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MATHEMATICS AND PROGRAMMING

Using Mathematical Apps with Reluctant Learners

Nigel Calder[1] · **Anthony Campbell**[2]

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Abstract This paper reports on a research project that examined the beliefs and attitudes of reluctant 16–18 year old learners when using apps in their numeracy and literacy programmes. Specifically, it is concerned with the student engagement and cognitive aspects of the numeracy section of the study. The use of apps enhanced student engagement with the numeracy tasks and transformed their attitudes towards mathematics from generally negative dispositions to positive and frequently enthusiastic ones. The visual and dynamic affordances of the apps influenced the nature of the student engagement. The use of apps shaped the learning experience in ways that differed from their prior experiences and engendered confidence and differentiation of the individual engagement. While there were greater than expected gains in mathematical understanding, this was conditional on the apps used and the pedagogical processes undertaken.

Keywords Digital technology · Mathematical apps · Numeracy · Foundation studies · Reluctant learners · Beliefs and attitudes · Affordances

The use of digital technologies in mathematics education offers new ways of engaging with mathematical concepts and processes. Some educators contend that they provide an opportunity to re-envisage aspects of mathematical education, along with alternative ways to facilitate understanding (e.g., Borba and Villareal 2005; Calder 2011). For instance, the visual and dynamic elements of engaging mathematical thinking through digital technologies reposition both the types of knowledge and understanding required and the ways in which learning emerges, which simultaneously shape the learning experience in a range of inter-related ways.

✉ Nigel Calder
n.calder@waikato.ac.nz

Anthony Campbell
Anthony.Campbell@twoa.ac.nz

[1] University of Waikato, Private Bag 12027, Tauranga 3143, New Zealand

[2] Te Wānanga o Aotearoa, Te Awamutu, New Zealand

The features of digital learning environments enable alternative ways to encounter, process, investigate and explain mathematical ideas. Being able to manipulate large amounts of information instantaneously and simultaneously view changes in multi-representational forms (e.g., numerical, symbolic and visual) permits learners to investigate real-life problems more easily (Calder 2011). Researchers have identified that the dynamic, interactive nature of the learning experience through dynamic geometry software facilitates student's experiences and understandings in geometry (Laborde 1998; Ng and Sinclair 2015). Likewise, the opportunities afforded for the exploration and transformation of data with digital technology reveals fresh approaches to analysing statistics (e.g., Forbes and Pfannkuch 2009).

In the New Zealand context, Māori and Pasifika students are over-represented in the statistics related to educational underachievement (MoE 2013a, b; TEC 2009) This includes leaving high school with minimal or no formal qualifications (MoE 2010, 2014). Policies and associated programmes have emerged recently which aim to address this issue, not only at the emergent learner level but also for those who have no qualifications and want to gain employment or to re-engage in education. Two critical areas that have been identified to enhance the success of both that re-engagement and employment opportunities are literacy and numeracy. A need for foundational literacy and numeracy has been strongly associated with adult learning and employment success (TEC 2009). *Youth Guarantee* is the name of a government-funded programme that aims to improve achievement, retention and transition of young people in the education system with foundational literacy and numeracy understanding identified as key aspects of this intention.

While there can be an eclectic range of reasons for students to become reluctant learners, some common elements have been identified. Protheroe (2004) has described reluctant learners as those that do not complete tasks, avoid challenges and are satisfied with just getting by. They may be disengaged and disruptive (Jackson 2011), frequently demonstrating low self-esteem and low efficacy (Sanacore 2008), while lacking in confidence and feeling frustrated (Downton 2006).

Students enrolled in these Youth Guarantee programmes have left school without any formal qualifications. Many have negative dispositions towards learning and school, with a significant number leaving before the end of compulsory schooling. This negative attitude towards school, coupled with low self-efficacy in mathematics, has engendered a reluctance to engage in mathematical activity. Hence, the term 'reluctant learner' was used to describe their disposition towards mathematics learning, one confirmed in our initial student group interviews.

Te Wānanga o Aotearoa (TWoA) is a tertiary education institute that has a core aim of developing creative education solutions to support the goals of *whanau* (extended families), *iwi* (tribes), communities and government. We examined TWoA Youth Guarantee programmes in three locations where iPad apps were introduced into student learning programmes. Our intention was to reshape the learning experience, to differentiate it from their previous unsuccessful engagements and to utilise the particular affordances of iPad apps when used as a digital pedagogical medium.

Affordances of Digital Technologies: Potentialities for Action

The notion of affordance implies the complementarity of the learner and the environment. These are not just abstract physical properties (Gibson 1977), but rather potential relationships between the user and the 'artefact' (Brown 2006). Important in this discussion is the symbiotic relationship between the digital medium and the user. While the former exerts influence on the student's approach, and hence any understanding that evolves, it is his/her existing knowledge that guides the way the technology is used and, in a sense, that shapes the technology. The student's engagement is influenced by the medium, but also influences the medium (Hoyles and Noss 2003).

