Use Your Brain! Arithmetic 3PC for Any Modulus with Active Security

Hendrik Eerikson
Cybernetica AS, Tartu, Estonia
hendrik.eerikson@cyber.ee

Marcel Keller
CSIRO's Data61, Eveleigh, Australia
mks.keller@gmail.com

Claudio Orlandi
Department of Computer Science, DIGIT, Aarhus University, Denmark
orlandi@cs.au.dk

Pille Pullonen
Cybernetica AS, Tartu, Estonia
pille.pullonen@cyber.ee

Joonas Puura[1]
Institute of Computer Science, University of Tartu, Estonia
joonas.puura@gmail.com

Mark Simkin
Department of Computer Science, DIGIT, Aarhus University, Denmark
simkin@cs.au.dk

Abstract

Secure multiparty computation (MPC) allows a set of mutually distrustful parties to compute a public function on their private inputs without revealing anything beyond the output of the computation. This paper focuses on the specific case of actively secure three-party computation with an honest majority. In particular, we are interested in solutions which allow to evaluate arithmetic circuits over real-world CPU word sizes, like 32- and 64-bit words. Our starting point is the novel compiler of Damgård et al. from CRYPTO 2018. First, we present an improved version of it which reduces the online communication complexity by a factor of 2. Next, we replace their preprocessing protocol (with arithmetic modulo a large prime) with a more efficient preprocessing which only performs arithmetic modulo powers of two. Finally, we present a novel "postprocessing" check which replaces the preprocessing phase. These protocols offer different efficiency tradeoffs and can therefore outperform each other in different deployment settings. We demonstrate this with benchmarks in a LAN and different WAN settings. Concretely, we achieve a throughput of 1 million 64-bit multiplications per second with parties located in different continents and 3 million in one location.

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1 Introduction

Secure Multiparty Computation (MPC) is an umbrella term for a broad range of cryptographic techniques and protocols that enable a set of parties $\mathrm{P}_1, \ldots, \mathrm{P}_n$ to compute some function f of their private inputs $x_1, \ldots, x_n$ without revealing anything beyond the output $f(x_1, \ldots, x_n)$ of the computation. Most importantly, an actively misbehaving participant should not be able to bias the outcome of the computation (except by choosing their input) or learn anything about the inputs of the honest parties (except for what is leaked by the output itself). MPC started out as a purely theoretical research field in the 90ies, but has developed into a science on the brink of practical deployment. The number of of real-world use cases, MPC framework implementations, and startups is constantly increasing (see [4] for a survey).

The landscape of MPC protocols is broad and diverse, and protocols differ greatly in many parameters such as the number of involved parties, the corruption threshold, the adversarial model, and the network setting. We focus on a popular model of *three-party computation with an honest majority*. This model has been used in different real-world applications [15, 14, 10, 11, 2], often in the so-called *client-server* scenario where a possibly large number of clients secret share their inputs to three computation servers who then perform the computations [33] and return the result to the clients. A major advantage of the honest majority setting is that one can obtain protocols which do not rely on computationally expensive cryptographic operations (e.g. exponentiations, oblivious transfer), but typically only use light-weight arithmetic operations and achieve information theoretic security.

Existing implementations of three-party computation protocols for the honest-majority case fall into two broad categories. VIFF [24] and its successors [40] only support arithmetic computations over prime order fields. Sharemind's protocol suite [12, 13] can be used to evaluate arithmetic circuits with arbitrary word sizes, but is only secure against passive adversaries that follow the protocol faithfully. This means that one has to either settle for rather weak security guarantees or to develop applications specifically tailored to rather unnatural word sizes instead of using the common 32- and 64-bit word sizes that dominate real-world system architectures. In particular, this means that a developer has to match the needs of the MPC framework rather than the framework meeting the needs of the developer.

The main barrier to constructing actively secure protocols for evaluating arithmetic circuits with arbitrary word sizes lies in the fact that known approaches to achieving active security, like *information checking* techniques [39], require prime order fields. Up until recently it has been an open question to design protocols for arithmetic circuits with active security for arbitrary word sizes. In a recent work Damgård et al. [27] addressed this question by presenting an information theoretically secure protocol compiler that transforms passively secure protocols into actively secure ones that can tolerate up to $\mathcal{O}(\sqrt{n})$ corruptions and only have a *constant* overhead in storage and computational work.

Our Contributions. We consider the class of protocols produced by compiler of Damgård et al. [27], and we improve such protocols in several ways. The main idea behind Damgård et al.'s compiler is to let the *real parties* "emulate" *virtual parties* that execute the desired computation on behalf of the real parties. The crucial point is that the virtual parties can

execute[2] a passively secure protocol in a way that prevents any real party from actively misbehaving. Every time that a virtual party $\mathbb{P}_i$ is supposed to send a message to another virtual party $\mathbb{P}_j$ in the passively secure protocol, every real party that is emulating $\mathbb{P}_i$ computes the same message redundantly and sends it to every real party emulating $\mathbb{P}_j$. Each real party emulating $\mathbb{P}_j$ therefore receives a set of messages and aborts in case the received messages are not all equal. Intuitively this approach ensures active security as long as there is at least one honest real party in every virtual party, since any malicious party either follows the protocol (in which case we effectively only have passive corruptions) or sends a message that disagrees with the message that is sent by at least one honest party (in which case the honest receiving party and consequently all other parties abort the protocol). This approach heavily relies on the fact that *all* messages are sent redundantly, thus incurring a multiplicative blow-up in the bandwidth overhead of the protocol.

We present an improved compiler that significantly reduces the number of redundant messages that need to be sent during the execution. The idea is to elect *one* real party in each virtual party to be the "brain", which sends all messages on behalf of its virtual party to *all* real parties in the receiving virtual party. The other real parties, the "pinkies", still receive messages from the brains and thus can locally follow the protocol execution. At the end of the protocol, right before the output is released, we let all parties perform a single check that guarantees that all messages sent by the brains during the protocol are consistent with the messages all the pinkies would have sent. It is clear that if any of the brains cheated during the protocol execution, then it must have sent a message that is inconsistent with the view of at least one pinky, thus the protocol would abort during the checking phase. On the downside our new compiler now imposes a stronger security requirement on the protocol it starts with. Honest brains continue the protocol execution up to the checking phase even if a malicious brain misbehaves, which means that we need a protocol that does not leak any private information even if cheating during the computation phase occurs, e.g. protocol with weak privacy [32]. Thankfully, most passively secure secret sharing based protocols provide such security guarantees. More concretely, these protocols follow a compute-then-open structure, where the output of the computation is only revealed in the last round and any cheating during the preceding computation can only affect the correctness of the output, but not the privacy of the inputs. Thus, performing the consistency check at the end of the computation phase and *before* the output phase, ensures that no information is leaked. We formally present our new compiler and prove its security in Section 3. For the specific three-party case, our compiler produces a protocol, which is roughly twice as efficient as the protocol produced by the compiler of Damgård et al., since in the three party case each virtual party is emulated by one pinky and one brain.

Our second contribution is an improved preprocessing protocol for generating secret-shared multiplication triples. Damgård et al. generate both triples modulo a prime and triples modulo a power of 2, followed by a check-and-sacrifice step. We replace this by a preprocessing phase which does not perform any arithmetic in the larger prime field and solely uses computation modulo a slightly larger power of 2, thus improving on efficiency. While the sacrifice step is not performed in a field anymore, security follows using similar arguments as in the recent work on SPDZ over rings [21].

We show that it is possible to completely avoid the preprocessing phase if one wishes to do so. Recall that our underlying protocols are assumed to preserve privacy until the outputs are opened. We exploit this security property by running the multiplication protocols

[2] Note that virtual parties do not physically exist. "Virtual parties execute a protocol" means that the real parties simulate the virtual parties protocol execution.

optimistically and then, prior to opening the outputs, perform a single combined check. The protocols with preprocessing and with postprocessing therefore offer different efficiency tradeoffs. The protocol with preprocessing has a leaner online phase, whereas the protocol with postprocessing has a better overall performances. Descriptions of our protocols are given in Sections 4, 5, 6. In Section 7, we provide extensive performance benchmarks of our framework, both in the LAN as well as different WAN settings. Our protocols have been integrated in two of the leading MPC frameworks, namely the Sharemind MPC protocol suite and MP-SPDZ. As described in Section 7, we achieve the most efficient implementation of a three-party computation protocol for arithmetic circuits modulo 2^{64} with active security.

Other Related Work. The SPDZ family of protocols [8, 28, 25] efficiently implements MPC with active security in the *dishonest majority* setting. These protocols are split up into a slower, computationally secure *offline phase* in which correlated randomness (Beaver's triples) is generated and a faster, information-theoretically secure *online phase* in which these triples are consumed to compute the desired functionality. Active security in the online phase is achieved using information theoretic message authentication codes (MACs), which until recently limited the SPDZ approach to computation over fields. In a recent work [21], this limitation has been lifted, allowing to perform computation modulo 2^k (by defining the MACs modulo to be $2^{k+\lambda}$ where λ is the security parameter, thus introducing an overhead proportional to the security parameter). An implementation (and optimizations) of [21] was presented in [23]. In addition, [18] follows up [21] with a two-party protocol that uses homomorphic encryption and efficient zero-knowledge proofs in the precomputation phase.

