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Using DEA to Evaluate the State of Society as Measured by Multiple Social Indicators

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Abstract—This paper presents an approach for discussing the state of society, which is measured by multiple social indicators, using data envelopment analysis (DEA). Replacing inputs and outputs in DEA with negative and positive social indicators respectively, we analyze the desirability of living in the 47 prefectures of Japan. This is also a proposal for the potential use of DEA in multi-dimensional evaluation analysis other than the standard DEA efficiency analysis. The results using eight social indicators identify 26 DEA desirable prefectures out of the 47 and present other useful knowledge and information. It is concluded that DEA, which can avoid uniform evaluation by an *a priori* weighting system, provides availability as a comprehensive evaluation tool different from traditional ones.

INTRODUCTION

Social indicators have been developed to discuss the state of society, which cannot be fully analyzed in terms of economic indicators alone. There exist many social indicators, each of which reflects some aspect of society. In order to grasp the state of society appropriately, we should thus evaluate the multiple aspects comprehensively, implying the simultaneous use of many social indicators.

This approach requires, in turn, use of the indicators' weighted sum as an integrated measure. But it is difficult to define such an *a priori* weighting because of the complexity and variety of human preference. If we employed this type of weighting system, resulting discussions might thus lead to uniform evaluation of societies with varying characteristics.

We therefore propose the use of *data envelopment analysis* (*DEA*) as a comprehensive evaluation capable of meeting the above-stated conditions.

DEA has been proposed as a method for measuring relative *efficiencies* of *DMUs* (*Decision Making Units*) [3] (see [2, 12, 13] for overviews). (Mathematical models of DEA are presented in the Appendix.)

As is well known, DEA examines how efficiently DMUs convert multiple *inputs* into multiple *outputs*. That is, any DMU producing more outputs with fewer inputs is judged relatively efficient (*DEA efficient*). However, DEA models do not necessarily assume such organic relationships between inputs and outputs as those in production (see Appendix). Thus, replacing inputs with *negative* evaluation items (the smaller the value, the better) and outputs with *positive* evaluation items (the greater the value, the better), yields a combined evaluation of these items. This is a comprehensive evaluation different from traditional ones in that it replaces a uniform evaluation using an *a priori* weighting system with a flexibly defined weighting system corresponding to each DMU.

While it should be possible to apply DEA to fields beyond efficiency analysis, we found but a few "non-standard" applications. These included preferential rankings aggregation [6], scholastic improvement measurement [7], computer printer comparison [8], baseball batters evaluation [9], and examination applicants selection [10]. In the current paper, we focus on DEA as a multi-dimensional evaluation tool, and seek its application to the state of society as measured by multiple social indicators.

APPLYING DEA TO THE STATE OF SOCIETY

In this study, we use DEA to evaluate *desirability* of living in the prefectures of Japan, thus defining the prefectures as DMUs. As inputs and outputs in DEA, we apply negative and positive social indicators, respectively. For the negative indicator, the smaller the value, the better; for the positive indicator, the greater the value, the better. The prefecture with greater positive and smaller negative indicators than others is thus judged relatively desirable (*DEA desirable*).

DMUs involved in DEA must be comparable, i.e. their essential differences should be expressed by their inputs and outputs. Although prefectures as DMUs differ fundamentally in terms of population density, economic functions, etc., these factors can be considered reflected in their social indicators. We thus propose that DEA using social indicators can lead to a better understanding of the desirability of living in Japan's 47 prefectures.

However, this does not imply that we seek a comprehensive social indicator of the prefectures. As is mentioned later, there is little value in the indiscriminate comparison of DEA measures less than 1. The DEA measure itself cannot, therefore, be a comprehensive indicator in the traditional sense. On the other hand, it is not meaningful to uniformly evaluate the desirability of living in all prefectures because each has different characteristics. We thus think it fair and reasonable to use DEA, since it has a flexible weighting system that can vary by prefecture.

We should note here how the various perspectives in the current study differ from that of production efficiency in the usual DEA sense. In production, there exist organic relationships between inputs and outputs. On the other hand, we may choose social indicators without considering such relationships between negative and positive indicators. There can be no outputs without inputs in production. Yet, in this study, it is possible, for example, to have only negative or only positive social indicators.

DMUs, inputs and outputs

The 47 prefectures of Japan, considered here to be the DMUs, are shown in Fig. 1.

As data for evaluating desirability of living in the prefectures, we apply the following eight social indicators:

Public safety
- Crime rate* (per capita)
- Rate of traffic accidents* (per capita)

Health
- Number of hospital beds (per capita)
- Suicide rate* (per capita)

Economic stability
- Average income (per capita)
- Bankruptcy rate* (per company)

Natural environment and housing conditions
- Water quality (proportion of water resources achieving national standard)
- Average house space (per capita)

(* Negative indicator)

Although this indicator system might seem somewhat arbitrary, we chose the individual components based on the following rationale:

Referring to the National Life Preference Survey [11], people in Japan remain vitally interested in improving their fundamental living state, even though the country's GNP is one of the highest in the world. We therefore measure the desirability of living in the prefectures in terms of fundamental living states. The National Life Preference Survey defines these states in terms of four dimensions: public safety, health, economic stability, and natural environment and housing conditions.