One aspect that has often been associated with digital environments is the notion of multiple representations. The ability to link and explore visual, symbolic and numerical representations simultaneously in a dynamic way has been acknowledged extensively in research. Borba and Confrey (1996), for example, contend that this aspect, particularly for topics including functions, facilitates the co-ordination of established representations, enriching the conceptualisation, and the way functions can be understood.

Ainsworth et al. (1998) suggested that multiple representations promote learning for the following reasons: (a) they highlight different aspects: hence, the information gained from combining representations will exceed that gained from a single representation; (b) they constrain each other, so that the space of permissible operators diminishes; (c) when required to relate multiple representations to one another, the learner has to engage in activity that promotes understanding. Meanwhile, Sacristán and Noss (2008) have illustrated how engagement in computational tasks in a carefully designed microworld can lead to different representational forms, a process they call *representational moderation*. Similarly, in various studies involving DGS, the dynamic, visual representations served to enhance understanding of functions (e.g., Mariotti et al. 2003). In a study of students' understanding of key aspects of geometric transformations when engaged with *The Geometer's Sketchpad*, Hollebrands (2003) reported the development of understanding of transformations as functions.

Others have indicated that these affordances, when facilitated appropriately by the teacher, may lead to students exploring powerful ideas in mathematics, learning to pose problems and create explanations of their own (e.g., Baker et al. 1993; Sandholtz et al. 1997). They report improved high-level reasoning and problem solving linked to learners' investigations in digital environments. Ploger et al. (1997) concluded from their study that students learnt to pose their own problems and create personal exploration through investigating in a digital environment. Attributes, such as the interactive nature of the engagement and the multi-representation of data, when coupled with appropriate teacher intervention, enable the learner to not only explore problems, but also to make links among different content areas that might otherwise have developed discretely. They allow students to model in a dynamic, reflective way, enhancing students' ability to model mathematically (e.g., Borba and Villareal 2005). They also foster risk taking and experimentation (Calder 2011), allowing space for students to explore.

Digital technologies, if used appropriately, enable mathematical phenomena to be presented and explored in ways that afford opportunities to initiate and enhance mathematical thinking and make sense of what is happening. They allow the learner potential to look through the particular to the general (Mason 2005). When the learning experience differs with the involvement of digital technology, we can assume that learning trajectories and understanding will also alter. The digital technology does not operate in isolation, however. Its influence is inextricably linked to the pre-conceptions of the user, other societal and cultural discourses, and the nature of the learning process. In the next sub-section, we examine a version of how these contributing aspects might facilitate a re-organisation of mathematical thinking.

Borba and Villareal (2005) discuss the notion of humans-with-media, which they see as collectives of learners, media (in various often collaborating forms) and other environmental aspects – e.g., mathematical phenomena, other humans and other technologies. They utilised Tikhomirov's (1981) perspective, one that claims the computer plays a mediating role in the reorganisation of thinking and, thus, understanding, one comparable with while not the same as Vygotsky's (1978) orientation that language mediates thinking.

Borba and Villareal see understanding as emerging from the reconciliation of re-engagements of the collectives of learners, media and environmental aspects with the mathematical phenomenon, viewing these collectives in a dynamic way, one where the collective not only influences the approach to the mathematical phenomena, but is itself transformed by that engagement. As each engagement reorganises mathematical thinking, and initiates a fresh perspective, this in turn transforms the nature of each subsequent engagement with the task. This also suggests that the process is on-going, and echoes the hermeneutic circle (Ricoeur 1981). The engagement with the task, and the tension or opportunities this evokes, reorganises the thinking through the ensuing dialogue and action.

They contend that because of the unpredictable nature of the learner's interpretive perspective, media might *influence* the way we think, but not *govern* it. Digital technology influences the engagement and ensuing dialogue in particular ways, ones that can lead to a reorganisation of the learner's prevailing discourse in that domain. The learner, through self-reflection, through dialogue with others or through a combination of both, then resets his/her sub-goals and re-engages with the task from the newly situated perspective. This iterative process continues until there is resolution of some form.

Learning Through Apps

The affordances of digital technologies for mathematics education are well documented (Brown 2006). Learning through apps offers potential affordances for learning that are similar to those identified within other digital technologies (Calder 2015). Apps offer the opportunity for students to engage dynamically with mathematical concepts, while gaining immediate feedback to input. Moreover, they can link various forms of information or data and transform them simultaneously. Apps predominantly present the mathematical ideas and processes in a game context and provide visual, interactive learning environments.

There are indications that the use of apps has a positive influence on student motivation in a variety of settings (Attard and Curry 2012). While this is a key attribute in both the engagement of learners and in their subsequent learning, optimising learning is also contingent on the appropriateness and quality of the task through which the learner is being engaged. An analysis of mathematics apps available through the App Store has indicated that they are predominantly designed for drill-and-practice, are highly variable in quality, and often inaccurately labelled in terms of the cognitive elements they are claiming to address (Larkin 2013). Many of these are for the practice of particular skills in hierarchical, rewards-based games (see also Highfield and Goodwin 2013). Attard and Curry found that app use encouraged students to engage both behaviourally and affectively, but also acknowledged that this did not necessarily translate to mathematical cognitive engagement. Other research (e.g., Pelton and Pelton 2012) contends that few apps exemplify current best practice in mathematics education, nor do they always integrate visual and dynamic affordances to model mathematics situations that support mathematical sense-making.

