USING PRIVILEGED INFORMATION TO MANIPULATE MARKETS: INSIDERS, GURUS, AND CREDIBILITY*

ROLAND BENABOU AND GUY LAROQUE

Access to private information is shown to generate both the incentives and the ability to manipulate asset markets through strategically distorted announcements. The fact that privileged information is noisy interferes with the public's attempts to learn whether such announcements are honest; it allows opportunistic individuals to manipulate prices repeatedly, without ever being fully found out. This leads us to extend Sobel's [1985] model of strategic communication to the case of noisy private signals. Our results show that when truthfulness is not easily verifiable, restrictions on trading by insiders may be needed to preserve the integrity of information embodied in prices.

I. INTRODUCTION

This paper shows that many types of insiders have both the ability and the incentives to manipulate public information and asset prices through strategically distorted announcements or forecasts. It also examines the extent to which the public's efforts to learn over time whether such an individual is trustworthy or not may limit, in the long run, his influence on the market.

The idea that allowing insiders to trade may lead them to manipulate stock prices can be traced back at least to the 1920s. It is one of the main arguments made at that time against insider trading which Manne [1966] examines. He rejects that argument (as he does all others), on the grounds that manipulators would quickly be discovered and lose all credibility, in addition to facing severe penalties. But his assertion loses much of its validity when it is recognized that private information is typically noisy, so that predictions which turn out to be incorrect can always be ascribed to honest errors. Manipulation will still hurt an insider's reputation, but much more gradually and reversibly, through a process that he can partially control by mixing truth and lies over time to suit his best interests.[1]

The interactions between imperfect private information, strategic communication, and learning are therefore central to the

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1. A related argument of Williamson [1969] is that by distorting information, hence prices, managers can partially elude the disciplinary role of the stock market.

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problem. This leads us to extend Sobel's [1985] model of credibility to the case of noisy private signals. This extension is not special to financial manipulation and could be applied in other areas, such as information-sharing between oligopolists or the credibility of government policies.

We have in mind three kinds of informed agents whose announcements influence prices. First, there is the journalist who writes a financial column, and can trade directly or through namesakes; the Winans-*Wall Street Journal* is an obvious example. Second, there is the "guru" who issues forecasts or newsletters, but is also in the business of trading, for his own account or some investment firm. This case provides the most dramatic illustration of prices reacting to someone's announcements:

> In the nervous market of 1987, Mr. Prechter has emerged as both prophet and deity, an adviser whose advice reaches so many investors that he tends to *pull the market* the way he has *predicted* it will move. . . . Mr. Prechter's words carry such power because he appears to have called the broad outlines of the bull market right since 1982, although he has made some bad calls on short-term moves [*International Herald Tribune,* October 3, 1987; emphasis added].

Finally, probably the most widespread case is that of a corporate executive who owns or trades stock in his company, and by the very nature of his job periodically makes prospective reports to stockholders and financial analysts.

We develop a model where manipulation and learning operate as follows. Rational speculators attach credibility to a manager's, a journalist's, or a guru's announcements for three reasons. First, they know or believe that he has superior information. Second, there is a chance that he is honest and reports it truthfully. Finally, even if he is not honest, he may still reveal genuine information out of concern for his reputation; but of course he will more often mislead the market and reap large profits. Over time, speculators use his track record to reassess the credibility of his predictions. Given the opportunity to accumulate a very large number of observations on the same individual, they will eventually uncover an opportunistic manipulator. Since he periodically lies, his forecasts will be incorrect more often than for an honest type, and his reputation will tend to deteriorate. But if the person making forecasts changes over time, or if his incentives do, learning will not limit market manipulation significantly even in the very long run.

As usually understood, insider trading describes individuals with private information who trade without disclosing it. Indeed, in most of the recent literature (e.g., Grossman and Stiglitz [1981] and Kyle [1985]), information is only disseminated through trading. To focus on market manipulation and credibility, we consider on the contrary insiders who trade without being detected, but influence the price through public announcements or forecasts.[2] In this respect, our paper bears some relationship to that of Admati and Pfleiderer [1986]. They consider an informational monopolist who sells information to market participants but is not allowed to trade. They show that he internalizes and limits the leakage of his signal through prices by charging enough so that relatively few people buy it, and by adding to it an unbiased noise, so as to lower its average accuracy.

This paper departs from the previous literature by recognizing that privately informed individuals can generally gain more by *both* speculating and spreading information, that an insider is often in a position to do so, and that this gives him an incentive to manipulate the market through *biased* messages. Suppose, for instance, that he learns that the return on an asset is likely to be high. He can engage in standard, or "silent" insider trading, and secretly buy large quantities of it. But to the extent that his announcements are believed, he can do much better by forecasting a low return and then buying the valuable asset at a depressed price. Only intrinsic honesty, a sufficient fear of the law, or concern for his reputation will prevent him from manipulating the price through his reports. That these factors are not always strong enough can be illustrated by the following examples.

The first one is that of the Texas Gulf Sulphur Company (see Jaffe [1974]). In late 1963 drillings by company engineers struck huge mineral deposits. Between November 1963 and mid-April 1964, company officials engaged in a large-scale effort to convince the public that the opposite was true, by falsifying evidence (such as drill cores), while accumulating company shares and options. On April 12, 1964, they even issued a press release stating that the technical evidence was inconclusive; four days—and many thousands of shares—later, the company admitted that deposits had in fact been found.

Similar events, but on the "down" side, are alleged to have taken place at the Emerson Radio Corporation, according to a

2. We thus assume, for simplicity, that our informed agent's trades have a negligible effect on the market. In doing so, we indirectly appeal to the results of Kyle [1985, 1989] showing that in an imperfectly competitive market, an insider can limit the leakage of his information into the price by restraining his trades, so as to hide behind noise traders. Similarly, in Laffont and Maskin [1988], he is able to induce a pooling equilibrium.

recent lawsuit (reported by the *New York Times* [November 22, 1989]). Top executives knew by December 1986 that incoming orders were shrinking, inventories were up, and sales of VCRs, a key product for the company, were softening. They nonetheless continued to make optimistic forecasts to shareholders and Wall Street analysts until late May 1987, when the company announced the sales downturn. In the meantime Dean Witter raised its estimate of Emerson earnings, and the stock price rose, while seven of the executives, thirteen of their employees, and various friends and relatives were selling 1.3 million shares.

Again, it is important to note what distinguishes such schemes from usual, or "silent," insider trading: the insiders do not just trade in anticipation of future price movements, but also distort public information and prices. In addition to misleading announcements, this may take the form of withholding accurate information,[3] or even require real resources to falsify evidence.

If the insiders' information was perfect, one could easily tell ex post whether or not they had been truthful. In this case they could lie at most once, and sanctioning fraud would eliminate the problem. This is in fact the main argument used by Manne [1966] to dismiss the idea that letting insiders trade may lead to manipulation. But of course in reality even private information is not fully reliable, so that the possibility of honest mistakes makes it very difficult to establish fraud conclusively. This clearly makes a crucial difference in insiders' ability to deceive the public repeatedly, hence in the profitability of market manipulation. It also implies that more effective ways to prevent manipulation may be to require some types of insiders to disclose their trades promptly, or even to prohibit them from trading certain assets; most likely, trading is easier to verify than truthfulness.

To avoid unnecessary complications, we focus on the issue of credibility and do not explicitly model the adverse efficiency consequences of insider manipulation that would justify such restrictions. Several recent papers have identified efficiency losses from insider trading in the usual framework where the only channel of dissemination of information is through prices. Manove [1989] shows that insider trading can discourage corporate investment by appropriating part of its returns. Ausubel [1990] shows

3. For instance, having both good and bad news but disclosing only one of them.

that if insiders derive benefits from investment by outsiders, this can even make all agents worse off. Fishman and Hagerty [1989] show that insider trading may also lead outsiders to underinvest in the acquisition of information. The harmful effects of the kind of market manipulation we consider arise through a different channel: asset prices reflect strategically biased signals, and clearly this will also distort decisions at the investment stage. For instance, if outsiders know that the price of their firm's shares can be manipulated by someone else later on, they are likely to find investment less attractive. Moreover, managers may be tempted to select projects whose characteristics, such as the variability and predictability of returns, make it easier for them to manipulate the price.

We formalize credibility and manipulation through a model of strategic information transmission [Crawford and Sobel, 1982; Sobel, 1985] in which a "sender" observes the state of nature and then transmits a message to a "receiver," who then chooses an action that determines payoffs. In our asset market game there are many receivers (the public), and their aggregate reaction is materialized in the market-clearing price; it can also be random due to noise traders. But the essential difference, and our technical contribution to this literature, is that we generalize Sobel's [1985] model to a sender with noisy information. This allows him to engage in manipulation repeatedly and makes the receivers' learning problem nontrivial. It also results in a reputation that fluctuates up and down in a realistic manner, rather than increase until the first opportunistic move brings it down to zero forever, as occurs in most games of reputation (e.g., Kreps and Wilson [1982] and Milgrom and Roberts [1982]). In that respect our model presents a similarity with Holmström's [1982] model of repeated moral hazard with learning about ability, and with Cukierman and Meltzer's [1986] model of reputation and ambiguity about a government's changing preferences.

Section II describes the asset market and identifies two types of manipulations. Section III examines information transmission and manipulation in the one-period game. Section IV deals with the infinitely repeated game, where the public attempts to learn the informed individual's type over time. Section V characterizes the long-run behavior of reputation and prices. Section VI presents extensions of the basic model. All proofs are gathered in the Appendix.

II. The Short-Run Model

A. Information

A continuum of agents, indexed by a in $[0,1]$, trade a financial asset whose return is contingent upon the state of nature $\tilde{n}$. The asset pays \$1 if $\tilde{n} = +1$ and zero if $\tilde{n} = -1$. Each of these outcomes has probability one half, and only becomes publicly observable at the end of the period. At the beginning of the period a single agent (say $a = 0$) called the "journalist," or the "sender," privately observes a signal $\tilde{s}$ that predicts the state of nature with probability $p > 1/2$:

$$\text{(1)} \qquad \text{prob}\,[\tilde{n} = n \,|\, \tilde{s} = n] = p > \tfrac{1}{2} \qquad \text{for all } n \text{ in } \{-1,1\}.$$

This information structure is common knowledge.[4] During the trading period (say, at the midpoint) the informed individual can send a message $\tilde{m}$ to "the public," i.e., to agents a in $(0,1]$. The assumption that messages are costless and announced rather than sold is quite appropriate for a financial journalist, who does not control the price of the newspaper hosting his column, or for an executive, whether he makes official reports or spreads rumors. For a guru, who generally sells his information, they are just a simplification. We assume that the informed individual reports his signal truthfully ($\tilde{m} = \tilde{n}$) or untruthfully ($\tilde{m} = -\tilde{n}$) using a *symmetric* mixed strategy:

$$\text{(2)} \qquad q = \text{prob}\,[\tilde{m} = s \,|\, \tilde{s} = s], \qquad \text{for all } s \text{ in } \{-1,1\}.$$

Thus, the message space is $\{-1,+1\}$ itself, and the probability that the report is truthful is independent of the private signal.[5] The public is uncertain about the journalist's "honesty," and has prior probability ρ that he always truthfully reports $\tilde{m} = \tilde{s}$, and $1 - \rho$ that he opportunistically maximizes his expected utility gains from trade, manipulating information to his advantage. Honesty can be interpreted either behaviorally—an honest type is one who strictly adheres to a code of ethics under which he pledges to always tell the

4. Apart from those of Section V, the paper's results actually apply to a "pure guru" as well as to a true insider, i.e., require only that all agents, including agent zero, *believe* with probability one that $\tilde{s}$ is correlated with $\tilde{n}$ as in (1), even if in reality prob$[\tilde{n} = n \,|\, \tilde{s} = n] = 1/2$.

