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# UTILIZATION AND COMPARISON OF MULTI ATTRIBUTE DECISION MAKING TECHNIQUES TO RANK BAYESIAN NETWORK OPTIONS

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AMIN KARAMI, 820601-T937
a10amika@student.his.se

Supervisor: Dr. Maria Riveiro
maria.riveiro@his.se

Examiner: Dr. Ronnie Johansson
ronnie.johansson@his.se

**Utilization and Comparison of Multi Attribute Decision Making Techniques to Rank Bayesian Network Options**

Submitted by Amin Karami to the University of Skövde as a dissertation towards the degree of M.Sc. by examination and dissertation in the School of Humanities and Informatics.

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I hereby certify that all material in this dissertation which is not my own work has been identified and that no work is included for which a degree has already been conferred on me.

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**Amin Karami**

# Abstract

A fusion system sometimes requires the capability to represent the temporal changes of uncertain sensory information in dynamic and uncertain situation. A Bayesian Network can construct a coherent fusion structure with the hypothesis node which cannot be observed directly and sensors through a number of intermediate nodes that are interrelated by cause and effect. In some BN applications for observing a hypothesis node with the number of participated sensors, rank and select the appropriate options (different combination of sensors allocation) in the decision-making is a challenging problem. By user interaction, we can acquire more and useful information through multi-criteria decision aid (MCDA) as semi-automatically decision support. So in this study, Multi Attribute Decision Making (MADM) techniques as TOPSIS, SAW, and Mixed (Rank Average) for decision-making as well as AHP and Entropy for obtaining the weights of indexes have been used. Since MADM techniques have most probably different results according to different approaches and assumptions in the same problem, statistical analysis done on them. According to results, the correlation between applied techniques for ranking BN options is strong and positive because of the close proximity of weights suggested by AHP and Entropy. Mixed method as compared to TOPSIS and SAW is ideal techniques; moreover, AHP is more acceptable than Entropy for weighting of indexes.

**Key words:** Multi Attribute Decision Making (MADM), Bayesian Networks, Sensor Allocation, AHP, Entropy, TOPSIS, SAW

# Contents

# 1 Introduction

There are different definitions of information fusion. A novel definition has been suggested by Boström et al. (2009) as "Information fusion is the study of efficient methods for automatically or semi-automatically transforming information from different sources and different points in time into a representation that provides effective support for human or automated decision making."

Several studies based on the Information Fusion system (IFS) characteristics and structures have been done, but according to Nilsson & Ziemke (2007) there is no general agreement regarding Information Fusion system classification. But a fusion system sometimes requires the capability which represents the temporal changes of uncertain sensory information in dynamic and uncertain situation (Zhang & Ji, 2006). To do this, Basic BNs are used for static situations and Dynamic BNs for dynamic situations. Hence, Bayesian Network is a solution which presents knowledge about domain variables in uncertain and unpredictable environments through numerical and graphical representation (Cobb & Shenoy, 2005). Moreover, a Bayesian Network can construct a coherent fusion structure with the hypothesis node which cannot be observed directly and sensors through a number of intermediate nodes that are interrelated by cause and effect. Likewise, to be able to handle the uncertainty of sensor reading, information variables may add an additional layer of variables which connects sensors to intermediate variables (Zhang & Ji, 2006).

According to Stevens & Sundareshan (2004), Bayesian Network applications can perform sensor configuration management in a collaborative sensor network through incorporation of expert knowledge. Likewise, they mentioned that in a target tracking case with set of stationary sensors for observation of a hypothesis variable (node), number of participated sensors and select the appropriate option (different combination of sensors allocation for node observation) in the decision-making is a challenging problem. Thus, it is important to rank and select sensors or subset of sensors which are more useful in order to help decision makers for their decisions making.

For better possibility of decision making to reach to a hypothesis variable in terms of temporal stresses, uncertainty, and the availability of massive amounts of unstructured information (Bossé et al., 2007), we will need to present better picture of options ranking and selection. Likewise, Bayesian Networks provide important support for decision-making, but in some situations we need to make decision and rank or re-rank the set of options based on multiple criteria such as those of multi-criteria decision aid (MCDA) (Fenton & Neil, 2001). To put it simply, Bayesian Networks provide a probabilistic and graphical framework for dealing with uncertain, imprecise, and complex problems based on probability theories. Probability theory provides inference mechanisms through subsets of evidence from intermediate variables to observe hypothesis (goal) nodes which are not observed (Besada-Portas et al., 2002). Obviously, by user interaction, we can manage different possible options based on criteria as semi-automatically decision support. Multi-criteria analysis tries to incorporate multiple and different types of information and human experience into a DSS. Integration of human expertise with a fusion-based DSS can enable suggestions and recommendations for actions through understanding of problems and problem solving skills within a specific domain (Nilsson & Ziemke, 2007). Hence, a Decision Support System can support decision makers to re-rank options more accurately in terms of user contribution (Berggren & Kylesten, 2009). Hence, re-ranking of options in Bayesian Network-based systems for achieving to a hypothesis/unobserved variable in terms of qualitative and quantitative criteria is one of the decision-making problems.

In recent decades, for complex decisions in terms of the consideration of multiple factors, researchers have been focused on Multi Criteria Decision Making (MCDM). MCDM is a well-established branch of decision making that allows decision makers to select and rank alternatives according to different criteria and is divided into two categories: Multi-Objective Decision-Making (MODM) and Multi-Attribute Decision-Making (MADM) (Pirdashti et al., 2009). MODM is the same as the classical optimization models with this difference that, instead of optimizing a goal function, it is focused on optimizing of several goal functions. To put it simply, MODM is a mathematical programming problem with multiple objective functions. In contrast, MADM which in this study is used, several alternatives according to some criteria are ranked and selected. Ranking and selecting will be made among decision alternatives described by some criteria (factors) through decision-maker knowledge and experience (Devi et al., 2009).

In this study, we are going to utilize and compare MADM techniques to re-rank Bayesian Network options. Applied decision-making techniques include TOPSIS, SAW, and Mixed (Rank Average) methods as well as AHP and Entropy methods for defining importance of indexes weights. Since MADM techniques have different approaches and assumptions for ranking and selecting in the same problem, there is more likely to have different results (Cheng, 2000). Finally, the results of the applied MADM techniques will be compared and analyzed.

This research is organized as follow. Section 2 considers related works about utilization and comparison of Multi Attribute Decision Making (MADM) techniques for alternative ranking and their results. The structure of section 3 is as follow: in subsection 3.1, decision making is described to introduce the overall definition. In subsection 3.2, Multi-Criteria Decision-Making (MCDM) approaches and techniques are considered to introduce categorization of decision making techniques and describe some techniques in detail in order to know about the differences of each other. In subsection 3.3, information fusion concepts as general domain of this thesis are presented. Finally, Bayesian Networks and the underlying concepts will be presented in subsection 3.4. Moreover, using of Bayesian Network in sensor allocation has been mentioned. In section 4, the aim, objectives, and research question of this thesis are presented. Section 5 will investigate objectives and the results of each objective will also be presented. Section 6 presents the conclusions and also suggestions for future work.

# 2 Related Work

This section includes some related works about utilization and comparison of Multi Attribute Decision Making (MADM) techniques for alternative ranking and their results.

Soltanpanah et al. (2010) have utilized and compared MADM Techniques for countries upon human development rate ranking. They have stated that Human Development Index (HDI) which is estimated in United Nations Development Program (UNDP) report for countries has become as useful tool for countries ranking in terms of human development. There are criticisms from researchers about using amount of HDI which is obtained from the arithmetic average amount of life expectancy at birth, education and GDP of each person indexes by the same weight. Hence, they utilized Entropy and AHP techniques for gaining the weights of HDI indexes and utilized SAW, TOPSIS techniques and Numerical Taxonomy analysis as replacement for arithmetic average method for re-ranking countries based on amount of human development. Their analysis according to re-ranking has revealed that TOPSIS model provides more acceptable results. In addition, about using of Entropy and AHP techniques for obtaining indexes weights, entropy method has distinctive power to rank entries.

Afshar & Mianabadi (2008) utilized and surveyed three MADM methods as Inducted Ordered Weighted Averaging (IOWA), Linear Assignment (LA), and TOPSIS to rank urban water supply schemes. Their results revealed that applied MADM methods in a same problem had significant differences in final ranking of alternatives. Therefore, selection of appropriate MADM methods for problem solving in a specific domain will be need to consider characteristics of the problem, type of data set, assessment criteria, and finally compare and analyze results. Then, final ranking and selection of alternatives in terms of different criteria can be made by evaluation of mentioned requirements by applying different decision-making methods to the problem.

Bernroider & Mitlöhner (2005) have presented the utilization of MADM techniques in Enterprise Resource Planning (ERP) software decisions. This study has tried to utilize MADM techniques in the context of ERP projects in terms of empirical insights based on 209 datasets originating from a primary, national and industry independent survey. The result reveals that the ERP decision problem can be structured in MADM techniques as a formal method. Desired expectations were fulfilled in the high level of magnitude in firms especially based on financial firm level impact and service quality supported by a formal MADM method.

Azar F. S. (2000) used the three different multi attribute ranking methods as Simple Additive Weighting (SAW) method, the Weighted Product Method (WPM), and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) in order to compare the performance of four imaging techniques for breast cancer detection. He found that the most robust method for new ranking of four imaging techniques seems to be SAW method. In TOPSIS method, cost factor had significant role to final ranking. The WPM method had extreme result and we should be careful when using this method due to use of weights as exponents in the mathematical calculations, because exponential functions has probably significant role for gained results.

# 3 Background

The structure of the background section is as follows: in subsection 3.1, decision making is described to introduce the overall definition. In subsection 3.2, Multi-Criteria Decision-Making (MCDM) approaches and techniques are considered to introduce categorization of decision making techniques and describe some techniques in detail in order to know about the differences from each other. In subsection 3.3, information fusion concepts as general domain of this thesis are presented. Finally Bayesian Networks and the underlying concepts will be presented in subsection 3.4. Moreover, using of Bayesian Network in sensor allocation has been mentioned.

## 3.1 Decision Making

According to Harris (2009) "Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered, and in such a case we want not only to identify as many of these alternatives as possible but to choose the one that (1) has the highest probability of success or effectiveness and (2) best fits with our goals, desires, lifestyle, values, and so on."

Decisions can be either formal or informal. Formal decisions are complex, non-routine and non-repetitive. Because there is no prior information, procedures, and methods about present problem and creativity has significant role. In contrast, informal decisions are routine, repetitive and there are prior information, knowledge, and procedures in order to assist to managers for better decision-making (Texas State Auditor's Office, 1993).

Simon (1960) categorized the decision-making process in three phases as intelligence (refers to the identification of the problem or opportunity), design (refers to the design or identification of alternative solutions to the problem or opportunity), and choice (process of the selection of one or a combination of alternatives). Intelligence phase can involve a wide variety of activities, such as listening to people (e.g. customers, employees, suppliers, etc.), brainstorming for gaps between current condition as ‘as is’ and some future condition as ‘what should be’, and SWOT (Strengths, Weaknesses, Opportunities, and Threats) analysis. Design phase can include brainstorming, reviewing the literature, conducting research, and benchmarking both within your industry and across industries. Choice phase is about decision making and choose the ‘best’ one or combination of alternatives as perhaps difficult part of the decision-making process (Forman & Selly, 2001).

## 3.2 Multi Criteria Decision Making (MCDM)

According to Xu & Yung (2001) Multiple Criteria Decision Making (MCDM) is an emerging discipline which supports decision makers who face multiple and usually conflicting criteria; in addition, it has a relatively short history with around three-decade and its extension is closely related to the advancement of computer science and information technology accomplishment especially in complex MCDM problems.

MCDM allows decision makers to select and rank alternatives according to different and conflicting criteria and is classified on the major components: Multi-Objective Decision-Making (MODM) and Multi-Attribute Decision-Making (MADM) (Pirdashti et al., 2009).

Zhou & Poh (2006) have classified Decision Analysis (DA) methods into the three main groups (Figure 3.1) as Single-Objective Decision-Making (SODM), Decision Support Systems (DSS), and Multi-Criteria Decision-Making (MCDM).

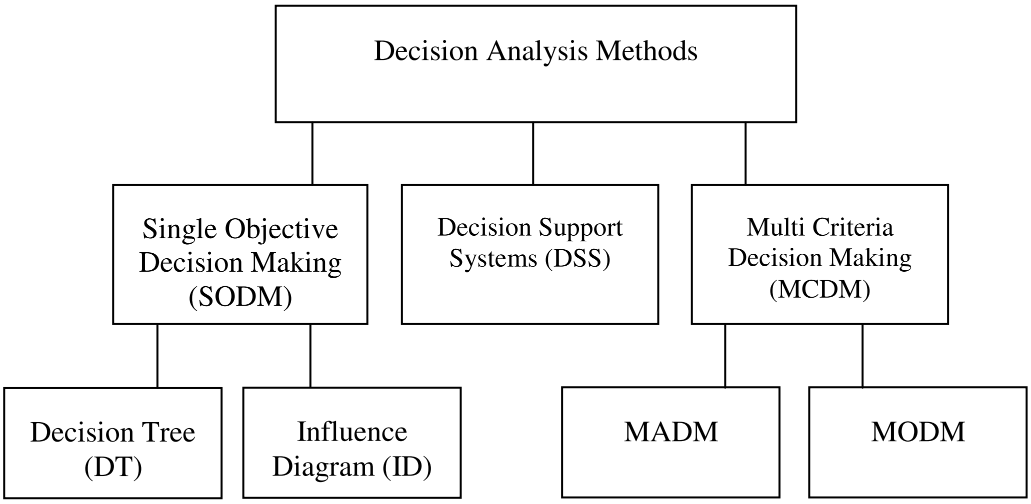


**Figure 3.1** Classification of decision analysis methods

MODM (a mathematical programming problem with multiple objective functions) is the same as the classical optimization models with the difference that, instead of optimizing a goal function, it is focused on optimizing several goal functions. In contrast in MADM, several alternatives according to some criteria are selected and ranked. To put it simply, selection and ranking will be made among decision alternatives described by some attributes through decision-maker information and experience (Devi et al., 2009).

The difference between MADM and MODM is based on criteria evaluation as attributes (the properties of elements in an applied system) and objectives (a statement about the desired and favorable state of the system), respectively. There is a classification of Multi-Criteria Decision Problem as figure 3.2 (Malczewski, 2006):

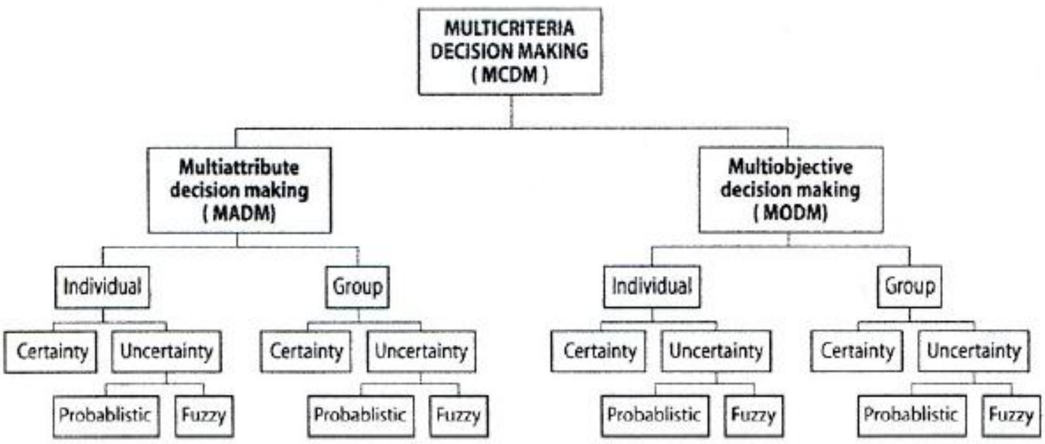


**Figure 3.2** Classification of Multi-Criteria Decision Problem

Another viewpoint is about comparison between MADM and MODM approaches in figure 3.3 (Yoon & Hwang, 1981, Starr & Zeleny, 1977):

| | MODM | MADM |
|---|---|---|
| Criteria defined by: | Objectives | Attributes |
| Objectives defined: | Explicitly | Implicitly |
| Attributes defined: | Implicitly | Explicitly |
| Constraints defined: | Explicitly | Implicitly |
| Alternatives defined: | Implicitly | Explicitly |
| Number of alternatives | Infinite (large) | Finite (small) |
| Decision maker's control | Significant | Limited |
| Decision modeling paradigm | Process-oriented | Outcome-oriented |
| Relevant to: | Design/search | Evaluation/choice |
| Relevance of geographical data structure | Vector-based GIS | Raster-based GIS |

**Figure 3.3** Comparison of MODM and MADM approaches

## 3.2.1 Multi Attribute Decision Making (MADM)

According to Devi et al. (2009) "MULTI-ATTRIBUTE decision making (MADM) is the most well-known branch of decision making. It is a branch of a general class of operations research models that deals with decision problems under the presence of a number of decision criteria. The MADM approach requires that the selection be made among decision alternatives described by their attributes. MADM problems are assumed to have a predetermined, limited number of decision alternatives. Solving a MADM problem involves sorting and ranking. MADM approaches can be viewed as alternative methods for combining the information in a problem's decision matrix together with additional information from the decision maker to determine a final ranking or selection from among the alternatives. Besides the information contained in the decision matrix, all but the simplest MADM techniques require additional information from the decision maker to arrive at a final ranking or selection."