Attard and Curry have also reported that iPad usage in primary-school mathematics programmes has led to greater reflective practice, enhanced engagement and higher-order thinking. Carr (2012), in a study with fifth-grade students learning mathematics through the use of iPads and apps, found that their use at times appeared to initiate higher order thinking and conceptual knowledge by enhancing the students' engagement, practice and reinforcement of concepts. She also reported that the students were more motivated and engaged compared with a control group not using mobile technology in their programme. O'Malley et al. (2013) additionally report that iPads also provide opportunities for the teacher to differentiate the learning for individuals or groups and foster independent learning. However, they additionally added that teacher professional development and technical support were essential parts of effective utilisation of the mobile technology.

Although situated within a study examining the use of the iPad in literacy learning, Hutchison et al. (2012) identified some advantages and considerations of using iPads that are more generic and would be applicable to learning in mathematics. They contend that iPads power on and off very quickly, so that it is easy to integrate them spontaneously, without disrupting learning. In addition, students were able quickly to learn how to navigate an iPad, and when they did encounter problems, they worked collaboratively to resolve them, leading to enhanced conversations. Given the specificity of available apps, and the ease of access to iPads in the class situation, teachers were more likely to integrate the iPads into their lessons spontaneously, thus enabling some dynamic, responsive differentiation of the learning for individual students. While educational apps are frequently game-based (Murray and Olcese 2011) and engaging, it is also important in mathematics education apps that mathematics learning opportunities are embedded seamlessly within the actual playing of the games (Masek et al. 2012; Moore-Russo et al. 2015). Carr (2012) also contends that multiple senses are incorporated with the use of apps, and that they might reinforce learning and support a variety of objectives.

There is also evidence that supports the use of apps in learning programmes and the contention that, if used appropriately, they enhance mathematical thinking (e.g., Li and Pow 2011). Carr recognised that apps could shape student academic success, and that game-based learning apps offered the potential to enable mathematical understanding and problem-solving processes. There are some excellent apps that foster mathematical learning, and there are teachers who have the knowledge and propensity to use them very effectively, but such learning is dependent on both these conditions for effective learning to occur. Enhancement of learning was seen to be conditional on the apps selected, on the purpose intended and, in particular, on the pedagogical processes within which they were used (Calder 2015). In addition, the proliferation of apps required proficiency in selection and use by the teacher.

Beliefs and Attitudes

Beliefs and attitudes are part of the affective domain and are influential in learners' engagement (Lomas et al. 2012; Zan et al. 2006). Beliefs are relatively stable positions which individuals perceive as truth. They are not demonstrated directly through observable actions, but by inference and interpretation of actions. Attitudes can be more directly attributed to responses or reactions to situations and phenomena, and are considered learnt behaviour (Schuck and Grootenboer 2004).

Leder and Grootenboer (2005) have proposed a theoretical model based on McLeod's (1992) reconceptualisation of the field, where beliefs are seen as more stable and less emotionally intense than attitudes, with emotions and feelings seen more as reactive behaviour, episodic in nature. This greater stability suggests that beliefs evolve slowly and incrementally, whereas attitudes might change for individuals through highly influential events or circumstances. Studies involving students' beliefs have indicated that they believe that the teacher plays a central role in the learning process which, coupled with the students' personal views on the nature of mathematics, exerted influence on their mathematical learning (e.g., Young-Loveridge and Taylor 2005).

Likewise, motivation is not observed directly, but rather is marked through behaviour and attitudes. Hannula (2006) described motivation as a preference towards doing some things and avoiding others. Motivation is related to personal interest (Wæge 2010) and plays a role in student achievement (Pintrich 2003). Others have reported positive correlation between students' positive attitudes and sense of autonomy, and enhanced learning and performance in school (e.g., Deci and Ryan 2000). There appears to be a relationship between motivation and engagement, with peer pressure and classroom culture influencing students' learning opportunities in mathematics (e.g., Sullivan et al. 2006). Positive student perceptions of the quality of instruction increases their enjoyment of the learning and alleviates feelings of anxiety towards mathematics (Frenzel et al. 2007), also suggesting that such perceptions influence engagement.

Studies into students' perceptions of their learning environments contend that these perceptions influence emotional and social behaviour (e.g., Anderman 2002). Hannula (2006) proposed that there was limited opportunity to meet students' needs, competence and autonomy in teacher-centred classrooms, implying a need to consider other types of learning environments. Student-centred group learning situations give opportunity for positive dispositions towards mathematics to develop, with enjoyment and increased effort reported (Hänze and Berger 2007; Schukajlow et al. 2012).

Learning environments are not just physical attributes, but include interactions (Frenzel et al. 2007), with classroom culture influential on the nature of them (Hunter and Anthony 2012). Frenzel and colleagues also reported that students' perceptions of their environment were related to achievement in mathematics, although they suggested that there were only tentative links between emotions and student achievement.

