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Authors
Peng, ChunYi
Zheng, Haitao
Zhao, Ben Y

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Utilization and Fairness in Spectrum Assignment for Opportunistic Spectrum Access

Chunyi Peng, Haitao Zheng, Ben Y. Zhao

***Abstract*— The Open Spectrum approach to spectrum access can achieve near-optimal utilization by allowing devices to sense and utilize available spectrum opportunistically. However, a naive distributed spectrum assignment can lead to significant interference between devices. In this paper, we define a general framework that defines the spectrum access problem for several definitions of overall system utility. By reducing the allocation problem to a variant of the graph coloring problem, we show that the global optimization problem is NP-hard, and provide a general approximation methodology through vertex labeling. We examine both a centralized strategy, where a central server calculates an allocation assignment based on global knowledge, and a distributed approach, where devices collaborate to negotiate local channel assignments towards global optimization. Our experimental results show that our allocation algorithms can dramatically reduce interference and improve throughput (as much as 12-fold). Further simulations show that our distributed algorithms generate allocation assignments similar in quality to our centralized algorithms using global knowledge, while incurring substantially less computational complexity in the process.**

***Index Terms*— Spectrum Management, Open Spectrum, User Collaboration, Resource Allocation.**

I. Introduction

Wireless devices are becoming ubiquitous, placing increasing stress on the fixed radio spectrum available to all access technologies. To eliminate interference between different wireless technologies, current policies allocate a fixed spectrum slice to each technology. This static assignment prevents devices from efficiently utilizing allocated spectrum, resulting in spectrum holes (no targeted devices in local area) and very poor utilization (6%) in other geographic areas [14]. Studies have shown that reuse of such "wasted" spectrum can provide an order of magnitude improvement in system capacity.

These results further motivate the *Open Spectrum* [2], [6], [12], [18], [25], [28] approach to spectrum access. Enabled by software defined radio (SDR) technology [5], [15], [23], Open Spectrum allows unlicensed (secondary) users to share spectrum with legacy (primary) spectrum users, thereby "creating" new capacity and commercial value from existing spectrum ranges. Based on agreements and constraints imposed by primary users, secondary users opportunistically utilize unused licensed spectrum on a non-interfering or leasing basis. Open spectrum system designs must also deal with *spectrum heterogeneity*, where spectrum available to secondary devices fluctuates with both location and time due to movement and traffic variations of primary users. A user seizing spectrum without coordinating with others can cause harmful interference with its surrounding neighbors, thus reducing available spectrum.

While maximizing spectrum utilization is the primary goal of open spectrum systems, a good allocation scheme also needs to provide fairness across devices. To the best of our knowledge, the question of how best to address these two goals in the context of spectrum allocation for open spectrum systems has not been previously addressed. In this paper, we describe our work in defining a general framework for spectrum allocation in open spectrum systems, and present centralized and distributed approaches to optimizing spectrum allocation for utilization and fairness. The key contributions of this paper are four-fold:

1) *Spectrum Allocation Framework and Utility.* We describe a graph-theoretic model that describes efficient and fair access in open spectrum systems. We also define three policy-driven utility functions that combine efficient spectrum utilization and fairness.
2) *Reduction to Graph Coloring and Lower-bound Proof.* We show how the optimal spectrum allocation problem can be reduced to a variant of the graph coloring problem, proving that it is NP-hard. We also prove a lower bound on the maximal utilization problem where fairness is not considered.
3) *Centralized and Distributed Approximation.* We describe a vertex labeling mechanism which we use to build both centralized and distributed approximation algorithms.
4) *Simulation of Efficiency and Complexity.* We use extensive simulations to quantify the impact of these spectrum allocation algorithms on network access, while comparing the distributed and centralized approaches in efficiency and complexity.

The rest of the paper is organized as follows. We begin in Section II by describing the context of open spectrum systems and its associated challenges. Next in Section III, we provide a mathematical modelling of open spectrum access, define three key utility functions and describe a reduction of the allocation problem to graph coloring. Then in Section IV, we describe a set of centralized and distributed approximation algorithms to optimize our utility functions. We describe our simulation results in Section V, and derive a theoretical lower bound for maximal spectrum utilization in Section VI. Finally, we summarize related works in Section VII, and conclude in Section VIII.