We fortunately also have an annual report [1] that supplies indicators of residents' perceptions of living in the 47 prefectures. We thus selected those (eight) social indicators that best represent the four dimensions of fundamental living states.

For health, we here adopted suicide rate rather than the more traditional life expectancy because of its small variance across prefectures. Moreover, we did not employ the more typical infant mortality and unemployment measures because of their low levels throughout the country.

In the system consisting of the eight social indicators, correlation coefficients among negative and among positive indicators are not large. The greatest absolute correlation coefficient is (−)0.667 between hospital beds and income; all others are less than 0.5. Since we cannot consider

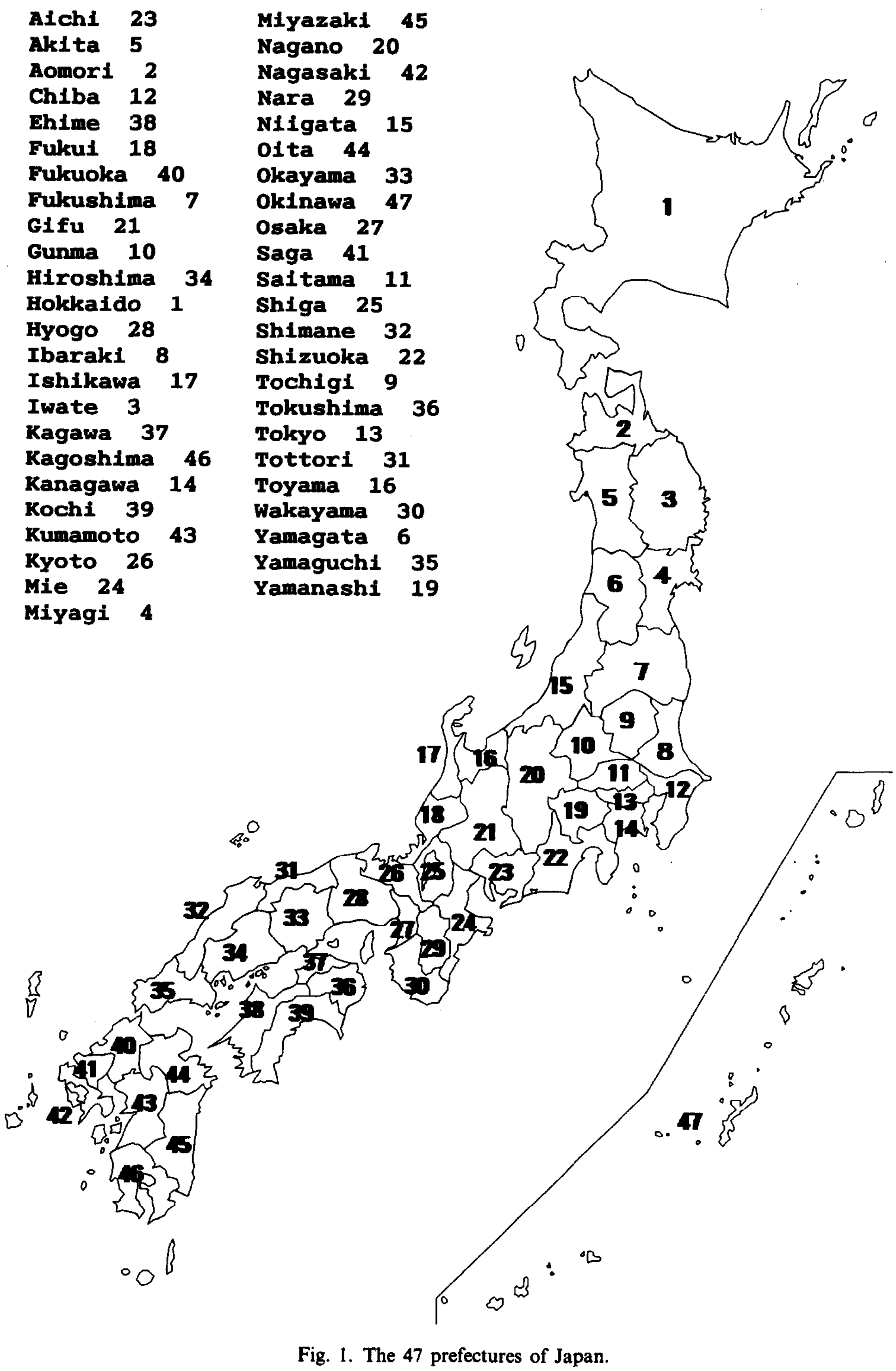

Fig. 1. The 47 prefectures of Japan.

