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Using Concept Maps to Assess Conceptual Knowledge of Function

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In this study I examine the value of concept maps as instruments for assessment of conceptual understanding, using the maps to compare the knowledge of function that students enrolled in university calculus classes hold. Twenty-eight students, half from nontraditional sections and half from traditional sections, participated in the study. Eight professors with PhDs in mathematics also completed concept maps. These expert maps are compared with the student maps. Qualitative analysis of the maps reveals differences between the student and expert groups as well as between the 2 student groups. Concept maps proved to be a useful device for assessing conceptual understanding.

Key Words: Assessment; Calculus/analysis; College mathematics; Conceptual knowledge; Functions

This study focused on the use of concept maps to assess conceptual knowledge, especially as this use relates to meaningful learning in mathematics. Cognitive psychologists seem to agree that the internal representation of knowledge resembles webs or networks of ideas that are organized and structured (Hiebert & Carpenter, 1992; Hiebert & Lefevre, 1986; Pintrich, Marx, & Boyle, 1993; Royer, Cisero, & Carlo, 1993). The more connections that exist among facts, ideas, and procedures, the better the understanding (Hiebert & Carpenter, 1992; Hiebert & Lefevre, 1986). Individuals whose knowledge within a particular domain is interconnected and structured will activate large chunks of information when they perform an activity in that knowledge domain (Fisher & Lipson, 1985; Prawat, 1989; Royer et al., 1993). A highly integrated knowledge structure signals the transition from novice to expert performance (Royer et al., 1993).

Concept maps are a direct method of looking at the organization and structure of an individual's knowledge within a particular domain and at the fluency and efficiency with which the knowledge can be used. Unlike assessment techniques used in heavily quantitative, primarily behavioral studies, cognitive techniques like the use of concept maps often employ both qualitative and quantitative measures. The rationale for using concept maps in this study was to maximize participant involvement and to minimize the researcher's intrusive role. In drawing and labeling the linking lines, the participants explicitly stated the relationships they saw regarding functions. Mathematical knowledge and structure do not always lend themselves to simple categorizations, but they can be depicted well by concept maps.

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At present, ways of constructing and using concept maps vary widely. In some instances, the participants draw the maps; in other cases, the researchers construct the map from participant protocols. Some researchers, such as Novak and Gowin (1984), who invented concept maps, require that maps be hierarchical. But researchers from the semantic-network tradition tend toward “spider maps” (Harnisch, Sato, Zheng, Yamagi, & Connell, 1994), that is, maps with a general concept in the center and with links coming out much like the spokes of a wheel.

Six recent studies have used concept maps to assess cognitive structure or conceptual understanding (Beyerbach, 1986; Coleman, 1993; Laturno, 1994; Markham, Mintzes, & Jones, 1994; Park, 1993; Wallace & Mintzes, 1990). Two of the studies (Markham et al., 1994; Wallace & Mintzes, 1990) supported the concurrent validity of concept maps as vehicles for documenting and exploring conceptual change in biology. Two others (Laturno, 1994; Park, 1993) used concept maps in studies pertaining to mathematics. Comparing concept maps with interviews concerning students’ understanding of relationships between concepts and with the academic progression of students in a remedial mathematics course, Laturno found that the maps showed indications of validity as a research tool. Park found a strong correlation between concept-map scores and post-achievement test scores in a study of a college computer lab calculus course.

The evaluation of concept maps as assessment tools reported here formed part of a larger study. Results from the larger study that focus more specifically on the concept image of function and on differences between students in nontraditional (reform) calculus and traditional calculus sections as measured with other instruments can be found in Williams (1994).

METHOD

A top-tier state university of over 20 000 students served as the setting for this study. For the school year of this study (1993-1994) the university had two 3-quarter sequences of first-year calculus; one was traditional and the other was called *reform* because of its emphasis on modeling and technology. Both students and professors participated in the study.

