Useful Estimation Procedures for Critical Gaps

by

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ABSTRACT

Many different methods for the estimation of critical gaps at unsignalized intersections have been published in the international literature. This paper gives an overview of some of the more important methods. These methods are described by their characteristic properties. For comparison purposes a set of quality criteria has been formulated by which the usefulness of the different methods can be assessed. Among these one aspect seems to be of primary importance. This is the objective, that the results of the estimation process should not depend on the traffic volume on the major street during the time of observation. Only if this condition is fulfilled, the estimation can be applied under all undersaturated traffic conditions at unsignalized intersections. To test the qualification of some of the estimation methods under this aspect a series of comprehensive simulations has been performed. As a result the maximum likelihood procedure, as it has been described by Troutbeck, and the method developed by Hewitt can be recommended for practical application.

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1.   INTRODUCTION

The estimation of critical gaps from observed traffic flow pattern is one of the most difficult tasks in empirical traffic engineering science. Miller (1), in his classical paper, could refer to nine different estimation methods, which also in his time did not cover the whole range of possible procedures to be obtained from international literature. Today it would be easy to find more than 20 or 30 methods for the estimation of critical gaps which are published around the world. All these methods produce different results. Therefore, the important question is: which of these procedures being recommended by different authors reveals a correct estimation? And the other question is: How can we find out if an estimation is valid or not?

Before we can answer these questions we should first of all discuss the fundamental definitions. Here we concentrate on the most simple case of an unsignalized intersection. This is a crossroad of two one-way streets (fig. 1). Here two movements are allowed:

- one major stream (= priority movement) of volume q_p

- one minor stream of volume q_n

According to traffic rules each major stream vehicle can pass the intersection without any delay. A minor street vehicle, however, can only enter the conflict area, if the next major vehicle is *far enough* away to allow the minor vehicle for a safe passage of the whole conflict area. "Far enough" is defined as: The next major street vehicle is arriving at the intersection at an instant which will happen t_c seconds after the previous major stream vehicle or t_c seconds after the minor vehicle's arrival. This value t_c is called the critical gap. In other words:

t_c = critical gap = minimum time gap in the priority stream which a minor street driver is ready to accept for crossing or entering the major stream conflict zone.

Another limiting factor for the minor street vehicles is the fact that they can not enter the conflict area during a short while after the previous minor street vehicle has entered. This is due to the physical length of the vehicles and the necessary headways. Thus, as the second variable for the characterization of minor street driver's behaviour we use the move-up time t_f.

t_f = move-up time = time gap between 2 successive vehicles from the minor street while entering the conflict area of the intersection during the same major street gap.

It is obvious that t_c and t_f differ from driver to driver, from time to time and between intersections, types of movements and traffic situations. Due to this variability there is no doubt that the gap acceptance process is of a stochastic nature. Thus the t_c and t_f can be regarded as random variables. Moreover, the parameters of the distribution functions for these variables may be subject to different external influences. It is, therefore, necessary to define some type of representative characteristics to model the usual behaviour of drivers. Therefore, the estimation of critical gaps and move-up times tries to find out values for the variables t_c and t_f as well as for the parameters of their distributions, which represent typical driver behaviour at the investigated intersection during the period of observation.

For this paper we concentrate our derivations on the critical gap t_c.

In unsignalized intersection theory it is generally assumed that drivers are both consistent and homogeneous.

- Consistent drivers are expected to behave the same way every time in all similar situations. This means: A driver with a specific t_c-value will never accept a gap less than t_c and he will accept each major stream gap larger than t_c. However, within a population of several drivers, each of which behaves consistent, different drivers could have their own t_c-values. These t_c-values are then treated as a random variable with a special statistical density function $F_c(t)$ and a cumulative distribution function $F_c(t)$.

- The population of drivers is homogeneous if each sub-group of drivers out of the population has the same functions $f_c(t)$ and $F_c(t)$.

It is clear that in reality drivers are neither completely consistent nor homogenous. A completely inconsistent driver would apply a new t_c-value for each gap. This would also include that the applied t_c-value which is compared with one major street gap is completely independent from the t_c used for the previous major stream gap by the same driver some seconds before in the same queuing situation. This is, however, not expected to be the case in reality. Instead it is assumed that a rather careful driver will always demand a rather large gap or that a risky driver will always be prepared to accept of rather narrow gaps. Therefore, we assume that real driver behavior is closer to consistency than to completely inconsistent behavior.

For the estimation of critical gaps t_c from observations, a long series of methods has been proposed. An overview on the status of English literature at the late 60ies was given by Miller (1). Meanwhile many more proposals have been made. For the preparation of this paper a selection of candidate procedures has been made which by far does not represent everything what is published. The selection has been made on the background that these procedures have been used or been recommended also by other authors than the original sources. In the first part of this paper these methods are described.

2. ESTIMATION TECHNIQUE FOR SATURATED CONDITIONS: SIEGLOCH'S METHOD

Siegloch (2) proposed a consistent framework for the theory of capacities of unsignalized intersections. This should be mentioned here to emphasized how the critical gaps are used within subsequent mathematical modeling. Let $g(t)$ be the number of minor street vehicles which can enter the conflict area during one minor stream gap of size t. The expected number of t-gaps within the major stream is $q_p \cdot h(t)$ where $h(t)$ is the statistical density function of all gaps (or headways) in the major stream. Thus, the amount of capacity which is provided by t-gaps per hour is $q_p \cdot h(t) \cdot g(t)$. To get the total capacity c we have to integrate over the whole range of possible major stream gaps t. Thus we get

$$c = q_p \cdot \int_{t=0}^{\infty} h(t) \cdot g(t)dt \tag{1}$$

This is the equation for the capacity of unsignalized intersections which is the fundament for the whole gap-acceptance theory. Almost each of the different analytical capacity estimation formulas which is known from international literature is based on this concept, also in cases where the original authors where not aware of this fact.

The consequence of this equation is, that for capacity calculations we need to know the major stream headway distribution $h(t)$ and the function $g(t)$. Siegloch, as a consequence of this theory proposes a regression technique for the derivation of $g(t)$ from field observations. For this estimation technique we need to observe saturated conditions, i.e. continuous queuing on the minor street. Only under these conditions we can observe realizations g for the function $g(t)$ by counting the number of minor street vehicles which enter major street gaps of size t. Of course, the realizations g are always integer numbers. The observation results can be plotted into a graph of the style of fig. 2. In almost all cases investigated by the authors (and that is quite a large number) the observation points are arranged such that a linear approximation for the representation of measurement points is justified. Therefore, a linear regression function is used to represent the observation data where t is the dependent variable and g is the independent variable:

$$t = a + b \cdot g \qquad\qquad (s) \qquad\qquad (2)$$

where the parameters a and b are the outcome of the regression analysis. It is useful to calculate the average t_g (from the observed t-values) for each observed g-value before starting the regression. Thus, for every g-value within the sample, only one t-value ($= t_g$) is used. Otherwise the more numerous observations for smaller g would govern the whole result. Experience shows that in almost every case the average t_g-values show only small deviations from a straight line.

The straight line for t = function (g) would be exactly correct, if t_c and t_f were constant values. In that case eq. 2 could be written as

$$g(t) = \begin{cases} 0 & \text{for } t < t_0 \\ \dfrac{t - t_0}{t_f} & \text{for } t \geq t_0 \end{cases} \qquad\qquad (3)$$

where
$$t_0 = t_c - \frac{t_f}{2} \qquad\qquad (s)$$

Therefore, t_c and t_f can be evaluated from the regression technique directly. Some authors (e.g. (9)) have classified this technique for critical gap estimation as deterministic, which is not correct. Instead this technique fully considers the stochastic nature of gap acceptance.