A MADM problem with $m$ criteria and $n$ alternatives can present according to $C_1 \dots C_m$ and $A_1 \dots A_n$ as criteria and alternatives, respectively. Moreover, A MADM methodology is shown as 'decision table' (table 3.1). Each row and column presents the alternatives and criteria, respectively. The score $a_{ij}$ describes the value and amount of alternative $A_j$ against criterion $C_i$. In addition, weights $W_1 \dots W_m$ should be assigned to every criterion. Weight presents the importance of criterion $C_i$ to the decision, and is assumed to be positive. After filling the decision table by decision-maker experience, a MADM technique must be selected in order to rank and select alternatives.

**Table 3.1** The decision table

| | $W_1$ | . | . | $W_m$ |
|---|---|---|---|---|
| | $C_1$ | . | . | $C_m$ |
| $A_1$ | $a_{11}$ | . | . | $a_{n1}$ |
| . | . | . | . | . |
| . | . | . | . | . |
| $A_n$ | $a_{n1}$ | . | . | $a_{mn}$ |

The multiple attribute-based decision problems should be solved with one of the many methods; moreover, the availability to the large number of MADM problem-solving techniques provides the paradox between selections of MADM methods (Triantaphyllou, 2000). There are different MADM methods for solving decision-making problems. Different applied methods will be provided different results in a same problem domain. These contradictions may come from differences in use of weights, the selection approach of the 'best' solution, objectives scaling and introduction of additional parameters (Lezzi, 2006).

Multiple criteria decision support systems are provided to assist decision makers with an explicit and comprehensive tool and techniques in order to evaluate alternatives in terms of different factors and importance of their weights. Five common Multi-Attributes Decision-Making (MADM) techniques are (Cheng, 2000):

- Simple Additive Weighted (SAW)
- Weighted Product Method (WPM)
- Cooperative Game Theory (CGT)
- Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)
- ELECTRE with complementary analysis

In addition, other well known MADM techniques are PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) by Brans & Vincke (1985) and AHP (Analytical Hierarchy Process) by Thomas L. Saaty (1995).

#### 3.2.1.1 SAW

SAW (Simple Additive Weighting) model is also known as Weighted Sum Model (WSM) or Scoring Method (SM) and most often used in multi-attribute decision-making techniques. To do this, the normalized value of the criteria for the alternatives must be multiplied with the weight of the criteria. Then, the best alternative with highest score is selected as the preferred alternative.

According to Janic & Reggiani (2002) "The SAW (Simple Additive Weighting) method consists of quantifying the values of attributes (criteria) for each alternative, constructing the Decision Matrix A containing these values, deriving the normalized Decision Matrix R, assigning the importance (weights) to criteria, and calculating the overall score for each alternative. Then, the alternative with the highest score is selected as the preferred (best) one. The analytical structure of the SAW method for N alternatives and M attributes (criteria) can be summarized as follows:

(Equation 1)

$$S_i = \sum_{j=1}^{M} w_j r_{ij}$$

For $i = 1, 2, \dots, N$

Where:

$S_i$: is the overall score of the i$th$ alternative;

$r_{ij}$: is the normalized rating of the i$th$ alternative for the j$th$ criterion which:

$r_{ij} = \frac{x_{ij}}{max_i x_{ij}}$ for the benefit and $r_{ij} = \frac{1/x_{ij}}{max_{i\,(1/x_{ij})}}$ for the cost criterion representing an element for the normalized matrix;

$x_{ij}$: is an element of the decision matrix, which represents the original value of the j$th$ criterion of the i$th$ alternative;

$w_j$: is the importance (weight) of the j$th$ criterion;

N is the number of alternatives;

M is the number of criteria."

### 3.2.1.2 TOPSIS

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) technique is suggested by Yoon & Hwang in 1981. Any problems of the type of the multi-attribute decision with M alternative and N criteria can be evaluated in a geometric system with 'm' points in 'n' dimensional space. Based on the idea the best alternative should have the shortest distance from a positive ideal solution (the best possible) and the longest distance from negative ideal solution (the worst possible).

The TOPSIS method consists of the following steps:

1) Normalize the decision matrix: the normalization of the decision matrix is done using the below transformation for each $n_{ij}$:

$$n_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}}$$ (Equation 2)

Then, weights should be multiplied to normalized matrix.

2) Determine the positive and negative ideal alternatives:

(Equation 3)

$$A^+ = \{v_1^+, v_2^+, \dots, v_n^+\} = \{(max_i V_{ij} \mid j \in J), (min_i V_{ij} \mid j \in J' \mid i = 1,2,\dots,m)\}$$

$$J = \{ j = 1,2,\dots,n \mid j\ for\ positive\ attributes\}$$

Positive attribute: the one which has the best attribute values (more is better).

$$J' = \{ j = 1,2,\dots,n \mid j\ for\ negative\ attributes\}$$

Negative attribute: the one which has the worst attribute values (less is better).

In addition, the weighted normalized decision matrix should be calculated with multiplying the normalized decision matrix by its associated weights. The weighted normalized value $V_{ij}$ is calculated as: $V_{ij} = w_{ij} r_{ij}$

Where $w_j$ represents the weight of the $jth$ attribute or criterion.

$$A^- = \{v_1^-, v_2^-, \dots, v_n^-\} = \{(min_i V_{ij} \mid j \in J), (max_i V_{ij} \mid j \in J' \mid i = 1,2,\dots,m)\}$$
$$J = \{ j = 1,2,\dots,n \mid j \; for \; positive \; attributes\}$$
$$J' = \{ j = 1,2,\dots,n \mid j \; for \; negative \; attributes\}$$
(Equation 4)

3) Obtain the separation measure (based on Euclidean distance) of the existing alternatives from ideal and negative one (The separation between alternatives will be found according to distance measure called normalized Euclidean distance (Szmidt & Kacprzyk, 2000)):

(Equation 5)

$$d_{i^+} = \left\{ \sum_{j=1}^{n} (V_{ij} - V_j^+)^2 \right\}^{0.5} \quad ; \quad i = 1,2,\dots,m$$

(Equation 6)

$$d_{i^-} = \left\{ \sum_{j=1}^{n} (V_{ij} - V_j^-)^2 \right\}^{0.5} \quad ; \quad i = 1,2,\dots,m$$

4) Calculate the relative closeness to the ideal alternatives:

(Equation 7)

$$cl_{i^+} = \frac{d_{i^-}}{(d_{i^+} + d_{i^-})} \quad ; \quad 0 \leq cl_{i^+} \leq 1 \quad ; \quad i = 1,2,\dots,m$$

5) Rank the alternatives: based on the relative closeness to the ideal alternative, the most is the $cl_{i^+}$, the better is the alternative $A_i$.

#### 3.2.1.3 AHP

AHP (Analytical Hierarchy Process) is the one of the well known MADM techniques which has been developed by Thomas L. Saaty (1995). It is a popular MADM technique and widely used, especially in military problems (Coyle, 2004). AHP reflects the natural behavior of human thinking. This technique examines the complex problems based on their interaction effects.

The AHP procedure consists of the following steps (Kasperczyk & Knickel, 2006, Ariff et al., 2008):

1. **Define the problem:** this step is to decompose a decision problem into different parts as problem goal in topmost level, criteria (it is possible to break to lower levels as sub-criterion) at the intermediate levels, and options in the lowest level (Figure 3.4).

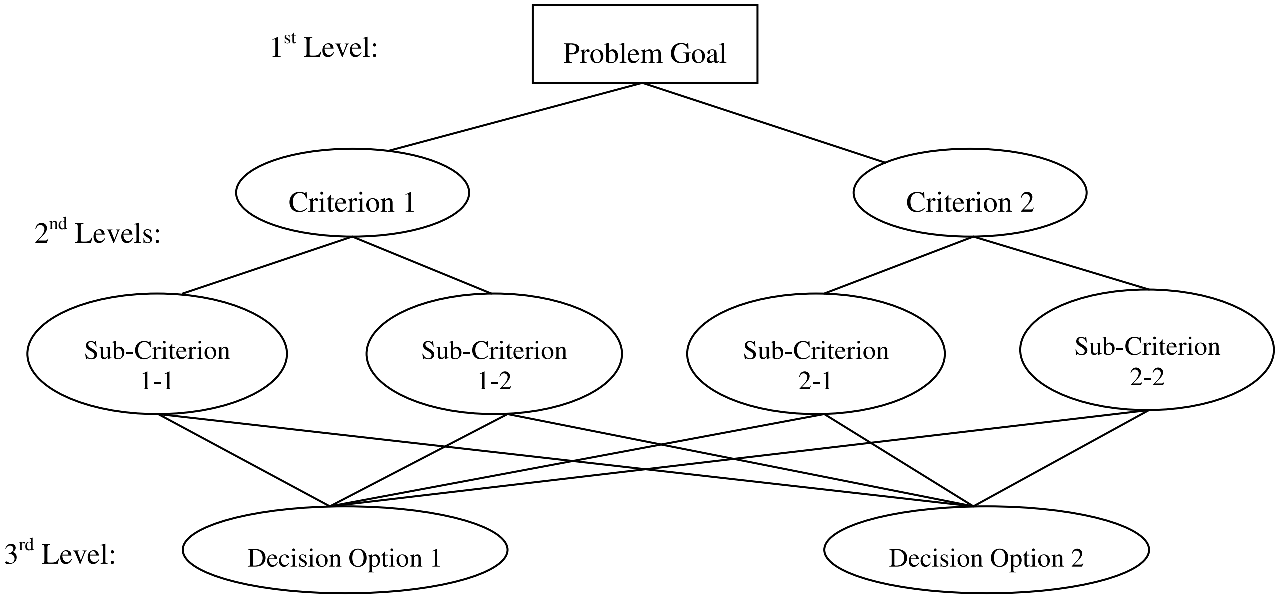


**Figure 3.4** A hierarchy of decision problem

2. **Construct a Pair-wise Comparison Matrix (weighing):** according to Kasperczyk & Knickel (2006), the decision maker should be answered to a question such as 'How important is criterion A relative to criterion B?' It must be performed for each pair of criteria. In continue rating the relative priority of the each paired criteria is done by assigning a weight between 1 as 'equal importance' and 9 as 'extreme importance'. To do comparative judgment, Saaty's 9-point scale is used (Table 3.2).

**Table 3.2** 9-point scale for comparative judgments

| Intensity of importance | Definition | Explanation |
|---|---|---|
| **1** | Equal Importance | I*th* option has equal importance as compared with J*th* option or has not any preference to each other |
| **3** | Moderate Importance | I*th* option is a little important than J*th* option |
| **5** | Strong Importance | I*th* option is important than J*th* option |
| **7** | Very Strong Importance | I*th* option is much more preferable than J*th* option |
| **9** | Extreme Importance | I*th* option is most important and incomparable to J*th* option |
| **2, 4, 6, 8** | For compromise between about values | Intermediate values between above preferred values shows. E.g., 8 shows the importance of higher than 7 and lower than 9 for I*th* |

3. **Judgment of Pair-wise Comparison of options on each criterion (scoring):** according to Kasperczyk & Knickel (2006) "For each pairing within each criterion the better option is awarded a score, again, on a scale between 1 (equally good) and 9 (absolutely better), whilst the other option in the pairing is assigned a rating

equal to the reciprocal of this value. Each score records how well option 'x' meets criterion 'Y'. Afterwards, the ratings are normalized and averaged."

4. **Integration of relative weights:** in order to rank the decision alternatives, at this point, the relative weight of each element must be multiplied to above elements to the final weight obtained. By doing this step for each option, the value of the final weight will be obtained.

The final stage of AHP technique is calculation of Inconsistency Ratio (IR) to measure of the logical rationality of the pair wise comparisons. If the IR is less than 0.10, pair wise comparison is generally considered acceptable. AHP evaluations are based on the assumption that a decision maker is rational. For example, if A is preferred to B and B is preferred to C, then A is preferred to C (Zaim et al., 2004).

In real life decision problems, pair wise comparison matrices are rarely consistent. But decision makers would like to reach in the level of the consistency of the judgments, because inconsistent judgments may lead to meaningless decisions (Bozóki & Rapcsák, 2007). Saaty (1980) proposed the formula for calculating inconsistency:

$$CI_n = \frac{\lambda_{max} - n}{n - 1}$$ (Equation 8)

Where:

$CI_n$ is Consistency Index,

$\lambda_{max}$ is the Eigenvalue and $\lambda_{max} > n$, and

$n$ is number of comparison.

Next, $\lambda_{max}$ must be calculated: $[Ax = \lambda_{max} x]$

Where:

$A$ is the comparison matrix of size $n \times n$, for $n$ criteria, and

$x$ is the Eigenvector of size $n \times 1$.

$$CR_n = \frac{CI_n}{RI_n}$$ (Equation 9)

$CR_n$ is Consistency Ratio which Saaty (1980) concluded that if the value of Consistency Ratio is smaller or equal to 10%, the inconsistency is acceptable. If the Consistency Ratio is greater than 10%, we need to revise the subjective judgment, and

$RI_n$ is Random Consistency Index which $n$ is number of comparison (Table 3.3).

**Table 3.3** Random Consistency Index (*RI*)

| **n** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **RI** | 0 | 0 | 0.58 | 0.9 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 |

### 3.2.1.4 Mixed Method

Decision-makers usually use more than one decision-making technique in important decisions. Obviously, different decision-making techniques may provide different results according to their approaches and assumptions. In order to overcome to this problem, mixed method as Rank Average Method is used. Since mixed method involves average of methods results and their specifications, it can be ideal method in some problems (Soltanpanah et al., 2010).

$$Average = \frac{sum\ of\ elements}{number\ of\ elements} \quad \text{(Equation 10)}$$

An example of Mixed method according to Equation 10 is in table 3.3. For instance, numbers of 2nd, 3rd, and 4th columns have been calculated from some MADM methods for ranking the five options. Likewise, last column is average of different results of MADM methods for every option.

**Table 3.3** An example of Mixed method

| Options | MADM method 1 | MADM method 2 | MADM method 3 | **Mixed** |
|---|---|---|---|---|
| O1 | 3 | 4 | 2 | (3+4+2)/3=3 |
| O2 | 2 | 1 | 1 | (2+1+1)/3=1.33≈1 |
| O3 | 1 | 3 | 3 | (1+3+3)/3=2.33≈2 |
| O4 | 4 | 5 | 5 | (4+5+5)/3=4.66≈5 |
| O5 | 5 | 2 | 4 | (5+2+4)/3=3.66≈4 |

#### 3.2.1.5 Entropy

Entropy is the one of the important concept in social science, physics, and information theory. Shannon's entropy method is suitable for finding the appropriate weight for each criterion in MADM problems (Andreica et. al, 2010). According to this method, whatever dispersion in the index is greater, the index is more important. Entropy steps are as follow:

Step1) Calculate $P_{ij}$ as the normalization to eliminate anomalies with different measurement units and scales.

$$P_{ij} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}} \quad ; \forall j \quad \text{(Equation 11)}$$

Step2) Calculate the entropy of $E_j$

$$E_j = \left(\frac{-1}{\ln(m)}\right) \sum_{i=1}^{m} [P_{ij} ln P_{ij}] \quad ; \forall j \quad \text{(Equation 12)}$$

Step3) Calculate of uncertainty $d_j$ as the degree of diversification

$$d_j = 1 - E_j \quad ; \forall j \quad \text{(Equation 13)}$$

Step4) Calculate of weights ($W_j$) as the degree of importance of attribute $j$

(Equation 14)

$$W_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \quad ; \forall j$$

Where:

$a_{ij}$ is value of i*th* option (entry) for j*th* index;

$P_{ij}$ is the value-scale of j*th* index for i*th* option (entry).

## 3.3 Information Fusion

The concept of *fusion* is not new. This means that humans and animals have been using a combination of different senses to survive. Humans employ combination of different senses such as sight, touch, smell, hearing, and taste for information deduction via interaction with environment (Hall & McMullen, 2004).

As noted by Steinberg & Bowman (2001), data fusion is the process of data or information combination in order to estimate or predict the state of the entity/object.

There are different definitions of information fusion. A novel definition has been suggested by Boström et al. (2009) as "Information fusion is the study of efficient methods for automatically or semi-automatically transforming information from different sources and different points in time into a representation that provides effective support for human or automated decision making." As suggested by Mastrogiovanni et al. (2007) "the aim of data fusion process is to maximize the useful information content acquired by heterogeneous sources in order to infer relevant situations and events related to the observed environment". The purpose of the information fusion is to gather relevant information from various sources (e.g. sensor and database) in order to support decision-making (Bossé et al., 2006). Torra & Narukawa (2007) have stated that information fusion is used to reduce noises in environment, increase accuracy, information extraction and summarization, and finally provide decision-making process.

### 3.3.1 The development of JDL model

The data fusion model has developed by U.S. Joint Directors of Laboratories (JDL) in 1985 with subsequent revisions. It is widely used to facilitate common ground, understanding, and communication among managers, researchers, designers, evaluators, and users. As figure 3.5, the JDL model breaks functions into different and separated levels (Hall & Llinas, 2001).