5. The first restriction just simplifies the exposition; the message space could as well be arbitrary. The symmetry assumption is helpful in the infinite horizon game; however, we can show that allowing asymmetric strategies in any finitely repeated version of the (symmetric) game still results in a unique, symmetric equilibrium.

truth—or in terms of payoff uncertainty. A journalist faces penalties if caught lying, but only he knows his probability of escaping discovery. The public is thus uncertain whether the expected penalty is sufficient to deter lying. This situation is formalized in subsection VI.A. Alternatively, the unknown characteristic could be the ability to carry out trades without being detected. In any case, the honest journalist always plays $q = 1$, so we shall reserve the notation q for the strategy of the opportunistic type.[6]

B. Inference

The public comprises both well-informed, rational speculators, a in $(0,\alpha]$, $0 < \alpha < 1$, and traders with noisy information, a in $(\alpha,1]$. Rational agents use their prior on the journalist's type, their knowledge of each type's strategy, and Bayes's rule to infer the *credibility* of his prediction, i.e., the probability $\pi \in [1 - p,p]$ that it will be realized:

$$\pi = \rho p + (1 - \rho)[pq + (1 - p)(1 - q)]. \tag{3}$$

By symmetry π is independent of m: agents' confidence (or lack thereof) in the journalist's forecasts of the asset's value is the same whether these are optimistic or pessimistic. If he announces $\tilde{m} = 1$, they update their belief about the asset's being valuable, from 1/2 to a common *posterior belief* $\beta = \pi$. If he announces $\tilde{m} = -1$, the posterior is $\beta = 1 - \pi$. This makes clear the basic mechanism of market manipulation: the journalist uses his message to affect the public's belief about the asset's value, but his ability to do so is subject to how credible they judge him to be. Formally,

$$\beta \equiv \text{prob}\,[\tilde{n} = 1 | \tilde{m} = m] = \tfrac{1}{2} + m(\pi - \tfrac{1}{2}). \tag{4}$$

There is transmission of information (true or false) unless $\pi = 1/2$, in which case the situation reduces to that of a traditional insider who can trade on his information but not manipulate the market.

"Noise" traders are agents who do not correctly receive or take into account the journalist's message, and act instead according to a common belief $\tilde{\beta}$, drawn from a common knowledge distribution with support in $[1 - p,p]$. As shown on Figure I, all public signals, namely the journalist's message $\tilde{m}$ and the shock $\tilde{\beta}$, arrive in the market at the same time and are immediately incorporated into

6. We implicitly focus on what Sobel [1985] calls "honest equilibria," in which a sender who does not have any incentive to lie is assumed to behave honestly.

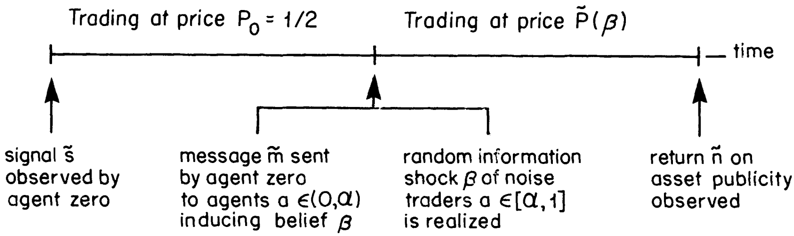

FIGURE I
Timing of Moves and Information

beliefs and prices. The simplest interpretation of $\tilde{\beta}$ is as a "fad," but it could also stand for a liquidity shock.[7]

Noise traders play two roles in this market. First, they ensure that the journalist cannot be identified as the sole agent willing to trade at the prevailing price: for this minor role an arbitrarily small number $1 - \alpha$ of noise traders would be sufficient. Second and more important, they prevent riskless manipulation: the smaller α, the less the journalist can predict how the market will react to his announcements.

C. Prices and Payoffs

We now turn to the determination of the asset's price following the announcement, considering in turn preferences, endowments, and beliefs. Agents trade so as to maximize the expected utility of their wealth at the end of the period, given their information. Each agent a has initial wealth $w_a > 0$ and utility function u_a which is increasing, concave, twice continuously differentiable, and satisfies the Inada conditions on R_+; furthermore, u_a exhibits nonincreasing absolute risk aversion.[8] When no confusion results, we shall omit the subscript zero from the journalist's wealth w_0 and utility u_0.

We shall concentrate on the case of a *purely speculative market,* in which the total supply of the asset, as well as each agent's initial endowment of it, is zero. This is of course restrictive but allows us to make the conflict between the journalist's and the

7. Noise traders are just a convenient way of introducing risk into the asset's price. In general, there could also be a shock before the journalist's announcement. Since we focus on the innovation that interferes with this message, we assume that the first shock has a negligible impact on the market.

8. Also, both marginal utility u'_a and w_a are assumed to be continuous in a. Note that u_a could as well be defined on R, with $u'_a(-\infty) = +\infty$.

public's interests more clear-cut. It also simplifies the model considerably, ensuring in particular that the demand curve for the asset slopes downward.

At the beginning of the period, the journalist has observed his signal and knows what message he will send, hence what beliefs β about the likelihood of the "good" state he will induce in the rational public. Because of noise traders, however, the price that will prevail following his announcement is still a random variable, which we denote by $\tilde{P}(\beta)$, with cumulative distribution M_β.

PROPERTY 1. Given any belief of rational traders about the asset's value, there is a unique (random) price that clears the market. The more optimistic traders are, the higher the price, stochastically. Formally, for any β in $[1-p,p]$, there is a unique market price $\tilde{P}(\beta)$. This price lies in $[1-p,p]$, and if $\beta \geq \beta'$, then $M_\beta \leq M_{\beta'}$.

As noted above, the two signals that the journalist can send induce Bayesian beliefs that are symmetric around 1/2. To preserve the symmetry of the problem in the presence of noise traders, we assume the following.

ASSUMPTION 1 ("SYMMETRIC NOISE"). The price distributions M_β and $M_{1-\beta}$ are symmetric with respect to one another around the uninformative price $P_0 = 1/2$:

$$M_\beta(P) + M_{1-\beta}(1-P) = 1, \text{ for all } \beta \text{ in } [1-p,p] \text{ and } P \text{ in } [0,1].$$

This assumption can easily be related to symmetry in noise traders' belief. For instance, if all agents have logarithmic utility, one can show that $\tilde{P}(\beta) = \alpha\,\beta + (1-\alpha)\,\tilde{\beta}$, while with common CARA preferences $\tilde{P}(\beta) = [1 + ((1-\beta)/\beta)^{\alpha}.((1-\tilde{\beta})/\tilde{\beta}))^{1-\alpha}]^{-1}$. In both cases, if the noise $\tilde{\beta}$ is distributed uniformly on $[1-p,p]$, Assumption 1 holds.

D. The Two Types of Market Manipulation

As shown in Figure II, the market offers a different speculative opportunity in each subperiod for a privately informed agent with some influence on the public's beliefs. The first scheme, from now on called "*pre-announcement speculation,*" consists of trading in anticipation of the announcement's effect; for instance, first buy the asset, and then announce good news ($\tilde{m} = 1$) on its future value to raise the portfolio's value at midperiod. Note that the profitability of this scheme depends solely on the credibility of announce-

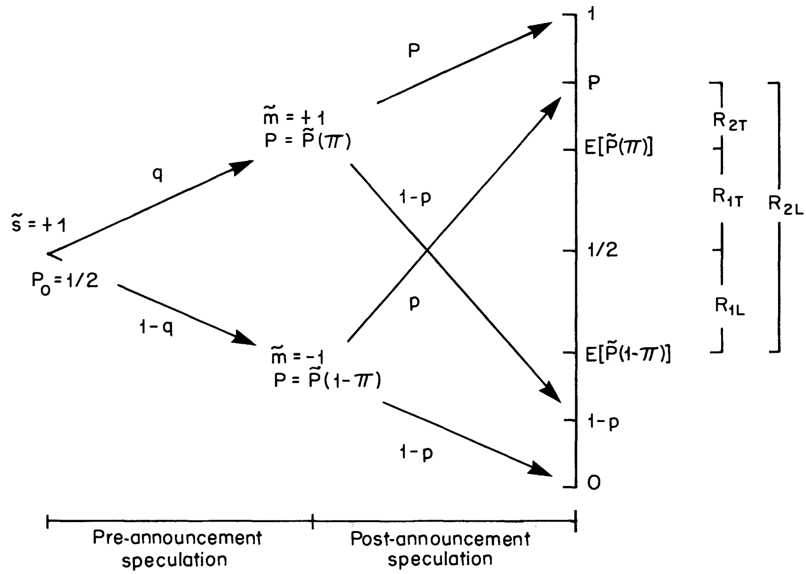

FIGURE II

The Two Market Manipulation Schemes (for $\tilde{s} = +1$)

R_i is agent zero's expected return (as of the beginning of the period) from speculation in subperiod $i = 1,2$, when he decides to report truthfully (R_{iT}) or to lie (R_{iL}).

ments and not on the actual signals that are privately received: $R_{1T} = R_{1L}$ in Figure II. The second scheme, called "*post-announcement speculation,*" consists first of inducing erroneous beliefs by a misleading announcement, and then of trading on the basis of private information. Thus, if $\tilde{s} = 1$, the journalist can depress the price in the short term by announcing $\tilde{m} = -1$ and then buy the very undervalued asset, expecting large profits when it is liquidated. Note that the expected return per share $R_{2L} = R_{1T} + R_{2T} + R_{1L}$ is larger than under the first scheme;[9] also, the "usual" type of insider trading is a particular case where the insider has no credibility ($\pi = 1/2$). Finally, the two schemes can be combined to make money both on the way "up" and on the way "down": say, if the news is bad, $\tilde{s} = -1$, first buy the asset, announce good news, $\tilde{m} = +1$ to induce a high price, and then sell short and wait for the price to collapse.

9. This is also true in terms of rates of return.

The first type of speculation was considered by Hirshleifer [1971] and corresponds to the Winans case; the Texas Gulf Sulphur and Emerson examples belong to the second. But the two types of market manipulation are perhaps best contrasted by the following cases.[10]

Pre-announcement Speculation. In 1814 a group of people secretly bought large numbers of British government securities and then arranged for false reports of the death of Napoleon to reach London. The price rose sharply, and they made large profits by selling out their holdings (Rex vs. de Berenger, quoted by King [1977]).

Post-announcement Speculation. During the battle of Waterloo (1815) the banker Nathan Rothschild, who was known to have superior information from the continent due to a system of carrier pigeons, walked around the city looking dejected, spreading the news that the battle was going badly, and had his agents openly sell British government securities. Meanwhile, he was secretly buying much larger quantities of these securities, taking advantage of the depressed price and of his actual knowledge of an impending victory.

As mentioned above, only the second scheme really makes use of private signals, and affects the integrity of information embodied into prices. It is also much more profitable than the first (riskiness, however, may go either way). For these reasons, but essentially because it simplifies the analysis considerably, we shall mostly focus on the second scheme, by assuming the following.

ASSUMPTION 2. The journalist only trades after his announcement.

This restriction is maintained until subsection VI.B, where we allow both types of speculation. This is where noise traders will become truly important, by making pre-announcement speculation risky; if there are enough of them, all the results derived under Assumption 2 will remain unchanged. Until subsection VI.B, however, the number of noise traders is inessential for the results.

E. The Journalist's Expected Trading Gains

The utility $U(s,\beta)$ which the journalist can expect from his trades in the second subperiod depends only on the signal s he received and on the belief β which he plans to induce through his

10. We are indebted to Mervyn King for providing us with these examples.

message. For instance, if $\tilde{s} = 1$,

$$(5) \quad U(1,\beta) = \int_{1-p}^{p} \max_B \{pu(w + (1 - P)B) + (1 - p)u(w - PB)\} \times dM_\beta(P),$$

and his incentive to manipulate the price is clear-cut. Since the bracketed term decreases in P, the journalist would like to induce the lowest possible price, so as to buy the asset cheap and make large profits at liquidation time. By Property 1, this means that he will send a message that makes traders pessimistic; i.e., that induces a low value of β. Conversely, when $\tilde{s} = -1$, he would like traders to be as optimistic as possible.