Other recent works have considered active security in the three-party setting. [31] uses correlated random number generation to achieve efficient preprocessing and replication to achieve security. The protocol was originally presented only for Boolean circuits, but it was then noticed that the approach generalizes to general rings [36]. They mention actively secure protocols in this setting, but do not give detailed protocol descriptions and only implement semi-honest versions of their protocols. For finite fields, [20] achieves active security by running two copies of the computation, respectively with real and random inputs, and uses the latter to verify correctness (this approach can be used for more than three parties). Boyle et al. [16] recently presented a protocol that achieves no asymptotic communication overhead over a semi-honest protocol in the same setting. However, their benchmarks suggest that the computation of their protocol might be rather limiting in some network settings, see Section 7.3 for more information. After the first version of our paper appeared online, a very different protocol for the same three party honest majority setting was presented in [19]. They combine two linear secret sharing schemes, one between two and other between three parties where the former is used to share a component of the latter sharing. This allows them to create a circuit dependent precomputation phase where all the two party sharings of random values are precomputed based on the circuit structure. The online phase focuses on computing modifiers to turn the random precomputed sharings to real outputs. Moreover, novel techniques for honest-majority MPC over rings have very recently been deployed in [1]. It is however still unclear whether this can lead to protocols which are efficient in practice.

2 Preliminaries

We write $v \leftarrow \mathcal{X}$ to denote the sampling of a uniformly random value v from set $\mathcal{X}$. Throughout the paper λ denotes the security parameter. Given n parties $\mathrm{P}_1, \ldots, \mathrm{P}_n$, we write P_{i+1} to denote the party after P_i and we implicitly assume a wrap around of the party's index. That is $\mathrm{P}_{n+1} = \mathrm{P}_1$ and $\mathrm{P}_0 = \mathrm{P}_n$.

We define security using the UC framework [17]. In particular, we require the notion of *"Weak Privacy" against active adversaries* introduced in [32, Definition 5.11]. We include background definitions in Appendix A. Throughout the paper, we assume a synchronous communication network, a rushing adversary, and secure point-to-point channels.

2.1 Auxiliary Ideal Functionalities

We will make use of the following basic auxiliary ideal functionalities in this paper: The broadcast with *individual* abort functionality $\mathcal{F}_{\mathsf{bcast}}$ (Figure 1) allows a sender S to send a value v to a set of parties $\mathbb{P}$. The functionality guarantees that either a party aborts or it agrees on a consistent value with the other parties. Such a functionality is weaker than detectable broadcast [30], which requires that either all players agree on the same value or that all players unanimously abort. The functionality can easily be instantiated by letting the sender S send v to all parties in $\mathbb{P}$. Every party in $\mathbb{P}$ echoes the received value to all other parties in $\mathbb{P}$. Parties that receive consistent values output that value, parties that receive inconsistent values abort.

Functionality $\mathcal{F}_{\mathsf{bcast}}$ Functionality with sender S, who has input v, parties P_1, $\ldots$, P_n, and adversary $\mathcal{A}$.

1. S sends $(v, \mathbb{P})$ to $\mathcal{F}_{\mathsf{bcast}}$, where $v \in \{0,1\}^*$ and $\mathbb{P} \subset \{\mathrm{P}_1 \ldots \mathrm{P}_n\}$.
2. If either S or a party from $\mathbb{P}$ is corrupt, then $\mathcal{A}$ receives v and can decide which parties from $\mathbb{P}$ abort and which receive the output by sending a $|\mathbb{P}|$ long bit-vector b to the ideal functionality. For $\mathrm{P}_i \in \mathbb{P}$:
   a. If $b_i = 1$, then $\mathcal{F}_{\mathsf{bcast}}$ sends v to P_i.
   b. If $b_i = 0$, then $\mathcal{F}_{\mathsf{bcast}}$ sends $\perp$ to P_i.

Figure 1 Broadcast functionality.

The message checking functionality $\mathcal{F}_{\mathsf{check}}$ (Figure 2) allows a receiver R, who holds a vector of messages, to check whether all other parties $\mathrm{P}_1, \ldots, \mathrm{P}_n$ hold the same vector of messages. The functionality can be instantiated by letting each party P_i send its input to R. However, in this case the communication overhead is $\Theta(n\ell)$ messages, where ℓ is the number of messages in a vector. Assuming the existence of collision-resistant hash functions, one can obtain a more communication efficient solution by simply letting all parties hash their message vectors into small digests before sending them to R. The communication overhead of this is $\Theta(n\lambda)$ bits if we assume that the output length of the hash function is $\Theta(\lambda)$.

2.2 Additive Secret Sharing

We recall what additive secret sharing is and how to perform some basic operations on it. We will use this type of secret sharing in our three-party protocol in Section 4 and the modulus 2^m defines the word size over which computations will be performed. For example, for arithmetic computations over 64-bit integers, one can set $m = 64$. For the sake of concreteness, we restrict our attention to the three-party case.

If party P_i wants to share a value $a \in \mathbb{Z}_{2^m}$, it picks uniformly random $a_1, a_2 \leftarrow \mathbb{Z}_{2^m}$, sets $a_3 = a - a_1 - a_2 \mod 2^m$, and sends a_j to P_j. We use $[a]_m$ to denote additive secret sharing of a modulo 2^m. For a prime p, we will abuse notation and use $[a]_p$ to denote a secret sharing of a modulo p. To open a value $[a]_m$, every party P_i sends its value a_i to P_{i-1} and P_{i+1}.

Functionality $\mathcal{F}_{\mathsf{check}}$ The functionality runs with receiver R, parties $\mathrm{P}_1, \ldots, \mathrm{P}_n$, and adversary $\mathcal{A}$. Party $\mathrm{P}_i \in \{\mathrm{P}_1, \ldots, \mathrm{P}_n\}$ has input $\left(m_{(1,i)}, \ldots, m_{(\ell,i)}\right)$ and receiver R has $(m_1, \ldots, m_\ell)$.

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1. All parties send their inputs to the $\mathcal{F}_{\mathsf{check}}$.
2. $\mathcal{A}$ can decide to continue or to abort.
   a. If $\mathcal{A}$ continues, then $\mathcal{F}_{\mathsf{check}}$ checks whether all inputs are the identical. It outputs same if this is the case, and different otherwise, to the receiver R (in the latter case, the functionality gives the inputs of all honest parties to $\mathcal{A}$).
   b. If $\mathcal{A}$ aborts, then $\mathcal{F}_{\mathsf{check}}$ sends $\perp$ to all parties.

Figure 2 Message checking functionality.

To add constant c to $[a]_m$, i.e., compute $[b]_m$ with $b = c + a \mod 2^m$, P_1 locally computes $b_1 = a_1 + c \mod 2^m$, while P_2 and P_3 just set $b_i = a_i$. To compute $[c]_m$, where $c = a + b \mod 2^m$, every party P_i locally adds its shares, i.e., computes $c_i = a_i + b_i \mod 2^m$.

Given a secret shared multiplication triple $([x]_m, [y]_m, [z]_m)$ with $z = x \cdot y \mod 2^m$ and two secret shared values $[a]_m$ and $[b]_m$, we compute $[c]_m$ with $c = a \cdot b \mod 2^m$ as follows:

1. Open $e = [x]_m + [a]_m$ and $d = [y]_m + [b]_m$
2. Compute $[c]_m = [z]_m + e \cdot [b]_m + d \cdot [a]_m - ed$

2.3 Additive Replicated Secret Sharing

We will use additive replicated secret sharing in our preprocessing protocol in Section 5 because it allows for efficient multiplication. Since our preprocessing protocol focuses on the three-party case, we will also restrict our attention to this case here.

If party P_i wants to share a value $a \in \mathbb{Z}_{2^m}$, it sets $a_i = 0$ and samples $a_{i+1}, a_{i-1} \leftarrow \mathbb{Z}_{2^m}$ under the constraint that $a = a_1 + a_2 + a_3 \mod 2^m$. It then sends a_{j-1} and a_{j+1} to P_j.[3] We write $[\![a]\!]_m$ to denote an additive replicated secret sharing of a modulo 2^m. We will abuse notation and write $[\![a]\!]_p$ to denote the additive replicated secret sharing modulo a prime p.

Generating a random shared value is a subroutine which will be useful in later protocols. If the parties want to generate shares of a random value they can do it in the following two ways. For unconditionally secure randomness each party P_i picks a random $s_{i-1} \in \mathbb{Z}_{2^m}$ and sends it to P_{i+1} while at the same time receiving s_{i+1} from P_{i-1}. For computationally secure randomness the parties run the unconditionally secure version, at the beginning of the protocol, once and for all, and interpret their shares as PRF keys K_1, K_2, K_3 such that party P_i knows K_{i-1} and K_{i+1}. When they want to generate the j-th random share, the parties define their shares $s^j_{i-1} = F_{K_{i-1}}(j)$ and $s^j_{i+1} = F_{K_{i+1}}(j)$.

To reveal a secret shared value $[\![a]\!]_m$, each party P_i sends a_{i-1} to P_{i-1} and a_{i+1} to P_{i+1}. Each P_j receives a_j from P_{j-1} and P_{j+1}, checks consistency of the received values, and outputs $a = a_1 + a_2 + a_3 \mod 2^m$ if the check passed. *Computational Security:* Opening

[3] This is a small yet non-trivial optimization of the protocol of [27], where a_i is also a random share and the other two parties have to check consistency of this value. By setting $a_i = 0$ we save communication of 4 ring elements per input gate, and one additional round of communication. Note that this change has no impact on security. If P_i is corrupt we need that the two other parties receive the same value of a_i, and this is trivially achieved by setting the value to 0. If one of the other two parties is corrupt, they would learn a_i anyway so whether it is random or a constant value has no security impact.

several values $a^{(1)}, \ldots, a^{(n)}$ can be optimized as follows: Each P_j only receives $a_j^{(l)}$ from P_{j-1} and computes $a^{(l)} = a_1^{(l)} + a_2^{(l)} + a_3^{(l)} \mod 2^m$. In addition the parties broadcast hashes of $(a^{(1)}, \ldots, a^{(n)})$ and abort in case of a mismatch.