Methods

In the study that this article describes, the instruction included the teaching associated with the use of mobile devices. As they were a central part of the learning environment, it is reasonable to suggest that the apps, with the associated pedagogy, might influence student engagement. As mentioned above, several studies have reported that the use of iPads has enhanced student engagement with mathematics, while Attard also reported that the use of mobile technologies facilitated increased engagement with mathematics at home. In this study, the learner environment, including iPads, the apps and the pedagogical processes associated with them, was central when identifying any changes in student attitudes or transitions in engagement with mathematics learning.

This research project was undertaken at *Te Wānanga o Aotearoa*, a tertiary institute conceived and developed under the cloak of New Zealand Māori *kaupapa* – a set of values, principles and plans that underpin its philosophy. TWoA offers a range of Youth Guarantee foundation programmes that, at the time of this project, included introductory Sport and Leisure, and Contemporary Māori Arts programmes with embedded literacy and numeracy. At this time, Youth Guarantee programmes were exclusively focused on 16–18-year-olds who had left school without any formal qualifications and who frequently had negative attitudes towards school. While our research examined the influence of iPad apps on the beliefs and attitudes of Youth Guarantee students towards both numeracy *and* literacy, this article is only concerned with student engagement and changes in attitude towards numeracy.

An interpretive research methodology was used with a mixed-method approach. This lens resonates with a contemporary hermeneutic frame that helps to unpack the mathematical learning process when the mathematics is engaged through digital media (Calder and Brown 2010). The preconceptions that each learner brings to engaging with mathematical phenomena are derived from a specific cultural domain that the learner inhabits (Gallagher 1992). From this viewpoint, mathematical thinking emerges from a process of interpretation, with understanding and concepts perceived as temporary fixes of on-going development, rather than permanent realities. This resonates with the notion of the hermeneutic circle with learning and understanding always in transition (Ricoeur 1981).

Our understanding of mathematical ideas emerges through cyclical engagement with the phenomena and the continual drawing forward of previous encounters and understandings. Hence, mathematical concepts are not considered as fixed realities, but more formative processes that are enhanced as the learner interprets the mathematical phenomena from new, ever-evolving perspectives. The mathematical task, the pedagogical medium, the preconceptions of the learners and the dialogue that these encounters evoke are co-formative, while their relationship with the learner engenders understanding (Calder et al. 2006). Here, understanding is the learner's interpretation of the mathematical situation through those various interdependent filters. It is in continual transition and emerges from cycles of interpretation, with each fresh engagement made from a modified perspective (Calder and Brown 2010).

The research project involved a case study approach with three different Youth Guarantee classes. We assumed that learning is mediated by language and the use of tools. Not only does the dialogue of the teacher and the learners in the classroom act as a mediator, the app itself acts as a mediating tool. The learner's preconceptions of the pedagogical media, in conjunction with the opportunities and constraints offered by the media themselves, promote distinct pathways in the learning process. That is, mathematical activity is inseparable from the pedagogical means, derived as they are from a particular understanding of social organisation. Hence, the device will inevitably influence the mathematical ideas developed and is more than an environment. It is imbued with a complexity of relationships evoked by the users and the influence of underlying discourses.

Participants

There were 41 student (*tauira*) and eight teacher (*kaiako*) participants altogether in the original interview groups. A number of students had left by the time of the second interviews, due to shifting, finding work or other training, or not being present on the days of the interviews at their campus. They were all 16–18 years old and came from a variety of settings. By the nature of Youth Guarantee, they had no formal literacy and numeracy qualifications and a large proportion had left school without any qualifications. There was a mixture of ethnicities, but the great majority were Māori and Pasifika.

Student Tasks Involving Apps

The apps that the students used were selected in the year prior to the research being undertaken. The teachers were responsible for selecting the apps, in conjunction with a contracted external consultant. They were predominantly consumer apps; pre-determined environments that used hierarchical game contexts designed with the intention to develop particular numerical skill, such as whole number multiplication or operations with fractions. They used rewards such as verbal and visual feedback, with success at one level enabling transition to the next higher one. This usually meant having to do similar problems, but at a more difficult level and/or with the pace of the game increasing. Some of the mathematics apps used were *King of Maths, Math Blaster, Match the Fraction* and *Slide 1000*.

Initially, the apps were introduced and used individually, in a self-directed way, with the students generally quietly engaged with the games, but as their experience and that of the teachers grew, the students began to work collaboratively. The teachers introduced whole-class challenges and competitions between groups working collaboratively to solve problems or tasks. The competitions that were between the groups usually involved accuracy or speed. By engaging with the tasks through the app, the students had opportunity to reflect on the outcome, and reorganise their thinking, before re-engaging with the next task from a potentially modified perspective. The discussion with peers and their teachers, allied with the verbal and visual feedback from the app, mediated their emerging understanding.