PROPERTY 2. The sender is better off, the more the public's belief differs from his private signal. For any s in $\{-1,+1\}$, $s.U(s,\beta)$ is continuous and decreasing in β on $[1 - p,p]$.

The interests of the message sender in this game of strategic information transmission are thus diametrically opposed to those of the receivers, as in Sobel [1985], and in contrast to Crawford and Sobel [1982], where the disagreement concerns only the degree to which both parties want to move in a common direction.[11] This monotonicity of the journalist's payoff plays a crucial role in most of our results, together with the symmetry inherited from noise traders' information (Assumption 1).

PROPERTY 3. The sender's expected utility $U(s,\beta)$ is symmetric in the following sense:

$$U(1,\beta) = U(-1,1 - \beta), \text{ for all } \beta \text{ in } [1 - p,p].$$

This symmetry means, for instance, that a journalist with a positive signal facing a pessimistic public ($\beta < 1/2$) and a journalist with a negative signal facing a correspondingly optimistic public ($\beta' = 1 - \beta > 1/2$) are equally well off. While Property 2 says that the journalist has an incentive to mislead the public, Property 3 implies that it is independent of his own information. As shown in Figure II, his message and trading strategy following a negative signal are just the reverse of those following a positive one, and yield the same expected payoff: $U(1,\pi)$ if the report is truthful and

11. The assumptions that the market is purely speculative and that the journalist does not trade before his announcement are essential for this property. As they are restrictive and not very appropriate in the case of a manager trading his company's shares, we shall relax them in subsection VI.B.

$U(1,1-\pi)$ if not, where π is his credibility, as defined in (3). Our extension of Sobel's [1985] model to noisy private information applies to *any game* where $U(s,\beta)$, the sender's expected utility when he has observed the signal s and sent a message that induces the belief β, satisfies Properties 2 and 3.

III. Information Transmission and the Single-Period Game

Given the signal he observed and the inference process of rational speculators, the opportunistic journalist chooses his message as a Stackelberg leader; the public, however, is conscious of this behavior when assessing the message's credibility and updating the outcome probabilities.

Suppose first that $U(1,\pi) = U(-1,1-\pi) > U(1,1-\pi) = U(-1,1-\pi)$. This implies that whatever signal he observes, the opportunistic type prefers to tell the truth; i.e., $q = 1$. Since the honest type always tells the truth, the credibility of announcements must be $\pi = p$. If the reverse inequality holds, agents know that the opportunistic type always lies, i.e., $q = 0$; therefore $\pi = \rho p + (1-\rho)(1-p)$ necessarily. Finally, if there is equality instead of inequality, the opportunistic type is indifferent and randomizes his messages with a probability $q \in [0,1]$ that is known in equilibrium; equivalently, credibility is some $\pi \in [1-p,p]$ associated with q by (3). We can therefore eliminate the signaling rule and define an equilibrium as a credibility π in $[1-p,p]$ which is consistent, or self-fulfilling, in the following sense:

$$(6) \qquad \pi = \rho p + (1-\rho)\ \text{prob}\,[\tilde{n} = 1 \,|\, U(\tilde{s},\pi) \geq U(\tilde{s},1-\pi)].$$

This is just a rational-expectations-type fixed point requirement on beliefs. It states that the credibility that informed traders attach to announcements is correct in expectation over the unknown journalist's type, given that the opportunistic type chooses his messages so as to induce the beliefs that are most profitable for him. Property 2 implies the following result (as in Sobel [1985], but with $p < 1$).

Proposition 1. The single-period game has a unique equilibrium. If the probability ρ that the sender is honest is no greater than $1/2$, his messages are not credible, and there is no transmission of information: $\pi = 1/2$. If $\rho > 1/2$, his messages are believed to convey information, even though the opportunistic sender always lies: $\pi = \rho p + (1-\rho)(1-p) > 1/2$.

Thus, having privileged information is not enough to manipulate the market. The public must also have sufficient confidence in the sender's honesty; this allows the opportunistic type to "hide" behind the honest one and manipulate public beliefs. The intuition behind these results is quite simple. If the journalist has credibility $\pi > 1/2$, the market will react positively to his messages, pushing the price up when $\tilde{m} = 1$ and down when $\tilde{m} = -1$. If he is opportunistic, he will exploit this behavior by lying systematically ($q = 0$). Because rational agents realize this, messages will be credible only to the extent that the journalist is thought to be honest: $\pi = \rho p + (1 - \rho)(1 - p) = 1/2 + (2p - 1)(\rho - 1/2)$. Thus, $\rho > 1/2$ is necessary for $\pi > 1/2$. As to $\pi < 1/2$, it is impossible. If there is more than a 50 percent chance that the prediction will be wrong, rational traders will just do the opposite of what it prescribes (buy when $\tilde{s} = -1$, sell when $\tilde{s} = 1$), so as to be right more than half the time. But the opportunistic type will then systematically mislead them by always telling the truth ($q = 1$); this in turn means that his messages are in fact very credible ($\pi = p > 1/2$) and should be followed, a contradiction.

In this simple case, the journalist or insider can manipulate the price only if agents believe that there is at least a 50 percent chance that he is honest. Otherwise, they will not even listen to him. We shall see below that multiperiod interaction offers considerably more room for hiding behind honest types to influence the market.

IV. The Infinite Horizon Reputation Game

The most interesting question is indeed that of the transmission of information and the evolution of reputation when the market operates repeatedly. On the one hand, the journalist has many opportunities to manipulate the price; on the other, the public has many opportunities to reassess his credibility and discover whether he is honest or not. This section examines the interplay of these two effects.

At the beginning of the game, $t = 1$, a journalist is selected randomly, to be in charge of the journal in all the future periods, $t = 1, 2, \ldots$ [12] It is known that with probability ρ_1 he is honest and that with probability $1 - \rho_1$ he is opportunistic. When opportunistic, he maximizes the discounted (at rate δ) expected sum of his instantaneous utilities. The market for the one-period asset oper-

12. The stochastic renewal of the journalist is considered in subsection V.B.

ates at every date as described previously. For simplicity, we assume that all traders, including the journalist, start each period with constant wealth endowments w_a and consume their end-of-period wealth.[13] The public therefore behaves as successive generations of identical, one-period traders. The only intertemporal link is the journalist's reputation ρ_t, which is revised at the end of every period in view of whether his forecast was realized or not. We examine the sequential equilibria of the game in stationary, symmetric Markov strategies, where the opportunistic type's reporting rule, hence also the credibility of a sender of unknown type, are functions $q(\rho)$ and $\pi(\rho)$ of the current reputation.[14]

A. Inference Within and Across Periods

If the reputation at date t is ρ_t, then the opportunistic journalist's report is truthful with probability $q(\rho_t)$, and the credibility of any message is, as before:

$$\pi(\rho_t) = \rho_t p + (1 - \rho_t)[pq(\rho_t) + (1 - p)(1 - q(\rho_t))]. \tag{7}$$

A message $m_t \in \{-1,1\}$ then induces posterior belief $\beta_t = 1/2 + m_t(\pi(\rho_t) - 1/2)$ that the asset is valuable. At the end of period t, the true state of nature is observed, the journalist receives $U(s_t,\beta_t)$, and his reputation is updated. From the public's point of view the probability of a correct forecast is $\pi(\rho_t)$ by definition, and the probability that the journalist is honest and makes a correct forecast is $\rho_t p$. For an incorrect forecast the corresponding probabilities are $1 - \pi(\rho_t)$ and $\rho_t(1 - p)$, so the updated reputation is

(8)

$$\rho_{t+1} = \begin{cases} \rho_{t+1}^{+} \equiv \dfrac{p\rho_t}{\pi(\rho_t)} & \text{after a correct forecast } (m_t = n_t) \\[2ex] \rho_{t+1}^{-} \equiv \dfrac{(1 - p)\rho_t}{1 - \pi(\rho_t)} & \text{after an incorrect forecast } (m_t = -n_t). \end{cases}$$

Since $\pi(\rho_t)$ is in $[1 - p,p]$, reputation strictly increases with each

13. Keeping track of the cumulative wealth of each agent, or even of the sole journalist, would needlessly complicate the problem, and obscure the basic incentives through uninteresting effects on risk aversion.

14. Our game therefore belongs to the class of games with one long-run and a sequence of short-run players considered by Fudenberg, Kreps, and Maskin [1987] and by Fudenberg and Levine [1988]. These authors provide general bounds for the limits of equilibrium payoffs as δ goes to one, by focusing on "commitment-type" strategies. We consider a more restrictive class of games, but provide results on the equilibrium itself, for any δ. Note finally that both $\tilde{m} = 1$ and $\tilde{m} = -1$ always have positive probability (when $\rho > 0$) so that a sequential equilibrium here is simply a Bayesian perfect equilibrium.

correct prediction and decreases with each incorrect one, unless the sender is known for sure to be honest or dishonest ($\rho_t = 0$ or $\rho_t = 1$), or to act honestly ($q_t = 1$). When the sender has perfect information ($p = 1$), as in Sobel [1985] and nearly all the literature, his reputation improves monotonically until the first incorrect message, after which $\rho_t = 0$ forever and the game stops. When his signal is noisy ($p < 1$), his reputation evolves more realistically, with both ups and downs but never reaching zero, and the game continues indefinitely. Since ρ_t embodies all information available to the public at the beginning of period t (denoted Ω_t), it follows, from their *subjective* point of view, a martingale:

$$(9) \qquad E[\rho_{t+1}|\Omega_t] = \pi(\rho_t)\rho_{t+1}^{+} + (1 - \pi(\rho_t))\rho_{t+1}^{-} = \rho_t.$$

The *actual,* or objective process of ρ_t along an equilibrium path depends on which type of journalist is actually playing; it will be characterized in Section V.

B. Reputation Equilibrium

The opportunistic journalist must now take into account the effect of his messages on his future reputation, as well as on his short-run profit. Let his reputation be ρ and his signal $\tilde{s} = 1$. If he reports the truth, $\tilde{m} = 1$, the public puts probability $\pi(\rho)$ on the asset's being valuable. Also, with probability p his prediction will be correct, and his reputation in the next period will increase to $p\rho/\pi(\rho)$. With probability $1 - p$ it will be wrong, and his reputation will decrease to $(1 - p)\rho/(1 - \pi(\rho))$.