The combination of eq. 1 and 3 together with the assumption that $h(t)$ can be described by the exponential distribution leads to the well-known Siegloch-formula for the capacity of an unsignalized intersection of the simple type as shown in fig. 1:

$$c = \frac{3600}{t_f} \cdot e^{-p \cdot t_0} \qquad\qquad (s) \qquad\qquad (4)$$

The advantage of Siegloch's procedure for the estimation of t_c and t_f is its close relation to the subsequent capacity theory. The drawback for practical application is the fact that this

method can only be applied for saturated conditions which are difficult to find in many practical cases.

3.   ESTIMATION TECHNIQUES FOR UNDERSATURATED CONDITIONS

3.1   The lag method

It is more complicated to estimate the critical gap t_c from traffic observations with undersaturated conditions. One simple method could be based on lags. A lag is the time from the arrival of the minor vehicle until the arrival of the next major vehicle. We now assume

- consistent drivers
- independence of the minor street vehicle arrival time and the traffic situation on the major street.

Then the proportion $p_{a,lag}(t)$ of drivers which accept a lag of size t is identical with the probability that a driver has a t_c-value smaller than t. Thus we can state

$$P_{a,lag} = F_c(t) \quad . \tag{5}$$

From this consideration we could derive the first method of critical gap estimation for undersaturated conditions.

From traffic observations at an unsignalized intersection all lags should be measured. Also the fact if this lag has been accepted or rejected has to be noted. Then the time scale is divided into W segments of size Δt, e.g. $\Delta t = 1s$. For each interval i we look at

N_i =   number of all observed lags within interval i
A_i =   number of accepted lags within interval i
a_i = A_i / N_i

If t_i is the time at the center of interval i then

$$F_c(t_i) = a_i \quad . \tag{6}$$

which is an approximation of the cumulative distribution function of critical gaps. The mean critical gap then is

$$t_c = \sum_{i=1}^{W} t_i \cdot \left[F_c(t_i) - F_c(t_{i-1}) \right] \quad . \tag{7}$$

where
W   =   number of intervals of size Δt

Similarly also the standard deviation for the distribution $F_c(t)$ could be estimated.

For practical application this method has some drawbacks. For the method, in each interval i a sufficiently large sample should be available. This demands for very long observation periods since

- with low major street traffic flow it takes a while to observe enough smaller lags,

- with large major street volumes most minor street vehicles have to queue before they can enter the conflict zone. Thus, although a large number of driver's decisions has been observed, there will only be very few lags which can be used for this estimation procedure.

Therefore, an estimation procedure is needed which makes use of observed rejected and accepted gaps (i.e.: not only lags) since they also contain information about the size of the critical gap for the drivers who have been observed.

Another disadvantage of this method is, that it only regards rather relaxed situations where no queuing occurs. An additional problem could be, that the critical value for the lags might be systematically different from that for the gaps. Due to all of these problematic aspects the lag method is not used in practice. It provides us only some insight from a theoretical point of view.

3.2   Fundamental considerations for further methods

Due to the reasons mentioned before, what is really needed for critical gap evaluations under undersaturated conditions is a procedure which also extracts information from those drivers who accept a gap after queuing. The proportion p_a, $\text{lag} + \text{gap}(t)$ of accepted lags and gaps is no longer the distribution of the t_c. The reason is that a driver who accepts a gap has selected among several gaps which were provided to him. In this case the distribution of all major stream gaps affects the distribution of accepted gaps. Moreover, among the rejected gaps those drivers with large t_c are overrepresented since they experience much more rejections compared with drivers who apply small t_c-values. Therefore, some more considerations are necessary.

If we observe a driver on the minor street and his gap acceptance / rejection decisions, we can state: his t_c is greater than the maximum rejected gap and t_c is smaller than the gap he accepts. This is true if the driver behaves consistently (see above). If we observe a series of accepted gaps t_a, then these accepted gaps can be described by an empirical statistical distribution function $F_{a(t)}$ (cf. fig. 3).

On the other hand we can observe the distribution $(F_r(t))$ of rejected gaps. Here it is, however, a question which types of rejected gaps are included into this distribution. Three different definitions are in use:

a)   Only the largest rejected gap for each driver is taken into account.

If a minor street driver was able to accept the first lag, he has not rejected any gap. In this case it could be defined to withdraw this driver out of the sample (= case **a1**); i.e.: also his accepted lag is not evaluated for the estimation procedure.
Or it could be defined to use 0 as the size of the rejected gap for this driver (= case **a2**).

b)   All observed rejected gaps are taken into account. This includes that for one individual minor street driver, who was waiting for a while, a longer series of rejected gaps is included. Also here the distinction of case a) has to be made:

case **b1**: drivers accepting a lag are omitted

case **b2**: for drivers accepting a lag the largest rejected gap is defined as 0

Regardless of the definition a) or b) being used, we know that this distribution $F_r(t)$, in a plot like fig. 3, must be left of the desired distribution $F_c(t)$. On the other hand the function $F_a(t)$ of accepted gaps must be on the right of the $F_c(t)$ distribution. This is due to the fact that for each individual consistent driver $t_r < t_c < t_a$.

Since the distribution function $F_c(t)$ can not be observed directly, it is the purpose of all of the following procedures to estimate the function $F_c(t)$ as valid as possible or to estimate at least its typical parameters like the expectation or the median or - in addition - the variance.

3.3 Raff's method

The earliest method for estimating critical gaps seems to be that by Raff (3). His definition translated to our terminology - means: t_c is that value of t at which the functions

$$1 - F_r(t) \qquad \text{and} \qquad F_a(t)$$

intercept. Miller (1) gave some additional mathematical interpretations for this method. He also points out that the results of this t_c-estimation are sensitive to the traffic volumes under which they have been evaluated. Raff's method has been used in many countries in earlier times, e.g. Retzko's work (4) introduced this procedure to Germany.

3.4 Ashworth's method

Under the assumption of

a)   exponentially distributed major stream gaps with statistical independency between consecutive gaps

b)   normal distributions for t_a and t_c,

Ashworth (5, 6, 7) found that the average critical gap t_c can be estimated from μ_a (= mean of the accepted gaps t_a; in s) and σ_a (= standard deviation of accepted gaps) by

$$t_c = \mu_a - p \cdot \sigma_a{}^2 \qquad\qquad \text{(s)} \qquad\qquad \text{(8)}$$

with p = major stream traffic volume (veh/s). If t_a is not normal distributed the solution might become more complicated. However, also for a gamma distribution or a log-normal distribution of t_a and t_c eq. 4 is still a very close approximation. Miller (1) provides another correction method for the special case that the t_c are gamma distributed. Then the two equations apply

$$t_c = \mu_a - p \cdot \sigma_c{}^2$$
$$\sigma_c = \sigma_a \cdot \frac{t_c}{\mu_a} \qquad\qquad \text{(s)} \qquad\qquad \text{(9)}$$

from which t_c and σ_c are to be obtained by substitution.

For our evaluations we used eq. 8.