Table 1. DEA desirable prefectures

Aichi	Iwate	Nara	Shizuoka
Akita	Kagawa	Oita	Tochigi
Chiba	Kanagawa	Okayama	Tokushima
Fukui	Kochi	Okinawa	Tokyo
Gunma	Mie	Saitama	Yamagata
Ibaraki	Miyagi	Shiga	
Ishikawa	Nagasaki	Shimane	

direct relationships, even, for example, between hospital beds and income, we feel that this indicator system helps avoid most overlapping information.

DEA EVALUATION OF PREFECTURE DESIRABILITY

Applying 1990 data from the 47 prefectures of Japan with the four negative and four positive social indicators, the model of (A.3) finds that 26 prefectures have a DEA measure of $h_{j_0} = 1$. As with all the social indicators, we employed normalized index numbers as the national averages are 100. Since each of the 26 identified prefectures has a maximum $\sigma_{j_0} = 0$ in model (A.4), each is on the *efficient frontier*, implying that no prefectures are on the *extended* frontier [4, 5]. That is, 26 of the 47 total prefectures are judged DEA-desirable while the remaining 21 are judged DEA-undesirable.

Table 1 shows the DEA desirable prefectures. Table 2 shows the DEA undesirable prefectures in order of their DEA measures, with their reference sets and combination coefficients, optimal solutions, λ_j, to model (A.3). The reference set of (DEA undesirable) prefecture j consists of those prefectures that have a DEA measure 1 in terms of the weights optimal for prefecture j. That is, the reference set composes a part of the frontier (facet) that involves a reference point comparison of the DEA undesirable prefecture. For example, Saga prefecture is compared with the facet composed by its reference set, Nagasaki, Ishikawa and Tokushima prefectures. The DEA measure 0.9920 is compared to 1.0, the supposed value of the reference point. This indicates that DEA undesirable prefectures can aim at the facets in order to attain a DEA measure 1 in terms of their current optimal weights.

Finding 26 of the 47 prefectures DEA desirable, we sought to test the stability of this result. We thus performed a sensitivity analysis by deleting the social indicators. Table 3 shows the number of DEA desirable prefectures in each case where one of the eight social indicators was deleted. Results of the sensitivity analysis show that the number of DEA desirable prefectures is quite stable except for the case where bankruptcy is rejected. In this regard, we note the singularity of the indicator of bankruptcy.

DEA desirable prefectures

Table 4 shows the frequency with which each of the 26 DEA desirable prefectures appears in the reference sets of DEA undesirable prefectures. The prefecture with the greatest frequency implies that it attains the maximum DEA measure 1 many times with the optimal weights for DEA undesirable prefectures. It does so in that case where the weights maximize the prefectures' DEA measures. This prefecture can thus be defined as "representative" with general characteristics. Ishikawa prefecture appears in approximately one half of the reference sets of the 21 DEA undesirable prefectures. We would therefore define it as representative of Japan for the year 1990.

On the other hand, there exist DEA desirable prefectures appearing no times in the reference sets (they are called *self evaluators* [2]). A self evaluator is a prefecture that has no DEA undesirable prefectures aiming at the facet it composes. It may thus be viewed as a DEA desirable, but peculiar, prefecture. Note that we have six self evaluators in Table 4. Surprisingly, four (Chiba, Gunma, Saitama and Tochigi) of the six are located near Tokyo prefecture. It is said that the recent excessive concentration in Tokyo has greatly influenced its surrounding areas. In this regard, the number of times that a DEA desirable prefecture appears in the reference sets might be a simple indicator of that prefecture's desirability.

As an alternative tool, we consider the *cross efficiency* (here, *cross desirability*) *matrix* [2, 14]. This is an $n \times n$ matrix (n = the number of DMUs; here, $n = 47$), where its ij element is the DEA

Table 2. DEA undesirable prefectures and their reference sets

DEA measure	Prefecture	Reference set and combination coefficient																			
		Ishikawa	Tokushima	Iwate	Miyagi	Nagasaki	Fukui	Okayama	Shizuoka	Akita	Kanagawa	Mie	Oita	Aichi	Tokyo	Okinawa	Ibaraki	Kagawa	Kochi	Nara	Shiga
0.9920	Saga	0.20	0.00			**0.79**															
0.9841	Toyama	**0.78**		0.04	0.04		0.11			0.09											
0.9809	Kyoto		0.23								**0.39**			0.21				0.18			
0.9798	Fukuoka		**0.79**												0.18						
0.9796	Hokkaido	0.26			0.13	**0.51**							0.12		0.04						
0.9708	Yamaguchi	**0.43**	0.08	0.11		0.16		0.21													
0.9668	Gifu	0.15			0.17				0.14			**0.49**								0.11	
0.9666	Kumamoto		**0.53**			0.52															
0.9611	Fukushima		0.19	0.08			**0.35**	0.20				0.15									
0.9566	Niigata	0.33		**0.34**			0.31			0.01											
0.9520	Yamanashi	0.28					0.17		**0.39**	0.11											
0.9424	Aomori	**0.40**			0.07	0.11	0.23			0.18											
0.9170	Hyogo				0.06		0.05		0.06		0.21	**0.41**		0.22							
0.9057	Miyazaki			0.11		**0.75**										0.14					
0.8924	Hiroshima	**0.43**							0.16		0.20		0.18								
0.8887	Nagano	**0.71**		0.00	0.09							0.05									0.12
0.8710	Kagoshima		0.35			**0.64**															
0.8621	Osaka		0.17					0.14			**0.32**		0.06		0.26						
0.8558	Tottori			0.09	**0.83**			0.14								0.01					
0.8484	Wakayama			0.18										**0.56**			0.09		0.18		
0.8311	Ehime		**0.48**		0.23			0.09	0.07				0.13								