The students in the study were 28 volunteers enrolled in the third quarter of calculus. Each student had been in either the reform sequence or the traditional sequence for all three quarters of calculus. Fifteen students from the reform sections volunteered. One was not officially enrolled and was excluded from the sample. By chance, half of the remaining 14 were men and half were women. Thirty students volunteered from the traditional sections, 7 women and 23 men. I chose all 7 women for the sample and then chose 7 men at random from the remaining volunteers.

Each student attended an instructional session on concept maps during which examples of concept maps were shown. Each map had concepts contained in ovals and linking words on the lines connecting the concepts. The examples included hierarchical maps, web or spider maps, and nonhierarchical maps. I instructed the students that they could draw their maps however they wished. Each student then drew up a list of terms related to functions and fashioned the terms into a concept map, adding other ideas when they arose. Each student completed the task in less than an hour.

Eight professors (with PhDs in mathematics) at two different universities participated in the study as experts. Four taught at the same large state research university the student participants attended. The other four taught at a small, private, West Coast university. The researcher explained concept maps to each expert individually in the same manner as to the students, showing them the same examples. Each expert drew up a list of starting terms before beginning the maps, just as the students had done. Each expert then completed two concept-map tasks. The unrestricted task was to draw a concept map for function from his or her perspective as a mathematician. The restricted task was to draw a concept map of function that represented what he or she would expect students completing the first-year calculus sequence to know (see Figure 1). Only the maps created by the experts during the restricted task were used in the analysis.

ANALYSIS

The analysis focused on differences among groups of participants. Did concept maps reveal differences in the concept of function held by students in reform sections (RS) of calculus from that held by students in traditional sections (TS) of calculus? What conceptual differences existed between the students and the professors?

Most researchers using concept maps design a scoring scheme to assign a numerical value to each map. The categories used for scoring often include valid propositions, levels of hierarchy, and cross-links. They occasionally include examples. However, the maps drawn by the participants in this study proved to be widely divergent and did not lend themselves to a numerical scoring scheme. Figure 1 shows an expert's map, Figure 2 an RS student's map, and Figure 3 a TS student's map. Although all of the expert maps were large and complex, the students' maps varied widely. The two student maps shown here are the most extensive of those from each student group. (The maps have been redrawn to improve legibility.) The maps illustrate the diversity and complexity of concept maps. Each participant's map contained the function concepts in ovals with the words denoting relationships among concepts on the lines linking the ovals. This common understanding of the process for drawing a concept map indicates that the wide diversity of maps derives mainly from participants' different concepts of function.

Because for this particular set of data the typical numerical scoring schemes did not appear to be valid, I looked at the concept maps as integrated wholes and searched for differences between the two student groups and between the experts and the students. The most striking observation about both the reform and traditional students' maps was that many of the students' concepts and propositions were trivial or irrelevant. For example, many of the maps showed an emphasis on letters used