3.5   Harders' method

Harders (8) has developed a method for t_c-estimation which since has become rather popular in Germany. The whole practice for unsignalized intersection in Germany is still based on t_c- and t_f-values which were evaluated using this technique. The method makes only use of gaps (i.e. case b1 mentioned above). The method is very similar to the lag method mentioned as paragraph 3.1. However, for Harders' procedure, as it is described in (8) lags should not be used in the sample. The time scale is divided into intervals of constant duration, e.g. $\Delta t = 0.5\,s$. The center of each interval i is denoted by t_i. For each vehicle queuing on the minor street we have to observe all major stream gaps which are presented to the driver and, in addition, the accepted gap. From these observations we have to calculate the following frequencies and relative values:

$$N_i \quad = \quad \text{number of all gaps of size i, which are provided to minor vehicles}$$
$$A_i \quad = \quad \text{number of accepted gaps of size i}$$
$$a_i \quad = \quad A_i / N_i \tag{10}$$

Now these a_i-values can be plotted over the t_i. The curve generated thus has the form of a cumulative distribution function. It is treated as the function $F_c(t)$. However, nobody has provided any conclusive mathematical concept that this function $a_i = \text{function} (t_i)$ has real properties of $F_c(t)$. Instead the approach might be a misunderstanding of the lag method.

Part of the method is, that each gap $t < 1s$ is assumed to be rejected and that each gap $t > 21s$ is assumed to be accepted. For practical application it is not guaranteed that $a_i = function \ (t_i)$ is steadily increasing over the t_i, which should be the case for $F_c(t)$. Therefore, the a_i-values are corrected by a floating average procedure, where each a_i is also weighted with the A_i-values. Finally, the estimation of t_c is given by the expectation of the thus formed $F_c(t)$-distribution function. From the descriptions it can be obtained that this method is a more pragmatic solution without a strong mathematical background.

3.6   Logit procedures

A couple of methods have been proposed (e.g. (9)) which can be summarized as logit models, since they provide similarities to the classical logit models of transportation planning (cf. (10)). In each case the models lead to a function of the logit type. One typical formulation for this family of models is like the following.

Each minor street driver waiting for a sufficient gap has to judge between the two alternatives

    i   =   accept the gap for the crossing or merging maneuvers
    j   =   reject the gap.

A driver in his decision situation d will expect a specific utility from his decision. This utility can be regarded as a combination of safety on one side and low delays on the other side. We

regard the total utility U_{id} as an additive combination of a deterministic term V_{id} and a random term ε_{id} :

$$
\begin{aligned}
U_{id} &= V_{id} + \varepsilon_{id} \\
U_{jd} &= V_{jd} + \varepsilon_{jd}
\end{aligned}
\tag{11}
$$

We assume that the deterministic component V_{id} can be computed from attributes which can be evaluated by objective measurement techniques. Here we use as one possible solution a linear utility function.

$$
\begin{aligned}
V_{id} &= \alpha + \beta_1 \cdot x_{id1} + \beta_2 \cdot x_{id2} + \dots + \beta_K \cdot x_{idK} \\
V_{jd} &= \alpha + \beta_1 \cdot x_{jdt1} + \beta_2 \cdot x_{jd2} + \dots + \beta_K \cdot x_{jdtK}
\end{aligned}
\tag{12}
$$

where

$\alpha, \beta_1, \beta_2, \dots \beta_K$	= parameters
x_{idk}	= value of the k - th attribute in situation d in case of acceptance
x_{jdk}	= value of the k - th attribute in situation d in case of rejection
K	= number of attributes

The random component ε_{id} includes all influencing factors which can not be evaluated precisely or which are due to really random elements of the decision process.

We do, however, assume that the drivers, on average, make rational decisions, i.e.: they make those decisions which provide the highest utility for them. Thus the probability $p_i(t)$ of acceptance of a gap by a driver is

$$
\begin{aligned}
p_i(t) &= p\left(U_{id} > U_{jd}\right) \\
p_i(t) &= p\left(\varepsilon_{jd} - \varepsilon_{id} \le V_{id} - V_{jd}\right)
\end{aligned}
\tag{13}
$$

For the random component ε_{id} we assume a Gumbel-distribution (cf. (10)). Then the difference
$\varepsilon_d = \varepsilon_{jd} - \varepsilon_{id}$ has a logistic distribution; i.e.:

$$
F_{\varepsilon_d}(x) = \frac{1}{1 + e^{-\mu \cdot x}}
\tag{14}
$$

$$
f_{\varepsilon_d}(x) = \frac{\mu \cdot e^x}{\left(1 + e^{-\mu \cdot x}\right)^2}
\tag{15}
$$

where

μ = parameter of the distribution

Therefore, eq. 13 and 14 can be written as

$$
p_i(t) = F_{\varepsilon_d}(V_{id} - V_{jd}) = \frac{1}{1 + e^{-\mu \cdot \left(V_{id} - V_{jd}\right)}}
\tag{16}
$$

Within the product $\mu \cdot \left(V_{id} - V_{jd} \right)$ the factor μ can be included into the parameters α and β_i (cf. eq. 12). For the special case that only one attribute is observed (K = 1) we get

$$p_i(t) = \frac{1}{1 + e^{-\beta \cdot \left(x_{id} - x_{jd} \right)}} \tag{17}$$

As attributes we can use e.g.: size of the presented gap, time which the minor street driver has spent in the queue, speed of the major street vehicle, driving direction of the next arriving major vehicle in case of a two-way street, etc.

So far the model formulation is much the same as within the classical logit models of transportation planning. If we, however, analyse eq. 16 we see that, of course, the x_{id} and x_{jd} (which could e.g. be the gap until the next arriving major street vehicle) are the same either if the gap is accepted (i) or rejected (j).

Therefore, not the difference of attributes is used within the equation. Instead the attribute itself is introduced into eq. 14 or 16. Thus eq. 16 becomes

$$p_i(t) = \frac{1}{1 + e^{\alpha + \beta_1 \cdot x_{d1} + \beta_2 \cdot x_{d2} + \dots + \beta_K \cdot x_{dK}}} \tag{18}$$

and eq. 17 (for $k = 1$ and attribut x_d = major stream gap t) gets the form

$$p_i(t) = \frac{1}{1 + e^{\alpha + \beta \cdot t}} \; F_r(t) \tag{19}$$

Now, to derive the critical gap t_c, we understand $p_i(t)$ = function (gap size t) (= probability that a driver in situation d accepts a gap of size t) as a statistical density function for a random variable T. Then the critical gap is defined as the median of this random variable T; i.e.

$$t_c = \text{value of } T, \text{ for which } \int_0^{t_c} p_i(t) dt = 0.5 \tag{20}$$

Finally the parameters $\beta, \beta_1, \beta_1 \dots \beta_k$ are estimated by a maximum likelihood technique. As an example Pant, Balakishan (11) have used this kind of logit model with $\alpha = 0$ and $K = 11$ different attributes.

To solve this model we have to determine the log-likelihood function. This, for the model formulation of eq. 19, is given by

$$L(\alpha, \beta) = \sum_{d=1}^{n} \left[\ln \left(\frac{1}{1 + e^{\alpha + \beta \cdot t_d}} \right) + \alpha + \alpha \cdot y_d + \beta \cdot t_d - \beta \cdot y_d \cdot t_d \right] \tag{21}$$

where

$$y_d \quad = \quad \text{1 if a driver in situation d accepted a gap}$$
$$= \quad \text{0 if a driver in situation d rejected a gap}$$
$$n \quad = \quad \text{no. of observed decisions}$$
$$t_d \quad = \quad \text{gap size offered to a minor street driver in situation d} \qquad \text{(s)}$$

The maximum of $L(\alpha, \beta)$ can be determined by forming the derivatives and setting both as sero:

$$\frac{\delta L}{\delta \alpha} = \sum_{t=1}^{n} \left[\ln\left(\frac{e^{\alpha + \beta \cdot t_d}}{1 + e^{\alpha + \beta \cdot t_d}} \right) + 1 - y_d \right] = 0 \tag{22}$$

$$\frac{\delta L}{\delta \beta} = \sum_{t=1}^{n} \left[\ln\left(\frac{e^{\alpha + \beta \cdot t_d}}{1 + e^{\alpha + \beta \cdot t_d}} \right) + t_d - t_d \cdot y_d \right] = 0 \tag{23}$$

These two equations could be solved iteratively. Instead, also eq. 21 could be maximized using a spreadsheet maximizing technique. This (using Quattro Pro, version 5) is the method which has been employed for the following analyses.