To add a public constant c to a secret shared value $[\![a]\!]_m$, i.e., to compute $[\![b]\!]_m$, where $b = c + a \mod 2^m$, we set $b_1 = a_1 + c$, $b_2 = a_2$, and $b_3 = a_3$. To add $[\![a]\!]_m$ and $[\![b]\!]_m$, i.e., to compute $[\![c]\!]_m$, where $c = a + b \mod 2^m$ every party P_i locally adds their shares. It computes $c_{i-1} = a_{i-1} + b_{i-1} \mod m$ and $c_{i+1} = a_{i+1} + b_{i+1} \mod 2^m$. To multiply $[\![a]\!]_m$ by constant c, i.e., to obtain $[\![b]\!]_m$ with $b = c \cdot a \mod 2^m$, every party P_i computes $b_{i-1} = c \cdot a_{i-1} \mod 2^m$ and $b_{i+1} = c \cdot a_{i+1} \mod 2^m$.

Given $[\![a]\!]_m$ and $[\![b]\!]_m$, we can compute $[\![c]\!]_m$, with $c = a \cdot b \mod 2^m$, optimistically (i.e. with potential error in case of cheating) as follows:

1. The parties generate a random value $[\![s]\!]_m$;
2. Each P_i computes $u_{i+1} = a_{i+1}b_{i+1} + a_{i+1}b_{i-1} + a_{i-1}b_{i+1} + s_{i-1}$ and sends u_{i+1} to P_{i-1};
3. P_i receives u_{i-1}, thus defining $[\![u]\!]_m$;
4. The parties compute $[\![c]\!]_m = [\![u]\!]_m - [\![s]\!]_m$.

2.4 Additive Replicated Secret Sharing with Redundant Shares

In some of our protocols we use a different kind of replicated secret sharing, which we denote as $[\![x]\!]_{m,\lambda}$. Those are sharing of values in $\mathbb{Z}_{2^m}$ but represented with shares in $\mathbb{Z}_{2^{m+\lambda}}$, where λ is a security parameter. Those shares work as the regular additive replicated secret sharings described in the previous sections (e.g., all basic commands are unchanged), but employ shares in a larger ring – this is useful for checking correctness of multiplication triples as we shall see. We describe some basic protocols that can be run with this kind of shares:

To convert $[\![x]\!]_m$ to $[\![x]\!]_{m,\lambda}$ each party simply interprets their shares as elements of the larger ring (in other words, the shares are padded with 0s in the λ most significant positions). Note that in general $\sum_i x_i \mod 2^{m+\lambda} \neq x$, that is the sum can be either equal to x or to $x + 2^m$ depending on the magnitude of the shares. However, since the semantic of our sharing is that the shared value is $\sum_i x_i \mod 2^m$ the protocol is indeed correct (and this notation allows us a simpler description of more advanced protocols).

To convert $[\![x]\!]_{m,\lambda}$ down to $[\![x]\!]_m$ each party P_i reduces their shares modulo 2^m as $x'_{i+1} = x_{i+1} \mod 2^m$ and $x'_{i-1} = x_{i-1} \mod 2^m$. Both conversions preserve the shared value because computing modulo 2^m and modulo $2^{m+\lambda}$ are commutative as 2^m divides $2^{m+\lambda}$ and both operations trivially preserve the replication of shares.

2.5 Additive Replicated Secret Sharing over the Integers

Finally, we recall the replicated secret sharing over integers from [27]. The authors observed that one can secret share a value $a \in \mathbb{Z}_{2^m}$ over the integers using shares with bit-length $m + \lambda$. The λ extra bits ensure that the statistical distance between the distributions of shares for any two values in $\mathbb{Z}_{2^m}$ is negligible in λ.

To share a value $a \in \mathbb{Z}_{2^m}$, P_i picks $a_1, a_2 \leftarrow \{0, \ldots, 2^{m+\lambda} - 1\}$ and sets $a_3 = a - a_1 - a_2$. The shares are distributed among the parties as above. We write $[\![a]\!]_{\mathbb{Z}}$ to denote an additive replicated secret sharing of a over the integers.

Optimistic multiplication of $[\![a]\!]_{\mathbb{Z}}$ and $[\![b]\!]_{\mathbb{Z}}$ is similar to its counterpart modulo p. Let B be a bound on the share amplitude. Optimistically compute $[\![c]\!]_{\mathbb{Z}}$ with $c = a \cdot b$ as follows:

1. The parties generate shares of a random value $[\![s]\!]_{\mathbb{Z}}$ as described above, but s_i are chosen in $\{0, \ldots, 2^{2\lceil \log B \rceil + \lambda + 2} - 1\}$ (if the information-theoretic version is used, parties also check that the received shares s_i are in this range);

2. Each P_i computes u_{i+1} as before but over $\mathbb{Z}$;
3. P_i receives u_{i-1} and checks $|u_{i-1}| \leq 2^{2^{\lceil \log B \rceil + \lambda + 3}}$;
4. The parties compute $[\![c]\!]_m = [\![u]\!]_m - [\![s]\!]_m$.

Other operations are analogous to their counterparts modulo m. For a more details see [27].

3 Extension of the Compiler by Damgård et al.

The compiler $\mathsf{COMP}_{\mathsf{old}}$ by Damgård et al. [27] takes an n-party passively $\left(t^2+t\right)$-secure protocol Π and transforms it into a protocol $\mathsf{COMP}_{\mathsf{old}}(\Pi)$ that is secure with abort against t active corruptions[4]. For $t = 1$, the compiler transforms a passively two-secure three-party protocol into a protocol that is secure against one active corruption. The high-level idea of the compiler is to let virtual parties execute the passively secure protocol on behalf of the real parties. Each virtual party $\mathbb{P}_i$ is simulated by $t+1$ real parties $\mathrm{P}_i, \ldots, \mathrm{P}_{i+t}$ in a way that prevents an active adversary, who controls at most t real parties, from corrupting any of the virtual parties. Meaning that corrupting t real parties allows the adversary to see the view of at most $\left(t^2+t\right)$ virtual parties running the passive protocol. In the following we will write $\mathrm{P}_j \in \mathbb{P}_i$ to denote that real party P_j is simulating virtual party $\mathbb{P}_i$.

The workflow of their compiler can be split into two phases. In the first phase, for each virtual party $\mathbb{P}_i$, all real parties $\mathrm{P}_j \in \mathbb{P}_i$ agree on a common input and randomness that will be used by $\mathbb{P}_i$ during the execution of the passively secure protocol Π. Having the same input and the same randomness, every $\mathrm{P}_j \in \mathbb{P}_i$ will be able to redundantly compute the exact same messages that $\mathbb{P}_i$ is supposed to send during the execution of Π. In the second phase, the virtual parties run Π to compute the desired functionality from the inputs and randomness that the virtual parties have agreed upon. Whenever $\mathbb{P}_i$ is supposed to send a message to $\mathbb{P}_j$ according to Π, *every* real party simulating $\mathbb{P}_i$ will send a separate message to *every* real party simulating $\mathbb{P}_j$. Each real party verifies that it receives the same message from all sending real parties and aborts if this is not the case.

Intuitively, the resulting protocol is secure against t active corruptions, since an adversary cannot misbehave on behalf of a virtual party it is simulating, and at the same time be consistent with at least one other honest real party in the same virtual party. From an efficiency point of view, every message from one $\mathbb{P}_i$ to some other $\mathbb{P}_j$ is sent redundantly from $t+1$ to $t+1$ real parties. That is, if the passively secure protocol Π sends ℓ messages during a protocol execution, then $\mathsf{COMP}_{\mathsf{old}}(\Pi)$ will send roughly $\mathcal{O}\left(\ell \cdot t^2\right)$ messages.

3.1 A New Compiler for Protocols with Weak Privacy

We present a new compiler $\mathsf{COMP}_{\mathsf{new}}$, which makes slightly stronger assumptions about the starting protocol Π, but compiles it into an actively secure protocol in a more communication efficient manner. $\mathsf{COMP}_{\mathsf{new}}$ takes as input a $\left(t^2+t\right)$-weakly private protocol Π and outputs a compiled protocol $\mathsf{COMP}_{\mathsf{new}}(\Pi)$ that is secure against t active corruptions. If Π sends ℓ messages in total, then our compiled protocol will only send $\mathcal{O}\left(\ell \cdot t + t^2\right)$ messages.

Our new compiler follows the approach of $\mathsf{COMP}_{\mathsf{old}}$. However, instead of verifying the validity of every single message between virtual parties as soon as it is sent, we will let the real parties simulate the virtual parties in a more optimistic and communication efficient fashion, where the correctness of all communicated messages is only verified once at the end of the computation phase, right before the opening phase of Π. Pushing the whole

[4] The authors also show how to achieve active security with guaranteed output delivery, but we focus on security with abort.

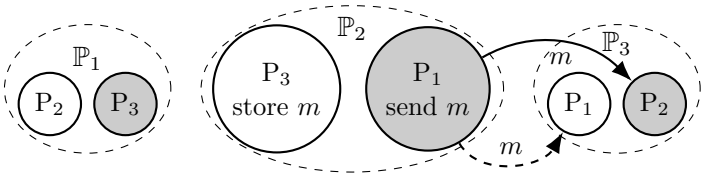

Figure 3 Simulation strategy for three parties with one active corruption. Dashed ellipses represent virtual parties and solid circles represent the real parties simulating it (brains are gray). Virtual party $\mathbb{P}_2$ is sending a message to virtual party $\mathbb{P}_3$. The arrows indicate that P_1, the brain of $\mathbb{P}_2$, sends one message to P_2 and one to P_1, which is omitted in reality, since it is sending a message to itself. P_3 stores this message in its transcript.

verification to the end of the computation phase allows us to reduce the total number or redundant messages that are sent. This new simulation strategy crucially relies on the weak active privacy of Π, since we are now allowing the adversary to misbehave up to the opening phase without aborting the protocol execution.