This work was conducted while C. Peng was a visiting student at MSR Asia. C. Peng is with the Automation Department, Tsinghua University, Beijing China. H. Zheng and B. Y. Zhao are with U. C. Santa Barbara, Santa Barbara, CA, USA. Contact authors at *pengchunyi@tsinghua.edu.cn*, *htzheng@cs.ucsb.edu* and *ravenben@cs.ucsb.edu*.

II. Access in Open Spectrum Systems

We begin by describing the spectrum allocation problem in the context of *Open Spectrum* systems. Open spectrum systems allow unlicensed devices (who we refer to as *secondary users*) to make use of spectrum unused by legacy spectrum devices (*primary users*), thereby "creating" new capacity and commercial value from existing spectrum. Secondary users utilize licensed bands on a non-interfering or leasing basis based on agreements and constraints imposed by primary users. They can detect predefined spectrum signatures or footprints of primary users automatically, through operator-initiated broadcasts, or by accessing a central database. A recent example of this approach is the FCC's recent report on the feasibility of allowing unlicensed devices to operate in TV broadcast spectrum ranges at locations and times when it is under-utilized. Secondary users can detect the presence of a sound carrier in NTSC (analog) TV systems or a pilot tone in ATSC (digital) TV systems, and operate without interfering with TV broadcasts (primary users in this case). While the goal is to maximize utilization, secondary users must not interfere with the normal operation of primary users.

In open spectrum systems, primary users' mobility and traffic variations result in the fact that the available spectrum observed by secondary devices fluctuates with both location and time. We call this property *spectrum heterogeneity*. In addition, the interference constraint and the reward (*i.e.* throughput, connectivity) obtained on each spectrum band could be different due to non-uniformly partitioned spectrum bands, differences in power constraints and associated technology. Spectrum heterogeneity also results from variations in device radio capabilities. For example, a new radio device might have integrated Ultra Wide Band (UWB) and IEEE 802.11a/b/g interfaces while an older device only supports 802.11a. In general, spectrum heterogeneity refers to variations in spectrum availability, interference constraints and rewards on each spectrum band.

The key to efficient utilization of open spectrum is to find an appropriate distribution of channels among secondary users while minimizing interference. When two simultaneous transmissions overlap in spectrum and physical location, both can fail [1]. Hence, a user seizing spectrum without coordinating with others can cause harmful interference with its neighbors and degrade overall spectrum usage. For a simple example, consider a ring of nodes around a center node. If the center node uses its entire available spectrum, its will interfere with and disrupt all transmissions coming from its neighbors. In contrast, network controlled spectrum access can optimize network-wide spectrum utilization by forcing secondary users to behave in a collaborative fashion. Specifically, the network needs to define and enforce a set of rules to encourage utilization and minimize interference. Finally, spectrum allocation should be fair to ensure that each device gets a certain amount of spectrum under normal conditions, *i.e.* avoid starvation.

In this paper, we consider the case where the collection

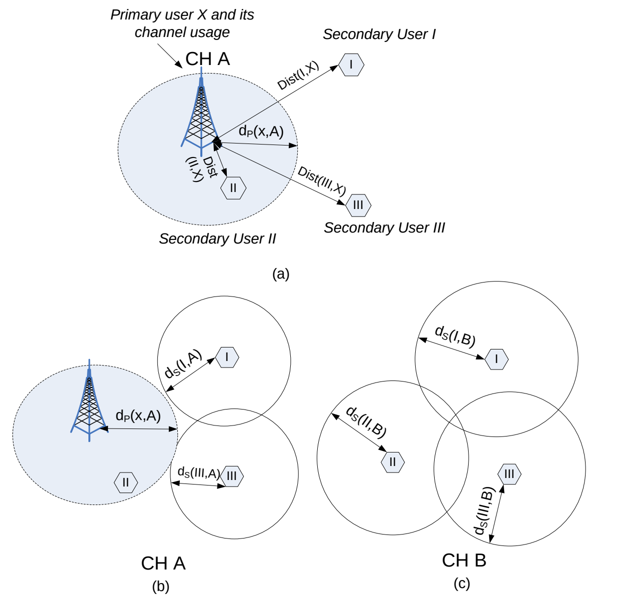


Fig. 1. Spectrum availability changing with the presence of primary users. (a) Topology (b) availability of channel/color A; (c) availability of channel/color B.

of available spectrum ranges forms a spectrum pool, divided into non-overlapping orthogonal channels [2]. Secondary users select communication channels and adjust transmit power accordingly to avoid interfering with primary users. Each secondary user keeps a list of available channels that it can use without interfering with neighboring primary users. The spectrum access problem becomes a channel allocation problem.