The maximization of $L(\alpha, \beta)$ reveals values for α and β in eq. 19. Since this is the distribution function of a logistic distribution, eq. 20 can be solved for t_c as the mean of this distribution, which is

$$t_c = \frac{\alpha}{\beta} \tag{24}$$

The variance of the critical gap thus can be estimated as

$$\sigma_{t_c}^2 = \frac{\pi^2}{3\beta^2} \tag{25}$$

Finally it should once again be noted that this family of models allows also the evaluation of other external effects on the critical gap by using eq. 18 instead of eq. 19. Then the log-likelihood function (cf. eq. 21) must be formed for this more complex model. As attributes we then have to include other external influencing parameters in addition to the major stream gap (see text below eq. 17).

As an example for this type of logit estimation fig. 4 should be regarded. It has been evaluated from the simulations mentioned below for case b1 (see above), i.e. situations when the driver could accept the first lag were omitted.

3.7   Probit procedures

Probit techniques for the estimation of critical gaps have been used since the 60ies (e.g. (12); cf. also references given by (1)). The formulation for this type of models is quite similar to the logit concept. In their original form these models do, however, not use the utility term. Instead the size of the critical gap t_c is directly randomized by an additive term ε. Thus we formulate for a consistent driver d:

$$t_{c,d} = \overline{t_c} + \varepsilon_d \tag{26}$$

where

$$t_{c,d} = \quad \text{critical gap for driver d} \qquad \qquad \text{(s)}$$

$$\overline{t_c} \quad = \quad \text{average critical gap for the}$$
whole population of drivers   (s)

$$\varepsilon_d \quad = \quad \text{deviation of the critical gap for driver d from } \overline{t_c} \quad \text{(s)}$$

The probability that a driver will accept a major street gap of size t is

$$p_a(t) = p\left(t \geq t_{c,d}\right) = p\left(t \geq \overline{t_c} + \varepsilon_d\right) \qquad . \tag{27}$$

For the probit model it is assumed that the random component ε_d is normal distributed with mean 0 and standard deviation σ_ε. Then eq. 27 can be further developed into

$$p_a(t) = N\left(t \mid \overline{t_c}, \sigma_\varepsilon\right) \tag{28}$$

where

$$N\left(t \mid \overline{t_c}, \sigma_\varepsilon\right) = \text{cumulative distribution function of a normal distribution with}$$
$$\text{mean} = \overline{t_c} \text{ and standard deviation} = \sigma_\varepsilon$$

Using the standardized form for the normal distribution this equation can be written as

$$p_a(t) = \Phi\left(\frac{t - \overline{t_c}}{\sigma_\varepsilon}\right) \tag{29}$$

where

$$\Phi(z) = \quad \text{value for the standarized}$$
cumulative normal distribution function at point z

The terms $\overline{t_c}$ and σ_ε are parameters of the model. They can be evaluated by regression techniques (1) for the probit if the proportion of accepted lags is used as an estimate for $p_a(t)$. With this technique the method is nearly identical with the lag method. If also gaps were included the technique has all the problems mentioned above (see paragraph 3.2). Therefore, Hewitt (13, 14) proposed a correction strategy to the basic probit method to account for the bias caused by multiple rejection of gaps by drivers applying a large t_c- value. This technique here is mentioned as a separate paragraph 3.8.

Another important contribution to probit estimation techniques has been given by Daganzo (19). Here a theory has been proposed which estimates t_c based on the whole history of rejected gaps and the accepted gap for each individual minor street driver. A normal distribution is applied for the t_c and its variance over the whole population of drivers as well as for the random term ε_d (cf. eq. 26). The model can only be solved by special software for multinominal probit estimation techniques. However, it may not be certain that a solution for the parameters will be found. Therefore, this approach seems to be too complicated for practical application.

Mahmassani, Sheffi (20) propose a probit model which allows to account for the influence of waiting time at the stop line on the gap acceptance behavior of drivers. The number of gaps which a driver has rejected before he accepts a gap is one parameter of the model. Here also a log-likelihood function is given which allows a maximum-likelihood estimation based on probit theory. The estimation leads to a solution in which the critical gap depends on the

number of rejected gaps. This type of solution could be useful as input for simulation models if the concept has proven to be realistic, based on ample empirical research. The solution containing the number of rejected gaps is, however, not useful for application in guidelines (e.g. chapter 10 of the HCM) or other analytical capacity calculations. Here the theories allow only for one typical fixed value of t_c to be introduced into further calculations.

One problem with all probit approaches is that the normal distribution seems not to be adequate to be applied for critical gaps since a significant skewness of the t_c-distribution must be expected. The concept of probit estimations has been included into our analyses via Hewitt's solution.

3.9.   Hewitt's method

Hewitt (13, 14, 15, 16) has published a series of papers on the estimation of critical gaps. For full explanation of the details of the different procedures the reader is refered to the original sources. However, a short characterization of the method should be given here.

Again the time scale is divided into intervals of constant duration, e.g. $\Delta t = 1$. The center of each interval i is denoted as t_i. The method is using an iterative procedure. As a first approach for the gap acceptance function $F_c(t)$ the lag method as it is described under paragraph 3.1 is used. However, for the purpose of analytical tractability $F_c(t)$ in the first step is estimated according to the probit method (cf. above). This leads to values for the probability that t_c is inside the interval i, which is denoted as $c_{i,0}$, where the index *0* stands for the 0th step of iteration.

Subsequent theoretical derivations lead to formulas for the expected number of accepted and rejected lags and gaps which are given in the following table (corresponds to eq. 30).

Expected no. of gaps or lags of duration t_i, which are used as following by drivers d with a critical gap of size $c, d \cong t_i$			
used :	as lags	as gaps	for
Accepted	$\beta \cdot N \cdot c_i \cdot f_j$	$\beta \cdot N \cdot c_i \cdot \dfrac{f_j \cdot F_i}{\left(1 - F_i\right)}$	$j \geq i$
Rejected	$\beta \cdot N \cdot c_i \cdot f_j$	$\beta \cdot N \cdot c_i \cdot \dfrac{f_j \cdot F_i}{\left(1 - F_i\right)}$	$j \leq i$

where   (30)

c_i =   probability that the critical gap is inside interval i

f_i =   probability that a major stream gap is inside interval i

(Distributions of lags and gaps are assumed to be identical as it is the case for the exponential distribution.)

F_i =   value of the cumulative distribution function for major stream gaps at the center of interval i

β =   1 for $j \neq i$

       =   0.5   for $j = 1$

Applying this set of formulas we can compute the number of accepted an rejected gaps and lags from a given set of $\{c_{i,0}\}$. From these putative values a new estimation of the $\{c_{i,1}\}$ can be computed, e.g by using a probit estimation technique. This set of $\{c_{i,1}\}$ is imbedding a new estimation for t_c. Again from these new $\{c_{i,1}\}$ (applying eq. 30) new numbers of accepted and rejected lags are calculated, which again are the basis for the $\{c_{i,2}\}$ and so on. This iteration is repeated, until the values of subsequent t_c become nearly unchanged by the next iteration.