Bold: the greatest combination coefficient for each DEA undesirable prefecture

Table 3. Results of sensitivity analysis

Indicator deleted	Number of DEA desirable prefectures	Indicator deleted	Number of DEA desirable prefectures
Crime	22	Hospital bed	21
Traffic accident	20	Income	23
Suicide	21	Water quality	22
Bankruptcy	13	House space	23

measure of prefecture j computed with the weights optimal for prefecture i. Here, the average of the DEA measures in each column (*average cross desirability*) shows how the prefecture associated with the column is rated by all the prefectures. The cross desirability matrix thus shows mutual evaluation of all prefectures while the frequency of appearance in the reference sets shows evaluation of DEA desirable prefectures by DEA undesirable prefectures.

Table 5 shows the average cross desirabilities of the 47 prefectures. Note that Ishikawa prefecture has the greatest value. It can thus be said that Ishikawa is an "all-round" prefecture since it attains the largest DEA measures with most of the weights optimal for different prefectures.

We should note that Miyagi, Okayama and Tokushima may also be considered "all-round" prefectures. Thus, Yamaguchi and Fukushima, which are DEA undesirable, have considerably greater average cross desirability values. They should therefore be evaluated as "near all-round" prefectures, but cannot be DEA desirable since there are more desirable prefectures of the same type (see Table 2).

On the other hand, Okinawa, Akita, Yamagata and Tokyo prefectures, which are DEA desirable, rank low in Table 5 (especially, Okinawa, which ranks lowest). Each of these prefectures thus attains DEA measure 1 with its own optimal weights, but cannot attain greater DEA measures with weights optimal for other prefectures. This implies that they depend on weights different from those of the others. It is noteworthy that Tokyo prefecture, the economic and political center of Japan, ranks rather low. That is, Tokyo is DEA desirable, but seems to have peculiar characteristics compared with other prefectures.

Virtual indicator values

We now examine with what weights these peculiar prefectures might attain DEA measure 1 using *virtual indicator values* [2], the product of social indicator values and the corresponding optimal weight. Virtual indicator values convey information on the importance a prefecture attaches to particular social indicators in order to attain its maximum DEA measure. They are used instead of social indicator weights since the actual weights are dependent on the scale of the associated social indicators. That is, virtual indicator values are normalized weights, by which we can see *feature* indicators of a prefecture.

Table 6 shows the virtual indicator values for DEA desirable prefectures. For such prefectures, the sum of virtual negative indicator values and the sum of virtual positive indicator values are both 1 [see model (A.2)], so that the individual virtual negative/positive indicator values show the contribution rates in negative/positive indicators to being DEA desirable. For example, Okinawa prefecture attains DEA measure 1 being evaluated in terms of traffic accidents and hospital beds, while Akita prefecture achieves this in terms of water quality, house space, and crime.

Yamagata prefecture is evaluated in terms of crime and suicide (negative indicators) and house space and water quality (positive indicators), while Tokyo is analyzed in terms of suicide (negative indicators) and the balance of the four positive indicators. These DEA desirable but peculiar prefectures thus each have featured characteristics.

Table 4. Frequency of appearance in the reference sets

10	Ishikawa	5	Shizuoka	2	Okinawa	0	Gunma
9	Tokushima	4	Akita	1	Ibaraki	0	Saitama
8	Iwate	4	Kanagawa	1	Kagawa	0	Shimane
8	Miyagi	4	Mie	1	Kochi	0	Tochigi
7	Nagasaki	4	Oita	1	Nara	0	Yamagata
6	Fukui	3	Aichi	1	Shiga		
5	Okayama	3	Tokyo	0	Chiba		