Thus, if $W(\rho)$ is the discounted expected sum of the opportunistic type's utility at the beginning of a period where his reputation is ρ, he gets, when he reports honestly (T stands for truth);

$$(10) \quad V_T(\rho) = U(1,\pi(\rho)) + \delta p W\left[\frac{p\rho}{\pi(\rho)}\right] + \delta(1 - p)\, W\left[\frac{(1 - p)\rho}{1 - \pi(\rho)}\right].$$

Similarly, if he misrepresents his signal (L stands for lie),

$$(11) \quad V_L(\rho) = U(1,1 - \pi(\rho)) + \delta p W\left[\frac{(1 - p)\rho}{1 - \pi(\rho)}\right] + \delta(1 - p)\, W\left[\frac{p\rho}{\pi(\rho)}\right].$$

The payoffs are the same when $\tilde{s} = -1$, by symmetry. The opportunistic type optimally weighs his profit-related *incentive to lie* $U(1,1 - \pi) - U(1,\pi)$ against his reputational *incentive to be truthful* $\delta(2p - 1)[W(\rho p/\pi) - W((1 - p)\rho/(1 - \pi))]$. If $V_T(\rho) > V_L(\rho)$, he always reports truthfully, $q(\rho) = 1$, so the credibility of messages is $\pi(\rho) = p$. If $V_T(\rho) < V_L(\rho)$, he lies systematically, $q(\rho) = 0$, and credibility is $\pi(\rho) = \rho p + (1 - \rho)(1 - p)$. Finally, when $V_T(\rho) = V_L(\rho)$, he randomizes with probability $q(\rho) \in [0,1]$; equivalently $\pi(\rho)$ takes a value in $[\rho p + (1 - \rho)(1 - p), p]$, from which $q(\rho)$ can be deduced by (7). Therefore,

PROPOSITION 2. An equilibrium of the dynamic game corresponds to a credibility for the sender as a function π: $[0,1] \rightarrow [1 - p, p]$ of his reputation and an associated value function W: $[0,1] \rightarrow R$ such that for all ρ:

(12)
$$\begin{cases} V_T(\rho) > V_L(\rho) & \text{implies that } \pi(\rho) = p, \\ V_T(\rho) = V_L(\rho) & \text{implies that } \rho p + (1 - \rho)(1 - p) \leq \pi(\rho) \leq p, \\ V_T(\rho) < V_L(\rho) & \text{implies that } \pi(\rho) = \rho p + (1 - \rho)(1 - p), \end{cases}$$

(13) $$W(\rho) = \max\,[V_T(\rho), V_L(\rho)],$$

where the functions V_T and V_L are defined from π and W by (10) and (11).

We shall concentrate on equilibria associated with a value function $W(\rho)$ of the opportunistic sender which is continuous and nondecreasing in his reputation; this makes intuitive sense. C_+ will denote the space of such functions on [0,1], endowed with the norm of uniform convergence.

The reader who wishes to skip the construction of the equilibrium can go directly to Theorem 1. We first solve the *short-term game,* or temporary equilibrium, where the valuation W of future reputations is taken as given. Then we impose the restriction that the function W indeed be the expected present value of future utilities.

PROPOSITION 3. The short-term game has a unique equilibrium, and the opportunistic type always lies with positive probability. He is better off, the higher his reputation and his end-of-period valuation. Formally; for any ρ in [0,1] and W in C_+, there exists a unique credibility $\pi^*(\rho, W)$ satisfying (12); π^* is continuous in both arguments, and for all $\rho < 1$ and all W:

$$\tfrac{1}{2} = \pi^*(0,W) \leq \pi^*(\rho,W) < \pi^*(1,W) = p.$$

The functions $V_T^*(\rho,W)$ and $V_L^*(\rho,W)$ on $[0,1] \times C_+$ associated with W and π^* by (10)–(11) are continuous and nondecreasing in both arguments.

To understand why the opportunistic type is never completely truthful ($\pi^*(\rho,W) < p$, so $q(\rho) < 1$), suppose that he were, for some reputation level ρ. His credibility would be maximal ($\pi(\rho) = p$), and the public would attribute any failure of his prediction to the imperfection in his signal. In fact, his reputation for honesty would be totally unaffected by the outcome of his prediction ($\rho_{t+1} = \rho$ in (8)). With no incentive at all to be honest, he would systematically lie—a contradiction.

Now let T be the mapping that associates to any end-of-period valuation W the beginning-of-period valuation resulting from the short-term game, i.e., $T(W)$: $\rho \in [0,1] \rightarrow \max[V_T^*(\rho,W), V_L^*(\rho,W)]$. The continuity and monotonicity properties of Proposition 3 allow us to use Blackwell's [1965] theorem and show that T is a contraction on C_+.

THEOREM 1. There is a unique symmetric Markovian equilibrium of the dynamic game, with a continuous, nondecreasing value function W.[15] In equilibrium the sender's messages always have some credibility: $\pi(\rho) > 1/2$ for all $\rho > 0$.

Thus, every period and reputation level involve the transmission of information, which becomes embodied in the market price. This allows the opportunistic type to take advantage of his private information and of the public's imperfect knowledge of his objectives to *manipulate the market* price through misleading announcements, during an *unbounded* length of time. This is in sharp contrast to the case where $p = 1$ [Sobel, 1985], where only one deception is possible, and also to the one-shot game of Section III, where the public only listens if the reputation is greater than a half.

C. Equilibrium Strategies and Credibility

We now examine how the journalist's strategy and credibility vary with his reputation, in particular for very good or bad reputations, which will play an essential role in the long run. Note from (7) and (8) that, in equilibrium, the revised reputation is much less sensitive to whether or not the prediction is confirmed

15. We could not rule out the existence of equilibria associated with a function W not in C_+, or which are asymmetric. However, a proof by induction using Proposition 3 shows that when the game (with no symmetry restriction on q) is played a finite number K of times, it has a unique equilibrium, which is symmetric and associated with a $W_K \in C_+$. If the infinite game's solution is to be in any sense the limit of the finite game's solution as K tends $+\infty$, W will be the limit of the W_K's and will therefore also belong to C_+.

when ρ is near zero or one than when it is well inside (0,1). The equilibrium incentive to be honest, $W(\rho_{t+1}^{+}) - W(\rho_{t+1}^{-})$, is therefore not monotonic, and in particular is zero at $\rho = 0$ and at $\rho = 1$. Near these extreme points, reputation effects thus become unimportant, and one should expect an outcome similar to that of the static game. Indeed,

PROPOSITION 4. The reporting strategy $q(\rho)$ and credibility $\pi(\rho)$ are continuous, with $q(0) = \pi(0) = 1/2$. Above a certain reputation $\bar{\rho} < 1$, the opportunistic sender always lies.

A journalist with high reputation will thus "milk" it down, in expected terms (when opportunistic). As to a journalist with very low reputation, he makes no significant attempt to rebuild it ($q \approx 1/2$ for $\rho \approx 0$); only a long sequence of correct forecasts could convince the public to listen to him again, and it is more profitable simply to trade on the inside information.

For $\rho > \bar{\rho}$, $\pi(\rho)$ clearly increases in ρ. The dependence of credibility on reputation when the message is randomized, i.e., when $V_T(\rho) = V_L(\rho)$, depends on the shape of the value function. Given π, a higher ρ raises both ρ_{t+1}^{+} and ρ_{t+1}^{-}. The net effect on the incentive for truthfulness, $W(\rho_{t+1}^{+}) - W(\rho_{t+1}^{-})$, has the sign of $\rho_{t+1}^{+}.W(\rho_{t+1}^{+}) - \rho_{t+1}^{-}.W(\rho_{t+1}^{-})$. It is positive, and π increases with ρ, if W has coefficient of relative risk aversion $-\rho.W''(\rho)/W'(\rho) < 1$ on $[(1-p)\rho/(1-\pi(\rho)), \rho p/\pi(\rho)]$. Conversely, if risk aversion is greater than 1, π decreases in ρ.[16] The same indeterminacy holds for $q(\rho)$. The first effect on q of an increase in ρ is to raise π (see (7)), thereby increasing the incentive to lie $U(1,1-\pi) - U(1,\pi)$. Thus, credibility $\pi(\rho)$ may increase while truthfulness $q(\rho)$ decreases. On the other hand, if $\pi(\rho)$ ever decreases in ρ, so does $q(\rho)$ by (7).

Given these effects, the issue of whether intermediate reputations are worth improving by "investing in truth," i.e., whether q rises above 1/2 before decreasing to zero, remains unresolved. Such behavior, however, will definitely occur when the journalist's payoff also depends on an exogenous, i.i.d. random variable (see subsection VI.A): when ρ is not too low but the payoff to dishonesty in the current state is very small, it is worth trying to upgrade the reputation while waiting for better days.

16. In the *two-stage* game with quadratic utility functions (leading to $U(1,\pi) = 1 - 4p + 8p\pi - 4\pi^2$), one does find an interval $[\rho_1, 1/2]$, where $\pi(\rho)$ decreases.

V. Market Manipulation and Learning: Reputation in the Long Run

A. Constant Type

By Theorem 1 the opportunistic type always retains some credibility, allowing him to mislead rational traders indefinitely. In the long run, however, their learning from his track record should place some limit on his ability to manipulate. We examine this question through the stochastic properties and long-run behavior of the journalist's reputation along the equilibrium path of the game. In particular, given that he can always rebuild his reputation by being truthful for a while, will he choose to maintain it around some target level, or run it down completely?

The reputation $\{\rho_t\}_{t\in N}$ is a complex Markov process (see (8)) that depends on the sequence of private signals and public outcomes $\{\tilde{s}_t,\tilde{n}_t\}_{t\in N}$, on the equilibrium signaling strategy q—or the belief function π—and crucially, on the actual type of the journalist. For the public, which does not know his type, the probability that he makes a correct forecast that will improve his reputation from ρ_t to ρ_{t+1}^+ is prob$[\tilde{n}_t = \tilde{m}_t| \Omega_t] = \pi(\rho_t)$, making $\{\rho_t\}$ a martingale with respect to $\{\Omega_t\}$, as in (9). But when the journalist is in fact honest (denoted by H), the true transition probability is prob $[\tilde{n}_t = \tilde{m}_t| \Omega_t,H] = p \geq \pi(\rho_t)$, and

$$(14)\qquad E[\rho_{t+1}|\Omega_t,H] = \rho_t\left[\frac{p^2}{\pi(\rho_t)} + \frac{(1-p)^2}{1-\pi(\rho_t)}\right] \geq \rho_t$$

with strict inequality when ρ_t is different from 0 or 1. In other words, his reputation tends to improve and is in fact a *strict* submartingale. For any initial condition ρ_1, it therefore converges almost surely to some stationary random variable on [0,1], which remains to be identified. Similarly, as the opportunistic journalist (denoted by 0) always lies to some degree ($q_t < 1$, or $\pi_t < p$), his reputation will on average deteriorate: prob$[\tilde{n}_t = \tilde{m}_t|\Omega_t,O] = pq_t + (1-p)(1-q_t) = (\pi(\rho_t)-\rho_t p)/(1-\rho_t) \leq \pi(\rho_t)$. Therefore, in this case ρ_t is really a supermartingale:

$$(15)\qquad E[\rho_{t+1}|\Omega_t,O] \leq \rho_t$$

with strict inequality for $\rho_t \in (0,1)$. We show the following.

THEOREM 2. The sender reveals his type asymptotically: from any initial ρ_1 in (0,1), the equilibrium reputation process $\{\rho_t\}_{t\in N}$

converges almost surely to 0 (respectively, to 1) as t goes to infinity if the sender is opportunistic (respectively, honest).

What enables the public to tell the two types apart (asymptotically) is that the opportunistic one periodically lies, so that his forecasts are inaccurate more often than for the honest type. Reputation is like a capital stock, representing claims to future profits from manipulation (in excess of those from silent insider trading); naturally, the opportunistic type will "spend it down" asymptotically, letting himself eventually be identified in the process.

This process is also reflected in market volatility. As an opportunistic journalist's reputation ρ_t gradually goes to zero, the price $\tilde{P}(\pi(\rho_t))$ converges to $\tilde{P}(1/2)$ by Proposition 4, becoming less and less responsive to his announcements. In the limit, he only engages in traditional, or "silent" insider trading. Conversely, as an honest journalist's reputation strengthens, volatility tends to increase since $\tilde{P}(\pi_t)$ converges to $\tilde{P}(p)$. That truth will ultimately prevail is cause for moderate optimism only, because this learning process may take a long time. In the meantime, when the journalist is dishonest, the price reflects erroneous information, and rational speculators are repeatedly deceived.