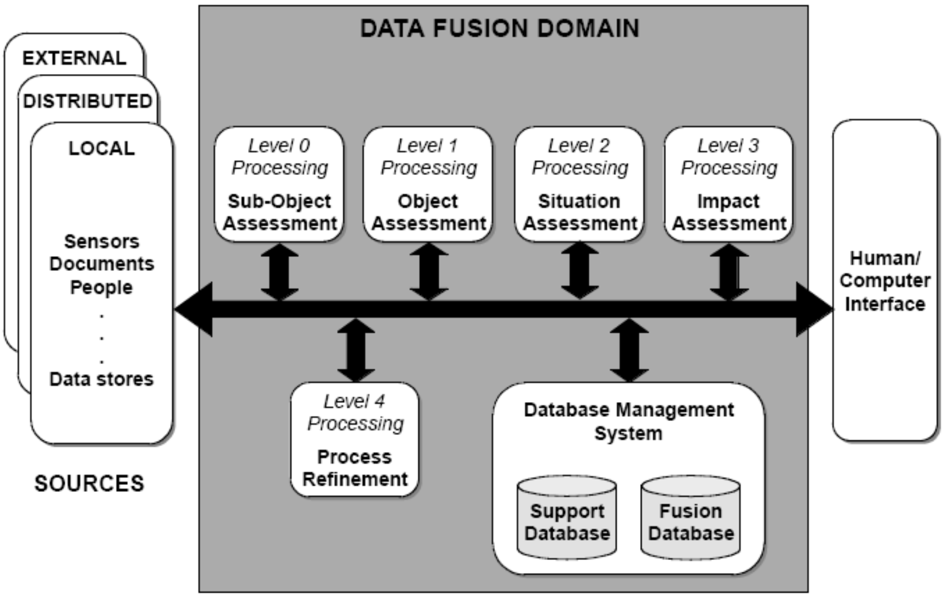


**Figure 3.5** Revised JDL data fusion model (Steinrberg et al., 1998)

- **Level 0 (Sub-Object Data Assessment):** estimation and prediction of signal/data/object as preprocessing level for further levels. It is included reducing data noise and jitter as well as filtering process. It is based on pixel/signal data association and characterization (Hall & Llinas, 2001).
- **Level 1 (Object Assessment):** estimation and prediction of entity/object states. In other words, entities/objects are tracked on the basis of inferences from observation. Such as entity kinematics estimation (e.g., speed and direction) and entity type estimation. For this kind of processing there are appropriate methods, and these are Detection, Kalman Filtering, Particle Filtering, and Multi Hypothesis Tracking (Hall & Llinas, 2001).
- **Level 2 (Situation Assessment):** estimation and prediction of entity states on the basis of relationships between entities and environments (Hall & Llinas, 2001). Different relationships can be considered, such as physical, informational, and perceptual.
- **Level 3 (Impact Assessment):** it is based on projection of the current situation into the future to investigate consequences. As noted by Hall & Llinas in 2001 third level is: "estimation and prediction of effects on situations of planned or estimated/predicted actions by the participants; to include interactions between action plans of multiple players (e.g. assessing susceptibilities and vulnerabilities to estimated/predicted threat actions given one’s own planned actions);"
- **Level 4 (Process Refinement):** fourth level exploits the optimization of the fusion process and the utilization of sensors to support missions and objectives; moreover, involves planning and control, not estimation (Hall & Llinas, 2001). E.g., changing/re-configuring sensing algorithms according to weather and lighting conditions.
- **Level 5 (Cognitive Refinement):** Hall et al. (2000) were suggested level 5 as "cognitive refinement" to the original JDL model. Level 5 investigates how to support the decision-making process through design an appropriate tools and

components; furthermore, it is the interaction between human and machine which is monitored and refined. Information visualization (it is about how to present information to the user under time pressure and information overload condition) and user aspects (a user as an active component in a fusion system for interacting with the system) are two important components in this level.

### 3.3.2 Information Fusion and Decision Making

In order to be able to decide based on correct information, the information must be present and accessible. It is important to know about availability of information, where it can be collected, how and in which format. By availability of information it is easier for decision makers to motivate their decisions in order to having better and robust decisions (De Vin et al., 2005). This is Information Fusion (IF) trait which can support and enhance user's decision making process in order to be as a Decision Support System (DSS). There are different researches which focus on user aspect of Information Fusion System (IFS) and its possibility to a function as DSS. It can be possible through integration of information from multiple and different sources into a usable format in order to support decision-making (Bossé et al., 2006, Bisantz et al., 1999). As noted by Bossé et al. (2006) information/data fusion from different sources can support decision-maker via reducing uncertainty, increasing accuracy and robustness.

Blasch (2003) presented that the user can contribute and aid the information fusion system with respect to JDL model levels, such as select incoming data (level 0), choose interest objects (level 1), define an area of coverage (level 2), defining the threats levels (level 3), and refining the location of sensor location (level 4). According to Nilsson & Ziemke (2007) "Fusion driven decision support systems are based on fused information from different sources such as sensors, databases and models, providing both automatic and semi-automatic fusion processes, i.e. enabling complex decision making from large amount of information (which may be conflicting/ contradicting or uncertain) without information loss (e.g. information is not just filtered, but, for instance, aggregated) with respect for the user decision making process." Fusion driven DSS types can be with either an automated or a semi-automated (interactive) fusion process (Nilsson, 2010). Nilsson (2010) continued that interacting with an IF based decision support is a challenging activity for humans. Challenges can come from time pressure, high stress, inconsistencies, imperfect and uncertain nature of the information.

For better understanding of Information Fusion System (IFS) functionality, Nilsson & Ziemke (2007) clarified Information Fusion System (IFS) component as follow:

- "*IFS* contains knowledge of the environment due to data from different sensors, and could sometimes predict future states
- *IFS* has the ability to acquire and store knowledge (information) from different sensors
- *IFS* could present knowledge and information in various ways
- *IFS* has the ability to fuse information, and present it to the user for further considerations
- *Users* of IFS could interact with the system influencing both the process and the result
- *IFS* could coordinate/facilitate interactions among multiple decision makers."

Moreover, there is no general agreement on Information Fusion System (IFS) characteristics, but basically IFS refers to a computer system which utilizes information or data from different sources to support decision makers. A schematic view of IFS components has shown is Figure 3.6 (Nilsson & Ziemke, 2007).

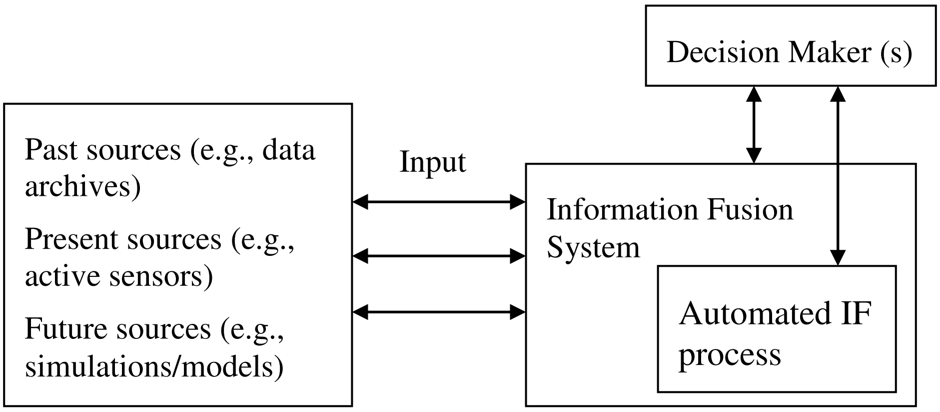


**Figure 3.6**: A schematic view of IFS components.

Admittedly, many information fusion applications (especially in military domains such as, target tracking and target identification) are often described as a high degree of complexity. This kind of complexity includes (Zhang et al., 2002):

- Data/information are gathered from different sensors by distinct degree of uncertainty, imprecision, and quality;
- Decision should be made quickly and
- Sensory observations and fused environments situation evolve over time.

Zhang et al. (2002) believed that provide a general framework to support decision-making with user cooperation and contribution in a fusion-based environment where decisions must be quickly and economically from different and disparate sources is crucial.

To put it simply, an Information Fusion System (IFS) does not only gather and combine information from different uncertain sources for aiding the users but also should provide automatic or semi-automatic process to support decision making. The benefits of considering IFS as DSS (Fusion-based DSS) include as follow (Nilsson & Ziemke, 2007):

- There is a lack of user perspective in an IF-based system and IFS as a DSS can naturally support it,
- Possible ensure the effectiveness of the system, and
- Provide a natural top-down perspective as holistic view in the IF process.

## 3.4 Bayesian Network

According to Jensen & Nielsen (2007) a Bayesian Network is a directed acyclic graphical (DAG) representation of the joint probability distribution to describe the combined set of variables which each variable has a finite set of mutually exclusive states. Probability distribution of every node is presented in a conditional probability table. The capabilities of Bayesian Networks (BNs) include representation of variables in a model by prior information in order to future state prediction and identifying the set of current situation based on its previous position. Moreover, analysis of problems is possible with ambiguous, inadequate, conflicting, and uncertain in both past and present situation. In addition, Bayesian Networks have been widely used for efficient probabilistic inference and reasoning (Pearl, 1998). In Bayesian Networks, the causal structure and the numerical values can be defined through two different approaches; these are, from an expert and learned from a dataset or data residing in a database (Nipat & Wichian, 2009).

A sample of a Bayesian Network (BN) representation is as follow (figure 3.7):

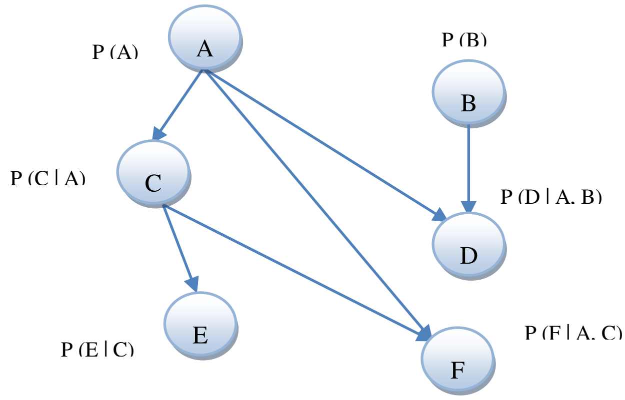


**Figure 3.7** A Bayesian Network Representation

The assumption about probability distribution of every node can be as follow:

For node 'C':

| A | C=0 | C=1 |
|---|---|---|
| 0 | 0.2 | 0.8 |
| 1 | 0.9 | 0.1 |

For node 'E':

| C | E=0 | E=1 |
|---|---|---|
| 0 | 0.4 | 0.6 |
| 1 | 0.8 | 0.2 |

For node 'D':

| A | B | D=0 | D=1 |
|---|---|---|---|
| 0 | 0 | 0.1 | 0.9 |
| 0 | 1 | 0.9 | 0.1 |
| 1 | 0 | 0.2 | 0.8 |
| 1 | 1 | 0.3 | 0.7 |

Moreover, chain rule is used for joint probability representation as a product of conditional probabilities between random variables in Bayesian Networks.

$$P(X1, X2, \dots, Xn) = \prod_{i=}^{n} P(X_i \mid Parents(X_i)) \qquad \text{(Equation 14)}$$

Where:

$\{X1, X2, \dots, Xn\}$ is the states of all variables;

$X_i$ represents the state of $i^{th}$variable;

$Parents(X_i)$ represents the states of the parents of $i^{th}$variable.

In figure 3.7, $P(A, B, C, D, E, F) = P(A)P(B)P(C|A)P(D|A, B)P(E|C)P(F|A, C)$

### 3.4.1 Bayes' theorem

Bayesian Networks (BNs) use Bayes' theorem in order to present the uncertainties. There are some terminologies of Bayes' theorem (Bohling, 2005):

P(A): Probability of occurrence of event A (marginal);

P(A,B): Probability of simultaneous occurrence of events A and B (joint probability);

P(A|B): Probability of occurrence of A given that B has occurred (conditional);

P(A,B) = P(A|B)P(B);

P(A, B) = P(B|A)P(A);

P(A|B)P(B) = P(B|A)P(A);

The Bayesian Network method is based on the efforts and studies of the mathematician Thomas Bayes in the end of the 18th century. This method is named as Bayes' theorem as follow:

$$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} \qquad \text{(Equation 15)}$$

Assume that event $B_i$ has occurred given that event A has been observed, then:

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_{j=1}^{n} P(A|B_j)P(B_j)} = \frac{P(A|B_i)P(B_i)}{P(A)} \qquad \text{(Equation 16)}$$

The advantage of Bayesian Network is its multipurpose application which includes (Langbein, 2011):

- *"Queries:* $P(Q|E=e)=?$
- *Reasoning:*
    - *Explanation:* diagnostic reasoning
    - *Prediction:* casual reasoning
- *Decision:* which action to execute next? $P(outcome|action, evidence)=?$ Also requires *utility* for *decision networks*
- *Value of information:* which evidence to seek next?
- *Sensitivity analysis:* which probabilities are most critical?"

Another important definition is that Bayesian Networks based on environmental variables provide a probabilistic and graphical framework for dealing with uncertain, imprecise, and complex problems based on probability theories. Probability theory provides inference mechanisms through subsets of evidence and intermediate variables to observe unobserved variables (Besada-Portas et al., 2002). When we model an uncertain domain with Bayesian Network, there will be different role of variables/nodes as sensory observation, informational/intermediate, and hypothesis/unobserved (goal) nodes. Obviously, there are different paths (set of candidate network or number of different routes in order to observe target variable) in terms of their probabilities from Bayes' theorem calculation in order to estimate state of a hypothesis node through informational/intermediate nodes (Figure 3.8).

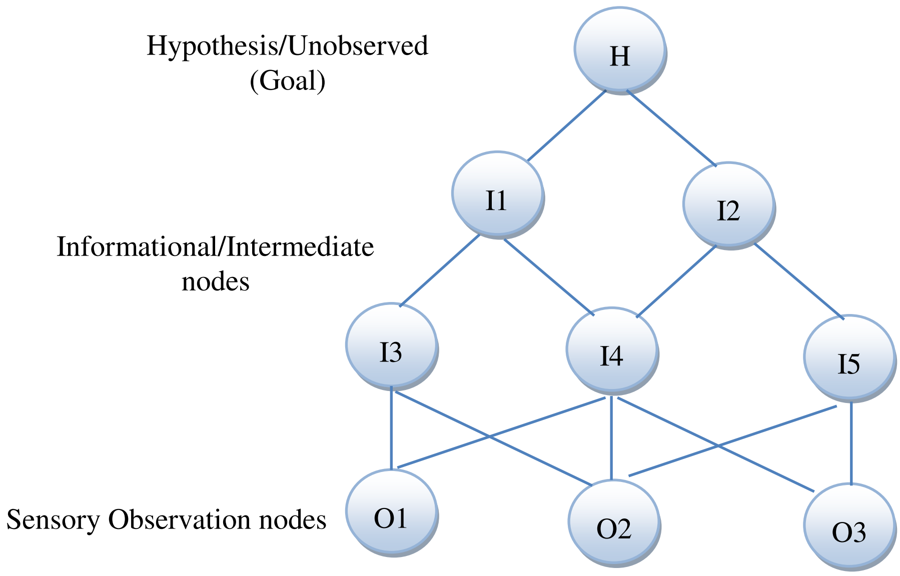


**Figure 3.8** A simple Bayesian Network with different roles of nodes

There are some Bayesian Network applications for intelligent decision aids and decision analysis. Because Bayesian Networks can provides (Buede et al., 2000):

- Knowledge of the structural relationship between situations, events, and event cues,
- Integrating the situations and events to make a holistic view of their meaning, and
- Framework for projecting and planning future events

Bayesian Network models are powerful tools for reasoning and decision-making under uncertainty. When a BN is structured, it can provide different inferences about the variables (nodes) in a domain. Since we can observe the values of some the observable variables, the corresponding variables are instantiated to the observed values. This includes belief updating from extended observations throughout the network to other nodes. Likewise, decision makers can update beliefs through manipulating the nodes value. Moreover, this joint distribution between network nodes changes with time and new information will be presented. Hence, BN inferences can be directly used for decision-making tasks as an important and a huge factor by dealing with various information (Watthayu & Yun Peng, 2004).

### 3.4.2 Bayesian Networks and Sensor Allocation

The sensor allocation problem has been considerably investigated in recent years. Two research issues of sensor allocation include decide where to physically install sensors, and decide which physical parameters should be measured by sensors. Also optimal sensor allocation is where to allocate sensor, which is closely related to decision-making objectives. To do this, a Bayesian Network is built to represent the casual relationships between the physical variables in order to determine which physical variables should be sensed (Li & Jin, 2009).

On the other hand, Bayesian Networks are used to find the prioritization of sensor configuration through value of obtained information from domain (Mullen et al., 2006). The recent approach to address the sensor configuration management problem is Bayesian Networks, which provide via expert knowledge interaction in the configuration process (Jensen, 2002). This means that, the Bayesian Network constructs an influence diagram to user incorporate with information of each sensor node in order to select appropriate ones. According to Stevens & Sundareshan (2004) “Unfortunately however, the use of influence diagrams is computationally intensive and is often prohibitive for real time applications, particularly when the number of sensors involved is large."