Data Generation

Methods used to generate the data included: student semi-structured group interviews (two groups in each class, pre-iPad intervention); student attitudinal surveys (post-iPad intervention); student semi-structured group interviews (two groups in each class, post-iPad intervention); teacher semi-structured group interviews; class observational data;

and before and after assessments using the Tertiary Education Commission (TEC) on-line diagnostic tool. The survey contained both quantitative and qualitative data with 19 questions using a five-point scale and three open-ended questions.

The TEC on-line adult assessment is designed to identify a learner's strengths and weaknesses in numeracy. It draws from a data base of problems set in adult contexts. Typically, students undertake about 30 questions, but this varies due to the adaptive, on-line nature of the assessment tool. The researchers only had access to the individual students' level scores, not the component parts.

Step one indicates the lowest conceptual understanding. The initial data gathered through the semi-structured group interviews and questionnaires gave a baseline perspective on the students and allowed the researchers to compare data from the second set of group interviews and questionnaires. These were undertaken after the period of working with the apps was completed. Comparison of the pre-intervention and post-intervention sets of data was made to see if there were any changes in their beliefs about, and attitudes towards learning in mathematics. The research was conducted in accordance with *Kaupapa Wānanga*: *Koha* (provided valued research); *Āhurutanga* (ensured the well-being and dignity of participants); *Kaitiakitanga* (acknowledgement of the contributions of all people associated with the research); *Mauri Ora* (the potential to improve student outcomes).

Results

This section traverses a number of inter-related aspects. First, the data related to students beliefs about the use of mathematics and its importance to being an informed, functioning citizen is considered, followed by their recollections of their school mathematics experiences. The purpose of the initial group interviews and questionnaires was to gain some insight into the nature of the learning approaches these particular students had encountered in their school experiences: what were their attitudes to mathematics; how did they remember their teachers and the learning experiences; what were their beliefs about the place of mathematics in everyday life outside of school?

This initial data produced baseline perspectives of the students for the researchers to compare with data from the second set of group interviews and questionnaires. It also identified that there was a need for the tutors to reshape the students' mathematical learning experience, so that it was not similar to their school experience. Finally, the data related the influence on the students' beliefs and attitudes when the learning experience was transformed through the inclusion of iPad apps is analysed.

Initial Beliefs and Attitudes

All the students agreed that numeracy was part of everyday life and that there was a connection between being functional in a range of everyday tasks and using numeracy. Typical responses to the question "Where do you use numeracy?" were:

> Charlie: Yeah, it pretty much revolves round everything […] you've got to know maths.

Jed: If you can't add or subtract, you're going to get ripped off by the shop-keepers, it's very important, you use it every day […] Yeah, bills and power bills and stuff like that.

In particular, they were referring to the elements of numeracy that they thought were beneficial for everyday living. While these were typical of the responses, generally they referred to the students' beliefs that numeracy was an area that they valued. As beliefs are relatively stable, it is unlikely that a 6-month intervention that occurred only during part of their tertiary study contact days would transform these beliefs.

In the questionnaire data, 95 % agreed or strongly agreed that mathematics was useful, while 90 % agreed that people use mathematics every day. Overall, they indicated a belief that mathematics was a valuable area to have understanding in. While the students were consistent in the acceptance of mathematics as a valuable aspect of being an informed citizen, their self-efficacy and attitude towards mathematics was often negative.

There is a tension here between their beliefs about numeracy and their attitude towards it. We would have expected, given the high value that they placed on the skills, that they would have been enthusiastic and motivated to learn and to understand pertinent processes and techniques. The data were analysed to gain insight into this apparent tension; their reasons for believing that numeracy was valuable, yet having negative attitudes towards it. The vast majority of the students found the maths at school too difficult, leading to a sense of failure and disengagement with the subject.

For example:

Yahni: Hard, don't really like maths. I didn't like those X + Y things.

Charlie: At school, it was just hard and fast – they didn't explain things, that's why I hated it at school.

Meanwhile, others indicated that it was the lack of time they were allowed that inhibited their understanding. Their perception was that they never had enough time to understand concepts and processes fully, or to use them in problems. They felt unsupported and their attitude towards learning mathematics in school was negative. Frequently, these comments related to particular teachers and episodes in the classroom.

A typical example was:

Job: Mr Peters, he put equations up on the board and gave the brainy ones time to do it, so when like the people that didn't excel at it were still stuck behind, he would rub it off the board quickly as and just get angry at us because we haven't finished it. He just didn't give us enough time.

In the questionnaire data, 90 % agreed that maths was a difficult subject for them. Interestingly, only 15 % thought that maths was something that only smart people do. They attributed some of their negative attitude to their teachers. It may be that they had negative attitudes towards other subjects and to school in general. Even so, we were concerned with their attitudes towards mathematics while they were in school and the majority reported negative perceptions of their experience, often hinged to their negative perceptions of the teacher. The students' perceptions of their teachers were accessed through a question asking them, "Is there a maths teacher you can remember? Tell me a bit about them". In general, this group retained negative perceptions of their relationship with their teachers, particularly in mathematics. Some felt they were treated inequitably, for instance:

> Nel and Jenni: The teachers didn't support the dumber students […]

> Mike: I always […] I never actually had a problem with literacy and numeracy, it was just all the teachers that spoiled it for me. Many students that have dropped out, it is all […] 90 % is because of the teachers. The way they teach […] yeah, some teachers mix it up and get you confused.