B. Changing Types

The preceding result rests on the assumption that the journalist's type is determined once and for all at $t = 1$. In practice, the public's assessment of the credibility of announcements is likely to be made even more difficult by the fact that this type fluctuates over time. For a given journalist or company executive, a stochastically changing type could capture variations in his (privately known) ability to trade without being detected, when he is forbidden to do so. Thus, in states where secret trades are not feasible, his payoff is $U^{H}(s,\beta) \equiv u(w)$, leading him to act honestly. When trades are feasible, it is $U^{O}(s,\beta) = U(s,\beta)$ given by (5). Alternatively, different journalists or executives could be in charge in different periods, with persistence in their types due to the fact that an imperfect monitoring and screening process is being used to detect and replace potentially dishonest agents.

To capture the basic aspects of these more realistic situations, assume now that the journalist's type $\tilde{J}_t \in \{H,O\}$ (honest or opportunistic) evolves according to a Markov process:

(16)

$$\text{prob}\,[\tilde{J}_{t+1} = H|\tilde{J}_t = H] = \phi, \qquad \text{prob}\,[\tilde{J}_{t+1} = O|\tilde{J}_t = O] = \psi,$$

where ϕ and ψ are in (0,1) and $\phi + \psi > 1$, expressing persistence.

The public still updates the journalist's reputation according to (8) at the end of each period, but then uses the transition probabilities (16) to form its beliefs about the new type at the beginning of period $t + 1$:

(17)

$$\rho_{t+1} = \begin{cases} \rho^+_{t+1} = \dfrac{p\rho_t}{\pi(\rho_t)}\,\phi + \left[1 - \dfrac{p\rho_t}{\pi_t}\right](1 - \psi) & \text{if } m_t = n_t \\[2ex] \rho^-_{t+1} = \dfrac{(1-p)\rho_t}{1-\pi(\rho_t)}\,\phi + \left[1 - \dfrac{(1-p)\rho_t}{1-\pi_t}\right](1 - \psi) & \text{if } m_t \neq n_t, \end{cases}$$

where $\pi_t = \pi(\rho_t)$ as before. The important point is that ρ^+_{t+1} (respectively, ρ^-_{t+1}) is increasing in $\rho_t/\pi(\rho_t)$ (respectively, in $\rho_t/(1 - \pi(\rho_t))$), due to the persistence of types, $\phi + \psi > 1$. This leaves the fundamental structure of the game unchanged, and allows the construction of Section IV to be replicated.[17]

THEOREM 3. There is a unique equilibrium of the game with stochastic renewal of the sender's type.[18] This type is never fully known; credibility does not vanish, and information is transmitted even in the long run.

The evolution of the equilibrium over time is described by the joint type-reputation process $\{\tilde{J}_t,\tilde{\rho}_t\}$ governed by (16)–(17). Standard compactness and continuity arguments show that, from any initial conditions, $\{\tilde{J}_t,\tilde{\rho}_t\}$ converges weakly to an invariant distribution; but we do not know whether this limiting distribution is independent of the initial conditions. In any case, renewal prevents the public from ever fully discovering the privately informed agent's current type, because for *any* invariant joint distribution, the marginal distribution over ρ cannot be degenerate, as is clear from (17). Therefore, the opportunities for manipulation, which

17. The dynamic programming problem is slightly different in the case of uncertain survival and in that of uncertain preferences. In both cases ρ^+_{t+1} and ρ^-_{t+1} must be substituted from (17) into $V_L(\rho)$ and $V_T(\rho)$. In the first case, δ is then simply replaced by $\delta\psi$; in the second, value functions $W^H(\rho)$ and $W^0(\rho)$ must be defined, leading to four equations in place of (10)–(11), and to a fixed point on the couple (W^H,W^L).

18. Meaning again: symmetric, Markovian, and with value functions in C_+.

journalists take advantage of when dishonest (in a proportion $(1 - \phi)/(2 - \phi - \psi)$ of periods) *do not disappear* over time but remain stationary. Whereas individual newsletters, executives, or gurus may be short-lived, market manipulation itself may well endure and flourish indefinitely.

VI. Extensions

A. State-Dependent Payoffs

Following Sobel [1985], let the informed sender's payoff in each period depend on some state variable $\tilde{\Theta}_t$, where the $\tilde{\Theta}_t$'s are independently and publicly drawn from a common distribution at the beginning of period t. All our results go through, simply adding $\tilde{\Theta}$ as an argument in strategies, beliefs, utility, and value functions. In particular, if an increase in $\tilde{\Theta}$ increases the incentive to lie, i.e. (see Property 2),

$$(18) \qquad s \cdot \frac{\partial^2 U(s,\beta;\Theta)}{\partial\Theta\partial\beta} < 0 \qquad \text{for all } s,\ \beta,\ \Theta,$$

then the equilibrium strategy $q(\rho;\Theta)$ and credibility $\pi(\rho;\Theta)$ will be non-increasing in Θ. In our asset market, intuition suggests that an increase in the journalist's *influence,* i.e., in the number α of rational agents who pay attention to his message, should have such an effect. Because changes in α also have other complicated effects through the equilibrium price distribution, we can only verify this intuition under restrictive assumptions.

Proposition 5. Assume that agents have logarithmic preferences, and that the number of rational speculators who receive the journalist's message in each period is an i.i.d. process $\{\alpha_t\}_{t\in N}$ in $[0,1/2]$. Then his truthfulness when opportunistic and his credibility are lower, the higher his "influence": $q(\rho;\alpha)$ and $\pi(\rho;\alpha)$ are non-increasing in α in $[0,1/2]$, for all ρ in $(0,1)$.

Similarly, suppose that a journalist who lies risks a *penalty,* with expected value $\tilde{K}$, to be subtracted from $U(s,\beta)$. $\tilde{K}$ reflects the authorities' toughness, but also his ability to hide. An honest journalist is one for which $\tilde{K}^H > U(1,1-p) - U(1,p)$ with probability one. An opportunistic journalist is one for which $\tilde{K}^0$ violates this inequality with positive probability. Only the journalist knows whether the expected penalties $\tilde{K}_t$ which he faces consist of independent draws distributed as $\tilde{K}^H$ or as $\tilde{K}^0$. Using (10) and

(18), it is easy to show that both the opportunistic type's truthfulness $q(\rho; K)$ and the credibility $\pi(\rho; K)$ of a journalist of unknown type increase with the current value of the penalty $\tilde{K}^0$.

In practice, however, it is often difficult to prove that someone lied, as opposed to making a mistake; other means of preserving the information content of prices will therefore be required. One could, for instance, require insiders who frequently make forecasts (managers, journalists, gurus) to promptly disclose all their trades; this would allow the public to check how those fit with the forecast. But because of inevitable delays in disclosures, and because these individuals may also trade for liquidity reasons (or argue that they did), one cannot really expect them to "put their money where their mouth is" at all times. Preventing manipulation might therefore require imposing restrictions on their right to trade certain assets altogether.

B. Combining the Two Types of Market Manipulation

As seen earlier, both pre-announcement and post-announcement speculative schemes are used in actual markets. Moreover, their combination can be particularly profitable, making money going both "up" and "down" (see Figure II). In particular, if a dishonest manager initially holds stock in his company, he will want to alter his position before his announcement. We therefore now lift Assumption 2 and allow for the two types of speculation.

In the first subperiod public trading is based solely on the prior $\beta = 1/2$ and results in an uninformative price $P_0 = 1/2$, with (almost) all traders $a \in [0,1]$ keeping their initial holdings of zero.[19] The market price $\tilde{P}(\beta)$ following the announcement therefore still satisfies Property 1 and Assumption 1.

As to the journalist, he now faces a two-stage decision problem. If B_1 and B_2 denote his asset holdings in the first and second subperiods, his interim wealth will be $w + (\tilde{P} - 1/2)B_1$, and his final wealth $w + (\tilde{P} - 1/2)B_1 + ((\tilde{n} + 1)/2 - \tilde{P})B_2$. To prevent riskless manipulation in the first subperiod, both $\tilde{P} > 1/2$ and $\tilde{P} < 1/2$ must have positive probability; i.e., $1/2$ must lie inside the support of M_β, for all β. This requires that there be enough noise traders, i.e., that α not be too large compared with the dispersion in the distribution of $\tilde{\beta}$. Thus, noise traders, whose role with respect

19. Recall that we restrict the noise in the first subperiod to be arbitrarily small, since we are primarily interested in the interaction of its innovation between the two subperiods with the journalists' message.

to post-announcement trading is mostly incidental, are crucial to *generate riskiness* in pre-announcement trades; we shall come back to this point later.

If the signal is (say) $\tilde{s} = 1$ and the planned message induces beliefs β, the optimal combined speculative scheme consists of pre-announcement trades $B_1^*(\beta)$ and post-announcement trade plans $B_2^*(w + (P - 1/2)B_1^*, P)$ at any interim price P, which maximize the expected payoff:

$$(19)\quad U^*(1,\beta) = \max_{B_1}\left[\int_{1-p}^{p} \max_{B_2}\left\{pu\left(w + \left(P - \frac{1}{2}\right)B_1 + (1 - P)B_2\right)\right.\right.$$
$$\left.\left. + (1 - p)u\left(w + \left(P - \frac{1}{2}\right)B_1 - PB_2\right)\right\} dM_\beta(P)\right].$$

The journalist can now speculate on his sole power to influence the market *irrespectively* of his information, and this complicates matters substantially. Whereas the profitability of post-announcement speculation rests on the extent to which the public was misled (how low $s.(\beta - 1/2)$ is), that of pre-announcement speculation rests on the degree to which their beliefs—hence the price—will be destabilized in *any* direction (how large $|\beta - 1/2|$ will be).[20]

Because it also embodies this second, U-shaped effect, the expected utility $U^*(s,\beta)$ at the time when the message is chosen may fail to satisfy Property 2. Since this monotonicity condition underlies all previous results, we examine under what conditions it still holds in the present case. Let us abbreviate $B_1^*(\beta)$ as B_1^*, and $B_2^*(w + (P - 1/2)B_1^*, P)$ as B_2^{**}. The necessary and sufficient condition for Property 2 to hold is (see Appendix)

$$(20)\quad \int_{1-p}^{p}\left[pu'\left(w + \left(P - \frac{1}{2}\right)B_1^* + (1 - P)B_2^{**}\right)\right.$$
$$\left. + (1 - p)u'\left(w + \left(P - \frac{1}{2}\right)B_1^* - PB_2^{**}\right)\right](B_2^{**} - B_1^*)\frac{dM_\beta}{d\beta}(P)\, dP < 0.$$

By symmetry, this is also the appropriate condition when $\tilde{s} = -1$. Its interpretation is the following: since $dM_\beta/d\beta \leq 0$, (20) holds if the journalist prefers to *hold more,* in an appropriately weighted

20. This difference is clearly apparent in the two examples of subsection II.D. Rothschild had to be quite confident of his information, while the scheme in Rex vs. de Berenger could have worked just as well in reverse, by selling British securities short before spreading rumors of a victory by Napoleon.

average sense, in the *second* subperiod than in the first; i.e., if the solution B_1^* to (19) is not too large.[21] The system (19)–(20) is too complicated to derive an explicit condition on the underlying parameters, but it suggests that monotonicity will obtain if speculation is "*safer*" in the *second* subperiod than in the first, i.e., if the journalist's information on the final value of the asset is reliable (p large enough) while his ability to move the intermediate price is limited, due to noise traders (α small enough).