There is multitude of sensors which are deployed in an array to cover a large area under surveillance. In decision-making process, these sensors need to be networked and configured for exchange of raw measurement or some decisions results from processing the data for the detection, discrimination, localization, and tracking the target of interest. Improving performance by sensor fusion and minimizing network latency in sensor configuration management are challenging problems (Stevens & Sundareshan, 2004). Moreover, the problem in the sensor planning include which appropriate sensor configuration must be selected in order to have a proper recognition (Saidi et al, 2007). According to Stevens & Sundareshan (2004), in a target tracking case with set of stationary sensors for observation of a hypothesis variable (node), number of participated sensors and select the appropriate combination of sensors in the decision-making is a challenging problem.

Santini (2010) introduced a novel heuristic sensor ranking technique in order to enable Coverage Configuration Protocol (CCP) for having a timely selection of the nodes as a guarantee for both carrying out of the coverage requirement and avoiding unnecessary node activations. These include ranking based on local density, ranking based on weighted local density, and load balancing.

According to Zhang & Ji (2006) with the hypothesis and sensors, we can construct a coherent fusion structure with a Bayesian Network (Figure 3.9). The root node of such a network would contain the hypothesis variable and the sensors are in the lowest level without any children. The hypothesis node is causally linked to the sensor nodes through intermediate nodes which are interrelated by cause and effect. Moreover, in the real world, a fusion system may receive incorrect information from sensors according to different reasons such as sensor noise and imprecise acquisition devices. Therefore, sensor readings include uncertainties which may reduce the reliability of a fusion system. To handle the uncertainty of sensor readings in a probabilistic network, we can add an additional layer of variables as 'information variables' which connect intermediate variables to sensors (Zhang & Ji, 2006). Evidences according to information variables are gathered through sensors and are fused by Dynamic Bayesian Network (DBN) inference. In figure 3.9, temporal links reflect the temporal causality and time *t* increases by one every time new sensor information arrives.

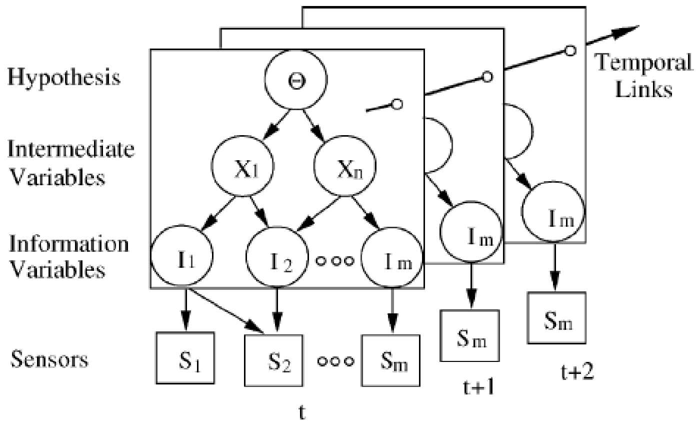


**Figure 3.9** A coherent fusion structure with a Bayesian Network (Zhang & Ji, 2006)

#### 3.4.2.1 A fusion structure with a Bayesian Network

An example of a fusion structure with a Bayesian Network has been presented with Johansson & Martensson (2010). This BN scenario includes a hypothesis variable (corresponding to a knowledge request and not directly observable) and information variables have been identified in Figure 3.10. This is an example of an intelligence model which is drawn with the GeNIe tool. In this BN example, only "will to attack", "capability to attack", "increased air movements", "increased radio", and "increased presence" can be observed. The hypothesis variable as "attack X-town" can normally not be observed.

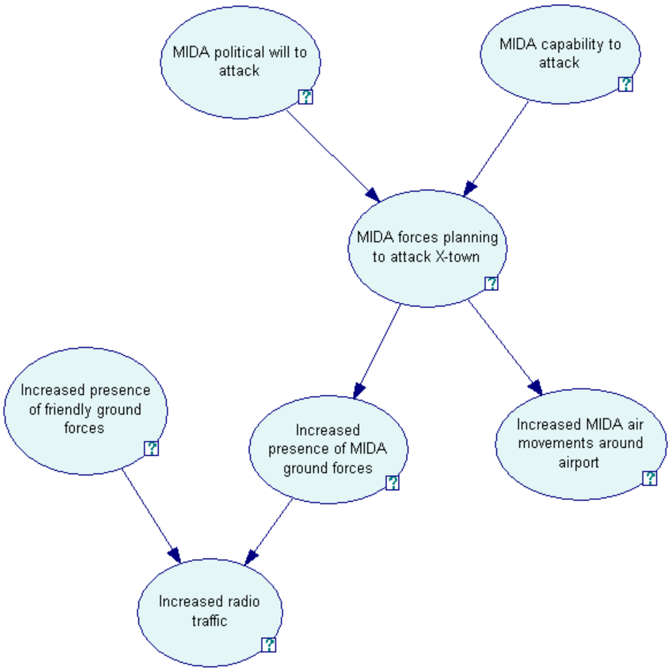


**Figure 3.10** MIDA scenarios with the GeNIe tool (Johansson & Mårtenson, 2010)

Johansson & Mårtenson (2010) have proposed a method to acquire information for general Bayesian Networks with uncertain observations. They have enumerated all possible options (allocations of sensing resources to Bayesian Network variables) and evaluated them according to their expected impact if an option was implemented. When evaluating an option, they considered what the most likely observations of the variables would be (based on the current state of the BN) and generated a set of virtual reports. Likewise, they input the reports into the BN and update BN. Then, they compare the current (real) state of the BN with the one updated with the virtual reports. The comparison is via Kullback-Leibler (KL) distance:

(Equation 17)

$$KL(p||q) = \int P(\theta) log \frac{P(\theta)}{q(\theta)} d\theta$$

Where, $q$ is the prior probability $p(H)$ and $p$ is the posterior $p(H|O)$.

Hence, the aim is to maximize the expected $KL( p(H|O) \,||\, p(H) )$, denoted $EKL(O)$. In presented case (figure 3.10), $p = p(H)$, i.e., the current probability function over the hypothesis variable, and $q=p(H|O)$, i.e., the probability function over the hypothesis variable given the virtual observations $O$. The expected $KL$, $EKL(O)$, is basically the $KL$ distance between $p(H)$ and $p(H|O)$ where $O$ is the set of the most likely observations (given the current state of the BN).

Using a BN, the true state of the hypothesis variable may be unattainable. Performance metric can be calculated as distance metric between the estimated hypothesis probability $PO(H|\boldsymbol{O})$ and $PI(H|\boldsymbol{I})$ (Johansson & Mårtenson, 2010):

$$Q(PO(H|\boldsymbol{O}), PI(H|\boldsymbol{I})) = \sum_{n=1}^{N} (PO,n - PI,n)^2 \quad \text{(Equation 18)}$$

Where, $PI(H|\boldsymbol{I})$ (all $\boldsymbol{I}$ known) is the best possible probability, $N$ is the number of states of the Hypothesis variable $H$, $PO,n$ and $PI,n$ are the probability values of the $n$th state for the probability functions. A low value of $Q$ is desirable.

So, the result of this scenario is set of options (sensor allocation) in seventy different allocations.
Structure of every option is as follow, e.g., ((2, 1, 1, 0, 0), 0.9939219115520805):

1st Number: 2 sensors/resources were assigned to the attack_will variable,

2nd Number: 1 sensor to attack_capability,

3rd Number: 1 sensor to air_movements,

4th Number: 0 sensors to presence_friendly,

5th Number: 0 sensors to radio_traffic, and

6th Number: 0.9939219115520805 is expected performance.

All seventy options include:

1. ((2, 1, 1, 0, 0), 0.9939219115520805)
2. ((2, 0, 2, 0, 0), 0.9223076920775368)
3. ((3, 0, 1, 0, 0), 0.8718746977093652)
4. ((2, 0, 1, 1, 0), 0.8312989387927261)
5. ((1, 2, 1, 0, 0), 0.8059858324317694)
6. ((2, 0, 1, 0, 1), 0.7137857627197801)
7. ((1, 0, 3, 0, 0), 0.6827651047694026)
8. ((1, 1, 1, 0, 1), 0.6435039429930585)
9. ((3, 1, 0, 0, 0), 0.6195506655132939)
10. ((1, 0, 1, 2, 0), 0.5869491656078953)
11. ((1, 0, 1, 1, 1), 0.5501478075261621)
12. ((4, 0, 0, 0, 0), 0.4810903088404924)
13. ((2, 1, 0, 0, 1), 0.4772914758547457)
14. ((1, 0, 1, 0, 2), 0.4617892427572136)
15. ((2, 0, 0, 2, 0), 0.4418689565698989)
16. ((2, 2, 0, 0, 0), 0.44186895656989883)
17. ((1, 3, 0, 0, 0), 0.42548417397137733)
18. ((2, 0, 0, 1, 1), 0.4079196081423548)
19. ((3, 0, 0, 0, 1), 0.3757925704767289)
20. ((0, 1, 3, 0, 0), 0.35019425135505694)

21. ((0, 1, 2, 1, 0), 0.3412000599785904)
22. ((2, 0, 0, 0, 2), 0.32756755991694725)
23. ((0, 3, 1, 0, 0), 0.3256578098471946)
24. ((1, 2, 0, 0, 1), 0.3233053348653976)
25. ((0, 0, 3, 0, 1), 0.32271610689772273)
26. ((0, 1, 1, 0, 2), 0.27237765157387667)
27. ((0, 1, 1, 2, 0), 0.2723776515738765)
28. ((1, 1, 0, 0, 2), 0.2720461360735274)
29. ((0, 1, 2, 0, 1), 0.2513506043476286)
30. ((0, 2, 1, 0, 1), 0.24724583216175416)
31. ((0, 1, 1, 1, 1), 0.2436014291657863)
32. ((1, 2, 0, 1, 0), 0.24301639033013472)
33. ((3, 0, 0, 1, 0), 0.24301639033013467)
34. ((1, 0, 2, 1, 0), 0.24301639033013467)
35. ((1, 0, 0, 3, 0), 0.24301639033013467)
36. ((1, 1, 0, 1, 1), 0.23154786143211725)
37. ((1, 0, 0, 2, 1), 0.22501382051328944)
38. ((1, 0, 0, 1, 2), 0.21181719308530067)
39. ((0, 0, 3, 1, 0), 0.17456188919017745)
40. ((0, 0, 1, 2, 1), 0.16900592216614813)
41. ((0, 0, 2, 2, 0), 0.16760195275679635)
42. ((0, 0, 2, 0, 2), 0.16760195275679635)
43. ((1, 1, 0, 2, 0), 0.15234274775967274)
44. ((1, 0, 0, 0, 3), 0.15146169303884252)
45. ((0, 0, 2, 1, 1), 0.14389216399492516)
46. ((1, 1, 2, 0, 0), 0.12008692252880501)
47. ((0, 2, 1, 1, 0), 0.11631757690928629)
48. ((0, 0, 1, 3, 0), 0.11631757690928618)
49. ((0, 0, 1, 1, 2), 0.11631757690928618)
50. ((1, 1, 1, 1, 0), 0.07088512321675969)
51. ((0, 4, 0, 0, 0), 0.06974364089486265)
52. ((0, 3, 0, 1, 0), 0.06936359488950794)
53. ((0, 2, 0, 2, 0), 0.06604231157179863)
54. ((0, 2, 2, 0, 0), 0.06604231157179855)
55. ((0, 0, 1, 0, 3), 0.052623137849170276)
56. ((0, 2, 0, 1, 1), 0.050543966517079264)
57. ((2, 1, 0, 1, 0), 0.0427146313291181)
58. ((0, 1, 0, 3, 0), 0.0427146313291181)
59. ((0, 1, 0, 1, 2), 0.02870123402213716)
60. ((0, 3, 0, 0, 1), 0.027755686680930383)
61. ((0, 2, 0, 0, 2), 0.020576632446827267)
62. ((0, 0, 0, 0, 4), 0.015439837608430526)
63. ((0, 1, 0, 2, 1), 0.011642925605669097)
64. ((0, 1, 0, 0, 3), 0.00794079629090609)

65. ((1, 0, 2, 0, 1), 0.0024325533525085125)
66. ((0, 0, 0, 1, 3), 0.0016053751114762302)
67. ((0, 0, 0, 2, 2), 0.0006484783031743789)
68. ((0, 0, 0, 3, 1), 0.0004437912996208375)
69. ((0, 0, 0, 4, 0), 1.1612421076329078e-16)
70. ((0, 0, 4, 0, 0), -5.80621053816454e-17)

In this research activity, main problem involves re-ranking of these generated options (different combination of sensors allocation) by user interaction. This research is going to re-rank the set of options via Multi Attribute Decision Making (MADM) techniques. By user interaction through applied MADM techniques, we can manage all possible options based on qualitative and quantitative criteria as semi-automatically decision support. Multi-criteria analysis tries to incorporate different types of information and human experience into a DSS. Integration of human expertise with a DSS can enable suggestions and recommendations for actions through understanding of problems and problem solving skills within a specific domain (Nilsson & Ziemke, 2007).

# 4 Problem

The structure of the problem section includes aim, objectives, research question, and research methods, respectively.

**Aim:** Comparison of Multi Attribute Decision Making (MADM) Techniques to re-rank Bayesian Network options.

**Objectives:**

1. To identify some general decision-making criteria for re-ranking the Bayesian Network options.

Decision alternatives should be measured to a set of predefined criteria as requirements or characteristics of decision-making situation. Since the research problem is about Bayesian Networks and sensor allocation, general decision-making criteria should be defined from these domains.

2. To compare and analyze Multi-Attribute Decision-Making (MADM) techniques for re-ranking the Bayesian Network options.

MADM techniques can select and rank a finite set of alternatives according to preference based on a number of attributes, but different techniques may provide different results. Hence, we are going to compare and analyze its results for re-ranking Bayesian Network options.

**Research Question:**

Can Multi-Attribute Decision-Making (MADM) techniques be utilized for ranking Bayesian Network options?

**Research Method:**

*First Objective: Literature analysis*

The first objective is based on literature analysis through journals/conferences articles and relevant books about decision making in order to identify some general decision making attributes.

*Second Objective: Descriptive method*

There are different MADM techniques for solving decision-making problems. Different applied methods will yield different results in the same problem domain. These contradictions maybe come from differences in use of weights, the selection approach of the 'best' solution, objectives scaling and introduction of additional parameters (Lezzi, 2006). Five common Multi-Attributes Decision-Making (MADM) techniques include Simple Additive Weighted (SAW), Weighted Product Method (WPM), Cooperative Game Theory (CGT), and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), ELECTRE with complementary analysis (Cheng, 2000) as well as PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) by Brans & Vincke (1985) and AHP (Analytical Hierarchy Process) by Thomas L. Saaty (1995).

Yeh (2002) investigated significant specifications of SAW, WPM and TOPSIS methods which include applicability for large-scale decision problems, simplicity in concept and computation, and applicability for hierarchical multi-level attributes. He also mentioned that AHP method is suitable when an attribute hierarchy has more than three levels. This means that, the overall goal of the problem on the top level, multiple criteria which define alternatives in the middle level, and competing alternatives in the bottom level. In spite of the mentioned specifications, he noted that there is no one best method as a general method and Acceptability of a method is based on the specific problem domain characteristics and its data set. So, in this study, we have selected three techniques as SAW (Simple Additive Weighting), TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) according to their ideal characteristics, and Mixed method. Since different methods provide different results, decision-makers use more than one technique in important decisions. In order to overcome to this problem, we have utilized a mixed method as Rank Average Method through makes average between applied techniques (Soltanpanah et al., 2010). Likewise, AHP and Entropy are two important weight methods which we will use for them. In this study, alternatives are ordered through average of ranks from four used methods. By three different techniques and two weight methods, we are faced to five different rankings as TOPSIS with AHP, TOPSIS with Entropy, SAW with AHP, SAW with Entropy, and Mixed method.

Yoon & Hwang (1981) developed Technique as Order Preference by Similarity to Ideal Solution (TOPSIS) based on the idea that the ideal solution and the negative ideal solution must be defined. Then chosen alternative should have the closest distance from the positive ideal solution and the longest distance from the negative ideal solution.

According to Afshari et al. (2010) "Simple Additive Weighting (SAW) which is also known as weighted linear combination or scoring methods is a simple and most often used multi attribute decision technique. The method is based on the weighted average. An evaluation score is calculated for each alternative by multiplying the scaled value given to the alternative of that attribute with the weights of relative importance directly assigned by decision maker followed by summing of the products for all criteria. The advantage of this method is that it is a proportional linear transformation of the raw data which means that the relative order of magnitude of the standardized scores remains equal."

Likewise, we will analyze and compare five presented methods with each other in order to know about applicability of MADM with respect to a Bayesian Network-based intelligence analysis fusion component (Johansson & Mårtenson, 2010). To put it simply, we are going to use Multi-Criteria analysis to provide multiple and different type of information and human experience into DSS in order to support decision makers to re-rank Bayesian Network options.

Since MADM methods have different approaches and assumptions for selecting/ranking alternatives in the same problem, it is likely that they yield different results (Cheng, 2000). Therefore, we will investigate applied MADM methods by statistical tests if these differences are significant. If the results are significantly different, utilization of MADM techniques in real cases may not be useful. We are going to use Kendall's tau-b factor, Spearman correlation coefficient, and Pearson correlation coefficient (because our study is about ranking data and data are quantitative). All statistical tests will be implemented by SPSS software.

# 5 Analysis and Results

In this section the results of each objective as well as discussion about objective results will be provided.