Table 5. Average cross desirability

Ranking	Average cross desirability	Prefecture	Ranking	Average cross desirability	Prefecture
1	0.949	Ishikawa[a]	25	0.775	Kyoto
2	0.927	Miyagi[a]	26	0.767	Hyogo
3	0.925	Okayama[a]	27	0.766	Chiba[a]
4	0.915	Tokushima[a]	28	0.764	Aomori
5	0.895	Mie[a]	29	0.750	Nagano
6	0.892	Fukui[a]	30	0.746	Hokkaido
7	0.870	Kagawa[a]	30	0.746	Saitama[a]
8	0.865	Nagasaki[a]	32	0.741	Gunma[a]
9	0.858	Yamaguchi	33	0.724	Ehime
10	0.854	Oita[a]	34	0.719	Kagoshima
11	0.852	Ibaraki[a]	34	0.719	Shimane[a]
12	0.846	Aichi[a]	36	0.718	Tokyo[a]
13	0.840	Fukushima	37	0.713	Yamanashi
14	0.829	Iwate[a]	38	0.708	Miyazaki
15	0.827	Toyama	38	0.708	Tottori
16	0.816	Kochi[a]	38	0.708	Yamagata[a]
17	0.812	Shizuoka[a]	41	0.683	Akita[a]
18	0.804	Saga	42	0.677	Hiroshima
19	0.798	Nara[a]	43	0.673	Niigata
20	0.796	Kumamoto	44	0.665	Fukuoka
21	0.793	Kanagawa[a]	45	0.658	Wakayama
22	0.788	Gifu	46	0.643	Osaka
23	0.784	Tochigi[a]	47	0.615	Okinawa[a]
24	0.780	Shiga[a]			

[a]DEA desirable prefecture

As shown in Table 6, DEA desirable prefectures are evaluated on a variety of attributes. Here, we can see a property peculiar to DEA vs other such comprehensive evaluation tools: multiple prefectures with various featured characteristics can attain the maximum measure 1. However, we should also note that the DEA model may yield alternative optimal solutions, which would lead to alternative virtual indicator values.

DEA undesirable prefectures

In terms of reference sets and combination coefficients, we can see (Table 2) the facets with which DEA undesirable prefectures are relatively compared and the target prefectures at which they should aim. That is, DEA models express the reference point on the facet of a DEA undesirable prefecture in terms of a non-negative linear combination of its reference set prefectures and slacks. The relative levels of combination coefficients show to which prefecture in the reference set the DEA undesirable prefecture is close. Moreover, finding prefectures with reference sets of the same kind, we can form comparison groups within which DEA undesirable prefectures should be compared with each other. Conversely, we can avoid the indiscriminate comparison of prefectures with different reference sets, i.e. prefectures of quite different types.

In Table 2, we find all the economic and political center prefectures of western Japan—Kyoto, Osaka, Hyogo, Hiroshima and Fukuoka, as DEA undesirable prefectures. This is in contrast to the center prefectures of eastern Japan—Miyagi, Tokyo, Kanagawa and Aichi, which are all DEA desirable. While it is said that there is a real economic gap between eastern and western Japan, we can here see the gap of fundamental living states between center prefectures of these two regions.

Of the center prefectures of western Japan, we here further analyze Osaka and Fukuoka. Table 7 (the upper part) shows that Fukuoka is close to Tokushima (DEA desirable), but Osaka has no such DEA desirable companions. Although Tokyo and Tokushima are commonly included in both reference sets, referring to the levels of combination coefficients, Osaka and Fukuoka would not be prefectures of the same type. There is thus little value in directly comparing DEA measures of these two prefectures.

Next, we examine those social indicator values needed to make Osaka and Fukuoka prefectures DEA desirable. Theoretically, any point on the efficient frontier can be such a set of targets. While DEA models can compute the reference point for a DEA undesirable prefecture, using this, we can define one out of an infinite set of targets. This can be considered the target under the condition that the prefecture keeps its character as unchanged as possible.

Table 7 (the lower part) shows the targets for Osaka and Fukuoka prefectures, which are negative