This perception, coupled with the lingering negativity towards mathematics, indicates the need to reshape the mathematics learning experience for these particular students. As well, we can only report on those aspects in the research instruments, with these related to the school experience. It is probable that other influences in the students' lives and their reactions or attitudes to events would have also had some influence on their negative perceptions.

Another interesting aspect from the questionnaire data was that 80 % agreed that they would avoid maths if they could. However, 45 % agreed or strongly agreed that maths is interesting. While they acknowledged the value of mathematics, their perception of the learning experience was usually negative.

Post-iPad Activity Attitudes

The use of iPad apps within the teaching and learning process led to some changes in student attitudes towards numeracy. Most students felt the change was positive and enjoyed learning in the visual interactive manner that learning through the apps evoked. Whereas the initial data indicated that the students' attitudes towards mathematics were predominantly negative, after the period of the intervention, students' comments in both the group interviews and the questionnaires were positive. They enjoyed the games context and perceptions of the learning experience were positive.

Their experience of using apps in their numeracy programme was for a 6-month period, using a range of apps and pedagogical approaches. This suggests that it was not a short-term attitudinal response related to a particular app or a particular experience, but rather an attitudinal transition. The comments regarding their attitudes frequently related to the affordances of the pedagogical media or pedagogical aspects associated with the learning experience. Some typical student comments were:

> Paora: It's visual. It's cool, good for each and all of us. We all had our own one to use. Gave you an easy answer, but was testing your speed and understanding. Made it a game and competition.

John: You play on them. They're active.

Whetu: […] and visual. Felt like it was more easy.

Tom: Yeah, it helped to focus, and with concentration.

John: Made me more confident.

The visual and interactive affordances, coupled with the game context, facilitated the positive attitudes towards the learning experience and enhanced the students' perceptions of the learning. Compared with their earlier experiences on learning mathematics in school, the data included statements related to greater confidence and more focused engagement. However, not all of the students were positive. Perhaps through the context of the games being too young, or the conceptual level of the games, a number of students found the games a bit repetitive as a learning approach.

Nel: Sometimes it gets boring.

Jenni: Yeah, sometimes.

In general, though, the changes in student attitudes and engagement were toward the positive, especially when the apps were integrated into the learning programme in an interactive way or as a class or group game or challenge. Teachers were positive about the learning experience for students, while also seeing potential learning opportunities. A prevalent teacher observation was that students were more engaged. They indicated that students enjoyed using the apps as part of the learning.

Anthony: Super-interactive, like when it came to maths, the maths games, everyone was so enthusiastic about it. And then the different games that we came up with. They are really good if you want to get team interaction games. Also, getting them into groups and working as a team. Everyone's just thoroughly enthusiastic.

They also indicated that these aspects led to greater student confidence.

Ben: They [the students] value themselves more when they are confident. These have helped.

The apps games were viewed positively as a context for engaging with mathematics. Students generally enjoyed them and the ensuing social interaction, identifying learning through games as a positive experience.

Manu: Now maths is fun, our teacher explains more. It helps learning when it's fun […] games, times tables; it's a fun way to learn.

In general, the students enjoyed the change in pedagogical medium, with learning through the games context, and indicated that they found the learning engaging. These transitions in attitude and engagement opened opportunity for enhanced cognition.

Cognitive Factors

In the teacher group interviews, the discussion about cognitive factors was often linked to student enhanced engagement and providing a challenge to the learning in a non-threatening way. Often, this was in the form of games and competitions.

> Ash: Very positive. It's good because it's more interactive, so we are able to utilise it as tutors to challenge them off against each other. You see who knows what. It's more 'hands on', more practical, so they are able to see the calculations and add it up on the spot as opposed to writing.

The comment 'You see who knows what' indicates that there is some assessment of the students' understanding. The context of the competitive games, with the class using the apps, enabled the teachers to gain insights into the students' understanding of the concepts that the apps involved for instance, multiplicative thinking or fractional number. This on-going formative assessment allows the opportunity to modify the games or levels, hence differentiating individual learning. It also suggests that the students are using and further developing their understanding in the conceptual areas that the apps involve. Other aspects of the teacher's comment, such as the learning is more 'interactive' and more 'hands on', suggest that the associated pedagogy has been influential with the students' engagement.

Some teachers discussed the way they introduced the apps so as to enhance engagement with learning. At times, they used students as peer tutors to introduce the apps.

> Paora: The next time we gave them a game, because there were a lot of apps, we gave one person a different app and got them to play with it and then they had to explain it to the rest of the group, how to play it, and what the key things of the game were, what to look out for.

> Anthony: The mathematics side of it is such a big help, because a lot of the stuff is visual, and then they can actually see it, explain it visually, especially fractions, when it comes to pizzas, perfect. Some of them didn't even know how to do fractions until it was broken down like that.