The first phase of $\mathsf{COMP}_{\mathsf{new}}$, where all parties agree on their inputs and random tapes, is identical to that of $\mathsf{COMP}_{\mathsf{old}}$ and is thus equally efficient. In the second phase, our new simulation approach works by selecting one arbitrary real party P_i in each virtual party $\mathbb{P}_j$ to be the brain $\mathrm{B}_j := \mathrm{P}_i$ of that virtual party. The brains will act on behalf of their corresponding virtual parties in an optimistic fashion and execute the computation phase of Π up to the opening phase. All other real parties, the pinkies, will receive the messages that their corresponding virtual parties should receive, which enables them to follow the protocol locally. However, the pinkies will not send any messages during the computation phase. They will only become actively involved in the opening phase to ensure that all brains behaved honestly during the computation phase. Once correctness is ensured, all parties will jointly perform the opening phase of Π. During the computation phase of Π, whenever virtual party $\mathbb{P}_i$ is supposed to send a message to virtual party $\mathbb{P}_j$, we let B_i send one message to each real party in $\mathbb{P}_j$. The receiving real parties do not perform any checks at this moment and just store the message. B_j will optimistically continue the protocol execution on behalf of $\mathbb{P}_j$ according to Π and the received message. This simulation strategy is illustrated in Figure 3.

At the end of the computation phase, all real parties jointly make sure that for each pair $(\mathbb{P}_i, \mathbb{P}_j)$, the sending virtual party $\mathbb{P}_i$ always behaved honestly towards the receiving virtual party $\mathbb{P}_j$. This is accomplished by using a message checking protocol (that implements $\mathcal{F}_{\mathsf{check}}$). If any of these checks output $\mathsf{different}$, then the protocol execution is aborted.

In the opening phase, after passing the previous check, every virtual party is supposed to send its last opening message to all other virtual parties. For each pair $(\mathbb{P}_i, \mathbb{P}_j)$, all real parties in $\mathbb{P}_i$ send the last message to all real parties in $\mathbb{P}_j$. Every receiving party checks that all $t+1$ received messages are consistent and aborts if this is not the case.

In our formal description, let $f(x_1, \ldots, x_n)$ be the n-party functionality that we want to compute. For the sake of simplicity and without loss of generality, we assume that all parties learn the output of the computation. Let $\mathbb{P}_i$ be the virtual party that is simulated by real parties $\mathrm{P}_i, \ldots, \mathrm{P}_{i+t}$. Let $\mathbb{V}_i$ be the set of virtual parties in whose simulation P_i participates.

Let f' be a related n-party functionality that takes as input $\left(x_1^i, \ldots, x_n^i\right)$ from every P_i and outputs $f(\sum_{i=1}^n x_1^i, \ldots, \sum_{i=1}^n x_n^i)$. That is, every party inputs one secret share of every original input. The functionality f' reconstructs the original inputs for f from the secret shares and then evaluates f on those inputs. Let $\Pi_{f'}$ be a passively $\left(t^2+t\right)$-secure protocol with weak privacy that securely implements $\mathcal{F}_{f'}$. The formal description of our compiler is given in Figure 4. Throughout our description we assume that honest parties consider message that they do not receive as malicious and act accordingly.

$\mathsf{COMP}_{\mathsf{new}}(\Pi_{f'})$

Inputs: Each party P_i has input x_i.

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1. **Input sharing:**
   a. Each P_i secret shares its input $x_i = x_i^1 + \cdots + x_n^i$.
   b. For $1 \leq j \leq n$, each P_i sends $\left(x_i^j, \mathbb{P}_j\right)$ to the broadcast functionality $\mathcal{F}_{\mathsf{bcast}}$.
   c. Each P_i receives $z_j := \left(x_1^j, \ldots, x_n^j\right)$ for each $\mathbb{P}_j \in \mathbb{V}_i$ from the broadcast functionality and aborts if any of the shares equals $\perp$.
2. **Randomness:** Each brain B_i chooses a uniformly random string r_i and sends $(r_i, \mathbb{P}_i)$ to $\mathcal{F}_{\mathsf{bcast}}$. The receiving real parties abort if they receive $\perp$.
3. **Computation phase:** All virtual parties jointly execute the computation phase of $\Pi_{f'}$, where each $\mathbb{P}_i$ uses input z_j and random tape r_i, as follows:
   - Whenever $\mathbb{P}_i$ is supposed to send message m to $\mathbb{P}_j$, the brain B_i sends m to all $\mathrm{P}_k \in \mathbb{P}_j$.
   - Whenever $\mathbb{P}_j$ receives message m, all parties store the message but only B_j continues sending messages according to $\Pi_{f'}$. The pinkies locally follow the protocol and compute the message that they would send.
4. **Check:** At the end of the computation phase, all parties, brains and pinkies, jointly check that the current transcript is valid. For each pair $(\mathbb{P}_i, \mathbb{P}_j)$, for each party $\mathrm{P}_k \in \mathbb{P}_j$, we invoke $\mathcal{F}_{\mathsf{check}}$, where P_k acts as the receiver and $\mathbb{P}_i$ act as the remaining parties. The input of P_k is the list of messages it received from $\mathbb{P}_i$ and the input of all parties from $\mathbb{P}_i$ is the list of messages that they would have sent. If any invocation outputs $\mathsf{different}$, then the protocol execution is aborted.
5. **Opening phase:**
   a. For each pair $(\mathbb{P}_i, \mathbb{P}_j)$, all real parties in $\mathbb{P}_i$ send the last message of $\Pi_{f'}$ to all real parties in $\mathbb{P}_j$.
   b. Every real party in $\mathbb{P}_j$ checks that all received messages are equal. If they are it obtains the output of the computation and otherwise it aborts.

Figure 4 Formal description of our compiler.

▶ **Theorem 1.** *Let $n \geq 3$. Assume $\Pi_{f'}$ implements n-party functionality $\mathcal{F}_{f'}$ with $\left(t^2 + t\right)$-weak privacy. Then, $\mathsf{COMP}_{\mathsf{new}}(\Pi_{f'})$ implements functionality $\mathcal{F}_f$ with active security under individual abort against t corruptions. If $\Pi_{f'}$ has a total bandwidth cost of ℓ messages, then $\mathsf{COMP}_{\mathsf{new}}(\Pi_{f'})$ has a total bandwidth cost of $\mathcal{O}\left(\ell \cdot t + t^2\right)$ messages.*

Proof. Our proof closely follows the proof of [27] for the $\mathsf{COMP}_{\mathsf{old}}$ compiler. Let $\mathbb{P}^*$ be the set of corrupted real parties and let $\mathbb{V}^*$ be the set of virtual parties that are simulated by at least one corrupt real party. Let $\mathcal{S}_{f'}$ be the simulator of the $(t^2 + t)$-weakly private protocol $\Pi_{f'}$. We will use this simulator to construct a simulator $\mathcal{S}$ for the overall actively secure protocol $\mathsf{COMP}_{\mathsf{new}}(\Pi_{f'})$. The simulator $\mathcal{S}$ works as follows:

1. For each party $\mathrm{P}_i \in \mathbb{P}^*$ and $j \in [n]$, the adversary $\mathcal{Z}$ sends $(x_i^j, \mathbb{P}_j)$ to the ideal functionality $\mathcal{F}_{\mathsf{bcast}}$, which is emulated by the simulator $\mathcal{S}$. For any invocation that involves a corrupted party, the environment decides which outputs are $\perp$ and which get delivered. For each $\mathbb{P}_j \in \mathbb{V}^*$ and each corrupt real party in $\mathbb{P}_j$, we send back $(x_1^j, \ldots, x_n^j)$, where x_i^j is either the share that was sent by $\mathcal{Z}$ if P_i is corrupt or otherwise a uniformly random share.

2. For each corrupted party $\mathrm{P}_i \in \mathbb{P}^*$, we reconstruct its input as $x_i = \sum_{j=1}^{n} x_i^j$.
3. $\mathcal{S}$ sends the inputs of the corrupted parties to $\mathcal{F}_f$ and receives back the output of the computation $z = f(x_1, \ldots, x_n)$.
4. For each $\mathbb{P}_i \in \mathbb{V}^*$ we consider two cases. If the brain B_i is corrupted, then it chooses a random tape r_i and sends it to $\mathcal{F}_{\mathsf{bcast}}$, which again is simulated by $\mathcal{S}$. If B_i is honest, then the simulator picks a uniformly random r_i and sends it back to $\mathcal{Z}$ on behalf of $\mathcal{F}_{\mathsf{bcast}}$. Again, the environment can decide that some of the outputs in this step will be $\perp$, which will then be handled accordingly by our simulator.
5. At this point, we know the inputs and the random tapes of all virtual parties $\mathbb{P}_i \in \mathbb{V}^*$. We can therefore compute the exact messages that we would expect from an honest party following the protocol. We initialize the simulator $\mathcal{S}_{f'}$ with parties $\mathbb{P}_1 \ldots \mathbb{P}_n$ and the set of corrupted players $\mathbb{V}^*$.
6. When $\mathcal{S}_{f'}$ queries $\mathcal{F}_{f'}$ for the inputs of the corrupted parties, we give it $(x_1^i, \ldots, x_n^i)$ for each $\mathbb{P}_i \in \mathbb{V}^*$.
7. We now describe how to simulate the computation phase of the protocol.
   - $\mathcal{S}$ queries $\mathcal{S}_{f'}$ for the messages that the honest brains send to the corrupted virtual parties. For each message m to some $\mathbb{P}_i \in \mathbb{V}^*$, we send m to each corrupted real party in $\mathbb{P}_i$ (unless the sender received $\perp$ in one of step 1 or 4 of this simulator in which case it sends nothing).
   - $\mathcal{Z}$ outputs the messages that the corrupt parties send to the honest ones. Since we know the input and random tape of each corrupted party, we can see which messages are honestly generated and which are not. Forward the message of the sending brain to $\mathcal{S}_{f'}$ as the message of $\mathbb{P}_i$.
8. At the end of the computation phase, we simulate the check protocol as follows. For each pair $(\mathbb{P}_i, \mathbb{P}_j)$, for each real party $R \in \mathbb{P}_j$, we have one invocation of the functionality $\mathcal{F}_{\mathsf{check}}$. The simulator $\mathcal{S}$ needs to simulate the ideal functionality towards the corrupted parties in each invocation that involves a corrupted party. Note that the inputs of all honest parties to each check are known from the previous part of the simulation. We, at this point, also know whether any of the corrupted brains cheated or not and if so which check invocation should fail. Furthermore, whenever a corrupted party sends a value to the check functionality, we know whether it's the correct one or not. Using the above observations it follows that the simulator always knows how to simulate each invocation of $\mathcal{F}_{\mathsf{check}}$ and when to return $\mathsf{different}$, when to return same, and when to abort the computation.
9. If all checks passed, meaning that the adversary did not misbehave at any point in time, then we continue the simulation. The simulator $\mathcal{S}$ knows all the last messages of each corrupted party and it knows the output of the functionality z. Since the opening phase is a linear reconstruction of the last messages, the simulator picks a uniformly last message for each honest party under the condition that the linear combination of all last messages results in z. The simulator faithfully executes the last step of the protocol compiler with the corrupted parties. For any simulated honest real party that receives incorrect messages from $\mathcal{Z}$, we will instruct $\mathcal{F}_f$ to make this party abort. For any honest real party that receives the correct last round messages, we instruct $\mathcal{F}_f$ to deliver the output of the computation.