Table 6. Virtual indicator values for DEA desirable prefectures

	Negative indicator				Positive indicator			
Prefecture	Crime	Traffic accident	Suicide	Bankruptcy	Hospital bed	Income	Water quality	House space
Aichi	0.107	0.710	0.019	0.163	0.129	0.869	0	0.003
Akita	0.381	0.151	0.196	0.271	0.159	0	0.532	0.309
Chiba	0	0.244	0.658	0.097	0.215	0.411	0.154	0.220
Fukui	0.243	0.126	0.554	0.078	0.315	0.250	0.437	0
Gunma	0.351	0.097	0.174	0.379	0.315	0.173	0.512	0
Ibaraki	0	0.277	0.240	0.482	0	0.130	0.398	0.473
Ishikawa	0.033	0.326	0.525	0.118	0.372	0.414	0	0.214
Iwate	0	0.379	0.447	0.174	0.048	0.224	0.093	0.635
Kagawa	0.044	0.320	0.285	0.352	0.413	0.217	0.235	0.136
Kanagawa	0	0.011	0.989	0	0.277	0.279	0.152	0.293
Kochi	0	0.320	0.124	0.558	0.620	0.312	0.068	0
Mie	0	0.545	0.261	0.194	0.036	0.301	0.091	0.573
Miyagi	0	0.112	0.882	0.006	0.350	0.120	0.226	0.305
Nagasaki	0.052	0.458	0.199	0.290	0.856	0	0	0.145
Nara	0	0.068	0.932	0	0.177	0.119	0.273	0.431
Oita	0	0.082	0.847	0.071	0.320	0.067	0.355	0.259
Okayama	0	0.218	0.297	0.485	0.367	0.177	0.247	0.209
Okinawa	0.273	0.638	0.023	0.065	0.556	0.298	0	0.146
Saitama	0.008	0.183	0.755	0.054	0.062	0	0.937	0
Shiga	0.016	0.042	0.860	0.082	0.061	0.205	0.071	0.664
Shimane	0.224	0.748	0	0.028	0.493	0.367	0	0.141
Shizuoka	0.789	0.210	0	0	0	0.892	0	0.108
Tochigi	0.306	0.119	0.194	0.381	0.138	0	0.863	0
Tokushima	0.111	0.256	0.237	0.397	0.451	0.156	0.319	0.073
Tokyo	0	0.086	0.830	0.084	0.243	0.230	0.321	0.207
Yamagata	0.509	0.072	0.419	0	0.035	0	0.458	0.506

indicator oriented as the main changes are to negative social indicators [2]. We should note that any positive scalar product of the (eight dimensional) target value vector in Table 7 can also be a target.

From Table 7, Osaka prefecture, which has a relatively small DEA value, must reduce all negative social indicators in order to be DEA desirable. In particular, the gap between target and actual values for crime is great, so that improving the crime rate would be needed. On the other hand, it would be necessary for Fukuoka prefecture to considerably reduce crime, bankruptcy and traffic accidents. We should also note that Fukuoka prefecture, which has a DEA measure closer to 1, must improve its social indicator values more than Osaka to be DEA desirable. This is the case for a DEA undesirable prefecture since its reference point is supposed to be valued at 1. When the reference point is on the extended frontier, the reference point itself is not DEA desirable, so that the DEA undesirable prefecture must further improve its social indicators by slacks to be DEA desirable. This implies that indiscriminate comparisons of DEA measures less than 1 are not meaningful.

Table 7. Target values of social indicators

Prefecture	Osaka		Fukuoka	
DEA measure	0.8621		0.9798	
Reference set and combination coefficient	0.3242 Kanagawa 0.2576 Tokyo 0.1710 Tokushima 0.1368 Okayama 0.0648 Oita		0.7886 Tokushima 0.1769 Tokyo	
Social indicator	Target value	Actual value	Target value	Actual value
Crime	93.6	[158.2]	83.8	[150.9]
Traffic accident	91.0	[105.5]	99.0	[131.4]
Suicide	84.2	[97.7]	98.0	[100.0]
Bankruptcy	101.4	[117.6]	83.9	[133.7]
Hospital bed	96.7	[96.7]	145.4	[145.4]
Income	110.9	[110.9]	81.1	[81.1]
Water quality	97.6	[97.6]	101.5	[99.1]
House space	89.9	[85.7]	100.8	[95.7]

Note: All social indicator values are normalized as the national averages are 100.

Table 8. DEA and DEA/AR measures

Prefecture	DEA measure	DEA/AR measure	Prefecture	DEA measure	DEA/AR measure
Miyagi	1	1 (27)	Toyama	0.9841	0.9234
Ishikawa	1	1 (21)	Akita	1	0.9212
Shizuoka	1	1 (10)	Yamaguchi	0.9708	0.9204
Kanagawa	1	1 (9)	Kagawa	1	0.9148
Mie	1	1 (9)	Kyoto	0.9809	0.9117
Nara	1	1 (7)	Hokkaido	0.9796	0.9096
Nagasaki	1	1 (4)	Hyogo	0.9170	0.8972
Tokyo	1	1 (4)	Aomori	0.9424	0.8921
Aichi	1	1 (2)	Yamanashi	0.9520	0.8833
Fukui	1	1 (0)	Nagano	0.8887	0.8832
Okinawa	1	1 (0)	Kumamoto	0.9666	0.8655
Saitama	1	1 (0)	Gunma	1	0.8625
Okayama	1	0.9939	Hiroshima	0.8924	0.8615
Oita	1	0.9926	Niigata	0.9566	0.8475
Tochigi	1	0.9909	Kochi	1	0.8462
Shiga	1	0.9758	Shimane	1	0.8437
Iwate	1	0.9747	Fukuoka	0.9798	0.8409
Chiba	1	0.9714	Tottori	0.8558	0.8274
Tokushima	1	0.9670	Miyazaki	0.9057	0.8229
Gifu	0.9668	0.9668	Kagoshima	0.8710	0.8157
Ibaraki	1	0.9509	Osaka	0.8621	0.8149
Yamagata	1	0.9461	Ehime	0.8311	0.7901
Saga	0.9920	0.9289	Wakayama	0.8484	0.7610
Fukushima	0.9611	0.9243			

Note: ( ) denotes frequency of appearance in the reference sets.