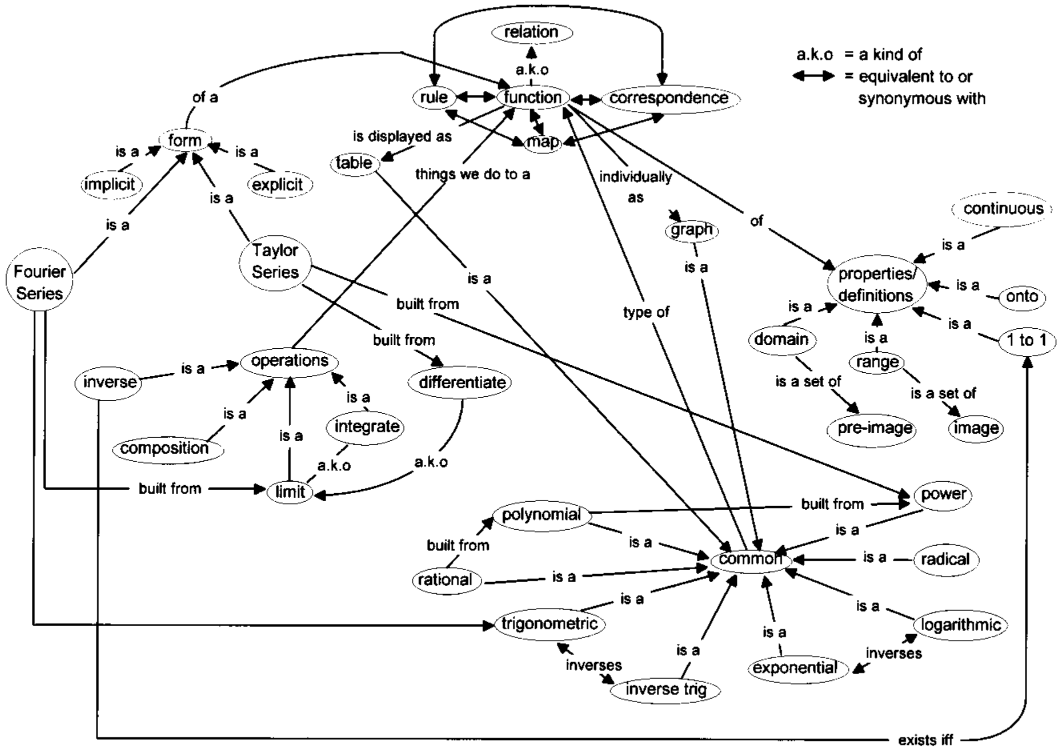

Figure 1. An expert's concept map of function (redrawn).

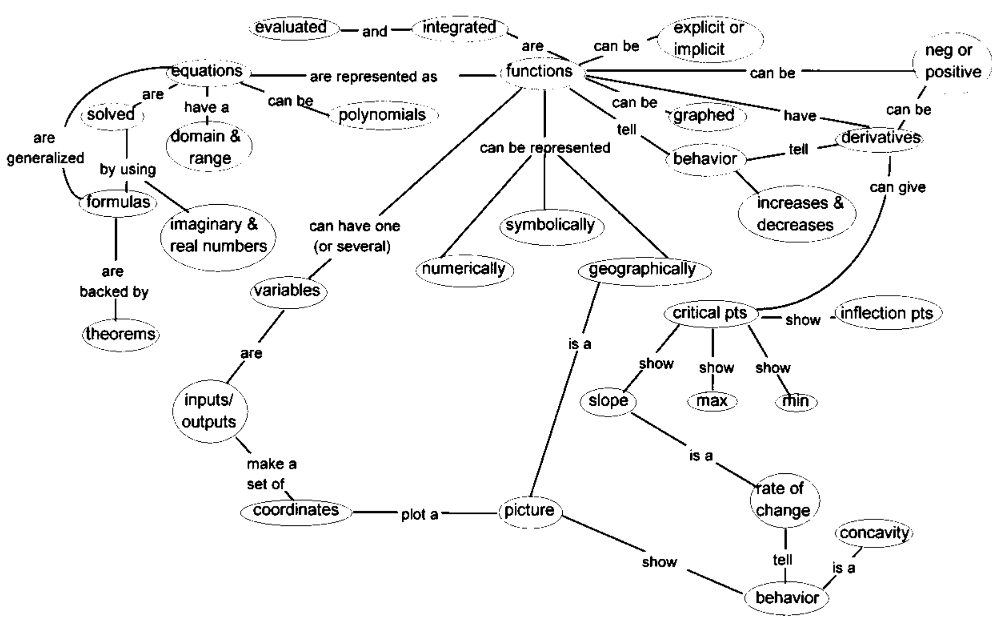

Figure 2. A reform student's concept map of function (redrawn).

for *variables*, even listing *x*, *y*, and *z* as concepts. Another student termed them *letters* and had three concepts under letters: *a-z*, *Greek*, and *x*'s and *y*'s. Often the students listed concepts closely tied to their class exercises, such as finding *maxima* and *minima* and classifying them as *absolute* (*global* for the reform classes) or *local*. One could say their preoccupation was with the trees rather than with the forest.

