an axial compressive stress of 182.0 MPa. The FEA stress result is 3.9% lower than pre-

Table 1. Comparison of simple vs complex element simulation.

| Element Type	Axisymmetric 4-node 8-DOF	Brick 8-node 24-DOF
Number of Elements	3200	3335
Element Classification	Simple by low DOF and 2D spatial representation	Complex by high DOF and 3D spatial representation
Total DOF	7,105	11,490
Event Time, microsecond	75	75
Computational Time, min:sec	03:26	09:33
Theoretical Stress, MPa	-189.09	-189.09
FEA Model Stress, MPa	-182.00	-171.49
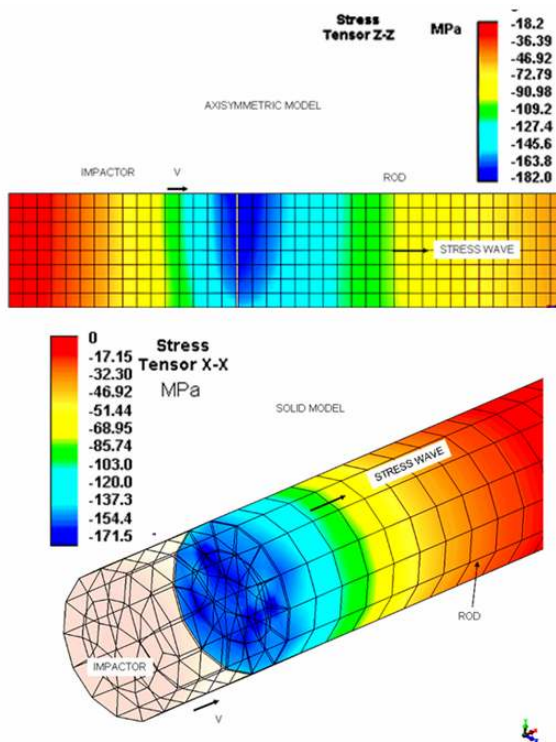
Fig. 11. FEA stress results of rod end impact (MPa).

dicted theoretically by Eq. (6). As for the propagation speed, an analysis of the FEA results using distances, times and an estimation of the position of the wave front yielded a propagation wave speed of 5418 m/s, within 7% the theoretical result of 5,038 m/s.

As a comparison to using the simpler axisymmetric elements, a 3D model of the impacted rod of Fig. 6 was modeled with 3335 8-node 24-DOF brick elements. These elements are considered complex due to high DOF and complex spatial representation. As seen in table 1, the computation effort was threefold of that of using the simple elements. The simple element model was within 4% of the closed-form solution while the complex element model was over 9%.
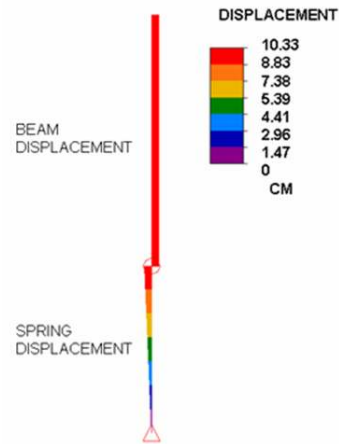

Fig. 12. FEA results of beam-impactor system.

4.3 Comparison of energy method with FEA approach to impact analyses

Using the energy method to calculate the benchmark results for the FEA analysis, the static deflection for the beam and spring compute as:

$$w_{st\_spring} = \frac{P}{2k} = 1.27\ cm$$

$$w_{st\_beam} = \frac{Pl^3}{48EI} = 0.006\ cm,$$

in which $P = 445$ N, $k = 175.2$ N/cm, $l = 152.4$ cm, $E = 206.8$ GPa, and $I = 270.6$ cm^4.

Adding the deflection of the spring plus the beam, the total system static deflection is 1.276 cm. With the system static deflection of the impactor known, the DLF maybe calculated from Eq. (9), for the 254 cm/s impact:

$$\frac{w_m}{w_{st}} = DLF = 1 + \sqrt{1 + \frac{v_0^2}{gw_{st}}} = 8.25\ .$$

The dynamic deflections and stresses are predicted by the energy method to be 8.25 times the respective static deflections due to the static force of the impactor alone. Calculating the dynamic deflections:

$$w_{m\_spring} = 8.25(1.27) = 10.48\ cm$$

$$w_{m\_beam} = 8.25(0.006) = 0.049\ cm$$

$$w_{m\_system} = 8.25(1.276) = 10.53\ cm\ .$$

Fig. 12 shows the results of this FEA analysis. The maximum deflection predicted by the FEA analysis for the system was 10.33 cm. The FEA results compare very favorable to the equivalent load theory prediction of 10.53 cm for the total system deflection. From the FEA analysis, the deflection of beam was predicted at 0.047 cm and the spring deflection was

predicted at 10.28 cm. The equivalent load theory predicted a beam deflection of 0.049 cm and a spring deflection of 10.53 cm. All FEA predictions were within 3% of the energy method theory. This demonstrates that FEA models constructed from simple finite elements are capable of emulating impact events of some complexity when proper modeling techniques are used. This method could be extended to examine the collision response and crush of 2-D analysis by using a series of general stiffness elements while greatly reducing the computational effort.

5. Case studies on FEM prediction of impact of mechanical systems

Two test cases are provided here to illustrate the use and the effectiveness of the FE-based methods for impact analyses of general mechanical systems.

5.1 Case study 1: Transverse impact of a sphere on a beam

The FEA emulation of two body system composed of a transverse beam under a 4572 cm/s impact by a steel ball of 1.27 cm diameter was designed and compared to experimental results. The event parameters under study, matching the experimental work of [25] are as defined by:

ρ = 7.83E-03 kg/cm^3)
A = 1.61 cm^2 (1.27 cm $\times$ 1.27 cm cross-section)
I = 13.50E-3 cm^4
E = 206.8 GPa
L = 76.2 cm
D_1 = 1.27 cm diameter impactor
m = 8.41 g
v_0 = 4572 cm/s

Fig. 13 diagrams the FEA half-model of a steel beam with a central impact by a 1.27 cm diameter steel sphere. Both the beam and sphere are isotropic elastic elements. The contact elements emulate the contact mechanics involved in the experimental impact. A 2-D half-model was used to reduce the computation effort. Appropriate boundary conditions were applied to the beam and sphere to enforce the half-model symmetry. The beam was meshed with (605) 4-node 8-DOF simple elastic plane stress elements with a thickness of 1.27 cm. The sphere was modeled with (151) 3-D elastic simple axisymmetric elements. As demonstrated in the figure, the mesh was refined at the contact regions of the beam and sphere to develop a better contact modeling. The contact interface was handed with 11 general contact elements. The contact modulus was 206.8 GPa for steel and the contact elements' areas were representative of the FEA elements' areas in order to achieve numerical results comparable to theory.

The FEA results for the bending stresses of the beam are shown in Fig. 14. The peak tensile stress occurred at mid-span of the bottom fiber directly below the impact region. The magnitude of this tensile stress measured 426.4 MPa from the
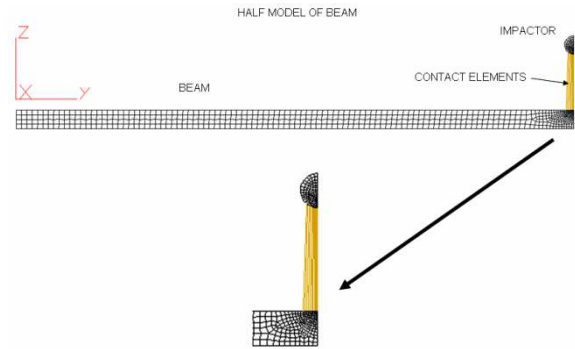

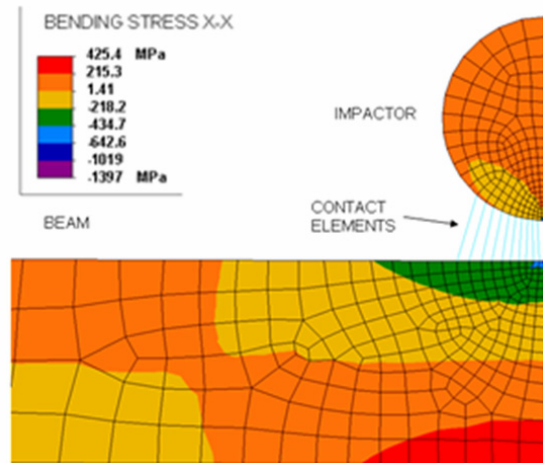Fig. 13. FEA model of transverse impact of beam.

Fig. 14. FEA results of beam impact.