DEA/AR analysis

DEA is able to define a weighting system for inputs and outputs corresponding to a target DMU. On this basis, the 26 prefectures of Table 1 were judged DEA desirable. This approach is in sharp contrast to the unified and uniform evaluation of using an *a priori* weighting system. A compromise between these two approaches is represented by *DEA/AR* (*DEA/Assurance Region*) *analysis* [15, 16]. In DEA models, the ratio of weights v_i (u_r) to negative (positive) social indicators is equal to the ratio of shadow prices for the negative (positive) indicators (see Appendix). Therefore, we can discriminate the importance of social indicators by bounding the ratios of weights. DEA/AR analysis aims at a more realistic analysis by incorporating experiences and expert opinions in the shape of constrained weight systems.

In this study, we perform a DEA/AR analysis bounding the ratios of weights v_i, u_r to social indicators as follows:

$$v[\text{Suicide}] \geqslant v[\text{Crime}] \geqslant v[\text{Bankruptcy}],$$

$$v[\text{Suicide}] \geqslant v[\text{Traffic accident}] \geqslant v[\text{Bankruptcy}],$$

$$u[\text{Water quality}] \geqslant u[\text{Income}] \geqslant u[\text{Hospital bed}],$$

$$u[\text{House space}] \geqslant u[\text{Income}].$$

For the negative social indicators, we assume that suicide is most heavily weighted since it seems to reflect an entirely negative quality of life. Moreover, we assume that bankruptcy is most lightly weighted because of its singularity in the earlier sensitivity analysis (see Table 3).

Amongst the positive indicators, water quality and house space are more heavily weighted than income under the assumption that the environment now takes precedence to the economy. Further, hospital beds is assumed to be least important since increasing this number does not necessarily yield an improvement in inhabitants' health. Table 8 shows results of the DEA/AR analysis listed by the DEA/AR values. Note that all 12 prefectures with DEA/AR measure $= 1$ were judged DEA/AR desirable.

It is noteworthy that the number of DEA/AR desirable prefectures is reduced to less than half the number of DEA desirable prefectures. We should here consider the 12 prefectures' desirability of living (Table 8). In light of earlier results, we view Ishikawa and Miyagi as representative prefectures of Japan for the year 1990. Of the 26 DEA desirable prefectures, Shimane, Kochi and Gunma have much smaller DEA/AR measures. Referring to the virtual indicator values in Table 6, the main causes of DEA/AR measure reduction appear to be the relative importance increases of suicide, water quality and house space for Shimane and Gunma prefectures, and the corresponding decreases of bankruptcy and hospital beds for Kochi prefecture.

In this way, DEA/AR analysis is useful in distinguishing DEA efficient DMUs in terms of *a priori* information. But we should note that in strictly bounding the weight ratios, DEA/AR analysis leads to a unified and uniform evaluation using a fixed weighting system. While we performed a DEA/AR analysis using the weight constraints above, there can certainly be other constraints based on alternative considerations.

SUMMARY AND CONCLUSIONS

This study analyzed the desirability of living in the 47 prefectures of Japan using DEA. We found that 26 of the 47 prefectures are DEA desirable, and that, among them, Ishikawa and Miyagi can be considered representative. For the DEA undesirable prefectures, we presented their DEA measures as well as their reference sets and combination coefficients. Further, we suggested those prefectures to target as well as those to be compared with. Using DEA/AR analysis, we identified twelve DEA/AR desirable prefectures.

From a methodological point of view, this study examined a field application of DEA beyond standard efficiency analysis. It thus applied DEA to a multi-dimensional evaluation analysis of the desirability of living in Japan's prefectures. We explored negative and positive evaluation items as inputs and outputs of the DEA model. In doing so, we showed that DEA, which can avoid indiscriminately unified comparisons as well as uniform evaluations by *a priori* weighting, is indeed a comprehensive evaluation tool with distinct advantages over alternative models.