The only information to be extracted from observations for each time interval i of duration Δt are:

- total number of gaps
- total number of lags

- number of rejected gaps
- number of rejected lags

For practical application some additional aspects have to be observed, if some of the time intervals are not filled up with sufficient empirical values. Then adjacent intervals have to amalgamated. Instead of a probit estimation procedure for the $\{c_i\}$, which assumes a normal distribution for $F_c(t)$, also a log-normal distribution could be applied. The whole estimation procedure is included into a set of computer programs called GAPTIM and PROBIT (Hewitt, 1995). These programs have been used for our investigations to analyse the Hewitt method.

3.9   Maximum likelihood procedures

Maximum likelihood techniques for the estimation of critical gaps seem to go back to Miller, Pretty (18) (for more detail cf. (1)). The method has been described in a more precise form also by Troutbeck (21). To understand the fundaments of this method let us assume that for one individual minor street driver d we have observed:

r_d =   largest rejected gap   (s)

a_d =   accepted gap   (s)

The maximum likelihood method then calculates the probability of the critical gap t_c being between r_d and a_d. To estimate this probability, the user must specify the general form of the distribution $F_c(t)$ of the critical gaps for the population of drivers and then assume that all drivers are consistent. The likelihood that the driver's critical gap will be between r_d and a_d is given by $F_a(a_d) - F_r(r_d)$. The likelihood L^* that within a sample of n observed minor street drivers the two vectors of the $\{r_d\}$ and $\{a_d\}$ have been obtained is given by the product:

$$L^* = \prod_{d=1}^{n} \left(F_a(a_d) - F_r(r_d) \right) \qquad\qquad\qquad (s) \qquad\qquad (31)$$

The logarithm L of the likelihood L^* is given by

$$L = \sum_{d=1}^{n} \ln\left(F_a(a_d) - F_r(r_d)\right) \qquad \text{(s)} \qquad (32)$$

In practice, the log-normal distribution is often used as the distribution of the critical gaps t_c. The mean critical gap within this distribution has been found to be an acceptable quantity for the representation of average driver behavior (1, 21).

The likelihood L^* is also maximized when the logarithm L of the likelihood is maximized. Appropriate values for the critical gap distribution parameters (the mean and the variance) are found by setting the partial derivatives of L with respect to these parameters, to zero. This leads to a set of two equations depending on the vectors of the observed $\{r_d\}$ and $\{a_d\}$. These two equations have to be solved by iterative numerical solution techniques. Troutbeck (21) describes a procedure for estimating the critical gap parameters using this maximum likelihood technique in more detail. This numerical method has been used to estimate t_c-values for the investigations described in this paper.

4. CRITERIA FOR THE CLASSIFICATION OF ESTIMATION METHODS

Before we can compare the different estimation procedures we have to define the requirements for a useful procedure to estimate critical gaps. These requirements can not be derived by mathematics. Instead we propose the following set of criteria.

1. Distribution

The critical gap t_c is not a constant value. Instead it is a variable term where a variation has to be expected between different drivers and - for each individual driver - over time. Therefore, the critical gaps which drivers apply for their decision making process at unsignalized intersections are distributed like a random variable. The distribution is characterized by

- a minimum value as the lower threshold, which is ≥ 0,
- an expectation μ_c (average critical gap or mean critical gap; which often is also denoted as "the critical gap"),
- a standard deviation σ,
- a skewness factor, which has to be expected as positive, i.e. a longer tail on the right side.

This distribution or its parameters can not be directly estimated because the critical gap can not be observed in an individual driving situation. What can only be measured are the rejected and accepted gaps. Therefore, procedures have to be established which try to estimate the distribution or their parameters as close as possible. Normally the accepted and/or rejected gaps are used as a basis for this estimation.

2. Consistency

Such an estimation procedure should be consistent. This means: If the minor street drivers within a specific composition of traffic streams have a given distribution of critical gaps then the procedure should be able to reproduce this distribution rather closely. The procedure should at least reproduce the average critical gap quite reliably.

These reproduction qualities should not depend on other parameters like

-         traffic volumes on the major street nor on the minor street
-         delay experienced by the drivers
-         other external influences.

Only if this consistency has been proven for a special procedure, then the method can be used to study influences of the external parameters on the critical gap. Otherwise the influences being found by empirical studies might be due to the inconsistency of the estimation procedure and those influences may not really be related to the external parameters being investigated.

For most of the well known estimation procedures for critical gaps this consistency has not been proven (as far as it is known to the authors). Therefore, there is a strong feeling that a great deal (if not the majority) of all relationships between critical gaps and other parameters (like traffic volumes, time in queue, delays at the stop line as service times of the imbedded queuing system, and also geometric characteristics of intersections which normally are studied at different sites under different traffic volumes) which can be found in the literature might be not existing in reality since the differences being found may just be due to this inconsistency.

3.   Robustness of the method

This aspect, as it is already mentioned by Miller (1), has been emphasized by Hewitt (17) discussing the experiences described in this paper. It means that the results of estimation procedures should not be too sensitive to the assumptions being made, e.g. assumptions about the distributions of critical gaps or of major stream headways.

Another factor which deserves mention is the size of the standard error of the mean critical gap. It is possible that a method which produces a slight bias, but a small standard error might be preferable to an unbiased method with a large standard error (17).

4.   Capacity model compatibility

The estimation of critical gaps is not an end in itself. Critical gaps are used in models for capacity computation at unsignalized intersections. Critical gaps estimated by different procedures could have different influences on the model's capacity output. It should be guaranteed that the estimated critical gap in conjunction with the move-up times t_f (and their estimation procedure) gives a reliable and realistic estimate of the capacity independent of the external parameters (mentioned above below 2.), especially the major street volume.

In no case, it is sensible to use a capacity computation model with critical gaps estimated in a way which is not related to the model. In other words: The estimation procedure for critical gaps and the capacity model (as well as the consecutive delay model) must form one entire integrated unity.

It has to be stated that this capacity model compatibility is only given for the Siegloch method, as shown above. For each of the other estimation procedures the method of critical gap estimation and the capacity (as well as delay) calculation methods have no theoretical connections. Some methods, however, produce results which can not be handled by the usual theoretical concepts for capacity calculations, e.g. some of the probit results (cf. above).

5.   SIMULATION STUDIES

5.1   Description of the simulation concept

These qualities of an estimation procedure - especially criterion 2 - can only be checked for by a simulation model since an analytical approach is not available. Therefore an extended simulation study for testing different t_c-estimation procedures according to requirement 2 has been performed. The two traffic streams - like illustrated in fig. 1 - have been generated by randomized procedures. For each simulation run a combination of constant traffic volumes q_p and q_n has been given. q_p was varied between 100 and 900 veh/h. q_n has been varied between 0 and the capacity c, which depends on q_p. Each such simulation run has been performed for constant traffic volumes over 10 hours. Thus, 46 different combinations of q_p and q_n (where q_n < capacity) have been performed for two different cases, which are combinations of t_c- and t_f-values. The first case may represent minor street right turner behavior whereas the second case could stand for driver behavior of left turners from the minor street.

	case "5.8"	case "7.2"	
critical gap t_c	5.8	7.2	s
minimum t_c-value	2.0	2.2	s
k-value for the Erlang-distribution	5	5	-
maximum t_c-value	12.5	15.5	s
move-up time t_f	2.6	3.6	s
minimum t_f -value	1.2	1.6	s
k-value for the Erlang-distribution	2	2	-
maximum t_f -value	7.2	10	s

The critical gaps t_c and the move-up times t_f for each simulated driver have been generated according to a shifted Erlang-distribution using the parameters mentioned before. Each driver is assumed to be consistent; i.e. he maintains his generated t_c-value until his departure. Generated t_c-values and t_f-values outside the margins of minimum and maximum values indicated above have been replaced by these given extreme values.