The simulation of the first protocol phase (steps 1-4) is perfect. The adversary sees uniformly random shares, random tapes, or the things it sent itself just like in a real execution. The simulation of $\mathcal{F}_{\mathsf{bcast}}$ is identical to a real execution. The indistinguishability of the simulation in step 6 directly follows from the security guarantees of $\mathcal{S}_{f'}$. As in the real execution, we do

not send anything from real honest parties that may have aborted during the first phase. Otherwise, in both the real and the ideal world, the protocol does not abort during the computation phase. During the computation phase $\mathcal{S}$ has access to the random tapes and inputs of the corrupted parties, thus always knows when and where cheating occurred. This enables to correctly determine when and where the protocol would abort and simulate the outcome of the check phase in step 8 correctly. ◀

Similar to [27], we proved our result for the case of active security with individual abort, where some honest parties may terminate, while some may not. As in their work, our result easily extends to unanimous abort with one additional round of secure broadcast.

4 Efficient Three-Party Computation

All of our protocols are fundamentally based on the seminal work of Beaver [5], who presented a conceptually simple and clean approach for passively secure circuit evaluation. We present two flavors of protocols. One with preprocessing and two different instantiations of the preprocessing phase and one without preprocessing, but some light postprocessing. The protocols that involve preprocessing have a larger *total* runtime, but a leaner online phase, whereas the postprocessing protocol has a smaller overall runtime. Our protocol with preprocessing is presented in this section and the two different preprocessing protocols are presented in Section 5. Protocol with postprocessing is presented in Section 6.

4.1 Beaver's Circuit Evaluation Approach

The circuit evaluation approach by Beaver [5] enables, in our case, three parties to evaluate an arithmetic circuit f over arbitrary rings $\mathbb{Z}_{2^m}$ with security against two passive corruptions. The protocol is split into a preprocessing and an online phase. During the preprocessing phase the parties jointly generate some function-independent correlated randomness in the form of additively secret shared multiplication triples $[a_i]_m$, $[b_i]_m$, $[c_i]_m$, where $c_i = a_i \cdot b_i \mod 2^m$. In the online phase these triples are then consumed to securely evaluate some desired function f. Beaver's online phase works in three steps. First, all parties additively secret share their input among the other parties. Then, all parties jointly evaluate the circuit in a gate-by-gate fashion on the secret shared values. Additions are performed locally, and multiplications require interaction as well as correlated randomness as explained in Section 2.2. In the last step, the parties jointly reconstruct the secret shared values of the output wires of the circuit. Note that the reconstruction phase is just a linear function of the messages received during the opening phase.

▶ **Proposition 2.** *Let f be an arithmetic circuit with N multiplication gates. Given N preprocessed multiplication triples, the three-party protocol* Beaver_f*, implements functionality $\mathcal{F}_f$ with 2-weak privacy and has linear reconstruction.*

4.2 Our Protocol with Preprocessing

We focus on the popular setting with three parties and one active corruption and obtain our protocol by applying Theorem 1 to Beaver's circuit evaluation approach. Let f be the three-party functionality that shall be computed, where each party P_i has an input $x_i \in \mathbb{Z}_{2^m}$. As before, let f' be the related three-party functionality that first recomputes the original inputs from the additive secret shares and then evaluates f. Let N be the number of multiplication gates in f and assume for the moment that all real parties have

$\mathsf{COMP}_{\mathsf{new}}(\mathsf{Beaver}_{f'})$

Inputs: Each party P_i has input $x_i \in \mathbb{Z}_{2^m}$ and they all share preprocessed triples $[\![a_j]\!]_m, [\![b_j]\!]_m, [\![c_j]\!]_m$ for $j \in \{1, \ldots, N\}$.

1. **Input sharing:**
   a. Each P_i picks $x_i^1, x_i^2 \leftarrow \mathbb{Z}_{2^m}$ and sets $x_i^3 = x_i - x_i^1 - x_i^2 \mod 2^m$.
   b. For $1 \le j \le 3$, each P_i sends $\left(x_i^j, \mathbb{P}_j\right)$ to the broadcast functionality $\mathcal{F}_{\mathsf{bcast}}$.
   c. Each P_i receives $z_{i-1} := \left(x_1^{i-1}, x_2^{i-1}, x_3^{i-1}\right)$ and $z_{i+1} := \left(x_1^{i+1}, x_2^{i+1}, x_3^{i+1}\right)$ via $\mathcal{F}_{\mathsf{bcast}}$ and aborts if any of the shares equals $\perp$.
2. **Randomness:** Each brain B_i chooses a uniformly random string r_i and sends $(r_i, \mathbb{P}_i)$ to $\mathcal{F}_{\mathsf{bcast}}$. The receiving real parties abort if they receive $\perp$.
3. **Computation phase:** All virtual parties now evaluate $\mathsf{Beaver}_{f'}$ in a gate-by-gate fashion, where each $\mathbb{P}_i$ uses input z_i as follows:
   - Multiplication gates are evaluated using correlated randomness.
   - Other gates are executed locally.
4. **Check:** For each pair $(\mathbb{P}_i, \mathbb{P}_j)$, we use $\mathcal{F}_{\mathsf{check}}$ to verify the correctness of the messages sent from $\mathbb{P}_i$ to $\mathbb{P}_j$.
5. **Opening phase:**
   a. For each output wire w, for each pair $(\mathbb{P}_i, \mathbb{P}_j)$, all real parties in $\mathbb{P}_i$ send their secret share of w to all real parties in $\mathbb{P}_j$.
   b. For all $\mathbb{P}_j$, all $\mathrm{P}_k \in \mathbb{P}_j$ check that all received are equal. If not, abort.
   c. If all received shares are consistent, then reconstruct the output and terminate.

Figure 5 Three-party arithmetic circuit evaluation in $\mathbb{Z}_{2^m}$ with active security with abort against one active corruption.

already shared this many *replicated* secret shares of multiplication triples $[\![a_i]\!]_m$, $[\![b_i]\!]_m$, $[\![c_i]\!]_m$ in a preprocessing phase[5]. Our concrete preprocessing protocol will be described in detail in Section 5. Since for $1 \le i \le 3$, virtual party $\mathbb{P}_i$ will be simulated by P_{i-1} and P_{i+1}, it holds that real parties holding replicated shares is equivalent to virtual parties holding additive secret shares. This way, one can think of those replicated shares as parts of the real parties' inputs that have already been shared correctly among the virtual parties during preprocessing. We state the compiled protocol $\mathsf{COMP}_{\mathsf{new}}(\mathsf{Beaver}_{f'})$ in Figure 5.

We explicitly state the concrete communication complexity of the protocol. Addition gates require no communication. Evaluating a multiplication gate requires sending 6 words of m bit each. The opening phase, including the checking protocol, requires sending 5 hash values (we choose 256 as the output of the hash) as well as the output shares giving a total of $1280 + 4m \cdot |\mathrm{out}|$ bits for out output gates.

5 Preprocessing

During the preprocessing phase we generate replicated secret sharings of multiplication triples $c = a \cdot b \mod 2^m$. We describe two versions of this phase. The first is obtained combining the preprocessing of Damgård et al. [27] with the batch verification technique of

[5] The functionalities f and f' have equally many multiplication gates, since reconstructing the inputs from additive secret shares does not require any multiplications.

SingleVerify

Inputs: Shared triples $[\![a]\!]_{\mathbb{Z}}, [\![b]\!]_{\mathbb{Z}}, [\![c]\!]_{\mathbb{Z}}$ and $[\![x]\!]_p, [\![y]\!]_p, [\![z]\!]_p$ with prime $p > c$

Interpret $[\![a]\!]_{\mathbb{Z}}, [\![b]\!]_{\mathbb{Z}}, [\![c]\!]_{\mathbb{Z}}$ as a triple in $\mathbb{Z}_p$.