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REFERENCES

1. Asahi Newspaper Co., Ltd. *Public Power '91* (in Japanese). Tokyo (1991).
2. A. Boussofiane, R. G. Dyson and E. Thanassoulis. Applied data envelopment analysis. *Eur. J. Opl Res.* **52,** 1–15 (1991).
3. A. Charnes, W. W. Cooper and E. Rhodes. Measuring the efficiency of decision making units. *Eur. J. Opl Res.* **2,** 429–444 (1978).
4. A. Charnes, W. W. Cooper and R. M. Thrall. Classifying and characterizing efficiencies and inefficiencies in data envelopment analysis. *Opns Res. Lett.* **5,** 105–110 (1986).
5. A. Charnes, W. W. Cooper and R. M. Thrall. A structure for classifying and characterizing efficiency and inefficiency in data envelopment analysis. *J. Product. Analy.* **2,** 197–237 (1991).
6. W. D. Cook and M. Kress. A data envelopment model for aggregating preference rankings. *Mgmt Sci.* **36,** 1302–1310 (1990).
7. A. Desai and A. P. Schinnar. Technical issues in measuring scholastic improvement due to compensatory education programs. *Socio-Econ. Plann. Sci.* **24,** 143–153 (1990).
8. J. R. Doyle and R. H. Green. Comparing products using data envelopment analysis. *Omega* **19,** 631–638 (1991).
9. A. Hashimoto. Evaluating baseball batters using DEA (in Japanese). *Comm. Opns Res. Soc. Japan* **38,** 146–153 (1993).
10. A. Hashimoto. A DEA selection system for selective examinations. Inst. Socio-Econ. Plann. Discussion Pap. Ser. 542, Univ. Tsukuba (1993).
11. Japan Economic Research Institute. *National Life Preference Survey* (in Japanese). Maruei, Tokyo (1987).
12. M. Norman and B. Stoker. *Data Envelopment Analysis: The Assessment of Performance.* Wiley, Chichester (1991).
13. L. M. Seiford and R. M. Thrall. Recent developments in DEA: the mathematical programming approach to frontier analysis. *J. Economet.* **46,** 7–38 (1990).
14. T. R. Sexton, R. H. Silkman and A. Hogan. Data envelopment analysis: Critique and extensions. In *Measuring Efficiency: An Assessment of Data Envelopment Analysis* (Edited by R. H. Silkman). Jossey Bass, San Francisco (1986).
15. T. Sueyoshi. Measuring the industrial performance of Chinese cities by data envelopment analysis. *Socio-Econ. Plann. Sci.* **26,** 75–88 (1992).
16. R. G. Thompson, F. D. Singleton Jr, R. M. Thrall and B. A. Smith. Comparative site evaluations for locating a high-energy physics lab in Texas. *Interfaces* **16,** 35–49 (1986).

APPENDIX

Charnes *et al.* [3] showed that the relative efficiency (DEA measure) of target DMU j_0, h_{j_0} $(0 \leqslant h_{j_0} \leqslant 1)$, can be obtained by solving the following fractional programming problem:

Maximize:

$$h_{j_0} = \frac{\sum_{r=1}^{t} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}}$$

subject to:

$$\frac{\sum_{r=1}^{t} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad j = 1, \dots, n \tag{A.1}$$

$$u_r, v_i \geqslant 0, \quad r = 1, \dots, t, \quad i = 1, \dots, m$$

where: y_{rj} = the amount of output r from DMU j; x_{ij} = the amount of input i to DMU j; u_r = the weight given to output r; v_i = the weight given to input i; n = the number of DMUs; t = the number of outputs; and m = the number of inputs.

DEA measures of all the DMUs can be found by solving problem (A.1) n times, setting each DMU as target DMU j_0 in turn. Here, DMUs j_0 with $h_{j_0} < 1$ are judged DEA inefficient.

The fractional programming problem (A.1) can be converted into the following linear programming formulation:

Maximize:

$$h_{j_0} = \sum_{r=1}^{t} u_r y_{rj_0}$$

subject to:

$$\sum_{i=1}^{m} v_i x_{ij_0} = 1 \tag{A.2}$$

$$\sum_{r=1}^{t} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \quad j = 1, \dots, n$$

$$u_r, v_i \geqslant 0, \quad r = 1, \dots, t, \quad i = 1, \dots, m.$$

Of course, instead of problem (A.2), we may solve the dual:

Minimize:

$$\theta$$

subject to:

$$\sum_{j=1}^{n} y_{rj}\lambda_j \geqslant y_{rj_0}, \quad r = 1, \dots, t$$

$$x_{ij_0}\theta - \sum_{j=1}^{n} x_{ij}\lambda_j \geqslant 0, \quad i = 1, \dots, m \tag{A.3}$$

$$\lambda_j \geqslant 0, \quad j = 1, \dots, n$$

(θ unconstrained).

Moreover, for each of DMUs j_0 with DEA measure $h_{j_0} = 1$, solving:

Maximize:

$$\sigma_{j_0} = \sum_{r=1}^{t} s_r^+ + \sum_{i=1}^{m} s_i^-$$

subject to:

$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{rj_0}, \quad r = 1, \dots, t \tag{A.4}$$

$$x_{ij_0}\theta - \sum_{j=1}^{n} x_{ij}\lambda_j - s_i^- = 0, \quad i = 1, \ldots, m$$

$$\theta = 1$$

$$\lambda_j, s_r^+, s_i^- \geqslant 0, \quad j = 1, \ldots, n, \quad r = 1, \ldots, t, \quad i = 1, \ldots, m$$

where s_r^+, s_i^- = slack variables, only DMUs with maximum $\sigma_{j_0} = 0$ are judged DEA efficient, while the other DMUs are DEA inefficient [4, 5].