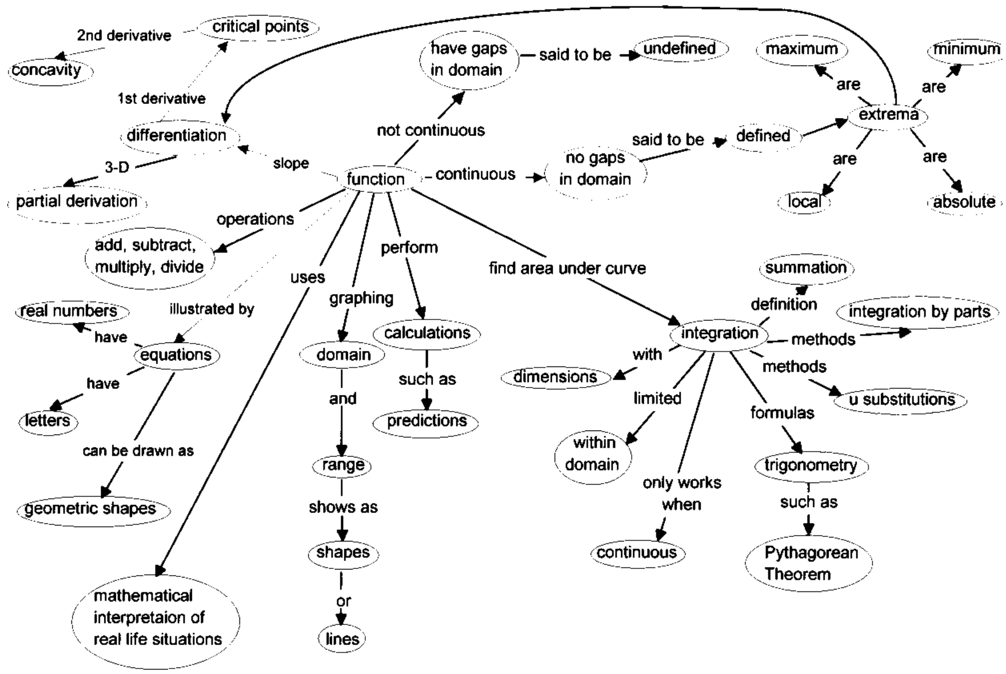

Figure 3. A traditional student's concept map of function (redrawn).

The second most noticeable characteristic of the students' maps was their algorithmic nature, particularly among the maps of the traditional students. Instead of naming concepts and the relationships connecting them, the students gave steps in a procedure. For example, one student had the chain "*function* can be *continuous* if *you can draw without lifting your pencil* or *they have no undefined points* such as *f(x)=1/x at x = 0* is *a hole at x = 0* which is *incontinuous*." She started out with the concepts of *function* and *continuous* but then drifted into procedural steps. Seven of the traditional students but only four of the RS students had groupings in which algorithms or processes were evident.

I studied the student maps to determine, if possible, each student's predominant view of function (e.g., an equation, a graph, a set of ordered pairs). The linking words were important in this analysis. For instance, one student's map had these propositions: "*function* consists of *variables*," "*function* can be *graphed*," "*function* can be *polynomial*." Use of the same type of link for *graphed* and *polynomial* indicates that the student viewed *graphed* and *polynomial* as being connected in the same way to function and that the connections indicated possibility rather than necessity. These propositions, along with heavily algorithmic portions of this student's map, indicated that this student held an equation view of function. Some students said, straightforwardly, "*functions* are *equations*."

Three students from the traditional group and four from the reform group made connections between function and real-life situations, but the two groups seemed to have different senses of these relationships. One student from the traditional group said "*functions* serve to *represent complex problems, e.g. velocity*." Another said "*function* uses *mathematical interpretation of real life situations*."