FEA contour plot. Note, the high compressive stress due to Hertzian contact pressures are also captured in the model. Experimental stress analysis [25] via resistance foil strain gauge was conducted on beam with the same dimensions and impact conditions resulted in a bottom fiber stress of 455.4 MPa directly beneath the impactor. The FEA emulation of the impact was within 6.3% of the experimental results. In addition, the contact times matched well between the experimental results and FEA analysis, 25 and 30 micro-seconds respectively.

5.2 Case study 2: Structural impact of a vehicle anticollision barrier/guard

To demonstrate the prediction capability and practicality of the FEA method using simple finite elements for impact analysis of a two-body system, an FEA model was developed for the authors' designed 55,590 N-m off center structural impact of a protection barrier used in the over the road trucking industry to protect against large animal strikes at highway speeds. The barrier was as a 254 cm deep welded space frame fabricated from 5.08 cm diameter 4130 CRS steel tubing with wall thickness varying from 0.024 cm to 0.048 cm. Fig. 15 presents a prototype barrier used for the impact test attached to

Fig. 15. Truck animal anti-collision barrier.

Fig. 16. FEA model of animal anti-collision barrier.

Fig. 17. Instrumentation of barrier with strain gages.

Fig. 18. The 39-meter temporary drop tower.

its test bed. Stresses and deflections predicted by the FEA model were compared to experimental stresses and deflections to assess the performance of the FEA using simple finite elements.

Fig. 16 presents the FEA model of barrier. The barrier was virtually impacted with flexible impactor consisting of (1,270) 24-DOF plastic brick elements. The impactor weighed 1419 N and was assigned an initial velocity of 99.8 km/hr representing impact energy of 55,590 N-m. This impactor represented the size, shape and stiffness of a modified drum used as the impactor in the actual test. The guard was modeled with (328) 12-DOF plastic beam elements. All material constitutes were of Von Mises isotropic hardening. The material properties for the tubing were modeled as 4130 CRS and for the impactor it was mild steel. The contact was managed with 198 contact elements with a contact modulus of 206.8 GPa representing steel.

The prototype barrier was instrumented with nine 350Ω resistance foil strain gages for experimental stress analysis during the structural testing. The strain gages were located in the areas that exhibited high stress levels observed in the FEA analyses. Fig. 17 presents a typical gage installation. The instrumented barrier was impacted with 55,590 N-m of energy delivered by dropping a 202.8 liter steel drum with ballast of 1419 N for 39.17 m acting as the impactor. Fig. 18 shows the drum on top of a grain elevator that was temporarily used as a drop tower. The impactor/drum was hoisted to the drop height

and filled with water for ballast pumped up by a fire truck. The drum was guided down to the guard via a guide wire that transverses through the drum center. The drum was released and allowed to free fall until it hit the barrier. The center of the strike was the roadside inner stanchion at the second row of horizontal tubes. The model exhibited 774 nodes, 1796 elements totaling to 3322-DOF predicted the barrier to perform as follows the 55,590 N-m off center impact:

Maximum Total Deflection: 4.8 cm
Plastic Set: 2.4 cm (left side of guard)
Energy Absorption: 84.7%
Contact Time: 6-7 milliseconds

Figs. 19 and 20 present sample results on predictions of the impact on the barrier by the FEA model.

Fig. 21 presents the guard after the impact. At the center of the strike, a plastic set was exhibited in the barrier. This set measured approximately 2.0 cm as shown in the figure. The FEA model prediction of plastic set was quite comparable at 2.4 cm. Fig. 22 presents the data from strain gage #3 during the impact event using a high speed data acquisition system (5000 Hz). The contact time measured from the strain gage data was in the 9-11 millisecond range. The FEA predicted a contact time of 6-7 milliseconds, which is comparable.
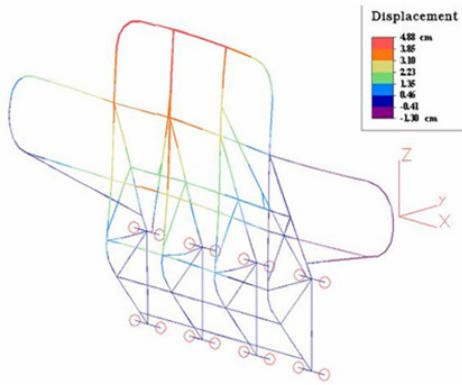
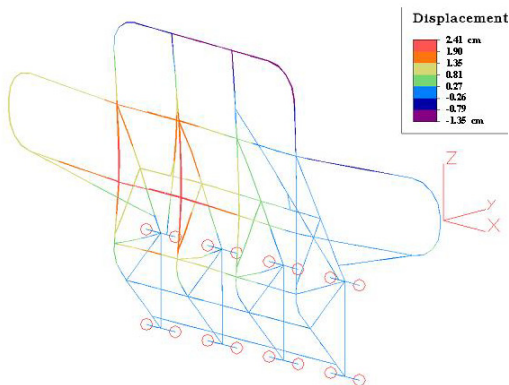Fig. 19. Maximum deflection during impact (cm).