Moreover, to achieve a rather realistic pattern of headways the so-called Hyperlang-distribution (according to (22), in the version of (23)) has been applied for the major stream traffic flow generation where traffic on one single lane has been assumed. Also the arrivals of minor stream vehicles have been generated according to this type of distribution. These complicated distributions generate traffic situations of great variability which are rather similar to realistic conditions.

Within each simulation run the critical gaps t_c have been estimated out of the simulated flow patterns according to the above mentioned estimation methods. Thus, a series of 46 estimations for t_c has been obtained for each method.

5.2 Results for the critical gap

To illustrate the type of results, fig. 5 is used. It presents all the t_c estimated by Hewitt's method for different q_p-values. Here they are plotted over the major stream traffic volume q_p which was generated by the simulation. This type of relation, after some comparisons, turned out to be the most sensible since results in relation to minor street flows q_n showed much less variation. Each cross represents one t_c-value. The crosses plotted one above the other were produced for different q_n. For both traffic flows the simulated volumes (not the predetermined values for the generation of headways) were used in the following graphs.

We see that the outcome of the Hewitt estimation process reveals t_c-values between 5.63 and 5.98 s for a correct value of 5.8 s and between 7.08 and 7.52 s for a correct value of 7.2 s. Thus a rather large variation of the results must be stated. For the t_c-values a regression analysis regarding their relation to q_p has been performed. In this case the regression line was rather horizontal and it was close to the correct t_c-value in both cases. That means: The Hewitt-method on average reproduces the correct t_c-value very well and its results do not depend on major street volumes. Thus, this method fulfills our most important quality criterion 2) with a rather high performance. We shall see that this is not the case with most of the other methods.

From all the other methods studied, only the maximum likelihood method (fig. 6; denoted as Troutbeck method) comes up to the same performance as the Hewitt method. Here again the regression line shows no relation to major street volumes and the correct t_c-values are on average close to the estimation results. Again a rather large variation of the results obtained for different minor street volumes can be recognized from the plots.

Nevertheless, these two methods fulfill the requirements formulated above, especially the most important criterion 2.

Similar plots for the illustration of simulation results have also been produced for the other methods studied. For the logit method (fig. 7) we see that the results become very different if we either use gaps and lags or only gaps. In our example (only $t_c = 5.8\,\text{s}$ has been evaluated) only the exclusive use of gaps (no lags have been used in the second sample) reproduces on average the correct t_c. The inclusion of lags into the estimation process leads to a remarkable underestimation of the critical gap. However, in each of the approaches the results show a significant dependency on the major street traffic volume. Thus the logit procedure fails the important criterion 2). Therefore, it can not be recommended as useful estimation procedure (in the form as it is described above; it could be that a correction to the procedure might become available).

For the Ashworth method (fig. 7) we see that most of the estimation results are smaller than the correct values. Since, thus, the results seem not to be correct, this procedure - at least in its uncorrected version (eq. 5) - are not to be recommended for application.

A rather disastrous result has evolved from the simulations both for the Raff and the Harders' procedures. The Raff method meets the correct values only for median major street traffic volumes whereas Harders' method leads to a tremendous overestimation of critical gaps. Only

if Harders' method included the lags into the evaluation process, some improvements for the average size of the resulting t_c-values are obtained. The even more important problem is, however, that the results coming out of both types of procedures have a strong relationship to the major street traffic volumes - a feature which should be avoided as our set of criteria indicates. Therefore, both methods should not be used for reliable critical gap estimations.

Finally, for the estimation method according to Siegloch we find again better performance. We see - similar as for the Hewitt and Troutbeck methods - that for both example values of t_c we get on average a correct replication of the true t_c-values. In addition, the regression lines are not too much related to major street traffic volumes; i.e. the major street flow does not systematically influence the estimation results. Thus the Siegloch method seems to be a useful estimation technique.

The simulation for the Siegloch-method allows us also to evaluate the capacity with the predetermined t_c- and t_f-values, since for the simulation a continuous queue has to be generated. Thus, the number of minor street vehicles, being able to pass the conflict zone, is exactly the capacity of the intersection. The results from a series of 10-hour simulation runs have been noted. They were described by a regression function (cf. fig. 10)

$$c = A \cdot e^{-B \cdot p} \hspace{4cm} (s) \hspace{2cm} (33)$$

which is of the same type as Siegloch's capacity function (eq. 4). The values for A and B have been estimated according to the principle of least squares using a spreadsheet technique without prior linear transformation. This type of function allows another type of estimation of t_c and t_f :

$$t_f = \frac{3600}{A} \hspace{1.5cm} t_c = B + \frac{t_f}{2} \hspace{3cm} (s) \hspace{2cm} (34)$$

The simulated capacity results and the regression function ("simulated") are represented in fig. 10. The regression has an extremely high r-value, which indicates a very good correlation. However, the t_c- and t_f-value obtained from the regression line are different from the true values. The relation between capacity and major flow, as it would be calculated from Siegloch's original formula with the true values for t_c and t_f is also indicated in the graph of fig. 10 ("calculated"). Both curves for the capacity show significant differences. Therefore, criterion 3 is not fulfilled. The reason is that the Siegloch formula (eq. 4) is only exactly valid for constant t_c- and t_f-values and poisson major street traffic flows whereas the simulation has been performed for more realistic circumstances especially with non-poisson major stream arrival patterns. This result indicates that the Siegloch estimation technique is sensitive to the major street headway distribution (cf. criterion 3).

Fig. 11 gives an overview of all the regression lines obtained.

5.2   Results for the variance of estimated critical gaps

The simulated 10 hours of constant flows do not represent a sample which could be observed in reality. Under practical circumstances only measurements for one or two hours can be taken. Therefore, we studied also the convergence behaviour of some of the methods. Here the results for the maximum likelihood method are mentioned. Fig. 12 shows the range of

minimum and maximum t_c-values obtained from observation intervals of different durations. We see that for 1 hour (and longer) intervals the variability is less then 0.2 s.

Of course, here also the traffic volume should be regarded as an additional influencing factor. Therefore, fig. 13 shows the standard deviation in relation to the true value as a function of the number of minor street vehicles simulated. On the horizontal axis a logarithmic scale has been used. We see that above a sample size of 100 minor street vehicles the standard deviation is less than 0.3 s. The relation shown in fig. 13 seems to be of more general validity compared to fig. 12.

5.   CONCLUSION

A review of publications about estimation of critical gaps reveals a lot of different proposed solutions. Here it is difficult to understand which procedure is reliable and which not. From the sample of methods, which have been tested by simulations for this paper, the maximum likelihood procedure and Hewitt's method gave the best results. Both were valid for the two cases which have been studied. This is the reason why the maximum likelihood method has been selected for the evaluation of critical gaps which are to be used for the next edition of the HCM, chapter 10, (24).

The investigation of the different theoretical concepts, however, shows that principles of the various methods could also be combined. Thus, in future also even more estimation techniques for critical gaps might be proposed.