1. Parties generate a random $[\![r]\!]_p$ and open r.
2. Parties compute $[\![e]\!]_p = r[\![x]\!]_p + [\![a]\!]_p$.
3. Parties compute $[\![d]\!]_p = [\![y]\!]_p + [\![b]\!]_p$.
4. Open e, d and compute $[\![t]\!]_p = de - rd[\![x]\!]_p - e[\![y]\!]_p + r[\![z]\!]_p - [\![c]\!]_p$
5. Open $[\![t]\!]_p$ and output success if $t = 0$ and fail otherwise.

Figure 6 Verification of triple $([\![a]\!]_{\mathbb{Z}}, [\![b]\!]_{\mathbb{Z}}, [\![c]\!]_{\mathbb{Z}})$ sacrificing one triple to check the other.

Ben-Sasson et al. [7]. The generation of multiplication triples is split in three steps. First, based on Damgård et al., we optimistically generate secret shared multiplication triples over the integers. Next, we interpret them as triples in a field $\mathbb{Z}_p$, for some sufficiently large prime p, and perform the batch verification protocol of Ben-Sasson et al. to ensure that all triples are correct. Lastly, we reduce all integer shares modulo 2^m to obtain shares of multiplication triples in our desired ring $\mathbb{Z}_{2^m}$[6].

The second version is inspired by Cramer et al. [21], where MACs modulo some prime are replaced with MACs modulo $2^{m+\lambda}$ with 2^m being the "plaintext space" and λ being a statistical security parameter. Hence, we replace the computation modulo prime in the correctness check with a check performed over $2^{m+\lambda}$, which still guarantees security. This is efficient because we avoid the computational complexity of computing modulo primes.

5.1 [27]-style Preprocessing

Optimistic generation of a multiplication triple over the integers is straightforward. First each party P_i uses replicated secret sharing over the integers to share random values $a_i, b_i \in \mathbb{Z}_{2^m}$. All parties jointly compute $[\![a]\!]_{\mathbb{Z}} = \sum_{i=1}^{3} [\![a_i]\!]_{\mathbb{Z}}$ and $[\![b]\!]_{\mathbb{Z}} = \sum_{i=1}^{3} [\![b_i]\!]_{\mathbb{Z}}$ and then use the optimistic multiplication of replicated secret shares from Section 2.5 to compute $[\![c]\!]_{\mathbb{Z}}$.

Given an optimistically generated triple $[\![a]\!]_{\mathbb{Z}}, [\![b]\!]_{\mathbb{Z}}, [\![c]\!]_{\mathbb{Z}}$, the verification of [27] proceeds as follows. First, optimistically generate another multiplication triple in $\mathbb{Z}_p$, where p is a prime such that $p > c$. Then parties interpret the multiplication triple over the integers as a triple in $\mathbb{Z}_p$ and employ the standard technique of "sacrificing" one triple to check the other [26]. Concretely, the authors sacrifice the triple in $\mathbb{Z}_p$ to check $[\![a]\!]_{\mathbb{Z}}$, $[\![b]\!]_{\mathbb{Z}}$, $[\![c]\!]_{\mathbb{Z}}$. The check, SingleVerify, is detailed in Figure 6. The rationale behind this approach is that if the multiplicative relation $a \cdot b = c$ holds over the integers, then it also holds in $\mathbb{Z}_p$ and vice versa since $p > c$ and thus no wrap-around due to the modulo operation happens.

Given N optimistically generated multiplication triples $[\![a_i]\!]_{\mathbb{Z}}$, $[\![b_i]\!]_{\mathbb{Z}}$, $[\![c_i]\!]_{\mathbb{Z}}$ over the integers, we would like to efficiently check that, for all $i \in \{1, \dots N\}$, the multiplicative relationship $a_i \cdot b_i = c_i$ holds. Checking every multiplication triple separately, would require us to generate N additional multiplication triples in $\mathbb{Z}_p$ and perform N invocations of SingleVerify.

Instead, we use a clever verification idea of Ben-Sasson et al. [7] to verify N triples with N additional optimistic multiplications and a *single* invocation of SingleVerify. The idea is to encode all multiplication triples $(a_1, b_1, c_1), \dots, (a_N, b_N, c_N)$ as three polynomials (f, g, h),

[6] Valid multiplication triples over integers are valid modulo 2^m.

where the relation $f \cdot g = h$ will hold iff all multiplication triples are correct. Then we will verify that the polynomial relation $f(x) \cdot g(x) \equiv h(x)$ holds.

More concretely, let f and g be degree N-1 polynomials over $\mathbb{Z}_p$ uniquely defined as $f(i) = a_i$ and $g(i) = b_i$. Since, we expect h to be $f \cdot g$ and thus of degree $2N - 2$, we require $2N - 1$ points to uniquely define it. For $i \in \{1, \dots N\}$, we set $h(i) = c_i$. For $i \in \{N + 1, \dots 2N - 1\}$, we set $h(i) = f(i) \cdot g(i)$, where the multiplication is performed optimistically. If all multiplication triples and optimistic multiplications are correct, then $f \cdot g = h$ holds and an evaluation at a random point z will always fulfill $f(z) \cdot g(z) \equiv h(z) \mod p$. If, however, some multiplication triple is not valid, then $f \cdot g \neq h$ and the two polynomials $f \cdot g$ and h can agree on at most $2N - 2$ many points. This means that for a uniformly random point $z \in \mathbb{Z}_p$, we have $\Pr[f(z) \cdot g(z) = h(z) \mid f \cdot g \neq h] \leq \frac{2N-2}{|\mathbb{Z}_p|}$.

This algorithm relies on the fact that we can interpolate and evaluate additively secret shared polynomials. Given shares of points $[\![a_i]\!]_p$ for $i \in \{1, \dots, N\}$ of polynomial f, we would like to evaluate $f(z)$. Define an extension of Kronecker delta $\delta_i^N(x)$ as

$$\delta_i^N(x) := \prod_{j=1, j\neq i}^{N} \frac{x-j}{i-j} = \begin{cases} 1 & x = i \\ 0 & x \neq i, \ x \leq N \end{cases} \text{ giving } [\![f(z)]\!]_p = \sum_{i=1}^{N} \left(\delta_i^N(z) \cdot [\![a_i]\!]_p\right)$$

where $f(x)$ is evaluated locally. Batch verification protocol is formalized in Figure 7. Its security directly follows from the security of the preprocessing of Damgård et al. and the batch verification protocol of Ben-Sasson et al. Let Π_{Triple} be the resulting preprocessing protocol that first optimistically generates N triples over the integers, then executes the batch verification, and finally reduces all shares modulo 2^m.

5.2 [21]-style Preprocessing

As in the previous subsection, optimistic generation of a multiplication triple is straightforward. This time, the parties (using replicated secret sharing), generate random sharings $[\![x]\!]_{m,\lambda}$, $[\![y]\!]_{m,\lambda}$ and then use the optimistic multiplication protocol to compute $[\![z]\!]_{m,\lambda}$.

We present a verified multiplication protocol in $\mathbb{Z}_{2^m}$ where, in order to mitigate zero divisors, most of the computation is executed in $\mathbb{Z}_{2^{m+\lambda}}$ for a statistical parameter λ. The techniques used in this approach are inspired by the protocol SPD$\mathbb{Z}_{2^k}$ [21]. However, here they are used in a very different context, since SPD$\mathbb{Z}_{2^k}$is a protocol for the dishonest majority case (and therefore their preprocessing phase requires expensive public-key operations), while our honest majority protocol can be instantiated using only cheap arithmetic operations.

Figure 8 presents the core of our protocol with replicated sharing with redundant shares. The protocol uses the sacrifice step of the MASCOT protocol [34]. Note that generating the share of a and the value r can be done non-interactively using PRSS. We now evaluate the properties of our protocol. The correctness follows from the correctness of optimistic multiplication and that the fact that $rz + c - ey = rxy + ay - (rx + a) \cdot y = 0 \mod 2^{m+\lambda}$.

Assuming that all openings are verified using $\mathcal{F}_{\mathsf{check}}$ (which ensures that a corrupt party cannot send different shares to different parties), the corrupt party can only deviate by adding an additive error in the optimistic products. We define the (potential) errors as $z = xy + \epsilon_z$ and $c = ay + \epsilon_c$. If the input tuple $[\![x]\!]_{m,\lambda}, [\![y]\!]_{m,\lambda}, [\![z]\!]_{m,\lambda}$ is incorrect then we have that $\epsilon_z \neq 0 \mod 2^m$. Inserting into the check equation, we get $rxy + r\epsilon_z + ay + \epsilon_c - (rx + a)y = r\epsilon_z + \epsilon_c \mod 2^{m+\lambda}$. Here $r\epsilon_z \mod 2^{m+\lambda}$ is uniform in a set of at least size 2^λ as r is a uniformly random λ-bit number. Since the ϵ_c is chosen by the adversary before r is sampled, the adversary will be able to make the protocol accept an incorrect tuple with probability at most $2^{-\lambda}$. This argument corresponds to the proof of Claim 6 of [21].

BatchVerify

Inputs: N preprocessed triples $[\![a_i]\!]_{\mathbb{Z}}$, $[\![b_i]\!]_{\mathbb{Z}}$, $[\![c_i]\!]_{\mathbb{Z}}$ for $i \in \{1, \ldots, N\}$ over the integers. And a uniformly random triple $[\![x]\!]_p, [\![y]\!]_p, [\![z]\!]_p$ in $\mathbb{Z}_p$.

Interpret$[\![a_i]\!]_{\mathbb{Z}}$, $[\![b_i]\!]_{\mathbb{Z}}$, $[\![c_i]\!]_{\mathbb{Z}}$ as a triple in the field $\mathbb{Z}_p$ for a sufficiently large prime p.