Fig. 20. Plastic set prediction post impact (cm).

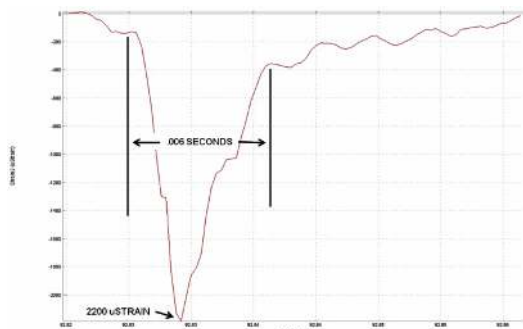Fig. 21. Actual plastic set post impact.
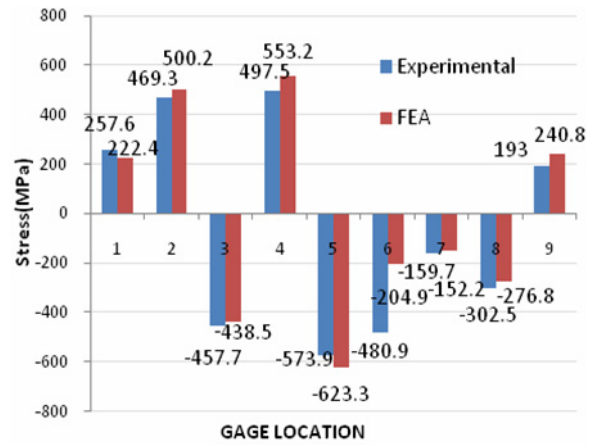

Fig. 22. Strain vs. time on strain gage #3.

Fig. 23. FEA predicted vs. experimental stress results.

As shown in Fig. 23, the FEA model predicted the peak stresses in the barrier within 15% of the strain gage measurements on the instrumented barrier at seven of nine instrumentation locations. Six of nine locations were within 10%. The FEA model predicted the impact event well within engineering expectations using simple finite elements.

6. Conclusions

A study was conducted to determine if an FEA analysis using simple (low ordered) finite elements would be robust enough to accurately predict impact events in terms of stresses and deflections in order to aid in the design of structures, mechanisms and products.

First, impact problems solved by the classical methods of stereomechanics, stress-wave and energy methods were emulated by the FEA method based on these approaches with simple elements. In addition, an experimental impact analysis found in literature was analyzed by the FEA method using low ordered elements. It was found that the proper elements, boundary conditions, contact elements and material constitutes selection that an FEA could indeed produce engineering acceptable results in predicting velocities, deflections and stresses involved in an impact without extraordinary hardware or computation times for reasonable design problems. FEA results of the above benchmarks were within 1% to 7% of the theoretical and/or experimental results. With this work, a small set of benchmarks for impact analysis by FEA was established.

Secondly, a 55,590 N-m impact event of a trucking impact protection barrier was modeled by the FEA method using simple plastic beams. The actual impact of the barrier was compared to the predictions of FEA. The plastic set predicted by the FEA model was within 0.4 cm of the actual permanent deformation. Stress levels predicted by the FEA model were generally within 10% of measured values, demonstrating that an FEA utilizing simple or low ordered finite elements can be used as a valid engineering tool for a medium sized manufacturer in for designing products exhibiting impact requirements.