**First Objective:** To identify some general decision-making criteria for re-ranking the Bayesian Network options.

The attributes we are seeking should be the most important ones which are more relevant to the final decision. They should preferably be mutually exclusive in order to avoid overlapping attributes according to their definition and nature. We have used literature review and recent experiences of some specialists in order to identify some general decision-making attributes for re-ranking the BN options as follows:

According to Bossé et al. (2007) the decision-maker can choose a specific alternative among variety of possible alternatives regarding the different and adopted alternatives which each person can postulate each strategy. Strategy selection will depend on three factors:

1. Characteristics of the choice (e.g., uncertainty, complexity, instability);
2. Environment (time and resource available, irreversibility of the choice, possibility of failure);
3. The decision maker (e.g., knowledge, strategies, expertise, motivation).

Moreover, in a Bayesian Network model, experts with different knowledge who work in a group for same project may have different solutions and opinions for identifying the causal relationship among variables, quantifying graphical models, and ranking on the set of alternatives in terms of numerical probabilities (Premchaiswadi & Jongsawat, 2010).

Jamieson (2007) proposed that a combination of contextual and informational decision factors will have effect on decision making. These factors are politics, power structure, trust, confidence, and time pressure for rapid selection decisions. In addition, tangible factors which include cost, risk, adherence to organizational technology standards and strategies, and informal external information sources with their relationship.

In order to the final identification of general attributes, we have used Delphi method as a structured communication technique which relies on a panel of experts. Ten experts that were familiar with Bayesian Networks, sensor allocation, and Decision Support Systems concepts were chosen. Firstly, experts answered to a questionnaire with some general attributes which were gained from literature review in above. They did score between 1 (very low importance) and 9 (very high importance) of each attribute. Then, those attributes score with less than the total average (for every member) have been eliminated and the number of attributes for the Delphi process remained. To increase reliability, besides attributes which had gained score higher than the average of all, all the attributes until moderate range were finally selected. Next, a questionnaire with obtained attributes from previous step provided. Every member did score again between 1 and 9 and attributes with score of greater than total average as final general attributes were selected which include:

1. **User knowledge:** user understanding and opinion of a situation which can gain through experience or education. User opinion based on his/her obtained

knowledge and experience according to previous decisions made in real cases (historical real cases) can effect on current decision making process. e.g., according to decisions made by a user in historical real cases shows that his/her knowledge about current 1st and 2nd options are good and high. In contrast, his/her knowledge about 10th and 11th ones are poor and low.

2. **User strategy:** an action plan for each contingent state of the situation.
3. **Time pressure:** the time shortage before something must be finished or executed. This means that, due to time pressure there is a risk that the implementation of the option will fail. Or time pressure might make it likely that the option will not be implemented in time.
4. **Resource availability:** it states how many resources are available at any time to do a job. This means that, is there sufficient resources for implementation of an option or not?
5. **External information sources:** extra information from environment and related objects.
6. **Possibility of failure:** the condition or fact of being unsuccessful, insufficient or disappointing. Possibility of failure can estimate the likelihood of occurrence of a hazardous event.
7. **Trust:** confident expectation of selecting or doing something. It is about decision-maker confidence to his/her option selection according to environmental information and personal experiences. e.g., a decision-maker may trust to 1st option more than 2nd, 3rd, 4th options according to environmental information and variables or his/her trust to 7th and 8th options are equal.
8. **Complexity:** the state or quality of a choice or option being complex or intricate. There is likely that state of an option be more complex or less complex by different reasons, e.g., lack of information for sensor controlling or lack of information about impact of external variables. However, it is likely to some more complex options can lead to failure.
9. **Cost:** estimates an amount of money, effort, time, risk or material that has to be paid for getting something.

And finally,

10. **Expected Performance:** It is calculated according to a proposed method by Johansson & Mårtenson (2010) in order to acquire information for general Bayesian Networks with uncertain observations (section 3.4.2.1).

**Second Objective:** To compare and analyze Multi-Attribute Decision-Making (MADM) techniques for re-ranking Bayesian Network options.

To reach this objective, a Bayesian Network scenario includes a hypothesis variable (corresponding to a knowledge request and not directly observable) and information variables have been identified in Figure 5.1 (Johansson & Mårtenson, 2010). This is an example of an intelligence model which is drawn with the GeNIe tool. In this scenario, hypothesis node is "MIDA forces planning to attack X-town" and there are multiple resources to observe this single node.

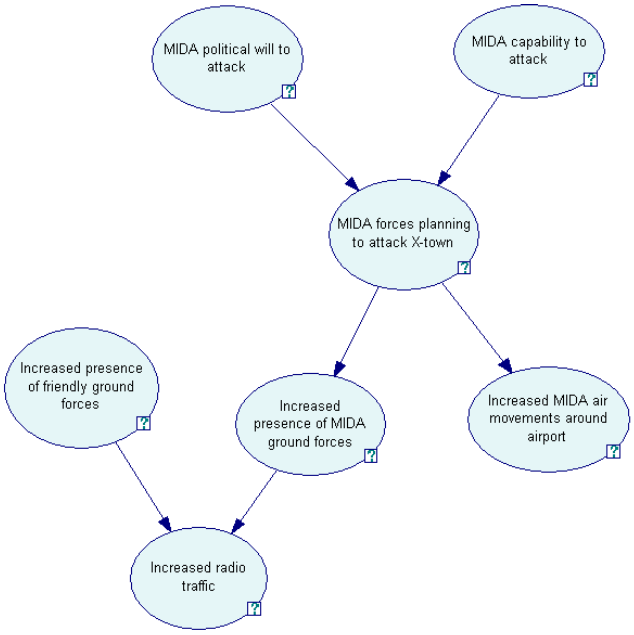


**Figure 5.1** MIDA scenarios with the GeNIe tool (Johansson & Mårtenson, 2010)

The result of this scenario is set of options (sensor allocation) in seventy different allocations. We will utilize first best twenty options in terms of the high expected performance in order to better possibility of filling decision matrix by expert user as well as analysis and comparison. The reason of using the expected performance is that, it was the only appreciable and available criterion from investigated options.

First twenty options include (for more detailed information such as, generation of options and numbers meaning refer to section 3.4.2.1):

1. ((2, 1, 1, 0, 0), 0.9939219115520805)
2. ((2, 0, 2, 0, 0), 0.9223076920775368)
3. ((3, 0, 1, 0, 0), 0.8718746977093652)
4. ((2, 0, 1, 1, 0), 0.8312989387927261)
5. ((1, 2, 1, 0, 0), 0.8059858324317694)
6. ((2, 0, 1, 0, 1), 0.7137857627197801)
7. ((1, 0, 3, 0, 0), 0.6827651047694026)
8. ((1, 1, 1, 0, 1), 0.6435039429930585)
9. ((3, 1, 0, 0, 0), 0.6195506655132939)
10. ((1, 0, 1, 2, 0), 0.5869491656078953)
11. ((1, 0, 1, 1, 1), 0.5501478075261621)
12. ((4, 0, 0, 0, 0), 0.4810903088404924)
13. ((2, 1, 0, 0, 1), 0.4772914758547457)
14. ((1, 0, 1, 0, 2), 0.4617892427572136)
15. ((2, 0, 0, 2, 0), 0.4418689565698989)
16. ((2, 2, 0, 0, 0), 0.44186895656989883)
17. ((1, 3, 0, 0, 0), 0.42548417397137733)
18. ((2, 0, 0, 1, 1), 0.4079196081423548)
19. ((3, 0, 0, 0, 1), 0.3757925704767289)
20. ((0, 1, 3, 0, 0), 0.35019425135505694)

Moreover, analysis of decision matrix should include quantitative values, but some criteria are qualitative. To solve this matter, Odd Bipolar Scaling will be used to convert qualitative variables to quantitative (Table 5.1) (Joseph et al., 2008).

**Table 5.1** Convert Qualitative to Quantitative

| **1** | **3** | **5** | **7** | **9** |
|---|---|---|---|---|
| Very Low | Low | Moderate | High | Very High |
| Very Small | Small | Medium | Large | Very Large |
| Unimportant | Little Important | Moderately Important | Important | Very Important |
| Very poor | Poor | Acceptable | Good | Very Good |

**Table 5.2** Decision Matrix

| | **C1(+)** | **C2(+)** | **C3(-)** | **C4(+)** | **C5(+)** | **C6(-)** | **C7(+)** | **C8(-)** | **C9(-)** | **C10(+)** |
|---|---|---|---|---|---|---|---|---|---|---|
| **O1** | 5 | 5 | 5 | 9 | 5 | 7 | 5 | 3 | 5 | 0.994 |
| **O2** | 3 | 9 | 3 | 7 | 3 | 5 | 9 | 9 | 1 | 0.922 |
| **O3** | 7 | 1 | 9 | 9 | 9 | 3 | 1 | 9 | 5 | 0.892 |
| **O4** | 5 | 9 | 3 | 3 | 9 | 5 | 9 | 5 | 3 | 0.831 |
| **O5** | 1 | 3 | 5 | 9 | 9 | 7 | 9 | 9 | 7 | 0.806 |
| **O6** | 1 | 5 | 3 | 3 | 7 | 7 | 7 | 3 | 5 | 0.714 |
| **O7** | 9 | 3 | 3 | 9 | 5 | 1 | 5 | 3 | 3 | 0.683 |
| **O8** | 5 | 1 | 5 | 7 | 9 | 7 | 9 | 7 | 7 | 0.643 |
| **O9** | 7 | 7 | 7 | 5 | 7 | 9 | 5 | 9 | 5 | 0.619 |
| **O10** | 3 | 5 | 3 | 5 | 3 | 5 | 7 | 7 | 3 | 0.587 |
| **O11** | 5 | 9 | 5 | 7 | 7 | 9 | 1 | 5 | 3 | 0.55 |
| **O12** | 5 | 9 | 1 | 5 | 9 | 3 | 7 | 7 | 5 | 0.481 |
| **O13** | 3 | 9 | 5 | 5 | 5 | 3 | 3 | 7 | 3 | 0.477 |
| **O14** | 5 | 5 | 5 | 3 | 9 | 7 | 5 | 9 | 5 | 0.462 |
| **O15** | 7 | 5 | 5 | 9 | 1 | 3 | 1 | 5 | 9 | 0.442 |
| **O16** | 3 | 7 | 5 | 1 | 1 | 9 | 7 | 5 | 7 | 0.442 |
| **O17** | 5 | 3 | 3 | 9 | 9 | 9 | 9 | 3 | 5 | 0.425 |
| **O18** | 3 | 9 | 1 | 5 | 7 | 9 | 3 | 3 | 3 | 0.408 |
| **O19** | 9 | 7 | 5 | 5 | 5 | 7 | 5 | 5 | 9 | 0.376 |
| **O20** | 1 | 3 | 7 | 1 | 7 | 7 | 1 | 5 | 5 | 0.35 |

Alternatives evaluation in front of each criterion in the form of decision matrix via a decision maker idea are presented (table 5.2). For example, possibility of failure (C8) is a negative criterion and its value for first and second options is 3 and 9, respectively. This means that the possibility of failure in the first option is low and for second one is very high. In contrast, user knowledge (C1) is a positive criterion and its value for first and second options is 5 and 3. This means that, the user knowledge for first option is acceptable and for second option is poor.

Since scales of indexes measurement are different, they should be expressed as non-scaling values. To do this, linear non-scaling method will be used as follow:

For positive indexes:

$$n_{ij} = \frac{r_{ij}}{max_i \ r_{ij}}$$

For negative indexes:

$$n_{ij} = \frac{1/r_{ij}}{max_i(1/r_{ij})} = \frac{min_i r_{ij}}{r_{ij}}$$

**Table 5.3** Non-scalar decision matrix

| | C1(+) | C2(+) | C3(-) | C4(+) | C5(+) | C6(-) | C7(+) | C8(-) | C9(-) | C10(+) |
|---|---|---|---|---|---|---|---|---|---|---|
| **O1** | 0.55556 | 0.55556 | 0.20000 | 1.00000 | 0.55556 | 0.14286 | 0.55556 | 1.00000 | 0.20000 | 1.00000 |
| **O2** | 0.33333 | 1.00000 | 0.33333 | 0.77778 | 0.33333 | 0.20000 | 1.00000 | 0.33333 | 1.00000 | 0.92757 |
| **O3** | 0.77778 | 0.11111 | 0.11111 | 1.00000 | 1.00000 | 0.33333 | 0.11111 | 0.33333 | 0.20000 | 0.89738 |
| **O4** | 0.55556 | 1.00000 | 0.33333 | 0.33333 | 1.00000 | 0.20000 | 1.00000 | 0.60000 | 0.33333 | 0.83602 |
| **O5** | 0.11111 | 0.33333 | 0.20000 | 1.00000 | 1.00000 | 0.14286 | 1.00000 | 0.33333 | 0.14286 | 0.81087 |
| **O6** | 0.11111 | 0.55556 | 0.33333 | 0.33333 | 0.77778 | 0.14286 | 0.77778 | 1.00000 | 0.20000 | 0.71831 |
| **O7** | 1.00000 | 0.33333 | 0.33333 | 1.00000 | 0.55556 | 1.00000 | 0.55556 | 1.00000 | 0.33333 | 0.68712 |
| **O8** | 0.55556 | 0.11111 | 0.20000 | 0.77778 | 1.00000 | 0.14286 | 1.00000 | 0.42857 | 0.14286 | 0.64688 |
| **O9** | 0.77778 | 0.77778 | 0.14286 | 0.55556 | 0.77778 | 0.11111 | 0.55556 | 0.33333 | 0.20000 | 0.62274 |
| **O10** | 0.33333 | 0.55556 | 0.33333 | 0.55556 | 0.33333 | 0.20000 | 0.77778 | 0.42857 | 0.33333 | 0.59054 |
| **O11** | 0.55556 | 1.00000 | 0.20000 | 0.77778 | 0.77778 | 0.11111 | 0.11111 | 0.60000 | 0.33333 | 0.55332 |
| **O12** | 0.55556 | 1.00000 | 1.00000 | 0.55556 | 1.00000 | 0.33333 | 0.77778 | 0.42857 | 0.20000 | 0.48390 |
| **O13** | 0.33333 | 1.00000 | 0.20000 | 0.55556 | 0.55556 | 0.33333 | 0.33333 | 0.42857 | 0.33333 | 0.47988 |
| **O14** | 0.55556 | 0.55556 | 0.20000 | 0.33333 | 1.00000 | 0.14286 | 0.55556 | 0.33333 | 0.20000 | 0.46479 |
| **O15** | 0.77778 | 0.55556 | 0.20000 | 1.00000 | 0.11111 | 0.33333 | 0.11111 | 0.60000 | 0.11111 | 0.44467 |
| **O16** | 0.33333 | 0.77778 | 0.20000 | 0.11111 | 0.11111 | 0.11111 | 0.77778 | 0.60000 | 0.14286 | 0.44467 |
| **O17** | 0.55556 | 0.33333 | 0.33333 | 1.00000 | 1.00000 | 0.11111 | 1.00000 | 1.00000 | 0.20000 | 0.42757 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **O18** | 0.33333 | 1.00000 | 1.00000 | 0.55556 | 0.77778 | 0.11111 | 0.33333 | 1.00000 | 0.33333 | 0.41046 |
| **O19** | 1.00000 | 0.77778 | 0.20000 | 0.55556 | 0.55556 | 0.14286 | 0.55556 | 0.60000 | 0.11111 | 0.37827 |
| **O20** | 0.11111 | 0.33333 | 0.14286 | 0.11111 | 0.77778 | 0.14286 | 0.11111 | 0.60000 | 0.20000 | 0.35211 |

## 5.1 Analysis with Entropy method

This method needs a decision matrix in order to evaluate weights. The idea of this technique is that more dispersion in index values, the more important it is. Based on this method steps, we can calculate weights of indexes as follow:

Step1) Calculate $P_{ij}$ (Section 3.2.1.5, Equation 11)

$$P_{ij} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}} \quad ; \forall j$$

Where $a_{ij}$ is an element of the decision matrix which represents the original value of the $j$*th* criterion of the $i$*th* alternative.

The raw data are normalized to eliminate anomalies with different measurement units and scales in order to allow for comparison of different criteria.