Students also commented on cognitive aspects:

> Ollie: Liked the maths games. We played it as a whole class. That maths thing with the facts helps me with my learning. It makes me brainy. It's interactive.

Here, the student identified that a particular app, *Maths Blaster*, in conjunction with the pedagogical approach, has enhanced his understanding. Not only did he feel his understanding was enhanced ("It makes me brainy"), but his confidence and attitude appear to have transitioned to a more positive frame ("Liked the maths games"). The app games also allowed the teacher to adapt to suit individual learning needs, to facilitate differentiated learning.

> Jaye: I think their attention was a lot better […] what helped is when you are doing maths with the whole class, students are finished beforehand, so they can go with things, while you are working with the ones who really need help.

TEC On-Line Assessment

The initial pre-app intervention assessment results for this group were low, with step 2 the most common level, with most students at steps 2 or 3. The steps (1–6) indicate the conceptual level across a range of numeracy progressions. These have been developed and analysed by the National Centre for Literacy and Numeracy for Adults. For instance, in the multiplication strategies progression, step 1 is: solve multiplication problems by counting all objects, while step 5 is: solve multiplication and division problems with multi-digit whole numbers, using partitioning strategies. The assessment aggregates the learning progressions with students expected to achieve at step 5 in numeracy overall.

In the post-app intervention assessment, steps 3 to 5 were the most common levels, with most students at steps 4 and 5. Overall, there was greater than expected improvement between the initial and final on-line numeracy assessments, although this cannot be attributed solely to the iPad app intervention. The mean of the initial on-line numeracy assessment steps was 3.3 while the mean of the final one was 3.8. In a hypothesis test for the difference in the two means, $z = 1.65$, which is significant at the 90 % level. So at this level, we can conclude that there was a difference between the two means. There was a complex array of interconnected contributing aspects that could have been influential in this improvement, including some outside of the learning environment.

The following graph compares the initial and final on-line assessment. The blue bars (on left of each pair) are the number of students at each step in the assessment taken before the iPad and app work, while the red bars (on the right of each pair) represent the on-line assessment result towards the end of the programmes. Each campus did the assessments on slightly different dates (Fig. 1).

The data suggest that the use of iPad apps in the numeracy programme influenced student mathematical understanding in a positive way. Overall, the data are indicative of a change in student attitude and engagement. For some, their integration into programmes, coupled with accompanying social interaction, resulted in students who had been very negative towards mathematics feeling confident and willing to try new approaches. While their cognition also developed over this period, this was not necessarily related to the use of iPad apps; hence the research was directed towards their beliefs and attitudes.

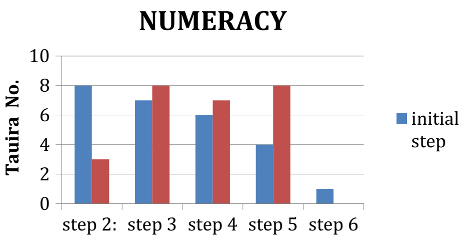

Fig. 1 Student step in numeracy on-line assessment

With reluctant learners, it would be difficult to change mathematical ability significantly over a short period of time, but if attitudes towards mathematics became positive, this in time will probably influence conceptual understanding. In a special issue on affect in mathematics education in *Educational Studies in Mathematics*, the interaction between affect and cognition was common to all the contributions (Zan et al. 2006). Deci and Ryan (2000) reported links between students' positive attitudes and sense of autonomy, and enhanced learning and performance in school. One way that affect and cognition were linked was through the constructs and processes of self-regulation, envisaged as combining features of powerful affect and cognition (Malmivuori 2006). She suggested that features such as "high personal agency with high self-awareness, positive self-appraisals and efficient self-regulation will empower students' mathematics learning and problem / processess [sic] solving" (p. 160).

Other influences both in the learning environment (e.g., relationship with teachers and peer group, the nature of the learning, institutional culture) and personal context (e.g., living situation, personal relationships, maturation) might also have evoked the perceived changes in attitude. The data were relatively cohesive across the three campuses, however, and indicated that student mathematical activity drawing on the apps was influential in the reshaping of the learning experience and in the change in student attitude towards mathematics.

Discussion and Conclusions

The data were consistent that the use of the iPad apps was received very positively by students and had been influential in transforming their attitudes towards numeracy. The initial interviews indicated a high proportion of negative attitudes, while the questionnaire likewise contained a relatively high proportion of responses that echoed those sentiments. The data indicated that student experiences in numeracy had been negative over a sustained period.

Beliefs and attitudes are episodic in their development and emerge through experiences that individuals respond to in varying ways. For these students, both relationships with their schoolteachers and the nature of the curriculum were influential in this disjuncture between perceiving something as important yet eventually not wanting to engage with it. Comments that indicated frustration, disengagement, negativity and, at times, hostility were articulated. To get even a small transition in attitudes would have been significant, but there was a high proportion of attitudinal change across the pre- and post-iPad-use data.