1. For $i \in \{1, \ldots, N\}$, define $[\![f(i)]\!]_p := [\![a_i]\!]_p$ and $[\![g(i)]\!]_p := [\![b_i]\!]_p$.
2. For $i \in \{N+1, \ldots, 2N-1\}$, evaluate

$$[\![f(i)]\!]_p := \sum_{j=1}^{N} \left(\delta_j^N(i) \cdot [\![a_j]\!]_p\right), \text{ and } [\![g(i)]\!]_p := \sum_{j=1}^{N} \left(\delta_j^N(i) \cdot [\![b_j]\!]_p\right)$$

3. For $i \in \{1, \ldots, N\}$, define $[\![h(i)]\!]_p := [\![c_i]\!]_p$.
4. For $i \in \{N+1, \ldots, 2N-1\}$, compute $[\![h(i)]\!]_p = [\![f(i)]\!]_p \cdot [\![g(i)]\!]_p$ optimistically.
5. Parties generate a random $[\![z]\!]_p$ and open z.
6. Parties evaluate the polynomials at z:

$$[\![\alpha]\!]_p = [\![f(z)]\!]_p := \sum_{j=1}^{N} \left(\delta_j^N(z) \cdot [\![f(j)]\!]_p\right), \; [\![\beta]\!]_p = [\![g(z)]\!]_p := \sum_{j=1}^{N} \left(\delta_j^N(z) \cdot [\![g(j)]\!]_p\right)$$

$$[\![\gamma]\!]_p = [\![h(z)]\!]_p := \sum_{j=1}^{2N-1} \left(\delta_j^{2N-1}(z) \cdot [\![h(j)]\!]_p\right)$$

7. Test SingleVerify $([\![\alpha]\!]_p, [\![\beta]\!]_p, [\![\gamma]\!]_p, [\![x]\!]_p, [\![y]\!]_p, [\![z]\!]_p)$.

Figure 7 Batch verification of multiplication triples.

SPD$\mathbb{Z}_{2^k}$-like check for correct multiplication

Inputs: Shared triple $[\![x]\!]_{m,\lambda}$, $[\![y]\!]_{m,\lambda}$, $[\![z]\!]_{m,\lambda}$

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1. Parties generate a random $[\![a]\!]_{m,\lambda}$ and execute an optimistic multiplication with $([\![a]\!]_{m,\lambda}, [\![y]\!]_{m,\lambda})$ to get $[\![c]\!]_{m,\lambda}$.
2. Parties jointly generate a random $r \in \mathbb{Z}_{2^\lambda}$.
3. Parties reveal $[\![e]\!]_{m,\lambda} = r[\![x]\!]_{m,\lambda} + [\![a]\!]_{m,\lambda}$.
4. Parties output the result of the equality check $r[\![z]\!]_{m,\lambda} + [\![c]\!]_{m,\lambda} - e[\![y]\!]_{m,\lambda} \stackrel{?}{=} 0$

Figure 8 Verification of a triple using redundant $[\![\cdot]\!]_{m,\lambda}$ sharing.

Similarly to other uses of this style verification, the check can be batched for an arbitrary number of triples. Batching makes the cost for generating the random number r negligible. Furthermore, the communication for the final step can be reduced: the parties hold $\{[\![x^{(j)}]\!]_m\}$ and would like to check if $x^{(j)} = 0$ for all j, every party P_i computes $x_i^{(j)} = 0 - x_{i-1}^{(j)} - x_{i+1}^{(j)}$. All parties then hash $\{x_0^{(j)}, x_1^{(j)}, x_2^{(j)}\}_{\forall j}$ and broadcast their result. If there is a mismatch, they abort. With these two optimizations, the asymptotic communication per triple is determined by the two optimistic multiplications and the opening of e. All involve sending one $m + \lambda$-bit value to one other party, so we arrive at $3(m + \lambda)$ bits per party and multiplication. If the

Protocol with Postprocessing

Input Each party P_i has input x_i.

1. Each party P_i secret shares its input x_i using the secret sharing with redundant shares as described in Section 2.4 to obtain $[\![x_i]\!]_{m,\lambda}$.
2. All parties jointly evaluate the circuit, where multiplications are performed optimistically as described in Section 2.3. Let $\{([\![x_i]\!]_{m,\lambda}, [\![y_i]\!]_{m,\lambda}, [\![z_i]\!]_{m,\lambda})\}_{i\in[N]}$ be the set of all the performed multiplications, where x_i, y_i are the left and right inputs and z_i is the output of the i-th multiplication.
3. For $i \in [N]$, compute a random $[\![a_i]\!]_{m,\lambda}$ and optimistically compute $[\![c_i]\!]_{m,\lambda} = [\![y_i \cdot a_i]\!]_{m,\lambda}$.
4. All parties generate a random value $r \in \mathbb{Z}_{2^\lambda}$.
5. Parties open $[\![e_i]\!]_{m,\lambda} = r[\![x_i]\!]_{m,\lambda} + [\![a_i]\!]_{m,\lambda}$
6. For each $i \in [N]$ the parties check $r[\![z_i]\!]_{m,\lambda} + [\![c_i]\!]_{m,\lambda} - e_i[\![y_i]\!]_{m,\lambda} \stackrel{?}{=} 0$ and abort if any of these checks fails.
7. If all checks passed, then the parties jointly open the outputs of the circuit.

Figure 9 Protocol for secure circuit evaluation that does not require a preprocessing phase.

protocol is used for preprocessing there are another m bits sent per party and multiplication during the online phase, otherwise $3(m + \lambda)$ is the total cost per multiplication. For the common choice of $m = 64$ and $\lambda = 40$, this gives a total of 312 bits.

6 Protocol with Postprocessing

Our postprocessing protocol is similar in spirit to our [21]-style preprocessing protocol in Section 5.2. We use similar building blocks, but in a different order, which allows us to reduce the total computation time at the cost of a slightly more expensive online phase. We describe our protocol in Figure 9 for the three party setting. For the sake of simplicity we describe the protocol with separate checks for each multiplication gate, but optimizations like the batch verification described in Section 5.2 can be applied equally well to this protocol. The security proof for the protocol is completely analogous to the security proof in Section 5.2.

7 Implementation and Evaluation

To help adoption and accessibility of our protocols, we implemented them using Sharemind [9] and the MP-SPDZ framework [29]. We provide extensive benchmarks in both LAN and WAN settings for both implementations as well as a theoretical analysis of the asymptotic communication. Throughout this section, we use a statistical security parameter $\lambda = 40$.

Sharemind already supported semi-honest computation in $\mathbb{Z}_{2^{32}}$ and $\mathbb{Z}_{2^{64}}$. We added [27]-style preprocessing with BatchVerify from Section 5.1 and postprocessing from Section 6.

MP-SPDZ already supported replicated secret sharing in $\mathbb{Z}_{2^{64}}$ and $\mathbb{Z}_p$ as well as the protocol for $\mathbb{Z}_p$ by Lindell and Nof [35]. Its use of C++ templating easily allows to add new protocols reusing existing components, and it provides an efficient implementation of $\mathbb{Z}_{2^k}$ arithmetic for any k. It also uses Montgomery representation for arithmetic modulo a prime. We have added the following protocols: [27]-style preprocessing with SingleVerify from Section 5.1, cut-and-choose preprocessing of triples similar to Araki et al. [3] (simple version), and [21]-style preprocessing as well as postprocessing from Section 5.2.

Table 1 Communication bits per party for $\mathbb{Z}_{2^{64}}$ multiplication.

	Offline	Online	Total
DOS18 preprocessing (single)	992	128	1120
DOS18 preprocessing (batch)	464	128	592
ABF+17 preprocessing (simple)	448	128	576
CDE+18 preprocessing	312	128	440
Postprocessing	-	312	312
Semi-honest	-	64	64
Malicious ASTRA [19]	448	85	553

7.1 Communication

Table 1 shows the communication complexity per multiplication in $\mathbb{Z}_{2^{64}}$ with the various protocols for $\lambda = 40$. While the numbers are obtained from running the protocols in batches of at least one million with rounding, they match the asymptotic cost one would expect from a manual analysis. For comparison, we have added the figures reported in a recent concurrent work by Chaudhari et al. [19] (averaged over the parties because their protocol is asymmetric) that also considers honest majority three party case.

One optimistic multiplication in $\mathbb{Z}_{2^m}$ requires sending m bits, and using Beaver multiplication in the data-dependent phase requires opening two masked values, thus sending $2m$ bits. A CDE+18-style sacrifice [21] requires two optimistic multplications and one opening in $\mathbb{Z}_{2^{m+\lambda}}$, while simple ABF+17 [3] asymptotically requires three optimistic multiplications and two classic sacrifices that require two openings each.[7] This comes down to $7m$ bits.[8] Finally, DOS18 preprocessing [27] with SingleVerify requires two optimistic multiplications in $\mathbb{Z}_p$ and two openings in $\mathbb{Z}_p$ as well as sending two $(m + \lambda)$-bit values for sharing over the integers, totalling in $2(m + \lambda) + 3 \log p$ bits. It roughly holds that $\log p > 7 + 2m + 3\lambda$, so for our choice of parameters $\log p > 255$.[9] The slight difference to the figure in the table comes from rounding up to multiples of eight. Using BatchVerify in Figure 7 allows to avoid the openings in $\mathbb{Z}_p$, bringing the complexity to slightly more than ABF+17 preprocessing.