**Table 5.4** Normalize the decision matrix

| | **C1(+)** | **C2(+)** | **C3(-)** | **C4(+)** | **C5(+)** | **C6(-)** | **C7(+)** | **C8(-)** | **C9(-)** | **C10(+)** |
|---|---|---|---|---|---|---|---|---|---|---|
| **O1** | 0.05435 | 0.04386 | 0.03227 | 0.07759 | 0.03968 | 0.03182 | 0.04630 | 0.08347 | 0.03809 | 0.08212 |
| **O2** | 0.03261 | 0.07895 | 0.05379 | 0.06034 | 0.02381 | 0.04455 | 0.08333 | 0.02782 | 0.19045 | 0.07617 |
| **O3** | 0.07609 | 0.00877 | 0.01793 | 0.07759 | 0.07143 | 0.07426 | 0.00926 | 0.02782 | 0.03809 | 0.07369 |
| **O4** | 0.05435 | 0.07895 | 0.05379 | 0.02586 | 0.07143 | 0.04455 | 0.08333 | 0.05008 | 0.06348 | 0.06865 |
| **O5** | 0.01087 | 0.02632 | 0.03227 | 0.07759 | 0.07143 | 0.03182 | 0.08333 | 0.02782 | 0.02721 | 0.06659 |
| **O6** | 0.01087 | 0.04386 | 0.05379 | 0.02586 | 0.05556 | 0.03182 | 0.06481 | 0.08347 | 0.03809 | 0.05899 |
| **O7** | 0.09783 | 0.02632 | 0.05379 | 0.07759 | 0.03968 | 0.22277 | 0.04630 | 0.08347 | 0.06348 | 0.05643 |
| **O8** | 0.05435 | 0.00877 | 0.03227 | 0.06034 | 0.07143 | 0.03182 | 0.08333 | 0.03577 | 0.02721 | 0.05312 |
| **O9** | 0.07609 | 0.06140 | 0.02305 | 0.04310 | 0.05556 | 0.02475 | 0.04630 | 0.02782 | 0.03809 | 0.05114 |
| **O10** | 0.03261 | 0.04386 | 0.05379 | 0.04310 | 0.02381 | 0.04455 | 0.06481 | 0.03577 | 0.06348 | 0.04850 |
| **O11** | 0.05435 | 0.07895 | 0.03227 | 0.06034 | 0.05556 | 0.02475 | 0.00926 | 0.05008 | 0.06348 | 0.04544 |
| **O12** | 0.05435 | 0.07895 | 0.16137 | 0.04310 | 0.07143 | 0.07426 | 0.06481 | 0.03577 | 0.03809 | 0.03974 |
| **O13** | 0.03261 | 0.07895 | 0.03227 | 0.04310 | 0.03968 | 0.07426 | 0.02778 | 0.03577 | 0.06348 | 0.03941 |
| **O14** | 0.05435 | 0.04386 | 0.03227 | 0.02586 | 0.07143 | 0.03182 | 0.04630 | 0.02782 | 0.03809 | 0.03817 |
| **O15** | 0.07609 | 0.04386 | 0.03227 | 0.07759 | 0.00794 | 0.07426 | 0.00926 | 0.05008 | 0.02116 | 0.03652 |
| **O16** | 0.03261 | 0.06140 | 0.03227 | 0.00862 | 0.00794 | 0.02475 | 0.06481 | 0.05008 | 0.02721 | 0.03652 |
| **O17** | 0.05435 | 0.02632 | 0.05379 | 0.07759 | 0.07143 | 0.02475 | 0.08333 | 0.08347 | 0.03809 | 0.03511 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **O18** | 0.03261 | 0.07895 | 0.16137 | 0.04310 | 0.05556 | 0.02475 | 0.02778 | 0.08347 | 0.06348 | 0.03371 |
| **O19** | 0.09783 | 0.06140 | 0.03227 | 0.04310 | 0.03968 | 0.03182 | 0.04630 | 0.05008 | 0.02116 | 0.03106 |
| **O20** | 0.01087 | 0.02632 | 0.02305 | 0.00862 | 0.05556 | 0.03182 | 0.00926 | 0.05008 | 0.03809 | 0.02892 |

Step2) Calculate the entropy of $E_j$ (Section 3.2.1.5, Equation 12)

$$E_j = \left(\frac{-1}{\ln(m)}\right)\sum_{i=1}^{m}[P_{ij} lnP_{ij}] \quad ; \forall j$$

Where, $\left(\frac{-1}{\ln(m)}\right)$ is the entropy constant.

**Table 5.5** Computation of Entropy

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Ej=** | 0.953461 | 0.957482 | 0.926392 | 0.960365 | 0.964491 | 0.917464 | 0.94418874 | 0.971673 | 0.941029 | 0.983549 |

Step3) Calculate of uncertainty $d_j$ as the degree of diversification (Section 3.2.1.5, Equation 13).

$$d_j = 1 - E_j \quad ; \forall j$$

**Table 5.6** Calculation of uncertainty

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **dj=** | 0.046539 | 0.042518 | 0.073608 | 0.039635 | 0.035509 | 0.082536 | 0.05581126 | 0.028327 | 0.058971 | 0.016451 |

Step4) Calculate of weights ($W_j$) as the degree of importance of attribute $j$ (Section 3.2.1.5, Equation 14).

$$W_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \quad ; \forall j$$

**Table 5.7** Calculation of weights

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 |
|---|---|---|---|---|---|---|---|---|---|---|
| | **User Knowledge** | **User Strategy** | **Time Pressure** | **Resource Availability** | **External Information Sources** | **Possibility of Failure** | **Trust** | **Complexity** | **Cost** | **Expected Performance** |
| **Wj=** | 0.096975 | 0.088596 | 0.153381 | 0.082589 | 0.073991 | 0.171984 | 0.11629657 | 0.059026 | 0.12288 | 0.03428 |

Since each criterion has a different meaning, it cannot be assumed that they all have equal weights. As a result, finding the appropriate weight for each criterion is one the main points in MADM techniques. To do this, the weights of criteria were determined by solving a mathematical model as Entropy without any consideration of the decision maker's preferences in table 5.7. The highest weights of criteria are sixth (possibility of failure) and third (time pressure) cases with 0.171984 and 0.153381, respectively. In contrast, the lowest weights of criteria are tenth (expected performance) and eighth (complexity) cases with 0.03428 and 0.059026, respectively. Obtained weights of criteria are prerequisite step in order to calculation of MADM techniques which will be used in sections 5.3.1 and 5.4.1.

## 5.2 Analysis with AHP technique

For constructing a pair-wise comparison matrix to determine important factors of each index, an expert user idea has been used. One important thing is about comparisons compatibility. This means that inconsistent expert judgment can be a factor when using the pair-wise comparison method (Udo et al., 2010). In addition, data will be analyzed by Expert Choice 2000 software which performs AHP formula and it calculates and displays the Inconsistency Ratio (IR) of the AHP technique in order to solve inconsistent expert judgment. The IR provides a measure of the logical rationality of the pair-wise comparisons, and IR value less than 0.10 is generally considered acceptable. The weight of each index is in table 5.8 (IR=0.09). A sample of implementation of AHP technique in Expert Choice 2000 software is shown in figure 5.2.

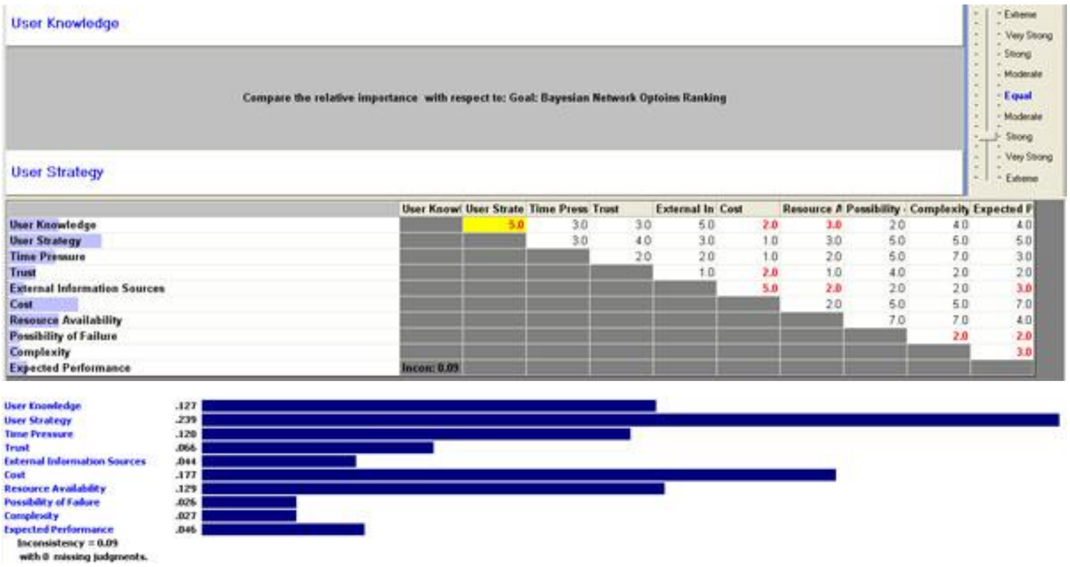


**Figure 5.2** Sample of AHP implementation steps in Expert Choice 2000 software

**Table 5.8** Weights of Indexes by AHP Technique

| Criteria | Weight |
|---|---|
| User strategy | 0.234 |
| Cost | 0.196 |
| Resource Availability | 0.128 |
| User Knowledge | 0.124 |

| | |
|---|---|
| Time Pressure | 0.109 |
| Trust | 0.066 |
| Expected Performance | 0.046 |
| External Information Sources | 0.044 |
| Possibility of Failure | 0.027 |
| Complexity | 0.026 |

The weights of criteria were determined with AHP method via expert choice software with consideration of the decision maker's preferences in table 5.8. The highest weights of criteria are user strategy and cost. In contrast, the lowest weights of criteria are complexity and possibility of failure. Obtained weights of criteria are prerequisite step in order to calculation of MADM techniques which will be used in sections 5.3.2 and 5.4.2.

## 5.3 Analysis with SAW technique

Results of this technique are dependent on the calculated weights of entropy and AHP methods in subsections 1 and 2.

### 5.3.1. Analysis of SAW technique with Entropy

According to SAW formula as equation 1 in section 3.2.1.1, the rank result based on Entropy method has been shown in table 5.9. For calculation, table 5.2 (decision matrix) should be multiplied to table 5.7 (weights of criteria).

**Table 5.9** SAW ranking with Entropy

| List of Options | SAW (Entropy) | Rank |
|---|---|---|
| **O1** | 0.05191217 | **8** |
| **O2** | 0.047332096 | **12** |
| **O3** | 0.051400567 | **10** |
| **O4** | 0.050340853 | **11** |
| **O5** | 0.059475669 | **3** |
| **O6** | 0.044064209 | **16** |
| **O7** | 0.038650188 | **20** |
| **O8** | 0.058727395 | **4** |
| **O9** | 0.063223955 | **1** |
| **O10** | 0.041240005 | **17** |
| **O11** | 0.052611899 | **7** |
| **O12** | 0.046831505 | **13** |
| **O13** | 0.041203651 | **18** |

| | | |
|---|---|---|
| **O14** | 0.053218364 | **6** |
| **O15** | 0.045546567 | **14** |
| **O16** | 0.051414143 | **9** |
| **O17** | 0.057675904 | **5** |
| **O18** | 0.044344747 | **15** |
| **O19** | 0.059797495 | **2** |
| **O20** | 0.040988616 | **19** |

### 5.3.2. Analysis of SAW technique with AHP

According to SAW formula as equation 1 in section 3.2.1.1, the rank result based on AHP method has been shown in table 5.10. For calculation, table 5.2 (decision matrix) should be multiplied to table 5.8 (weights of criteria).

**Table 5.10** SAW ranking with AHP

| List of Options | SAW (AHP) | Rank |
|---|---|---|
| **O1** | 0.051989364 | **9** |
| **O2** | 0.055967309 | **7** |
| **O3** | 0.040279727 | **16** |
| **O4** | 0.057826494 | **5** |
| **O5** | 0.057374202 | **6** |
| **O6** | 0.046551498 | **14** |
| **O7** | 0.040264245 | **17** |
| **O8** | 0.057839463 | **4** |
| **O9** | 0.06231822 | **1** |
| **O10** | 0.045171676 | **15** |
| **O11** | 0.05104522 | **11** |
| **O12** | 0.04961425 | **12** |
| **O13** | 0.040230901 | **18** |
| **O14** | 0.051140686 | **10** |
| **O15** | 0.037471967 | **19** |
| **O16** | 0.054335796 | **8** |
| **O17** | 0.061618687 | **2** |
| **O18** | 0.047145512 | **13** |
| **O19** | 0.057932684 | **3** |
| **O20** | 0.0338821 | **20** |

## 5.4 Analysis with TOPSIS technique

Results of this technique are included of calculated weights by Entropy and AHP methods in subsections 5.4.1 and 5.4.2.

### 5.4.1. Analysis of TOPSIS technique with Entropy

1) Normalize the decision matrix and multiple weights to normalized matrix (Section 3.2.1.2, Equation 2):

**Table 5.11** Normalized decision matrix with TOPSIS-Entropy

| | C1(+) | C2(+) | C3(-) | C4(+) | C5(+) | C6(-) | C7(+) | C8(-) | C9(-) | C10(+) |
|---|---|---|---|---|---|---|---|---|---|---|
| **O1** | 0.55556 | 0.55556 | 0.20000 | 1.00000 | 0.55556 | 0.14286 | 0.55556 | 1.00000 | 0.20000 | 1.00000 |
| **O2** | 0.33333 | 1.00000 | 0.33333 | 0.77778 | 0.33333 | 0.20000 | 1.00000 | 0.33333 | 1.00000 | 0.92757 |
| **O3** | 0.77778 | 0.11111 | 0.11111 | 1.00000 | 1.00000 | 0.33333 | 0.11111 | 0.33333 | 0.20000 | 0.89738 |
| **O4** | 0.55556 | 1.00000 | 0.33333 | 0.33333 | 1.00000 | 0.20000 | 1.00000 | 0.60000 | 0.33333 | 0.83602 |
| **O5** | 0.11111 | 0.33333 | 0.20000 | 1.00000 | 1.00000 | 0.14286 | 1.00000 | 0.33333 | 0.14286 | 0.81087 |
| **O6** | 0.11111 | 0.55556 | 0.33333 | 0.33333 | 0.77778 | 0.14286 | 0.77778 | 1.00000 | 0.20000 | 0.71831 |
| **O7** | 1.00000 | 0.33333 | 0.33333 | 1.00000 | 0.55556 | 1.00000 | 0.55556 | 1.00000 | 0.33333 | 0.68712 |
| **O8** | 0.55556 | 0.11111 | 0.20000 | 0.77778 | 1.00000 | 0.14286 | 1.00000 | 0.42857 | 0.14286 | 0.64688 |
| **O9** | 0.77778 | 0.77778 | 0.14286 | 0.55556 | 0.77778 | 0.11111 | 0.55556 | 0.33333 | 0.20000 | 0.62274 |
| **O10** | 0.33333 | 0.55556 | 0.33333 | 0.55556 | 0.33333 | 0.20000 | 0.77778 | 0.42857 | 0.33333 | 0.59054 |
| **O11** | 0.55556 | 1.00000 | 0.20000 | 0.77778 | 0.77778 | 0.11111 | 0.11111 | 0.60000 | 0.33333 | 0.55332 |
| **O12** | 0.55556 | 1.00000 | 1.00000 | 0.55556 | 1.00000 | 0.33333 | 0.77778 | 0.42857 | 0.20000 | 0.48390 |
| **O13** | 0.33333 | 1.00000 | 0.20000 | 0.55556 | 0.55556 | 0.33333 | 0.33333 | 0.42857 | 0.33333 | 0.47988 |
| **O14** | 0.55556 | 0.55556 | 0.20000 | 0.33333 | 1.00000 | 0.14286 | 0.55556 | 0.33333 | 0.20000 | 0.46479 |
| **O15** | 0.77778 | 0.55556 | 0.20000 | 1.00000 | 0.11111 | 0.33333 | 0.11111 | 0.60000 | 0.11111 | 0.44467 |
| **O16** | 0.33333 | 0.77778 | 0.20000 | 0.11111 | 0.11111 | 0.11111 | 0.77778 | 0.60000 | 0.14286 | 0.44467 |
| **O17** | 0.55556 | 0.33333 | 0.33333 | 1.00000 | 1.00000 | 0.11111 | 1.00000 | 1.00000 | 0.20000 | 0.42757 |
| **O18** | 0.33333 | 1.00000 | 1.00000 | 0.55556 | 0.77778 | 0.11111 | 0.33333 | 1.00000 | 0.33333 | 0.41046 |
| **O19** | 1.00000 | 0.77778 | 0.20000 | 0.55556 | 0.55556 | 0.14286 | 0.55556 | 0.60000 | 0.11111 | 0.37827 |
| **O20** | 0.11111 | 0.33333 | 0.14286 | 0.11111 | 0.77778 | 0.14286 | 0.11111 | 0.60000 | 0.20000 | 0.35211 |

2) Determine the positive and negative ideal alternatives (Section 3.2.1.2, Equations 3 and 4):

**Table 5.12** Positive and negative ideal alternatives with TOPSIS-Entropy

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **A+** | 0.037840 | 0.028262 | 0.009711 | 0.026085 | 0.021813 | 0.014367 | 0.038270 | 0.006768 | 0.009484 | 0.011996 |
| **A-** | 0.004204 | 0.003140 | 0.087399 | 0.002898 | 0.002424 | 0.129306 | 0.004252 | 0.020305 | 0.085354 | 0.004224 |

3) obtain the distance (based on Euclidean distance) of the existing alternatives from ideal and negative alternatives. This means that, the separation between alternatives will be found according to distance measure called normalized Euclidean distance (Szmidt & Kacprzyk, 2000), which shown in section 3.2.1.2 in equations 5 and 6.