The reasons articulated for this change were primarily because of the reshaping of the learning experience, the repackaging of the content and processes (e.g., Borba and Villareal 2005; Mackrell and Johnston-Wilder 2005; Mariotti et al. 2003). The iPad work was only one aspect of this, along with teacher pedagogical approaches and transformational practice. The students' learning was mediated by the use of the pedagogical device (the iPad) and the language associated with this usage. Most of the students and teachers responded that the iPad component of the programme was instrumental in the transition.

The reasons for this were based on engagement when using the maths apps, but also through the affordances of the digital pedagogical medium. The reasons articulated for this change primarily involved the repackaging of the content and processes. Comments such as the learning being visual, interactive and dynamic were recorded and resonate with other reports of learning experiences engaged through digital media (Carr 2012; Pelton and Pelton 2012). Many found the iPad apps less threatening and easier to learn from. Those apps that teachers played as a class challenge were received very positively.

The inclusion of the game-based apps in their numeracy programme made the learning more engaging and, in much of the data, students appeared to have increased enthusiasm and participation. Similar to Carr's (2012) findings, the students thought of strategies to become more accurate and faster, so as to progress more quickly through the stages. The findings of the study were also consistent with Attard's (2013) and Attard and Curry's (2012) conclusions that the use of apps in mathematics programmes enhanced student engagement. Some tasks which had previously been considered repetitive and boring, such as learning basic numeracy facts, were engaging for students within an apps game context. This needs to be tempered by comments that playing the same game repeatedly at times caused students to lose some motivation to play, and that several students commented that some of the games were too easy or the context too young.

Nevertheless, the vast majority of the data clearly indicated that in terms of the affective dimension of learning, the use of iPad apps in the numeracy programme led to more positive dispositions towards learning, increased engagement and enjoyment of the learning experiences in these areas. In general, this is in contrast with their attitudes towards numeracy prior to being enrolled in a Youth Guarantee programme. While increased engagement and a more positive disposition towards learning generally can transform and enhance cognitive understanding, this does not always manifest simultaneously and has to be considered within a tapestry of inter-related influences.

There is a complex range of integrated elements that influence thinking and understanding, so it is not feasible to attribute change directly to one particular intervention. Similarly, although increased understanding has been identified with learning through digital pedagogical media in various studies (Calder 2011; Li and Pow 2011), some maintain that users need to be fully immersed in working with the technology for a long period of time before it is realistic for a significant influence on cognitive development (Carr 2012).

Within that limitation, several key elements of the cognitive aspect were identified. The data illustrated that the learning is more enhanced when the use of the apps is part of the programme and the content is integrated with the other learning experiences, rather than presented in isolation. The use of apps was effective in initiating learning, but also proved to be productive in the development of conceptual knowledge by enriching the students' engagement, practice and reinforcement of concepts. The use of apps permitted the differentiation of the learning experiences, with activities able to be matched to particular students or groups to suit their identified needs, and through student self-identification and self-selection of apps. Differentiation of the learning allows more targeted facilitation of individual learning trajectories and was likewise reported in other research (e.g., O'Malley et al. 2013).

As well, interview and questionnaire data indicated that both teachers and students perceived that there were some cognitive gains in numeracy. In general, they also felt that the apps made the learning easier and more accessible. Mason's (1998) use of the notion of awareness gives an opportunity for the teacher to oscillate her or his attention between the various contributing influences on the learning, aligning teachers' and students' awareness of both potential affective and cognitive influences. In this way, teacher and student perceptions of the learning experience give insights and are informative.

The student and teacher perceptions of the cognitive influence of the apps were consistent with the student gains made between the initial and final on-line numeracy assessment. Overall, the data indicated that the iPad apps opened new opportunities for learning that enhanced understanding through their propensity to motivate and engage students, to facilitate learning in differing ways, and through the promotion of social interaction. On the other hand, the indiscriminate use of apps without teacher professional learning is most likely going to be ineffective in supporting learning objectives for the students. The research indicated considerable potential for iPad apps to influence the learning experience positively through their inclusion in numeracy programmes, but it also pointed towards some important considerations. The use of apps enhanced learning generally, but this was conditional on the apps selected, the purpose intended and the pedagogical processes used.

The research is relatively cohesive in its assertions regarding the appeal of game-based apps. Students found them engaging and motivational and advocated their inclusion in programmes. Teachers likewise reported perceptions of their influence on students' learning that echoed the students. Perhaps there was an element of novelty and a potential for interest without learning, but generally if students are motivated, more engaged and enjoying an element of learning, they will come to understand more readily. The challenge is to keep the apps as part of a varied programme, to ensure that they are relevant and appropriate for the students and for the development of apps to be on-going and responsive to critical review.

Today's learners are engaged in and generally engrossed by digital media and can use them effectively to communicate, investigate and process ideas and personal questions. However, just allowing these learners access to mobile technology is not sufficient, nor educationally ethical. It has to be resourced equitably and have both the learners and the teachers engaged in 'up-skilling' to enable effective use. While constraining the learning in particular ways, if used appropriately as part of a classroom programme, they add variety and can enhance mathematical thinking and understanding. If the interrelated pedagogical aspects and conceptual thinking are given primacy, then apps can certainly enhance the learning experience and understanding of students.

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