A third wrote "*equations* discover *natural phenomena* links *velocity and acceleration* example *balls falling in air*." Velocity and acceleration are typical examples of function in a traditional text. The least extensive map from the reform group contained the chain "*function* is found in *the real world* like *economics and engineering and medical field*." Another reform student connected three chains and conveyed the important concept of using functions to make predictions from collected data. A third map from the reform group had these propositional chains: "*functions* don't always involve *equations* some are about *real life situations* an example *death rate of a population as a function of time*" and "*functions* involve *modeling* an example *exponential decay* an example *interest rate for a savings account*." Although the evidence is not conclusive, the concept maps indicated that students from the reform group had a better understanding that functions may be used to model actual, real-life situations.

I further examined whether the maps reflected knowledge about the definition of function. In the group from the reform classes, one student had *domain* and *range* as concepts. Four others had *input* and *output.* None of the reform students indicated that each element of the domain can be paired with only a single range element, an essential part of the function definition. In the group from the traditional classes, four simply listed *domain* and *range* as concepts. Three others used *domain* and *range* and included on their maps the requirement about unique values for the range elements.

Did the maps reveal any differences in hierarchy and integration of concepts between the two groups? In a word, no. Few maps showed any significant hierarchical structuring. The number of concepts emanating directly from *function* ranged from 1 to 13, with an average of 7 for each student group. The branches that did have several levels generally delineated procedures rather than linked concepts. Integration of concepts, as shown by linking a concept to a concept in another branch (cross-links), virtually did not exist. Only two instances of cross-links showing an important connection, such as the inverse relationship of differentiation and integration, occurred. Although several students drew cross-links, most were trivial, for example, "*variables* can be *letters.*"

Complete analysis of the students' maps required comparison with the experts' maps. Unlike many of the students' maps, the experts' maps showed no hint of algorithms. Instead, they reflected many categorical groupings. For instance, five of the experts had a grouping that referred to *classes* or *common types* of functions, using terms such as *exponential*, *polynomial*, *trigonometric*, *logarithmic*, and *rational*. Just two students seemed to set up a class or type grouping of functions on their concept maps. Three experts had a cluster of terms involving *properties* of functions, *one-to-one*, *continuous*, *differentiable*, and having an *inverse.* Only one student used *one-to-one*, *continuous*, and *inverse*, whereas four others listed *continuous* alone. One student listed *inverse.* No student used *differentiable.* A third grouping using terms about operations one performs on functions appeared on four of the eight expert maps. The experts used terms such as *composition*, *differentiation*, *integration*, and "combining with *arithmetic opera-*

tions." No student's concept map showed any indication of an *operations* category. Finally, none of the experts demonstrated the students' propensity to think of a function as an equation. Instead, they defined it as a *correspondence*, a *mapping*, a *pairing*, or a *rule*. All experts incorporated a definition of function into their maps. Looking at the overall content and complexity, the experts' maps as a group showed much more homogeneity than the students' maps.

To summarize, qualitative analysis of the concept maps did indicate differences in conceptual understanding between the reform and traditional student groups in three areas: (a) their views of mathematics as algorithmic or not, (b) their preferences for representing a function, and (c) their abilities to connect functions to real-life situations. There were even more striking differences overall in conceptual knowledge as indicated by the students' maps and the experts' maps.

CONCLUSIONS

This study explored the use of concept maps as a research tool in the area of mathematics, particularly as the maps reflect conceptual understanding. The degree to which concept maps describe a person's actual mental representation is, of course, impossible to know. Nevertheless, the general homogeneity of the experts' maps and their distinct variance from the students' maps lend credibility to the conclusion that concept maps do capture a representative sample of conceptual knowledge and can differentiate well among fairly disparate levels of understanding. This study did not provide conclusive evidence that concept maps can differentiate more subtle levels of understanding. However, the qualitative analysis of the maps as a whole did suggest subtle differences between the groups of student participants. The analysis also provided information about students' understanding that is not readily gained from traditional pen-and-paper tests. Concept maps, therefore, provide important information about conceptual understanding and can play a useful role in the mathematics researcher's repertoire of tools.

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