This intuition can be formalized when utility functions are logarithmic. In this case, the expected utility from the optimal two-stage speculation scheme separates into a pre-announcement and a post-announcement component (see Appendix):

$$(21)\quad U^*(1,\beta) = \int_{1-p}^{p} \log\left(w + \left(P - \frac{1}{2}\right) B_1^*\right) dM_\beta(P) + \int_{1-p}^{p} \left[p \log\left(\frac{p}{P}\right) + (1-p)\log\left(\frac{1-p}{1-P}\right)\right] dM_\beta(P).$$

As expected, the second term decreases with β: the integrand is decreasing in P, while $\tilde{P}(\beta)$ increases stochastically in β. As argued above, the first term in U-shaped: it increases with β for $B_1^* > 0$, which corresponds to $\beta > 1/2$, and decreases with β for $B_1^* < 0$, which corresponds to $\beta < 1/2$.[22] For instance, let $\tilde{\beta}$ be uniformly distributed on $[1-p, p]$, so that $\tilde{P}(\beta) = \alpha\beta + (1-\alpha)\tilde{\beta}$ is uniformly distributed on $[\alpha\beta + (1-\alpha)(1-p), \alpha\beta + (1-\alpha)p]$. Assume also that $\alpha < 1/2$, so that for all β, this interval contains 1/2, making pre-announcement speculation risky. We then show that $B_1^* = 0$ if $\alpha = 0$,[23] and that, for small values of his influence α, the journalist's position in the first subperiod is

$$(22)\quad B_1^* \approx 3\alpha \frac{\beta - 1/2}{(p - 1/2)^2} w.$$

As intuition suggests, pre-announcement speculation $|B_1^*|$ is larger, the higher the ability to destabilize the price (i.e., credibility) and the less noise traders there are.[24] Conversely, for low enough α, (20) will hold.

21. This is always the case when the journalist lies, because $B_1^* < 0 < B_2^{**}$ for $\tilde{m} = -\tilde{s} = -1$; but when $\tilde{m} = \tilde{s} = 1$, both B_1^* and B_2^{**} are positive.

22. As long as noise traders' beliefs are unbiased on average, so that $M_{1/2}$ is symmetric around 1/2.

23. Surprisingly, this property does not hold for arbitrary utility functions.

24. As to the fact that an increase in p (given β) reduces $|B_1^*|$, it results from the specific assumption that the noise has support $[p,1-p]$.

PROPOSITION 6. If agents have logarithmic utilities and if there are enough noise traders in the market, with belief distributed uniformly on $[1-p,p]$, all previous results remain valid in the presence of both types of speculation.

VII. CONCLUSION

The main results of this paper are that many types of individuals with private information (corporate officers, financial journalists, or "gurus") can manipulate public information and asset prices through misleading announcements, and that their ability to do so is limited only in the long run by the public's constant reassessment of their credibility. Moreover, if different agents follow one another in these positions, learning remains incomplete even in the long run, leaving a constant scope for manipulation.

Our model of market manipulation is of course very specific, and does not capture all aspects of the problem. For instance, what happens when there are more than two states of nature, or when the asset is in positive net supply remain open questions. Another one, pertaining to the case of managers who trade their company's stock, is the extent to which our model with repeated trading of a short-term asset is an adequate representation of a market where the same security is traded over several periods, with persistent shocks affecting its value cumulatively. Nonetheless, the model makes clear the link between inside information and market manipulation, and the circumstances that contribute to or limit the latter. It also provides a rationale for requiring insiders to promptly disclose their trades, or even for restricting their right to trade, as a means of preserving the integrity of information communicated to markets and embodied in prices, when truthfulness itself is not easily verifiable and therefore not enforceable.

APPENDIX

Proof of Property 1. Let β_a denote agent a's beliefs over the final value of the asset after receiving the journalist's message. Given a price P in $(0,1)$, the asset demand of agent a is determined by maximizing over B in $(b^-,b^+) \equiv (-w_a/(1-\text{p}), w_a/P)$:

$$\Psi_a(B\colon P,\beta_a) = \beta_a u_a(w_a + (1-P)B) + (1-\beta_a)u_a(w_a - PB). \tag{A.1}$$

With our assumptions on u_a, Ψ_a is continuously differentiable in all its arguments and strictly concave in B. Moreover, for all $(P,\beta_a) \in (0,1)^2$, $\lim_{B\to b^-} \partial\Psi_a(B;\ P,\beta_a)/\partial B = -P(1-\beta_a)u'(-\infty) = -\infty$ and $\lim_{B\to b^+}\partial\Psi_a\ (B;\ P,\beta_a)/\partial B = (1-P)\ \beta_a\ u'(-\infty) = +\infty$, so agent a's demand $B_a(P,\beta_a)$ is uniquely given by

$$\text{(A.2)} \qquad \frac{u'_a(w_a + (1-P)B_a)}{u'_a(w_a - PB_a)} = \frac{1-\beta_a}{\beta_a} \cdot \frac{P}{1-P}.$$

Note that B_a has the sign of $\beta_a - P$. By the implicit function theorem, $B_a(.\,,.)$ is continuously differentiable and $\partial B_a(P,\beta_a)/\partial P$ has the sign of

$$\text{(A.3)} \quad \partial^2\Psi_a(B_a;P,\beta_a)/\partial P\partial B = -\beta_a u'_a(w_a + (1-P)B_a) - (1-\beta_a)u'_a(w_a - PB_a) + B_a\{-\beta_a(1-P)u''_a(w_a + (1-P)B_a) + (1-\beta_a)Pu''_a(w_a - PB_a)\}.$$

The first two terms are negative. To show that the third one is nonpositive, assume first that $P \leq \beta_a$, so $B_a \geq 0$; since u_a has non-increasing risk aversion,

$$-\frac{u''_a(w_a + (1-P)B_a)}{u'_a(w_a + (1-P)B_a)} \leq -\frac{u''_a(w_a - PB_a)}{u'_a(w_a - PB_a)}$$

so that the term multiplying B_a in (A.3) is no greater than

$$\frac{u''_a(w_a - PB_a)}{u'_a(w_a - PB_a)}[(1-\beta_a)Pu'_a(w_a - PB_a) - \beta_a(1-P)u'_a(w_a + (1-P)B_a)] = 0$$

by (A.2). A similar reasoning applies when $P > \beta_a$ and $B_a < 0$, so that $B_a(P,\beta_a)$ always decreases in P. Similarly, $\partial B_a(P,\beta_a)/\partial\beta_a$ has the sign of $\partial^2\Psi_a(B_a;P,\beta_a)/\partial\beta_a\partial B = (1-P)\ u'_a(w_a + (1-P)B_a) + P\,u'_a(w_a - PB_a) > 0$, so

$$\text{(A.4)} \qquad \frac{\partial B_a(P,\beta_a)}{\partial P} < 0; \qquad \frac{\partial B_a(P,\beta_a)}{\partial \beta_a} > 0.$$

Given the beliefs β and $\tilde{\beta}$ of rational and noise traders, and since agent zero's trades are negligible, a price P clears the market if

$$\text{(A.5)} \qquad \xi(P,\beta,\tilde{\beta}) \equiv \int_0^\alpha B_a(P,\beta)\,da + \int_\alpha^1 B_a(P,\tilde{\beta})\,da = 0.$$

By (A.4), $\xi(P,\beta,\tilde{\beta})$ decreases in P. If $P < 1 - p$, then $P < \beta$, and $P < \tilde{\beta}$, hence $B_a(P,\beta) > 0$ for all a in $(0,\alpha]$ and $B_a(P,\tilde{\beta}) > 0$ for all a in $(\alpha,1]$, so that $\xi(P,\beta,\tilde{\beta}) > 0$. Similarly, if $P > p$, $\xi(P,\beta,\tilde{\beta}) < 0$. Therefore, (A.5) has a unique solution $P^*(\beta,\tilde{\beta})$ in $[1 - p,p]$. Finally, by (A.4), $\partial\xi(P,\beta,\tilde{\beta})/\partial\beta > 0$, so that by the implicit function theorem:

$$\text{(A.6)} \qquad \partial P^*(\beta,\tilde{\beta})/\partial\beta > 0 \qquad \text{for all } \beta \text{ and } \tilde{\beta} \text{ in } [1 - p,p].$$

The random variable $\tilde{P}(\beta) = P^*(\beta,\tilde{\beta})$ is therefore increasing (in the sense of first-order stochastic dominance) in β, so that $M_\beta(.)$ is decreasing.

Q.E.D.

Proof of Property 2. Let us define p^s as $p^s = p$ for $s = +1$ and $p^s = 1 - p$ for $s = -1$. If the price is $P = P^*(\beta,\tilde{\beta})$, the journalist's demand is $B^s(P) \equiv B_0(P,p^s)$, given by (A.2) with $\beta_0 = p^s$. Note that $s.B^s(P) > 0$. His expected utility at the beginning of the period is by (5):

$$\text{(A.7)} \qquad U(s,\beta) = \int_{1-p}^{p} \hat{U}^s(P^*(\beta,\tilde{\beta}))\, dL(\tilde{\beta}),$$

where $\hat{U}^s(P) \equiv p^s\, u(w + (1 - P)B^s(P)) + (1 - p^s)\, u(w - PB^s(P))$ for all P in $[1 - p,p]$, and dL denotes the distribution of $\tilde{\beta}$. For all $\tilde{\beta}$, the function $P^*(.,\tilde{\beta})$ is differentiable and increasing (see (A.6)). Moreover, $\hat{U}^s(.)$ is also differentiable, and

$$\text{(A.8)} \qquad \hat{U}^{s\prime}(P) = -[p^s u'(w + (1 - P)B^s(P)) + (1 - p^s)u'(w - PB^s(P))]B^s(P),$$

which has the sign of $-B^s(P)$, i.e., of $-s$, hence Property 2.

Q.E.D.

For the sequel it will be useful to rewrite (A.7) as

$$\text{(A.9)} \qquad U(s,\beta) = \int_{1-p}^{p} [p^s u(w + (1 - P)B^s(P)) + (1 - p^s)u(w - PB^s(P))]\, dM_\beta(P).$$

Proof of Property 3. Equation (A.9) implies that (each maximum is taken over the appropriate domain of definition)

$$\begin{aligned} U(-1,\beta) &= \int_{1-p}^{p} \max_{B} \{pu(w - PB) \\ &\quad + (1-p)u(w + (1-P)B)\}\, dM_{\beta}(P) \\ &= \int_{1-p}^{p} \max_{B} \{pu(w - (1-P')B) \\ &\quad + (1-p)u(w + P'B)\}\, dM_{\beta}(1-P') \\ &= \int_{1-p}^{p} \max_{B'} \{pu(w + (1-P')B') \\ &\quad + (1-p)u(w - P'B')\}\, dM_{1-\beta}(P'), \end{aligned}$$

which, by Assumption 1, is equal to $U(1,1-\beta)$.

Q.E.D.

Proof of Proposition 1. The essence of the argument is given in the text. See Sobel [1985] for details (with slight modifications for $p < 1$).

Proofs for Section IV. For π in $[1-p,p]$, ρ in $[0,1]$ and W in C_+, define

$$\text{(A.10)} \quad V_{0T}(\pi;\rho,W) = U(1,\pi) + \delta p W\left[\frac{p\rho}{\pi}\right] + \delta(1-p)W\left[\frac{(1-p)\rho}{1-\pi}\right]$$

$$\text{(A.11)} \quad V_{0L}(\pi;\rho,W) = U(1,1-\pi) + \delta(1-p)W\left[\frac{p\rho}{\pi}\right] + \delta p W\left[\frac{(1-p)\rho}{1-\pi}\right]$$

$$\text{(A.12)} \quad \begin{aligned} F(\pi;\rho,W) &= V_{0T}(\pi;\rho,W) - V_{0L}(\pi;\rho,W) \\ &= U(1,\pi) - U(1,1-\pi) + \delta(2p-1)\left[W\left[\frac{p\rho}{\pi}\right] - W\left[\frac{(1-p)\rho}{1-\pi}\right]\right]. \end{aligned}$$

The following lemma is the key to the proof of Proposition 3.

LEMMA 1. For any ρ in $[0,1]$ and W in C_+, $F(.;\rho,W)$ is strictly decreasing on $[0,1]$. Moreover, $F(1/2;\rho,W) \geq 0$; $F(p;\rho,W) < 0$. The equation $F(\pi;\rho,W) = 0$ therefore has a unique root $\pi_1(\rho,W)$, which maps $[0,1] \times C_+$ into $[1/2,p]$ continuously. The function $V_1(\rho,W)$, defined on $[0,1] \times C_+$ by $V_1(\rho,W) =$

$V_{0T}(\pi_1(\rho,W);\rho,\ W)$ is nondecreasing in each of its two arguments.