**Table 5.13** Euclidean distance with TOPSIS-Entropy

| **List of Options** | $d_{i^+}$ | $d_{i^-}$ |
|---|---|---|
| **O1** | 0.03379 | 0.15250 |
| **O2** | 0.08454 | 0.12850 |
| **O3** | 0.05238 | 0.14109 |
| **O4** | 0.03857 | 0.14112 |
| **O5** | 0.03970 | 0.15764 |
| **O6** | 0.04831 | 0.14634 |
| **O7** | 0.12200 | 0.09349 |
| **O8** | 0.03246 | 0.15745 |
| **O9** | 0.02536 | 0.15908 |
| **O10** | 0.04593 | 0.13607 |
| **O11** | 0.04444 | 0.15112 |
| **O12** | 0.08632 | 0.11961 |
| **O13** | 0.05295 | 0.12922 |
| **O14** | 0.03476 | 0.15218 |
| **O15** | 0.05217 | 0.13955 |
| **O16** | 0.04246 | 0.15683 |
| **O17** | 0.03610 | 0.15383 |
| **O18** | 0.08985 | 0.13255 |
| **O19** | 0.02682 | 0.15811 |
| **O20** | 0.05811 | 0.15125 |

4, 5) First Calculate the relative closeness to the ideal alternatives, second rank the alternatives based on $cl_{i^+}$ (Section 3.2.1.2, Equation 7):

**Table 5.14** Rank the alternatives with TOPSIS-Entropy

| List of Options | $cl_{i^+}$ | Rank |
|---|---|---|
| **O1** | 0.81861 | **4** |
| **O2** | 0.60315 | **17** |
| **O3** | 0.72925 | **13** |
| **O4** | 0.78536 | **9** |
| **O5** | 0.79882 | **7** |
| **O6** | 0.75182 | **11** |
| **O7** | 0.43386 | **20** |
| **O8** | 0.82907 | **3** |
| **O9** | 0.86250 | **1** |
| **O10** | 0.74764 | **12** |
| **O11** | 0.77274 | **10** |
| **O12** | 0.58084 | **19** |
| **O13** | 0.70932 | **16** |
| **O14** | 0.81406 | **5** |
| **O15** | 0.72790 | **14** |
| **O16** | 0.78693 | **8** |
| **O17** | 0.80994 | **6** |
| **O18** | 0.59599 | **18** |
| **O19** | 0.85498 | **2** |
| **O20** | 0.72244 | **15** |

### 5.4.2. Analysis of TOPSIS technique with AHP

1) Normalize the decision matrix and multiple weights to normalized matrix (Section 3.2.1.2, Equation 2):

**Table 5.15** Normalized decision matrix with TOPSIS-AHP

| | C1(+) | C2(+) | C3(-) | C4(+) | C5(+) | C6(-) | C7(+) | C8(-) | C9(-) | C10(+) |
|---|---|---|---|---|---|---|---|---|---|---|
| **O1** | 0.55556 | 0.55556 | 0.20000 | 1.00000 | 0.55556 | 0.14286 | 0.55556 | 1.00000 | 0.20000 | 1.00000 |
| **O2** | 0.33333 | 1.00000 | 0.33333 | 0.77778 | 0.33333 | 0.20000 | 1.00000 | 0.33333 | 1.00000 | 0.92757 |
| **O3** | 0.77778 | 0.11111 | 0.11111 | 1.00000 | 1.00000 | 0.33333 | 0.11111 | 0.33333 | 0.20000 | 0.89738 |
| **O4** | 0.55556 | 1.00000 | 0.33333 | 0.33333 | 1.00000 | 0.20000 | 1.00000 | 0.60000 | 0.33333 | 0.83602 |
| **O5** | 0.11111 | 0.33333 | 0.20000 | 1.00000 | 1.00000 | 0.14286 | 1.00000 | 0.33333 | 0.14286 | 0.81087 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **O6** | 0.11111 | 0.55556 | 0.33333 | 0.33333 | 0.77778 | 0.14286 | 0.77778 | 1.00000 | 0.20000 | 0.71831 |
| **O7** | 1.00000 | 0.33333 | 0.33333 | 1.00000 | 0.55556 | 1.00000 | 0.55556 | 1.00000 | 0.33333 | 0.68712 |
| **O8** | 0.55556 | 0.11111 | 0.20000 | 0.77778 | 1.00000 | 0.14286 | 1.00000 | 0.42857 | 0.14286 | 0.64688 |
| **O9** | 0.77778 | 0.77778 | 0.14286 | 0.55556 | 0.77778 | 0.11111 | 0.55556 | 0.33333 | 0.20000 | 0.62274 |
| **O10** | 0.33333 | 0.55556 | 0.33333 | 0.55556 | 0.33333 | 0.20000 | 0.77778 | 0.42857 | 0.33333 | 0.59054 |
| **O11** | 0.55556 | 1.00000 | 0.20000 | 0.77778 | 0.77778 | 0.11111 | 0.11111 | 0.60000 | 0.33333 | 0.55332 |
| **O12** | 0.55556 | 1.00000 | 1.00000 | 0.55556 | 1.00000 | 0.33333 | 0.77778 | 0.42857 | 0.20000 | 0.48390 |
| **O13** | 0.33333 | 1.00000 | 0.20000 | 0.55556 | 0.55556 | 0.33333 | 0.33333 | 0.42857 | 0.33333 | 0.47988 |
| **O14** | 0.55556 | 0.55556 | 0.20000 | 0.33333 | 1.00000 | 0.14286 | 0.55556 | 0.33333 | 0.20000 | 0.46479 |
| **O15** | 0.77778 | 0.55556 | 0.20000 | 1.00000 | 0.11111 | 0.33333 | 0.11111 | 0.60000 | 0.11111 | 0.44467 |
| **O16** | 0.33333 | 0.77778 | 0.20000 | 0.11111 | 0.11111 | 0.11111 | 0.77778 | 0.60000 | 0.14286 | 0.44467 |
| **O17** | 0.55556 | 0.33333 | 0.33333 | 1.00000 | 1.00000 | 0.11111 | 1.00000 | 1.00000 | 0.20000 | 0.42757 |
| **O18** | 0.33333 | 1.00000 | 1.00000 | 0.55556 | 0.77778 | 0.11111 | 0.33333 | 1.00000 | 0.33333 | 0.41046 |
| **O19** | 1.00000 | 0.77778 | 0.20000 | 0.55556 | 0.55556 | 0.14286 | 0.55556 | 0.60000 | 0.11111 | 0.37827 |
| **O20** | 0.11111 | 0.33333 | 0.14286 | 0.11111 | 0.77778 | 0.14286 | 0.11111 | 0.60000 | 0.20000 | 0.35211 |

2) Determine the positive and negative ideal alternatives (Section 3.2.1.2, Equations 3 and 4):

**Table 5.16** Positive and negative ideal alternatives with TOPSIS-AHP

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **A+** | 0.048385 | 0.040832 | 0.006901 | 0.020845 | 0.012971 | 0.016374 | 0.077003 | 0.002981 | 0.002084 | 0.016098 |
| **A-** | 0.005376 | 0.004537 | 0.062110 | 0.002316 | 0.001441 | 0.147363 | 0.008556 | 0.008944 | 0.018755 | 0.005668 |

3) obtain the distance (based on Euclidean distance) of the existing alternatives from ideal and negative alternatives (Section 3.2.1.2, Equations 5 and 6):

**Table 5.17** Euclidean with TOPSIS-AHP

| List of Options | $d_{i^+}$ | $d_{i^-}$ |
|---|---|---|
| **O1** | 0.04568 | 0.14525 |
| **O2** | 0.04217 | 0.14850 |
| **O3** | 0.08483 | 0.12064 |
| **O4** | 0.03237 | 0.14987 |
| **O5** | 0.05150 | 0.15497 |
| **O6** | 0.05426 | 0.14488 |
| **O7** | 0.13919 | 0.07335 |

| | | |
|---|---|---|
| **O8** | 0.04344 | 0.15558 |
| **O9** | 0.03882 | 0.15292 |
| **O10** | 0.04740 | 0.13778 |
| **O11** | 0.07268 | 0.14783 |
| **O12** | 0.07095 | 0.12059 |
| **O13** | 0.07061 | 0.11896 |
| **O14** | 0.04781 | 0.14424 |
| **O15** | 0.08032 | 0.11885 |
| **O16** | 0.04481 | 0.15295 |
| **O17** | 0.03895 | 0.15750 |
| **O18** | 0.08343 | 0.13857 |
| **O19** | 0.03913 | 0.15032 |
| **O20** | 0.08815 | 0.13851 |

4, 5) First Calculate the relative closeness to the ideal alternatives, second rank the alternatives based on $cl_{i^+}$ (Section 3.2.1.2, Equation 7):

**Table 5.18** Rank the alternatives with TOPSIS-AHP

| List of Options | $cl_{i^+}$ | Rank |
|---|---|---|
| **O1** | 0.76073 | **8** |
| **O2** | 0.77885 | **6** |
| **O3** | 0.58715 | **19** |
| **O4** | 0.82239 | **1** |
| **O5** | 0.75055 | **10** |
| **O6** | 0.72751 | **12** |
| **O7** | 0.34511 | **20** |
| **O8** | 0.78173 | **5** |
| **O9** | 0.79755 | **3** |
| **O10** | 0.74403 | **11** |
| **O11** | 0.67040 | **13** |
| **O12** | 0.62957 | **14** |
| **O13** | 0.62753 | **15** |
| **O14** | 0.75106 | **9** |
| **O15** | 0.59674 | **18** |

| | | |
|---|---|---|
| **O16** | 0.77339 | **7** |
| **O17** | 0.80172 | **2** |
| **O18** | 0.62418 | **16** |
| **O19** | 0.79347 | **4** |
| **O20** | 0.61108 | **17** |

## 5.5 Results of the Study

Since different methods provide different results, decision-makers use more than one technique in important decisions. Consequently, in order to overcome to this problem, we have utilized a mixed method as Rank Average Method (Soltanpanah et al., 2010). The rank results are shown in table 5.19 and Figure 5.3.

**Table 5.19** Results of the study

| List of Options | SAW (AHP) | SAW (Entropy) | TOPSIS (AHP) | TOPSIS (Entropy) | Mixed Method |
|---|---|---|---|---|---|
| **O1** | 9 | 8 | 8 | 4 | 7 |
| **O2** | 7 | 12 | 6 | 17 | 11 |
| **O3** | 16 | 10 | 19 | 13 | 15 |
| **O4** | 5 | 11 | 1 | 9 | 6 |
| **O5** | 6 | 3 | 10 | 7 | 5 |
| **O6** | 14 | 16 | 12 | 11 | 12 |
| **O7** | 17 | 20 | 20 | 20 | 20 |
| **O8** | 4 | 4 | 5 | 3 | 4 |
| **O9** | 1 | 1 | 3 | 1 | 1 |
| **O10** | 15 | 17 | 11 | 12 | 13 |
| **O11** | 11 | 7 | 13 | 10 | 10 |
| **O12** | 12 | 13 | 14 | 19 | 14 |
| **O13** | 18 | 18 | 15 | 16 | 18 |
| **O14** | 10 | 6 | 9 | 5 | 8 |
| **O15** | 19 | 14 | 18 | 14 | 17 |
| **O16** | 8 | 9 | 7 | 8 | 9 |
| **O17** | 2 | 5 | 2 | 6 | 3 |
| **O18** | 13 | 15 | 16 | 18 | 16 |
| **O19** | 3 | 2 | 4 | 2 | 2 |
| **O20** | 20 | 19 | 17 | 15 | 19 |

The obtained re-ranking results from five different techniques are very different as compared to ranking only based on expected performance. This means that, by user interaction we could evaluate every twenty options with ten criteria as semi-automatically decision support. For example, first, second, third, and fourth options in initial ranking have been changed to seventh, eleventh, fifteenth, and sixth ranked in average re-ranking. Hence, the first five best options regarding to user interaction are ninth, nineteenth, seventeenth, eighth, and fifth options.

In spite of obtained different results from five different techniques, we cannot conclude which one is better and more acceptable than others. Because we need to investigate and evaluate these results with some historical real cases. Since there is no historical real case in order to investigate all MADM techniques with real decisions made, in this research two strong and important techniques according to their characteristics and positive results from previous researches have been used. Obviously, previous decisions made are helpful to evaluate which techniques are more close to real decisions and which ones are not.