Nomenclature——————————————————

A	: Area, in^2 (cm^2)
c	: Extreme beam fiber distance, in (cm)
d_i	: Diameter, inner, in (cm)
d_o	: Diameter, outer, in (cm)
E	: Elastic modulus, psi (Gpa)
e	: Coefficient of restitution
E_t	: Tangent modulus, psi (Gpa)
F	: Force, lbf (N)
G	: Shear modulus, psi (Gpa)
I	: Area moment of inertia, in^4 (cm)^4
J	: Impulse, lbf-sec (N-sec)
k	: Stiffness, lbf/in (N/cm)
L	: Length, in (cm)
m	: Mass, lbf-sec^2/in (g)
T	: Kinetic energy, in-lbf (N-cm)
x, u	: Displacement, in (cm)
U	: Total energy, in-lbf (N-cm)
v	: Velocity, ips (cmps)
w	: Beam transverse displacement, in (cm)
V	: Potential energy, in-lbf (N-cm)
Z	: Beam elastic section modulus
ΔV	: Change in velocity due to impact, ips (cmps)
ε	: Strain, in/in (cm/cm)
γ	: Specific density, lbf/in^3 (N/cm^3)
ρ	: Mass density, lbf-sec^2/in^4 (g/cm^3)
σ	: Stress, psi (Mpa)
ω	: Natural radial frequency, rad/sec
ν	: Possion's ratio
CRS	: Cold Rolled Steel
DLF	: Dynamic Load Factor
FEA	: Finite Element Analysis
DOF	: Degrees Of Freedom
HRS	: Hot Rolled Steel
PC	: Personal Computer
ips	: Inches per second
$cmps$	: Centimeters per second
fps	: Feet per second
mps	: Meters per second
f	: Final condition
$i, 0$	: Initial condition

References

[1] J. Zukas, *Impact dynamics*, John Wiley & Sons (1982).

[2] J. Zukus, Practical aspects of numerical simulations of dynamic events: effects of meshing, *Int. J. Impact Engineering* (2000) 925-945.

[3] S. Shiviaswamy, Modeling contact forces and energy dissipation during impact in mechanical systems, Ph.D. Dissertation, Wichita State University, Wichita, KS, USA (1997).

[4] A. Blake, *Practical stress analysis in engineering design*, 2nd edition, Dekker (1990).

[5] J. Faupel, *Engineering design*, 2nd edition, John Wiley & Sons (1981).

[6] R. Juvinall, *Stress, strain and strength*, McGraw-Hill (1967).

[7] R. Craig, *Structural dynamics*, John Wiley & Sons (1981).

[8] S. Faik, Modeling of impact dynamics: A literature survey, *International ADAMS User Conference* (2000).

[9] D. Halliday, *Fundamentals of physics*, 2nd edition, John Wiley & Sons (1981).

[10] L. Vu-Quoc, Modeling the dependency of the coefficient of restitution on the impact velocity of elasto-plastic collisions, *Int. J. of Impact Eng.* (2002) 317-241.

[11] K. Johnson, *Contact mechanics*, Cambridge University Press (1985).

[12] H. M. Lankarani and P. E. Nikravesh, Continuous contact force model for impact in multibody systems, *Nonlinear Dynamics* (1994) 193-207.

[13] F. Wu, Modeling crash dynamic responses of high energy absorbing materials by multibody non-linear finite element and optimization methods, Ph.D. Dissertation, Wichita State University, Wichita, KS, USA (1995).

[14] S. Timoshenko, *Vibration problems in engineering*, 5th Edition, Van Norstrand (1955).

[15] W. Goldsmith, *Impact*, Edward Arnold Publishing (1960).

[16] H. Bohnenblust, The behavior of long beams under impact loading, *Journal of Applied Mechanics* (1950) 27-36.

[17] M. Conroy, Plastic deformation of semi-infinite beams subject to transverse loading at the free end, *Journal of Applied Mechanics* (1956) 239-243.

[18] M. Conroy, Plastic-rigid analysis of a special class of problems involving beams subject to dynamic transverse loading, *Journal of Applied Mechanics* (1955) 48-52.

[19] P. Symonds, Dynamic load characteristics in plastic bending of beams, *Journal of Applied Mechanics* (1953) 475-481.

[20] H. Cox, On impacts on elastic beams, *Trans. Camb. Phil. Soc.*, 73 (1949).

[21] B. Creighton, Numerical resolution calculation for elastic-plastic impact problems, U.S. Army Research Laboratory, MR-3418 (1984).

[22] C. Nagtegall, On numerical accurate finite element solution in the fully plastic range, *Computer Methods Application, Mechanical Engineering* (1974) 153-178.

[23] D. Littlefield, A study on zoning requirements for 2-D and 3-D long rod penetrations, *American Institute of Physics* (1996) 1131-1134.

[24] Algor, Inc., 150 Beta Drive, Pittsburg, PA (2009).

[25] D. Cunningham, An experimental investigation of beam stresses produced by impact of a steel sphere, *Journal of Applied Mechanics*, 606 (1956).

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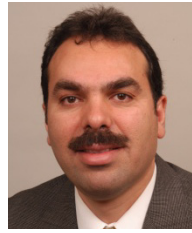
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