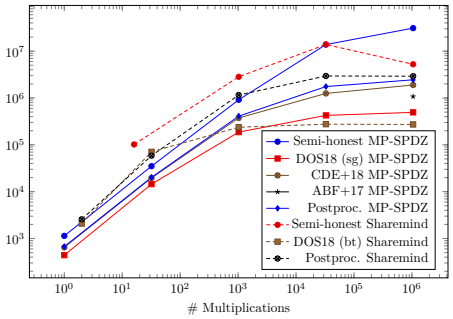
7.2 Benchmarks

We have run our implementations in SIMD fashion, that is, combining the communication of a varying number of multiplications in as few network messages as possible. All benchmarks in this section are averages over ten executions. Unsurprisingly up to a certain number the throughput increases. Figure 10a shows our benchmarks for various numbers of parallel multiplications in a LAN (AWS `c5.9xlarge` instances in the same region). We have 36 virtual CPUs, 72 GiB of RAM, and 10 Gbit/s network network connection. The figure for cut-and-choose is limited to 1048576 because the analysis by Araki et al. [3] mandates batches of at least this size. The plot shows that all malicious protocols perform similarly except the [27] protocol, and that the postprocessing protocol is slightly ahead as we expected.

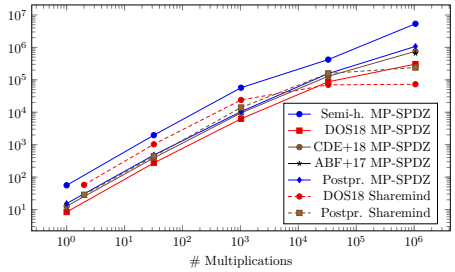
[7] Because of cut-and-choose we cannot use the trick used for DOS18-style sacrificing.

[8] The more sophisticated preprocessing of Araki et al. [3] would cost $5m$, which is still slightly more than a CDE+18-style sacrifice.

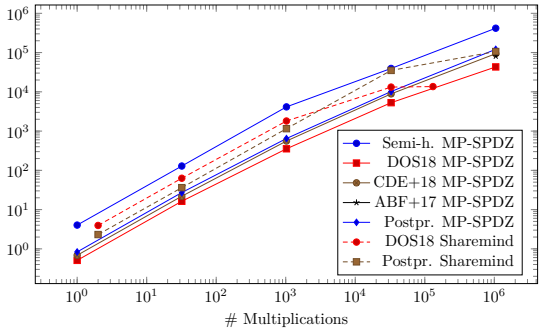
[9] According to Damgård et al. [27], $p > 100 \cdot 2^{2m+2\lambda}$, but a quick recalculation of $24 \cdot B^2 2^\lambda$ with $B = 2^{m+\lambda+1}$ shows that it should be 3λ instead of 2λ in the inequality for p.



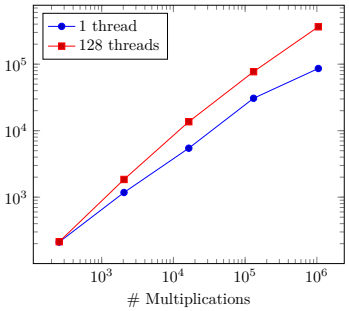
(a) Throughput in LAN.



(b) Throughput in continental WAN.



(c) Throughput in global WAN.



(d) Postprocessing on MP-SPDZ in global WAN.

Figure 10 Comparison of 64-bit multiplication throughput (multiplications/s).

Figure 10b shows our benchmarks for various numbers of parallel multplications in a continental WAN, that is one AWS `c5.9xlarge` instance in each of Frankfurt, London, and Paris. The results mirror the results in the LAN setting except for the fact that 2^{20} parallel multiplications perform better than 2^{15} for all protocols. This is most likely because of the increased network delay of up to 12 ms. Finally, Figure 10c shows benchmarks for a global WAN, that is one AWS `c5.9xlarge` instance in each of Frankfurt, Northern California, and Tokyo. The largest network latency we observed is 236 ms in this setting.

We generally found that our protocols do not use all bandwidth that is available. Figure 10d supports this by showing that increasing from a single thread to 128 increases the output while keeping same the number of parallel multiplications.

7.3 Comparison with Other Implementations

We provide a comparison with the most relevant previous implementations. The concurrent work of Choudhary et al. [19] does not provide throughput of multiplications for their offline phase, only for more complex computation such as AES evaluation. It is therefore hard to compare their implementation to ours. Furthermore, AES evaluation does not lend itself to computation in $\mathbb{Z}_{2^k}$ for $k > 1$, which makes a rather odd benchmark in this setting.

Three party honest majority actively secure multiplication with 61-bit Mersenne field is implemented in [20] where they measure that a circuit with 10^6 multiplication gates and depth 20 can be evaluated in 0.3 seconds in a single AWS region (presumably 10 Gbit/s networks). This amounts to a throughput of 3.3 million multiplications per second, while our postprocessing protocol with Sharemind in a LAN achieved 2.9 million for a slightly smaller batch size that $10^6/20$. This shows that our protocol is competitive despite the extra effort needed for rings as compared to fields.

For a 31-bit prime, [16] report a throughput of 1.7 million multiplications per second for the computation of their verification protocol on a single core of an AWS c5.9xlarge instance ($s = 128$ in Table 3 ibidem). In comparison, we achieve 2.9 million in a LAN setting including communicatio). In the WAN settings we benchmark below 1 million (continentally) or 0.2 million (globally). Also, note that their verification protocol for $\mathbb{Z}_{2^{64}}$ requires computation in the ring $(\mathbb{Z}_{2^{64}})[X]/f(X)$ with f being a polynomial of degree 47 while the benchmarked protocol for fields does not require an extension field. It is therefore likely that the throughput of their protocol for $\mathbb{Z}_{2^{64}}$ is significantly lower.

The batchwise multiplication verification is optimized in [37]. The authors estimate that their computation optimizations achieves up to 10^7 two-party verifications per second using multithreading for 64-bit primes and up to $5 \cdot 10^6$ with 128-bit prime. Their estimations are based on their implementation of the computations, they do not benchmark the protocol with communication. From a conceptual point of view, [37] uses similar verification as our batch verification, hence their work indicates that our implementation might also benefit from more optimized field arithmetic. However, we still need to use larger fields to accommodate the integer secret sharing meaning we need more communication to achieve triples modulo 2^k of the same length as their modulo prime triples.

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A Security Definitions

We define security using the UC framework of Canetti [17]. Protocols proven secure in this framework retain security even when composed arbitrarily and executed concurrently. Concretely, we use a flavour of the classical UC framework, proposed in [22]. We provide a short summary of the security framework here and refer the reader to [22] for more details. Security is defined by comparing a real and an ideal interaction. In the ideal interaction, we have a trusted party, called the ideal functionality $\mathcal{F}$, that receives inputs from all parties, computes the desired function, and returns the result to the parties. In the real interaction, the parties do not have $\mathcal{F}$, but rather interact with each other according to some protocol description Π. The protocol Π may make use of some other auxiliary ideal functionality $\mathcal{G}$. In both interactions, the environment $\mathcal{Z}$ chooses the inputs of all parties and acts as an adversary that may corrupt some subset of the parties passively or actively. We say that Π securely realizes $\mathcal{F}$ if an adversary in the real world can not do "more harm" than an adversary in the ideal world. Concretely, we require the existence of a simulator $\mathcal{S}$, aka. ideal world adversary, that simulates $\mathcal{Z}$'s view of a real interaction. $\mathcal{S}$ simulates the views of the corrupted players, the interaction with auxiliary functionality $\mathcal{G}$, and it may interact

with $\mathcal{F}$. At the end of a protocol execution $\mathcal{Z}$ outputs a single bit. Let $\mathsf{IDEAL}_\lambda[\mathcal{Z}, S, \mathcal{F}]$ and $\mathsf{REAL}_\lambda[\mathcal{Z}, \Pi, \mathcal{G}]$ be the random variables that represent $\mathcal{Z}$'s output bit in the ideal and real execution, respectively. We say that Π securely realizes functionality $\mathcal{F}$, if $\mathcal{Z}$ cannot distinguish real interaction from communicating with the simulator $\mathcal{S}$.

▶ **Definition 3.** Π *securely implements functionality* $\mathcal{F}$ *with respect to a class of environments* Env *in the* $\mathcal{G}$*-hybrid model, if there exists a simulator* S *such that for all* $\mathcal{Z} \in Env$ *we have*

$$|\Pr[\mathsf{REAL}_\lambda[\mathcal{Z}, \Pi, \mathcal{G}] = 1] - \Pr[\mathsf{IDEAL}_\lambda[\mathcal{Z}, S, \mathcal{F}] = 1]| \ \leq \mathsf{negl}(\lambda) \ .$$

We capture different security notions by specifying the environments. For passive security the environment $\mathcal{Z}$ can corrupt up to t parties. $\mathcal{Z}$ gets full read-only access to the corrupted parties internal tapes. All parties follow the protocol honestly. The simulator $\mathcal{S}$ is allowed to ask the ideal functionality $\mathcal{F}$ for the inputs of the corrupted parties. For active security the environment $\mathcal{Z}$ is allowed to corrupt up to t parties. $\mathcal{Z}$ gets full control of the corrupted parties. Once the ideal functionality $\mathcal{F}$ received inputs from all parties, it computes the output and sends it to $\mathcal{Z}$. The environment sends back a bit indicating whether the parties should obtain the output or $\perp$. A slightly weaker notion known as active security with individual abort allows the adversary to specify which honest parties abort and which do not.

We use the definition of weak privacy against active adversaries [32, Definition 5.11] (a slight variant of the same property was defined under the name "active privacy" in [38]), which captures the security properties offered by many existing protocols [6, 5] that follow the compute-then-open paradigm. These protocols are split into a computation and opening phase. The computation phase consists of multiple rounds of interaction, whereas the opening phase requires a single round of communication. Intuitively, weak privacy says that an active adversary cannot learn anything until the opening phase, and this is captured saying that there exists a simulator that can simulate the truncated view of the protocol up to the opening phase without having access to the inputs or outputs of any honest parties. Finally, these protocols are "linear", meaning that the output of the parties in the protocol is a linear function of the messages sent in the opening phase.