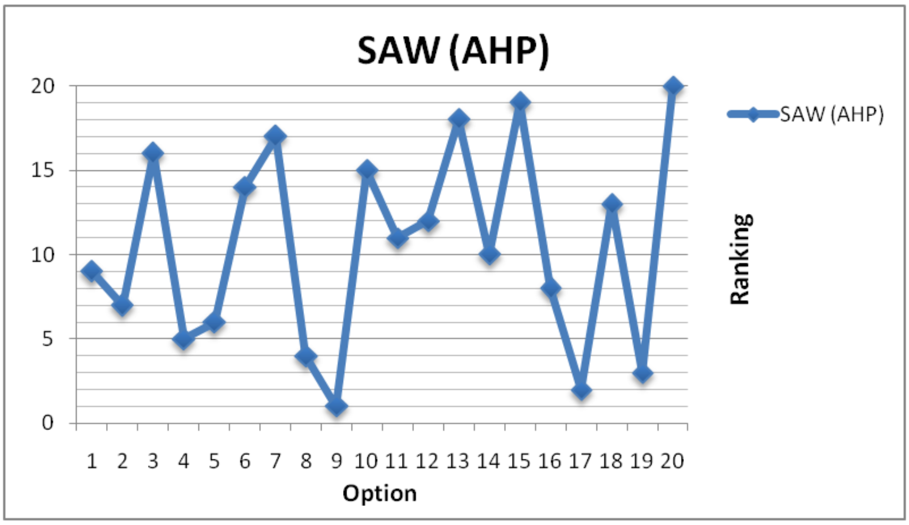


**Figure 5.3** Results of the rank from SAW (AHP) technique

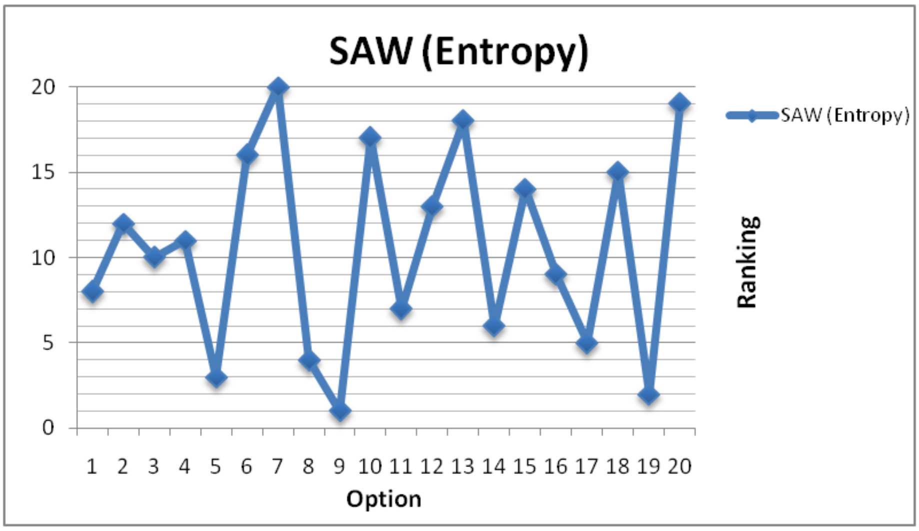


**Figure 5.4** Results of the rank from SAW (Entropy) technique

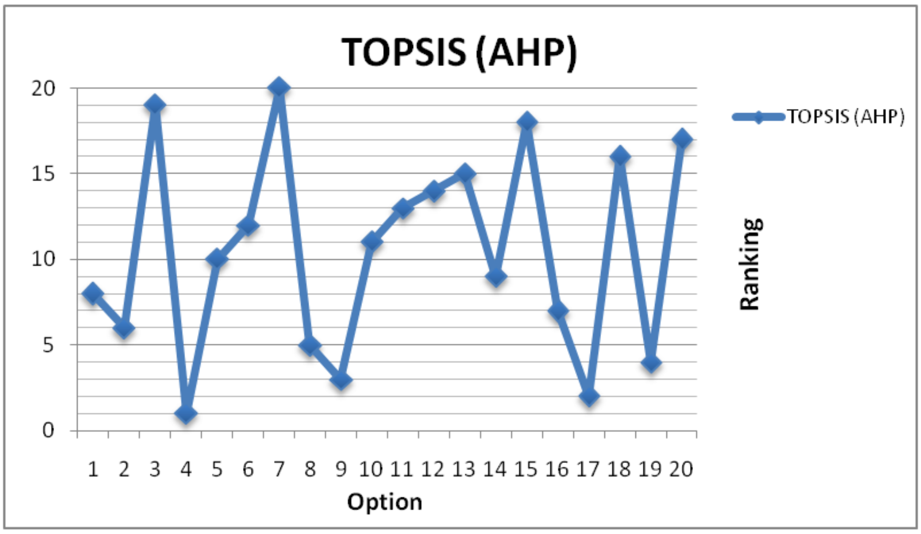


**Figure 5.5** Results of the rank from TOPSIS (AHP) technique

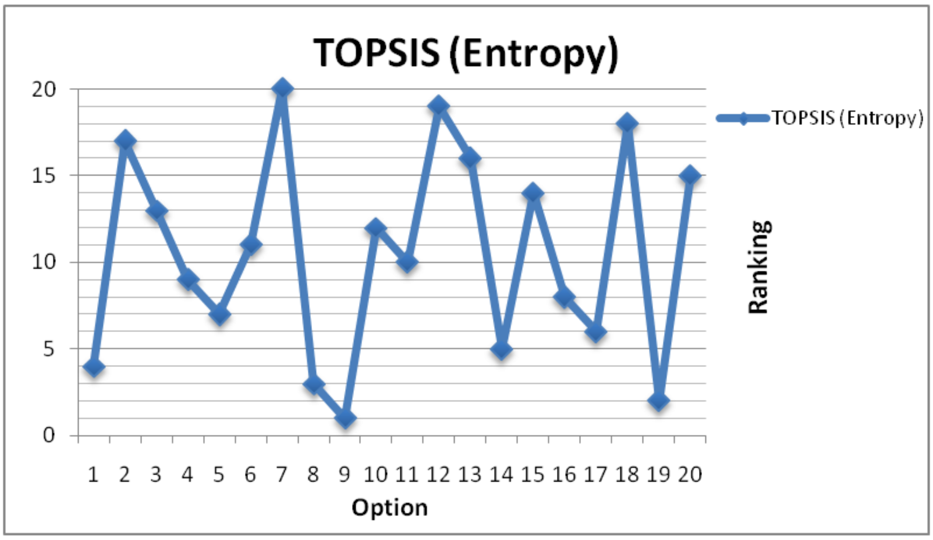


**Figure 5.6** Results of the rank from TOPSIS (Entropy) technique

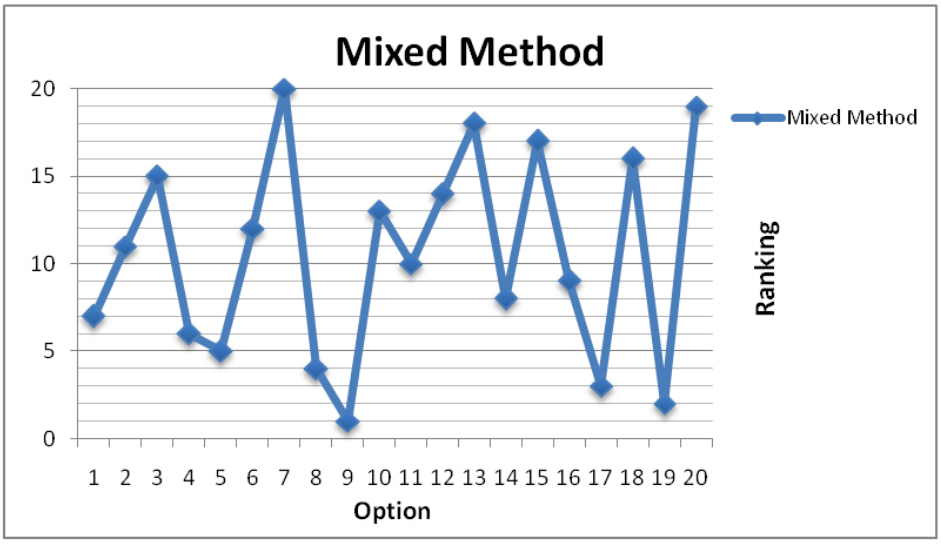


**Figure 5.7** Results of the rank from Mixed Method

The given line graphs (figures 5.3 to 5.7) illustrate information about the final results from five different techniques for ranking twenty options. Trend of variations in five applied techniques in line graphs are similar. For example, second option is ranked to 7th, 12th, 6th, 17th, and 11th from SAW-AHP (figure 5.3), SAW-Entropy (figure 5.4), TOPSIS-AHP (figure 5.5), TOPSIS-Entropy (figure 5.6), and Mixed Method (figure 5.7) respectively. Moreover, fourth option as the highest variation is ranked to 1st, 5th, 6th, 9th, and 11th from TOPSIS-AHP (figure 5.5), SAW-AHP (figure 5.3), Mixed Method (figure 5.7), TOPSIS-Entropy (figure 5.6), and SAW-Entropy (figure 5.4) respectively. In contrast, ninth option as one of the lowest variation is ranked to 3rd from TOPSIS-AHP and 1st from four other techniques. Comparison of results via statistical tests can be helpful in order to answer the research question. Because statistical tests can prove that is there strong correlation and relationship between different results from applied techniques or not?

## 5.6 Comparison of Results

In this study, five different results according to five different techniques have been obtained. So, differences between their results should be investigated. Hence, three statistical tests as Pearson Correlation, Kendall's tau, and Spearman Rank Correlation will be used in order to answer to research question. Pearson Correlation is widely used to measure the relationship degree between the two variables. It is same as the Spearman Rank Correlation which measures the strength of association of two variables. Kendall’s Tau-b rank correlation states the strength of the dependence in paired observations. If most of the ranked scores are same, Kendall correlation should be used. Kendall's tau provides value between [-1 +1] which a positive correlation indicates that the ranks of both variables increase together while a negative correlation indicates that the rank of one variable increases and the other one decreases. Moreover, in the most conditions the values of Spearman and Kendall's tau are very close to each other and will probably lead to the same outcomes.

Briefly, positive correlation indicates a positive relationship between two variables (the larger A, the larger B) while a negative correlation states a negative relationship between two variables (the larger A, the smaller B). Correlation is significant when *P*-value is less than hypothetical error level (0.01 or 0.05), so we can reject the null hypothesis of no association and state that there is an association between two compared techniques; otherwise, there is no association and null hypothesis is accepted. In this study, null hypothesis is that there is no relation between the applied techniques for ranking Bayesian Network options. All statistical analyses have been implemented in IBM SPSS Statistics version 19. Furthermore, there are different interpretations of SPSS statistic results and Pallant (2007) suggested that correlation results can be interpreted as follows:

- $r = 0.10$ to $0.29$ small correlation.
- $r = 0.30$ to $0.49$ medium correlation.
- $r = 0.50$ to $1.00$ high correlation.

In tables 5.20, 5.21, and 5.22, (a) a value for Pearson Correlation, Kendall’s tau_b, and Spearman's rho, (b) a Sig. (2-tailed) value and (c) a number ‘N’ value are shown. For example, you can find the Pearson’s r statistic on the top of each box in table 5.20. The Pearson’s r for the correlation between the TOPSIS_AHP and SAW_AHP variables is 0.913. Conclusion is that there is a strong relationship between two methods. However, we cannot make any other conclusions about this relationship, based on this value. In contrast, when correlation between two variables is close to 0, there is a weak relationship between two variables. This means that changes in one variable are not correlated with changes in the second variable.

Sig. (2-tailed) value tells us if there is a statistically significant correlation between two variables. If it is less than or equal to 0.05, conclusion is that there is a statistically significant correlation between two variables. So, increase or decrease in one variable is significantly related to increase or decrease in second variable. If the Sig. (2-tailed) value is greater than 0.05, conclusion is that there is no statistically significant correlation between two variables. So, increases or decreases in one variable is not significantly related to increases or decreases in second variable.

From obtained results, the Sig. (2-Tailed) values are 0, 0.001, and 0.003. These values are less than 0.05. So, we can conclude that there is a statistically significant correlation between compared paired techniques.

**Table 5.20** Results of Pearson Correlation

| | | TOPSIS_AHP | TOPSIS_ Entropy | SAW_AHP | SAW_Entropy | Mixed |
|---|---|---|---|---|---|---|
| TOPSIS_AHP | Pearson Correlation | 1 | .735** | .913** | .687** | .899** |
| | Sig. (2-tailed) | | .000 | .000 | .001 | .000 |
| | N | 20 | 20 | 20 | 20 | 20 |
| TOPSIS_Entropy | Pearson Correlation | .735** | 1 | .737** | .851** | .893** |
| | Sig. (2-tailed) | .000 | | .000 | .000 | .000 |
| | N | 20 | 20 | 20 | 20 | 20 |
| SAW_AHP | Pearson Correlation | .913** | .737** | 1 | .851** | .950** |
| | Sig. (2-tailed) | .000 | .000 | | .000 | .000 |
| | N | 20 | 20 | 20 | 20 | 20 |
| SAW_Entropy | Pearson Correlation | .687** | .851** | .851** | 1 | .913** |
| | Sig. (2-tailed) | .001 | .000 | .000 | | .000 |
| | N | 20 | 20 | 20 | 20 | 20 |
| Mixed | Pearson Correlation | .899** | .893** | .950** | .913** | 1 |
| | Sig. (2-tailed) | .000 | .000 | .000 | .000 | |
| | N | 20 | 20 | 20 | 20 | 20 |

**. Correlation is significant at the 0.01 level (2-tailed).

**Table 5.21** Results of Kendall's tau_b

| | | | TOPSIS_AHP | TOPSIS_ Entropy | SAW_AHP | SAW_Entropy | Mixed |
|---|---|---|---|---|---|---|---|
| Kendall's tau_b | TOPSIS_AHP | Correlation Coefficient | 1.000 | .537** | .747** | .484** | .716** |
| | | Sig. (2-tailed) | . | .001 | .000 | .003 | .000 |
| | | N | 20 | 20 | 20 | 20 | 20 |
| | TOPSIS_Entropy | Correlation Coefficient | .537** | 1.000 | .579** | .674** | .758** |
| | | Sig. (2-tailed) | .001 | . | .000 | .000 | .000 |
| | | N | 20 | 20 | 20 | 20 | 20 |
| | SAW_AHP | Correlation Coefficient | .747** | .579** | 1.000 | .674** | .821** |
| | | Sig. (2-tailed) | .000 | .000 | . | .000 | .000 |
| | | N | 20 | 20 | 20 | 20 | 20 |
| | SAW_Entropy | Correlation Coefficient | .484** | .674** | .674** | 1.000 | .768** |
| | | Sig. (2-tailed) | .003 | .000 | .000 | . | .000 |
| | | N | 20 | 20 | 20 | 20 | 20 |
| | Mixed | Correlation Coefficient | .716** | .758** | .821** | .768** | 1.000 |
| | | Sig. (2-tailed) | .000 | .000 | .000 | .000 | . |
| | | N | 20 | 20 | 20 | 20 | 20 |

**. Correlation is significant at the 0.01 level (2-tailed).

**Table 5.22** Results of Spearman's rho

| | | | TOPSIS_AHP | TOPSIS_Entropy | SAW_AHP | SAW_Entropy | Mixed |
|---|---|---|---|---|---|---|---|
| Spearman's rho | TOPSIS_AHP | Correlation Coefficient | 1.000 | .735** | .913** | .687** | .899** |
| | | Sig. (2-tailed) | . | .000 | .000 | .001 | .000 |
| | | N | 20 | 20 | 20 | 20 | 20 |
| | TOPSIS_Entropy | Correlation Coefficient | .735** | 1.000 | .737** | .851** | .893** |
| | | Sig. (2-tailed) | .000 | . | .000 | .000 | .000 |
| | | N | 20 | 20 | 20 | 20 | 20 |
| | SAW_AHP | Correlation Coefficient | .913** | .737** | 1.000 | .851** | .950** |
| | | Sig. (2-tailed) | .000 | .000 | . | .000 | .000 |
| | | N | 20 | 20 | 20 | 20 | 20 |
| | SAW_Entropy | Correlation Coefficient | .687** | .851** | .851** | 1.000 | .913** |
| | | Sig. (2-tailed) | .001 | .000 | .000 | . | .000 |
| | | N | 20 | 20 | 20 | 20 | 20 |
| | Mixed | Correlation Coefficient | .899** | .893** | .950** | .913** | 1.000 |
| | | Sig. (2-tailed) | .000 | .000 | .000 | .000 | . |
| | | N | 20 | 20 | 20 | 20 | 20 |

**. Correlation is significant at the 0.01 level (2-tailed).

For better evaluation of statistical tests from tables 5.20, 5.21, and 5.22, these results are shown in tables 5.23 and 5.24.

**Table 5.23** Results of Pearson and Spearman Correlation

| **Paired Comparison** | **Correlation Coefficient** |
|---|---|
| **SAW (AHP)-Mixed** | 0.950 |
| **SAW (Entropy)-Mixed** | 0.913 |
| **SAW (AHP)-TOPSIS (AHP)** | 0.913 |
| **TOPSIS (AHP)-Mixed** | 0.899 |
| **TOPSIS (Entropy)-Mixed** | 0.893 |
| **SAW (AHP)-SAW (Entropy)** | 0.851 |
| **SAW (Entropy)-TOPSIS (Entropy)** | 0.851 |
| **SAW (AHP)-TOPSIS(Entropy)** | 0.737 |
| **TOPSIS (AHP)-TOPSIS (Entropy)** | 0.735 |
| **TOPSIS (AHP)-SAW (Entropy)** | 0.687 |

**Table 5.24** Results of Kendall's tau-b Correlation

| **Paired Comparison** | **Correlation Coefficient** |
|---|---|
| **SAW (AHP)-Mixed** | 0.821 |
| **SAW (Entropy)-Mixed** | 0.768 |
| **TOPSIS (Entropy)-Mixed** | 0.758 |

| | |
|---|---|
| **TOPSIS (AHP)-SAW (AHP)** | 0.747 |
| **TOPSIS (AHP)-Mixed** | 0.716 |
| **SAW (AHP)-SAW (Entropy)** | 0.674 |
| **SAW (Entropy)-TOPSIS (Entropy)** | 0.674 |
| **SAW (AHP)-TOPSIS(Entropy)** | 0.579 |
| **TOPSIS (AHP)-TOPSIS (Entropy)** | 0.537 |
| **TOPSIS (AHP)-SAW (Entropy)** | 0.484 |

Since there is much data (20 entries), the results of Pearson and Spearman are convergent (Table 5.20 and 5.22). Moreover, we have utilized Kendall's tau statistics (as concordance coefficient) in order to know about agreement between ranked options from five applied methods. According to Table 5.20, the correlation between different applied techniques with 99% of confidence level is strong and positive which is statistically significant ($p<0.01$). This very high confidence level comes from statistical tests output in bottom of the tables 5.20, 5.21, and 5.22 as '*correlation is significant at the 0.01 level (2-tailed)*'. Correlation between TOPSIS and SAW techniques with AHP method (0.913) is stronger than with Entropy method (0.737). Moreover, correlation between TOPSIS, SAW, and mixed (0.899 and 0.95) with AHP are better than with Entropy (0.893 and 0.913).

Admittedly, statistically significant correlation between ranked options with five different techniques is because of the close proximity of weights by AHP and Entropy. According to Table 5.23 and 5.24, the highest relation is mixed method with SAW and TOPSIS by both AHP and Entropy. Since mixed method involves average of methods results, it is expected to have a stronger correlation as compared with others. When there is no historical real case for investigation of correlation between real decisions made and applied techniques, mixed method can be ideal technique among others. In contrast, the lowest relation is about TOPSIS (AHP) with SAW (Entropy) and TOPSIS (Entropy) with SAW (AHP). Hence, mixed method has provided better results with the most correlations among other paired comparisons.

The values for concordance coefficient from Kendall's tau_b results (Table 5.21) are close to +1; as a result, there is a large agreement between the ranks. Also, concordance coefficient between applied techniques with AHP is better than Entropy. Admittedly, all obtained results are based on the used example (research scenario) and results are not generally valid.

Since we could not find other results of applied MADM techniques about ranking the Bayesian Network options, it was not possible to compare this research results with others in order to provide discussion. However, if a decision maker is willing to use only one method for ranking or re-ranking, the methods should be evaluated according to their advantageous and disadvantageous. Obviously, by previous decisions made from historical real cases, evaluation of applied methods results can be more accurate.

# 6 Conclusions

Today, the use of the Multi Attribute Decision Making (MADM) techniques is increasing in decision-making processes and different areas. It is because of the simplicity and understandability of these techniques for various users. Unlike the mathematical models which cannot utilize qualitative variables, MADM techniques use different qualitative and quantitative variables. In this study, we used TOPSIS and SAW, and Rank Average (mixed) Method as decision-making techniques with AHP and Entropy as weighting methods. Obtained results are based on the used example (research scenario) and results are not generally valid. As we observed for re-ranking Bayesian Network options, there is a significant correlation (relation) between ranked options with five applied techniques because of the close proximity of weights by AHP and Entropy. This means that MADM techniques can be utilized for re-ranking Bayesian Network options in used research scenario. However, the concordance coefficient with AHP method is somewhat better than Entropy. In spite of simplicity of Entropy, AHP with usage of expert judgment is more reliable.

As we found, relation between TOPSIS and SAW techniques with AHP method is stronger than Entropy method according to results of statistical tests with stronger correlation in used research scenario. Relation between techniques of TOPSIS, SAW and Mixed with AHP are stronger than with Entropy. Moreover, the highest relation is between SAW (AHP) and Mixed and between TOPSIS (AHP) and SAW (AHP). In contrast, the lowest relation is between TOPSIS (AHP) and SAW (Entropy) and between TOPSIS (Entropy) and SAW (AHP). Hence, when there is no historical real case for investigation of correlation between real decisions made and applied techniques, mixed method has provided better results with the most correlations among other paired comparisons. Obviously, the use of the previous decisions made in some real cases will be helpful to evaluate which techniques are more close to real decisions and which ones are not. According to advantages of applied techniques, it is expected that TOPSIS technique and AHP method can provide closer results to real decisions made.

In spite of study results, decision making techniques only merge and convert qualitative and quantitative data to information. Since there is no acceptable pattern in order to which weighting method and decision making technique is optimal and superior, this information only aid to user to make optimum decision.

## 6.1 Future Work

First future work proposal is directed towards the use of previous decisions made by experts in order to compare and test every MADM techniques with real decisions. It will be helpful to examine which MADM techniques are closer to decision made. Second future work proposal involves concentrating on mixed-initiative interaction capabilities that rely on a collaborative interleaving of contributions by participants (decision makers). It allows alerting the decision makers if the ranking and selecting of options they are working on is infeasible. Finally, third proposal is using of fuzzy decision making methods. In these methods decision makers use the partial truth of each criterion in non-reliability situation which may range between completely true (1) and completely false (0).

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