Proof. By Property 2, $U(1,\pi) - U(1,1-\pi)$ is decreasing in π. Since W is in C_+, $W(p\rho/\pi) - W((1-p)\rho/(1-\pi))$ is non-increasing in π. Therefore, $F(\pi;\rho,W)$ is decreasing in π; it is also clearly continuous in all its arguments.

Since $p > 1/2$ and $W \in C_+$, $F(1/2;\rho,W) \geq 0$. From Property 2, $U(1,p) - U(1,1-p) < 0$, and consequently $F(p;\rho,W) < 0$. The equation $F(\pi;\rho,W) = 0$ therefore has a unique root $\pi_1(\rho,W)$ in $[1/2,p]$. Given this uniqueness, the continuity of $\pi_1(.\,,.)$ is a straightforward consequence of the continuity of F.

Finally, since $F(\pi_1(\rho,W);\rho,W) = 0$, $V_1(\rho,W) = V_{0T}(\pi_1(\rho,W);\rho,\ W)$ is also equal to $V_{0L}(\pi_1(\rho,W);\rho,\ W)$. Multiplying one of these expressions by p, the other by $1-p$, and subtracting therefore leads to two identities for $V_1(\rho,W)$:

$$\text{(A.13)}\quad V_1(\rho,W) = \frac{pU(1,\pi_1) - (1-p)U(1,1-\pi_1)}{2p-1} + \delta W\left[\frac{p\rho}{\pi_1}\right]$$

(A.14)

$$V_1(\rho,W) = \frac{pU(1,1-\pi_1) - (1-p)U(1,\pi_1)}{2p-1} + \delta W\left[\frac{(1-p)\rho}{1-\pi_1}\right],$$

where π_1 stands for $\pi_1(\rho,W)$. Consider now $(\rho^1,W^1) \geq (\rho^2,W^2)$, and let $\pi_1^i = \pi_1(\rho^i,W^i)$, for $i = 1,2$. If $\pi_1^1 \geq \pi_1^2$, by (A.14), Property 2 and the fact that W is nondecreasing, $V_1(\rho^1,W^1) \geq V_1(\rho^2,W^2)$. If $\pi_1^1 < \pi_1^2$, by (A.13) and the same argument, $V_1(\rho^1,W^1) \geq V_1(\rho^2,W^2)$.

Q.E.D.

Proof of Proposition 3. We first show that the unique solution to (12) is $\pi^*(\rho,W) = \max\{\rho p + (1-\rho)(1-p),\ \pi_1(\rho,W)\}$. Since $\pi_1(0,W) = 1/2$ and $\pi_1(1,W) < p$ by (A.12), this will prove the claimed inequalities. Let $I(\rho) \equiv [\rho p + (1-\rho)(1-p),p]$. When $\pi_1(\rho,W) \in I(\rho)$, Lemma 1 implies that $F(.;\rho,W)$ decreases on $I(\rho)$, from $F(\rho p + (1-\rho)(1-p);\rho,\ W) \geq 0$ to $F(p;\rho,W) < 0$, so the only solution to (12) occurs for $F(\pi;\rho,W) = 0$, i.e., $\pi^*(\rho,W) = \pi_1(\rho,W)$. When $\pi_1(\rho,W) < \rho p + (1-\rho)(1-p)$, $F(\pi;\rho,W) < 0$ on $I(\rho)$, so the only solution is $\rho p + (1-\rho)(1-p)$.

The continuity of $\pi^*(\rho,W)$ then follows from that of $\pi_1(\rho,W)$; in turn, it implies that $V_T^*(\rho,W) \equiv V_{0T}(\pi^*(\rho,W);\rho,\ W)$, $V_L^*(\rho,W) \equiv V_{0L}(\pi^*(\rho,W);\rho,W)$, and $T(\rho;W) \equiv \max[V_T^*(\rho,W),\ V_L^*(\rho,W)]$ are all continuous in (ρ,W). In the case where $\pi^*(\rho,W) = \pi_1(\rho,W) \geq \rho p +$

$(1-\rho)(1-p)$, we have $T(\rho,W) = V_1(\rho,W)$ with $V_1(.\,,.)$ defined in Lemma 1. In the case where $\pi^*(\rho,W) = \rho p + (1-\rho)(1-p)$, we have $F(\pi^*;\rho,W) \leq 0$ or $V_T^*(\rho,W) \leq V_L^*(\rho,W) = T(\rho,W)$. Thus, $T(\rho;W) = V_2(\rho,W)$, where we define $V_2(\rho,W) = V_{0L}(\rho p + (1-\rho)(1-p);\rho,W)$; i.e.,

$$\text{(A.15)}\quad V_2(\rho,W) \equiv U(1,1-\rho p-(1-\rho)(1-p)) + \delta p W\left[\frac{(1-p)\rho}{1-\rho p-(1-\rho)(1-p)}\right] + \delta(1-p)W\left[\frac{p\rho}{\rho p+(1-\rho)(1-p)}\right].$$

Note that $V_2(\rho,W)$ is nondecreasing in (ρ,W), due to Property 2, the fact that $W \in C_+$ and because its arguments in the last two terms are increasing functions of ρ. Summarizing, $T(\rho,W)$ is continuous and satisfies

$$T(\rho;W) = V_2(\rho,W).1_{\{\pi^*(\rho,W)\leq \rho p+(1-\rho)(1-p)\}} + V_1(\rho,W).1_{\{\pi^*(\rho,W)> \rho p+(1-\rho)(1-p)\}},$$

where $1_{\{.\}}$ denotes the indicator function and V_1 and V_2 are both nondecreasing in their arguments. To prove that T is nondecreasing in (ρ,W), consider $(\rho^1,W^1) \geq (\rho^2,W^2)$. If $\pi^{*1} = \pi_1^1 > \rho^1 p + (1-\rho^1)(1-p)$ and $\pi^{*2} = \pi_1^2 > \rho^2 p + (1-\rho^2)(1-p)$, then

$$T(\rho^1,W^1) = V_1(\rho^1,W^1) \geq V_1(\rho^2,W^2) = T(\rho^2,W^2).$$

If $\pi^{*1} = \rho^1 p + (1-\rho^1)(1-p)$ and $\pi^{*2} = \rho^2 p + (1-\rho^2)(1-p)$, the same argument holds, using V_2 instead of V_1. If $\pi^{*1} = \pi_1^1 \geq \rho^1 p + (1-\rho^1)(1-p)$ and $\pi^{*2} = \rho^2 p + (1-\rho^2)(1-p) \leq \pi_1^2$, by continuity there exists (ρ^3,W^3), with $(\rho^1,W^1) \geq (\rho^3,W^3) \geq (\rho^2,W^2)$ and $\pi^{*3} = \pi_1^3 = \rho^3 p + (1-\rho^3)(1-p)$. Now

$$T(\rho^1,W^1) = V_1(\rho^1,W^1) \geq V_1(\rho^3,W^3) = V_2(\rho^3,W^3) \geq V_2(\rho^2,W^2) = T(\rho^2,W^2).$$

The case $\pi^{*1} = \rho^1 p + (1-\rho^1)(1-p)$ and $\pi^{*2} = \pi_1^2$ is treated similarly.

Q.E.D.

Proof of Theorem 1. By Proposition 3, T maps C_+ continuously into itself, and is nondecreasing in W. Furthermore, by construction, $T(W+c) = T(W) + \delta c$, for any constant c. By

Blackwell's theorem, T is a contraction mapping, and since C_+ with the sup norm is complete, it has a unique fixed point W. Since $\pi(\rho) = \pi^*(\rho,W(\rho)) = \max\{\rho p + (1-\rho)(1-p), \pi_1(\rho,W)\}$, where $W \in C_+$ is the equilibrium value function, π is continuous in ρ by Proposition 3. The argument W in π_1 will now be omitted to lighten the notation.

We now show that $\pi(\rho) > 1/2$ for all $\rho > 0$. For $\rho > 1/2$ this is clear, since $\rho p + (1-\rho)(1-p) > 1/2$. For $\rho < 1/2$, $\pi(\rho) = \pi_1(\rho,W)$ so $\pi(\rho) = 1/2$ implies, by (A.12), that $W(2(1-p)\rho) = W(2p\rho)$; i.e., that W is constant on $[2(1-p)\rho,2p\rho]$. Expression (A.11) in turn requires that this constant be $U(1,1/2)/(1-\delta)$. But this value is just $W(0)$, since $\pi(0) = 1/2$, so W must be constant on all of $[0,2p\rho]$. To prove that $\pi(\rho) = 1/2$ for $\rho \leq 1/2$, it therefore suffices to show that W is strictly increasing in a neighborhood of $\rho = 0$.

Suppose not, so that there exists $\underline{\rho} > 0$ with $W(\rho) = W(0)$ on $[0,\underline{\rho}]$ and $W(\rho) > W(0)$ on $(\underline{\rho},1]$. If $\underline{\rho} > 1/2$, then $\pi(\underline{\rho}) > 1/2$, and by (A.11) $V_{0L}(\pi(\underline{\rho});\underline{\rho},W) > U(1,1/2)/(1-\delta) = W(0)$; but since $q(\rho) < 1$ for all ρ, $W(\underline{\rho}) = V_{0L}(\pi(\underline{\rho});\underline{\rho},W)$, hence a contradiction. If $\underline{\rho} \leq 1/2$, then $\pi(\underline{\rho}) = \pi_1(\underline{\rho},W)$, and $W(\underline{\rho}) = V_1(\underline{\rho},W)$. But $W(\underline{\rho}(1-p)/(1-\pi_1(\underline{\rho},W)) = W(0) = W(\underline{\rho})$ by definition of $\underline{\rho}$, so from (A.14) evaluated at $\rho = \underline{\rho}$ and the monotonicity of $U(1,\pi)$, we obtain $\pi_1(\underline{\rho},W) = 1/2$. But then (A.13) implies that $W(2p\underline{\rho}) = W(0)$, again a contradiction.

Q.E.D.

Proof of Proposition 4. The continuity of π was proved above. By (10), $\pi(1) = p$, so (A.12) yields $F(\pi(1);1,W) = U(1,p) - U(1,1-p) < 0$. By continuity of F, π, and W, this also holds on some interval $(\bar{\rho},1]$. Thus, by definition, $\pi(\rho) = \rho p + (1-\rho)(1-p)$, and $q(\rho) = 0$ on $(\bar{\rho},1]$. On $[0,\bar{\rho})$ (in fact on $[0,1)$), (7) implies that $(2p-1)q(\rho) = (\pi(\rho) - p\rho)/(1-\rho) - (1-p)$, so that q is continuous on all of $[0,1]$. Finally, $F(1/2;0,W) = 0$, so $\pi(0) = \pi^*(0,W) = 1/2$, and the above formula yields $q(0) = 1/2$.

Q.E.D.

Proof of Theorem 2. For ρ in $[0,1]$, let $f(\rho) \equiv E(\rho_{t+1} | \rho_t = \rho, H)$. By (14),

(A.16) $\quad f(\rho) \geq \rho$, with strict inequality for $\rho \in (0,1)$.