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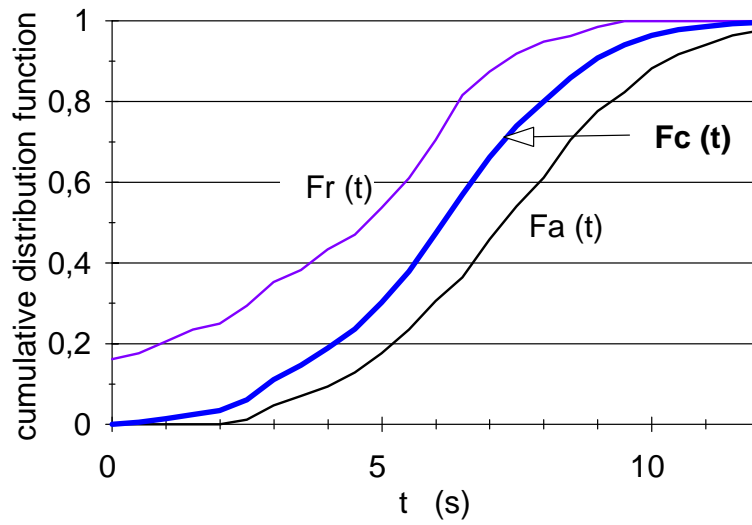17. Hewitt, R.H. (1995): personal correspondence with Werner Brilon

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minor stream

major stream

q_p

conflict area

Fig. 1:

Illustration of the basic queuing system

Fig. 2:   Illustration of Siegloch's method: The points illustrate the observed values for g. The circles represent the average t-values for each g. The line indicates the regression equation: $t = 4.8 + 2.9 \cdot t$,
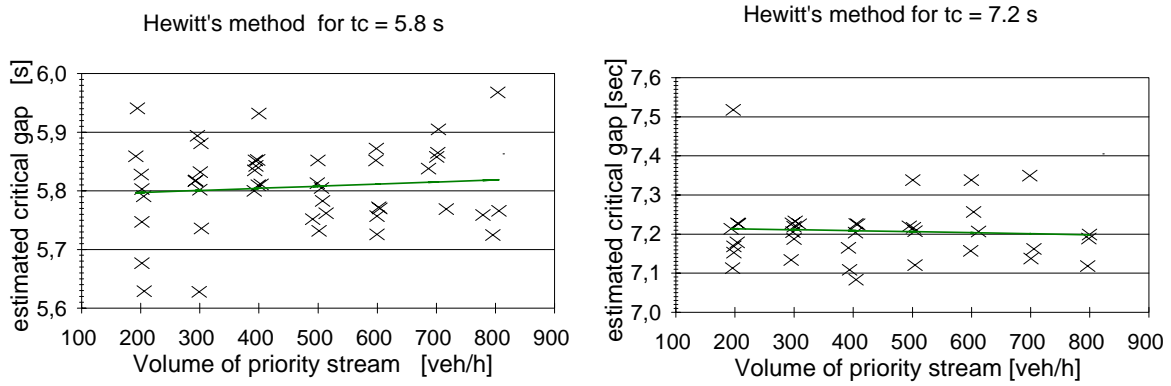from which we can obtain the estimations $t_c = 6.25\,s$ $t_f = 2.9\,s$.

Fig. 3: The distribution function $F_c(t)$ of the critical gaps must be situated between the distribution functions of rejected gaps $F_r(t)$ (here case a2): lags are treated as $t_r = 0$ and the distribution function $F_a(t)$ for accepted gaps.

Fig. 4: Example for the logit estimation obtained from the simulation runs mentioned below for $q_p = 800$ veh/h and $q_n = 200$ veh/h .

The resulting values are: $\alpha = 6.61$, $\beta = -1.01$, $t_c = 6.54$ s .

Fig. 5:   Results from the simulation runs for the Hewitt-method for two t_c -values.

Fig. 6:   Results from the simulation runs for Troutbeck's maximum likelihood method and for two t_c -values.

Fig. 7:   Results from the simulation runs for the logit method and Ashworth's method. For logit only the case for $t_c = 5.8$ s has been evaluated.
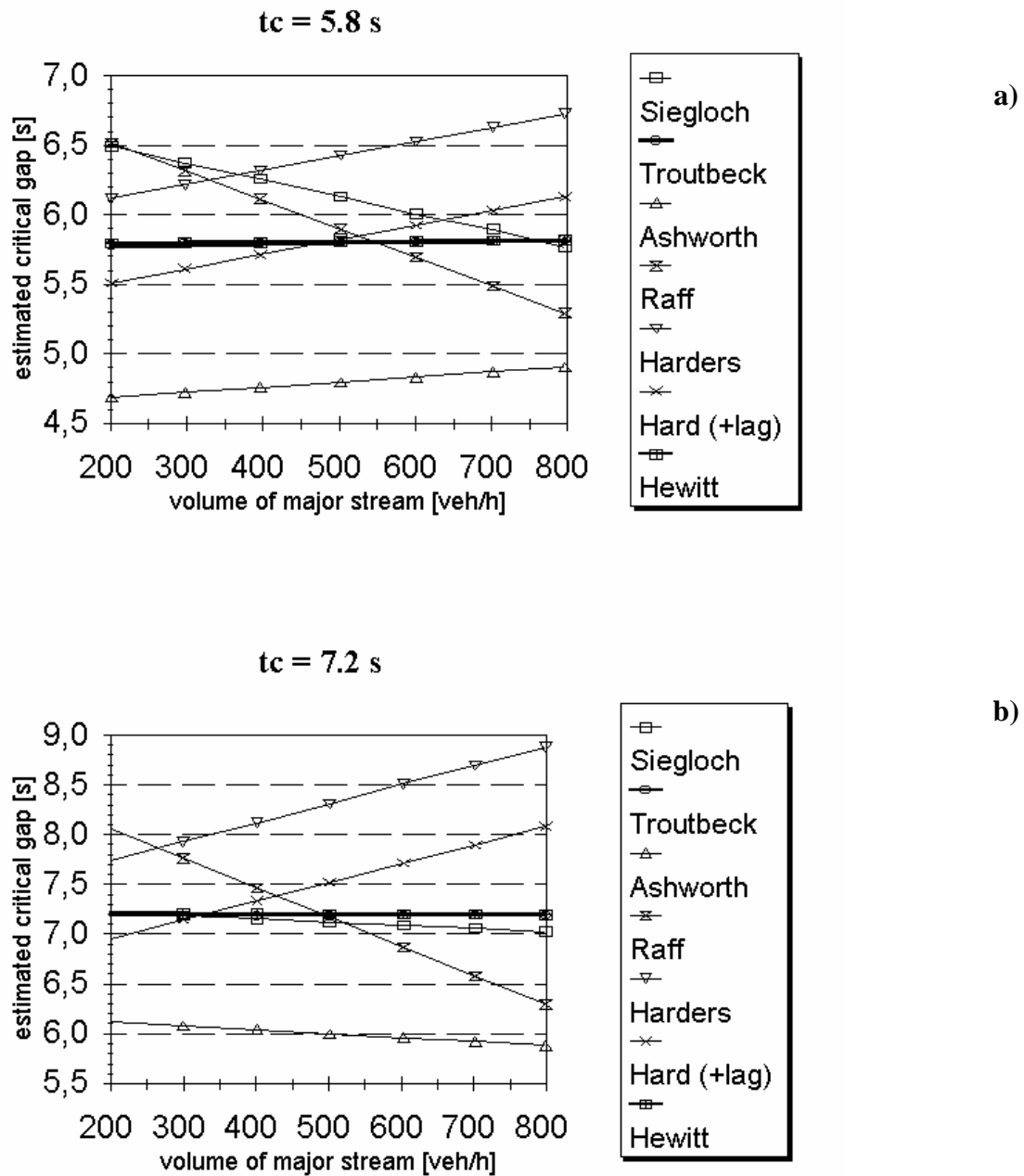
Fig. 8: Results from the simulation runs for the Raff and Harders methods and for two t_c-values.
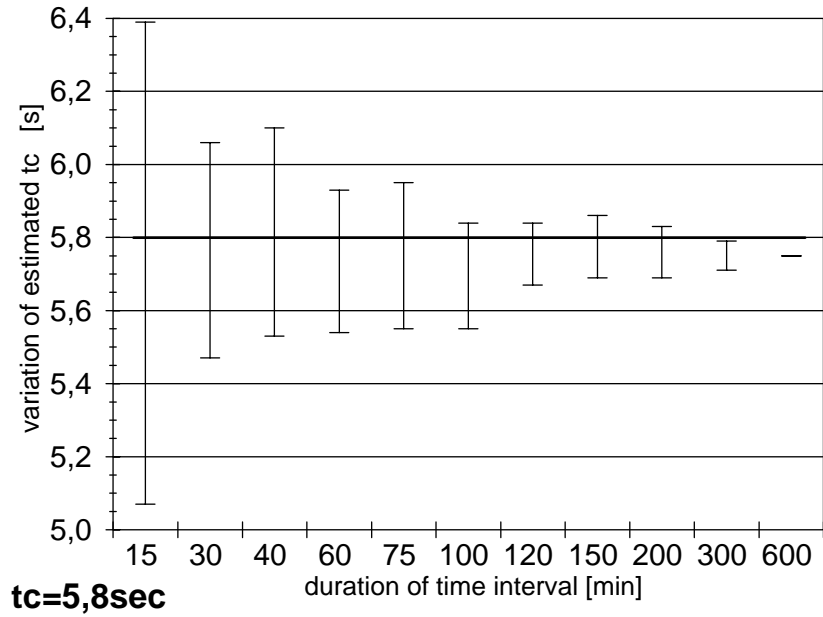
Fig. 9:   Results from the simulation runs for Siegloch's method.


