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Using recurrence plot analysis to distinguish between endogenous and exogenous stock market crashes

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ABSTRACT

Recurrence Plots are graphical tools based on Phase Space Reconstruction. Recurrence Quantification Analysis (RQA) is a statistical quantification of RPs. RP and RQA are good at working with non-stationarity and noisy data, in detecting changes in data behavior, in particular in detecting breaks, like a phase transition and in informing about other dynamic properties of a time series. Endogenous Stock Market Crashes have been modeled as phase changes in recent times. Motivated by this, we have used RP and RQA techniques for detecting critical regimes preceding an endogenous crash seen as a phase transition and hence give an estimation of the initial bubble time. We have used a new method for computing RQA measures with confidence intervals. We have also used the techniques on a known exogenous crash to see if the RP reveals a different story or not. The analysis is made on Nifty, Hong Kong AOI and Dow Jones Industrial Average, taken over a time span of about 3 years for the endogenous crashes. Then the RPs of all time series have been observed, compared and discussed. All the time series have been first transformed into the classical momentum divided by the maximum Xmax of the time series over the time window which is considered in the specific analysis. RPs have been plotted for each time series, and RQA variables have been computed on different epochs. Our studies reveal that, in the case of an endogenous crash, we have been able to identify the bubble, while in the case of exogenous crashes the plots do not show any such pattern, thus helping us in identifying such crashes.

1. Introduction

Stock market investment lies at the core of any modern market economy. The graphs charting movement of stock market indices are synonymous with the ECG of the economic heart of even a country like India. The fear of every investor is a sudden and steep drop of asset prices; the occurrence of a stock market crash. Crashes are rare but can happen even in mature markets. The study of critical phenomena like financial crashes has been the focus of much recent research work. One area of work studies the Stock Market Crashes considering stock price movement following a complex power law series only in case of endogenous crashes. Motivated by this, we then use a technique evolved from nonlinear time series analysis in the study of deterministic chaos to find out whether we can distinguish between endogenous and exogenous crashes.

There has been considerable research work towards modeling financial crashes, most suggesting that, close to a crash, the market behaves like a thermodynamic system which undergoes phase transition. Some propose a picture of stock market

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crashes as critical points in a system with discrete scale invariance. The critical exponent is then complex, leading to log-periodic fluctuations in stock market indices. This picture is in the spirit of the known earthquake-stock market analogy and of recent work on log-periodic fluctuations associated with earthquakes [1,2]. Some has shown that stock market crashes are caused by the slow build up of long-range correlations between traders, leading to a collapse of the stock market in one critical instant. A crash is interpreted as a critical point [3,4].

Mostly, these recent works have shown an analogy between crashes and phase transition [3–7]; as in earthquakes, log periodic oscillations have been found before some crashes [8,9], and then it was proposed that an economic index $y(t)$ increases as a complex power law, whose representation is

$$y(t) = A + B\text{In}(t_c - t)1 + C\text{Cos}[\omega \ln(t_c - t) + \phi] \tag{1.1}$$

where A, B, C, ω, ϕ are constants and t_c is the critical time (rupture time).

An endogenous crash is preceded by an unstable phase where any information is amplified; this critical period is called the speculative bubble.

Recurrence Plots are graphical tools elaborated by Eckmann, Kamphorst and Ruelle in 1987 and are based on Phase Space Reconstruction [10]. In 1992, Zbilut and Webber [11] proposed a statistical quantification of RPs and gave it the name of Recurrence Quantification Analysis (RQA). RP and RQA are good at working with non-stationarity and noisy data, in detecting changes in data behavior, in particular in detecting breaks, like a phase transition [12], and in informing about other dynamic properties of a time series [10]. Most of the applications of RP and RQA are at this time in the field of physiology and biology, but some authors have already applied these techniques to financial data [13–15]. We have used RP and RQA techniques for examining both endogenous and exogenous crash data to find out the distinction between the two phenomena. Then we have used these techniques in detecting critical regimes preceding an endogenous crash seen as a phase transition and hence give an estimation of the initial bubble time. We have worked with data from the US, Indian and Hong Kong Stock markets. To the best of our knowledge this combination of three different categories of market in terms of efficiency has not been analyzed together before. However, an important work on Indian Stock exchanges was carried out by Pan and Sinha [16].

2. Recurrence analysis

A recurrence plot (RP) is a graph that shows all those times at which a state of the dynamical system recurs. In other words, the RP reveals all the times when the phase space trajectory visits roughly the same area in the phase space.[1]

Natural processes can have a distinct recurrent behavior (e.g. periodicities (as seasonal or Milankovich cycles)) but also irregular cyclicities (as El Nino Southern Oscillation). Moreover, the recurrence of states, in the meaning that states are arbitrarily close after some time, is a fundamental property of deterministic dynamical systems and is typical for nonlinear or chaotic systems. The recurrence of states in nature has been known for a long time and has also been discussed in early publications (e.g. recurrence phenomena in cosmic-ray intensity, [17]).

Eckmann et al. [10] have introduced a tool which can visualize the recurrence of states x_i in a phase space. Usually, a phase space does not have a dimension (two or three) which allows it to be pictured. Higher dimensional phase spaces can only be visualized by projection into the two or three dimensional sub-spaces. However, Eckmann's tool enables us to investigate the m-dimensional phase space trajectory through a two-dimensional representation of its recurrences. Such recurrence of a state at time i at a different time j is marked within a two-dimensional squared matrix with one and zero dots (black and white dots in the plot), where both axes are time axes. This representation is called the recurrence plot (RP). Such an RP can be mathematically expressed as

$$R_{i,j} = \Theta(\epsilon_i - \|x_i - x_j\|), \quad x_i \in \diamond^m,\ i, j = 1, \ldots, N$$

where $R_{i,j}$ is the recurrence plot, N is the number of considered states x_i, ϵ_i is a threshold distance, $\| \diamond \|$ a norm and $\Theta(\diamond)$ the Heaviside function. The Heaviside step function is given by:

$$\Theta(x) = 0 \quad \text{if } x < 0$$
$$\Theta(x) = 1 \quad \text{if } x \geq 0.$$

The threshold distance is called the delay factor and the number of considered states is called the embedding dimension.

1 A phase space is a space in which all possible states of a system are represented, with each possible state of the system corresponding to one unique point in the phase space. In a phase space, every degree of freedom or parameter of the system is represented as an axis of a multidimensional space. For every possible state of the system, or allowed combination of values of the system's parameters, a point is plotted in the multidimensional space. Often this succession of plotted points is analogous to the system's state evolving over time. In the end, the phase diagram represents all that the system can be, and its shape can easily elucidate qualities of the system that might not be obvious otherwise. A phase space may contain very many dimensions.