When $\tilde{J} = H$, $\{1-\rho_t\}_{t\in N}$ is a positive supermartingale, and therefore [Neveu, 1975, p. 27] converges almost surely to a positive random variable $1 - \rho_\infty$. Thus, ρ_t, with distribution $d\mu_t$, converges almost surely to ρ_∞, with distribution $d\mu_\infty$. For all t,

$$E(\rho_{t+1}|H) = \int_0^1 E(\rho_{t+1}|H)\, d\mu_t(\rho) = \int_0^1 f(\rho)\, d\mu_t(\rho).$$

Taking limits as t tends to $+\infty$: $E(\rho_\infty) = \int_0^1 f(\rho)\, d\mu_\infty(\rho)$. Condition (A.16) then requires that $\rho_\infty = 0$ or $\rho_\infty = 1$ almost surely; otherwise, one would have $E(\rho_\infty) > \int_0^1 \rho d\mu_\infty(\rho) = E(\rho_\infty)$. Thus, $d\mu_\infty$ has all its mass concentrated on $\{0,1\}$.

We now show that there can be no positive mass on 0. Consider the process $\{y_t = 1/\rho_t\}_{t\in N}$. By (8) it is clearly a martingale, and therefore also a positive supermartingale. By the maximal inequality [Neveu, 1975, p. 23], $\sup_{t\in N}(y_t)$ is almost surely finite. Therefore, $\inf_{t\in N}(\rho_t) = (\sup_{t\in N}(y_t))^{-1}$ is almost surely strictly positive, hence the result when $\tilde{J} = H$. A similar argument holds when $\tilde{J} = 0$, because $\{\rho_t\}_{t\in N}$ is then a strict supermartingale and $\{z_t = 1/(1 - \rho_t\}_{t\in N}$ a martingale.

Q.E.D.

Proof of Proposition 5. We show that (18) holds for $\alpha \leq 1/2$. Using symmetry, let $s = 1$. From (A.7), omitting the superscript "+",

$$\text{(A.17)} \quad \frac{\partial^2 U(1,\beta)}{\partial\alpha\partial\beta} = \int_{1-p}^{p} \left[\hat{U}' \cdot \frac{\partial^2 P^*}{\partial\alpha\partial\beta} + \hat{U}'' \cdot \frac{\partial P^*}{\partial\alpha} \cdot \frac{\partial P^*}{\partial\beta}\right] dL(\tilde{\beta}).$$

A sufficient condition for α to satisfy (18) is therefore

$$\text{(A.18)} \quad \hat{U}'(P^*) \cdot \frac{\partial^2 P^*}{\partial\alpha\partial\beta} + \hat{U}''(P^*) \cdot \frac{\partial P^*}{\partial\alpha} \cdot \frac{\partial P^*}{\partial\beta} < 0, \qquad \text{for all } \beta,\tilde{\beta}.$$

With logarithmic utility the solution to (A.2) is $B_a(w_a,P,\beta_a) = w_a\cdot(\beta_a - P)/(P\ (1 - P))$, and (A.5) leads to $P^*(\beta,\tilde{\beta};\alpha) = \alpha\beta + (1 - \alpha)\tilde{\beta}$. Moreover, $B(P) = (p - P)/(P(1 - P))$, and $\hat{U}(P) = u(w) + p\log(p/P) + (1 - p)\log((1 - p)/(1 - P))$, so (A.18) becomes

$$\text{(A.19)} \qquad \alpha(\beta - \tilde{\beta})\left[\frac{p}{P^{*2}} + \frac{1-p}{(1 - P^*)^2}\right] < \frac{p - P^*}{P^*(1 - P^*)},$$

which always holds for $\beta \leq \tilde{\beta}$. Now let $\beta > \tilde{\beta}$, and rewrite (A.19) as

$$\alpha(\beta - \tilde{\beta})[P^{*2} + p(1 - 2P^*)] < pP^*(1 - P^*) - P^{*2}(1 - P^*)$$

or

$$p[P^* - \alpha(\beta - \tilde{\beta}) - P^*(P^* - 2\alpha(\beta - \tilde{\beta}))] > P^{*2}(1 - \tilde{\beta}),$$

i.e.,

$$p[\tilde{\beta}(1 - \tilde{\beta}) + \alpha^2(\beta - \tilde{\beta})^2] > (1 - \tilde{\beta})[\tilde{\beta}^2 + 2\alpha\tilde{\beta}(\beta - \tilde{\beta}) + \alpha^2(\beta - \tilde{\beta})^2]$$

or

$$\alpha^2[(\beta - \tilde{\beta})^2(p - 1 + \tilde{\beta})] - 2\alpha\tilde{\beta}(1 - \tilde{\beta})(\beta - \tilde{\beta}) + (p - \tilde{\beta})\tilde{\beta}(1 - \tilde{\beta}) > 0.$$

This second-degree polynomial in α has two positive roots, since $\beta - \tilde{\beta} > 0$. The smallest one is

$$\text{(A.20)} \qquad \alpha_- = \frac{\tilde{\beta}(1 - \tilde{\beta}) - \sqrt{\tilde{\beta}(1 - \tilde{\beta})p(1 - p)}}{(\beta - \tilde{\beta})(p - (1 - \tilde{\beta}))},$$

which is clearly a decreasing function of β. Therefore,

$$\text{(A.21)} \qquad \alpha_- > \frac{\tilde{\beta}(1 - \tilde{\beta}) - \sqrt{\tilde{\beta}(1 - \tilde{\beta})p(1 - p)}}{(p - \tilde{\beta})(p - (1 - \tilde{\beta}))} = \frac{z - \sqrt{yz}}{z - y},$$

where $z \equiv \tilde{\beta}(1 - \tilde{\beta}) > p(1 - p) \equiv y$. But $(z - \sqrt{yz})/(z - y) > 1/2$ for all $z > y$, so that $\alpha < 1/2$ implies (A.21), hence (A.19) and (18).

Finally, (18) implies that $U(1,\pi;\alpha) - U(1,1 - \pi;\alpha)$ decreases in α, for all $\pi > 1/2$. By (A.12), this implies that $\partial\pi_1(\rho,\tilde{W};\alpha)/\partial\alpha < 0$ for all $(\rho,\tilde{W})$ in $[0,1] \times C_+$; the same thus holds with weak inequality for $\pi^*(\rho,\tilde{W};\alpha) = \max\{\rho p + (1 - \rho)(1 - p), \pi_1(\rho,\tilde{W};\alpha)\}$. In equilibrium, $\pi(\rho;\alpha) = \pi^*(\rho,W(\rho);\alpha)$, and the equilibrium value function W is independent of the current state α of α_t (the α_t's being i.i.d.); therefore $\pi(\rho;\alpha)$ is non-increasing in α. Finally, $(2p - 1)q(\rho;\alpha) = \max[(\pi(\rho;\alpha) - p\rho)/(1 - \rho) - (1 - p), 0]$, so $\partial q(\rho;\alpha)/\partial\alpha \leq 0$, with strict inequality unless $q(\rho;\alpha) = 0$.

Q.E.D.

Proof of (20). Define, for all $(B_1,B_2;P)$ in $R^2 \times [1 - p,p]$, the function,

$$\Phi(B_1,B_2;P) = pu(w + (P - 1/2)B_1 + (1 - P)B_2) + (1 - p)u(w + (P - 1/2)B_1 - PB_2).$$

Its partial derivatives will be denoted as Φ_i or Φ_{ij}, $i,j = 1,2,3$. It is easily checked that Φ is strictly concave in (B_1,B_2). Then,

$$\text{(A.22)}\quad U^*(1,\beta) = \max_{B_1} \Gamma(B_1)$$
$$\equiv \max_{B_1} \int_{1-p}^{p} \Phi(B_1, B_2^*(w + (P - 1/2)B_1, P); P)\, dM_\beta(P),$$

where $B_2^* = B_2^*(w + (P - 1/2)B_1,P)$ is, by (A.2), the solution to

$$\text{(A.23)}\qquad \Phi_2(B_1,B_2^*;P) = 0, \text{ for all } B_1 \text{ and } P.$$

We first show that Γ is concave in B_1. Indeed, using (A.23),

$$\text{(A.24)}\quad \Gamma''(B_1) = \int_{1-p}^{p} (\Phi_{11} - \Phi_{12}\Phi_{21}/\Phi_{22})(B_1,B_2^*;P)\, dM_\beta(P) < 0$$

by concavity of Φ. The first-period problem (A.22) therefore has a unique solution $B_1^* = B_1^*(\beta)$. Provided that 1/2 is in the interior of the distribution M_β's support, B_1^* is finite and is the solution to

$$\text{(A.25)}\qquad \int_{1-p}^{p} \Phi_1(B_1^*,B_2^{**};P)\, dM_\beta(P) = 0,$$

where the function $B_2^*(w + (P - 1/2)\, B_1^*,P)$ is abbreviated as B_2^{**}. Moreover,

$$\text{(A.26)}\qquad U^*(1,\beta) = \int_{1-p}^{p} \Phi(B_1^*,B_2^{**};P)\, dM_\beta(P).$$

Integrating by parts and using (A.23) yields

$$\text{(A.27)}\quad U^*(1,\beta) = \Phi(B_1^*,0;p) - \int_{1-p}^{p} \Phi_3(B_1^*,B_2^{**};P)M_\beta(P)\, dP.$$

By Property 1, $M_\beta(P)$ is (for all P) monotonic in β, hence almost everywhere differentiable (a.e.d.) in β, with a derivative $dM_\beta(P)/d\beta \leq 0$. By (A.27), $U^*(1,\beta)$ is also a.e.d. in β, and, using $\Phi_{23} = 0$,

$$\text{(A.28)}\quad \frac{\partial U^*(1,\beta)}{\partial\beta} = -\int_{1-p}^{p} \Phi_3(B_1^*,B_2^{**};P)\left(\frac{dM_\beta(P)}{d\beta}\right) dP$$
$$+ B_1^*(\beta)[\Phi_1(B_1^*,0;p) - \int_{1-p}^{p} \Phi_{13}(B_1^*,B_2^{**};P)M_\beta(P)\, dP].$$

But integrating (A.25) by parts, using $\Phi_{12} = 0$, yields

$$\text{(A.29)}\qquad \Phi_1(B_1^*,0;p) = \int_{1-p}^{p} \Phi_{13}(B_1^*,B_2^{**};P)M_\beta(P)\, dP,$$

so that the last two terms in (A.28) cancel out, leaving only (20). At points β, where $M_\beta(P)$—hence also $U^*(1,\beta)$—is not differentiable, the same computations apply, but with $\partial U^*(1,\beta)/\partial\beta$ and $dM_\beta(P)/$

$d\beta$ representing generalized functions. Thus, in all cases, if (20) holds (in the appropriate sense), $\partial U^*(1,\beta)/\partial\beta < 0$ (in the appropriate sense); i.e., $U^*(1,\beta)$ decreases in β.

Q.E.D.

Proof of (21). As seen below (A.18), $\tilde{P}(\beta) = P^*(\beta,\tilde{\beta};\alpha) = \alpha\beta + (1-\alpha)\tilde{\beta}$, and $B_2^* = B_0(w_0 + (P - 1/2)B_1,P) = (w_0 + (P - 1/2)B_1)$. $(p - P)/[P(1 - P)]$ for all B_1 and P. Replacing in (A.26) with $B_1 = B_1^*$ yields (21).

Q.E.D.

Proof of Proposition 6. Let $[\underline{P}(\alpha,\beta),\overline{P}(\alpha,\beta)] \equiv [\alpha\beta + (1-\alpha) \times (1-p),\ \alpha\beta + (1-\alpha)p]$. With the assumptions made, (A.25) takes the form,

$$\text{(A.30)} \qquad \int_{\underline{P}(\alpha,\beta)}^{\overline{P}(\alpha,\beta)} \frac{P - 1/2}{w + (P - 1/2)B_1^*}\, dP = 0.$$

When $\alpha = 0$, $\underline{P}(\alpha,\beta) = 1 - p$, and $\bar{P}(\alpha,\beta) = p$ (for all β) are symmetric around 1/2, so the only solution is $B_1^* = 0$, and (20) is satisfied. By continuity, it still holds for small values of α, hence Proposition 6; a Taylor expansion of (A.30) yields (22).

Q.E.D.

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