Fig. 10:   Simulated capacities compared with results calculated from Siegloch's formula
                     (eq. 4) using the given critical gap and move-up time.

Fig. 11: Comparison of the regression lines for the relation between major street traffic flow and estimated t_c-values for $t_c = 5.8$ s (a) and $t_c = 7.2$ s (b).

tc=5,8sec

Fig. 12:   Minimum and maximum estimates for the critical gap in relation to the duration of the observation period for the maximum likelihood method and a given $t_c = 5.8$ s.
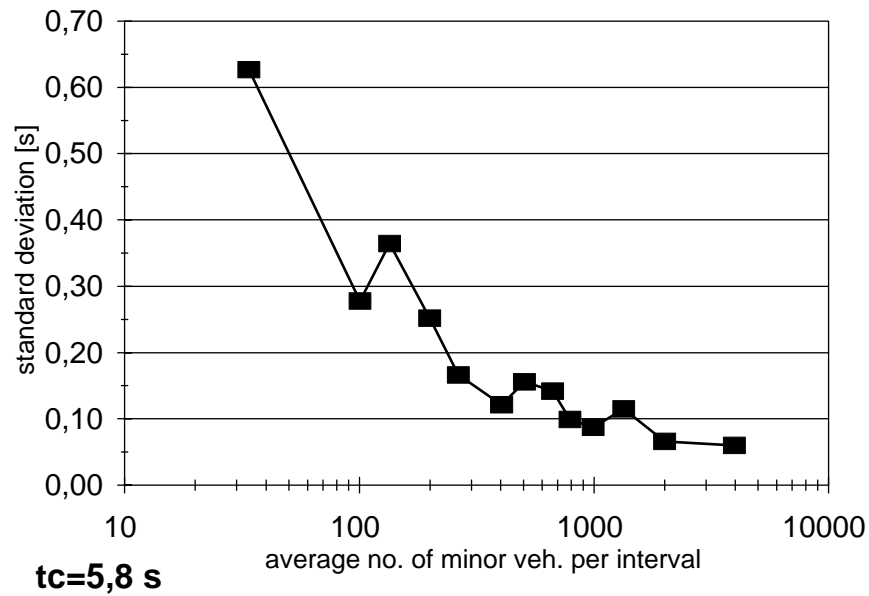
tc=5,8 s

Fig. 13:   Standard deviation of the estimated critical gaps t_c in relation to the sample size of minor street vehicles for the maximum likelihood method and a given $t_c = 5.8$ s .

Figure captions

Fig. 1: Illustration of the basic queuing system

Fig. 2: Illustration of Siegloch's method: The points illustrate the observed values for g. The circles represent the average t-values for each g. The line indicates the regression equation: $t = 4.8 + 2.9 \cdot t$,
from which we can obtain the estimations $t_c = 6.25$ s $\quad t_f = 2.9$ s .

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Fig. 11:   Comparison of the regression lines for the relation between major street traffic
                   flow and estimated t_c - values.

Fig. 12:   Minimum and maximum estimates for the critical gap in relation to the

                   duration of the observation period for the maximum likelihood method and a

                   given $t_c = 5.8$ s .

Fig. 13:   Standard deviation of the estimated critical gaps t_c in relation to the sample

                   size of minor street vehicles for the maximum likelihood method and a given

                   $t_c = 5.8$ s .

Definition of variables:

has not finally been adjusted to the text / **is not part of the text**

A_i = number of accepted gaps of size i (Harders method)

number of accepted lags of size i (lag method)

a_i = $a_i = \dfrac{A_i}{N_i}$ rate of acceptance for gaps of size t_i (Harders method)

$a_i = \dfrac{A_i}{N_i}$ rate of acceptance for lags of size t_i (lag method)

c = capacity = maximum number of minor street vehicles,

which can cross the major stream during one hour (veh/h)

Δt = length of time interval for the lag method or Harders method

$f_a(t)$ = statistical density function for the accepted gaps

$F_a(t)$ = cumulative distribution function for the accepted gaps

$f_c(t)$ = statistical density function for the critical gaps

$F_c(t)$ = cumulative distribution function for the critical gaps

$f_r(t)$ = statistical density function for the rejected gaps

$F_r(t)$ = cumulative distribution function for the rejected gaps

g = observed value for g(t)   (-)

$g(t)$ = number of minor street vehicles, which can enter into a

major street gap of size t   (-)

$h(t)$ = statistical density function for gaps (headways) between

vehicles in the major stream

L = Log-likelihood function

L^* = Likelihood function

μ_a = mean of the accepted gaps t_a (s)

μ_c = average critical gap

= expectation within the distribution function $F_c(t)$ (s)

n = number of observed minor street drivers

N_i = number of all gaps of size i , which are provided to

minor vehicles (Harders method)

number of all lags of size i , which are provided to

minor vehicles (lag method)

p	$=$	major (= "priority") street traffic volume $= q_p / 3600$	(veh/s)
q_n	$=$	minor street traffic volume	(veh/h)
q_p	$=$	major (= "priority") street traffic volume	(veh/h)
s_a	$=$	standard deviation of accepted gaps t_a	(s)
s_c	$=$	standard deriation of critical gaps	
	$=$	standard deriation within the distribution function $F_c(t)$	(s)
t	$=$	time	(s)
		index for drivers or decision situations (logit method)	
t_a	$=$	accepted gap	(s)
t_c	$=$	critical gap	(s)
t_f	$=$	move-up time	(s)
t_g	$=$	average t-value for each g (Siegloch method)	(-)
t_i	$=$	center of the i-th time interval for Harders method	(s)
t_r	$=$	rejected gap	(s)
W	$=$	number of time intervals (lag method)	(-)