A. Example Scenario

In this section, we illustrate the concept of spectrum heterogeneity in open spectrum systems with a sample scenario. We also demonstrate how the presence of a primary user impacts not only which channels are available to nearby secondary users, but also the power used on available channels, and the resulting range and throughput on those channels.

Figure 1 illustrates an example deployment where inactive broadcast (TV) spectrum is utilized to provide wireless connections to a residential community. The broadcast spectrum is divided into two channels (marked by A and B). In this example, broadcast stations (x) are primary users and wireless access points (I, II and III) are secondary users. Each primary user x occupies one channel m which is associated with a protection area with radius $d_P(x,m)$. Any radiation from secondary users falling into it would interfere with the primary user. Each secondary user n can adjust its interference range [3] $d_S(n,m)$ by tuning its transmit power on channel m to avoid interfering with primary users. We assume that a secondary user n can use the same channel m as a nearby primary user x only if $d_S(n,m) \leq Dist(n,x) - d_P(x,m)$, where $Dist(n,x)$ is the distance between n and x. In general, interference

[1] While multi-packet reception and other interference cancellation algorithms can minimize the impact of interference, in this paper we assume for simplicity that interference causes both transmissions to fail.

[2] Channel division can follow the format of TDMA, FDMA, CDMA or a combination of them.

[3] For our purposes, interference range is the same as the transmission range.

range d_S is bounded by the minimum and maximum transmit power, *i.e.* $[d_{min}, d_{max}]$. Note that in this paper we assume that each secondary user (wireless access point) can use technologies like Orthogonal Frequency Division Multiplexing Access (OFDMA) to utilize multiple channels to provide connections for devices within its coverage area.

In our example in Figure 1.a, primary user x uses channel A. Its protection area is shown as a dotted circle around x. Each secondary user adjust its d_S on channel A to avoid interfering with x. Secondary user II is within the protection range of x, and therefore cannot use channel A. If II was outside of x range, but its $d_S < d_{min}$, it still cannot use channel A. Figure 1.b shows the case when no primary users are present on channel B.

For each secondary user, tuning its transmission power to adjust d_S directly impacts its range or coverage. For example, the coverage area of a wireless access point is proportional to d_S^2. Increasing the range with a larger d_S value also increases the probability of interfering with a neighboring secondary user. For each channel, if two secondary users' interference areas overlap, then they conflict and cannot use the channel simultaneously. In this paper, we assume that secondary users use a fixed power control scheme to adjust their transmit power to the maximum permissible level to avoid interfering with primary users. Thus we see how the presence of primary users on a channel can impact secondary users' channel availability and transmission power, which in turn defines its coverage, throughput and interference condition with neighboring secondary users. This is the full impact of *spectrum heterogeneity*. Note that the secondary user can be a wireless access point or a transmission link in an ad hoc network. Note that there is no power control among secondary users, and interference mitigation is done through conflict free spectrum allocation. The interaction of power control and spectrum allocation will be investigated in a future study.

Finally, in order to adjust its d_S correctly to avoid interference with primary users, secondary users need to detect if and how much its transmission range overlaps with that of a primary user. Detecting this dynamically is a challenging open problem, since a secondary user can only listen for carrier signals inside the primary user's transmission range. Here we assume secondary users can use out-of-band mechanisms to get the location and power of primary users, and use that to calculate its ideal d_S. Similarly, secondary users can use similar mechanisms to get the location and power of neighboring secondary users, and use it to determine whether it will interfere with neighboring transmissions.

III. Optimized Allocation for a Fixed Topology

The two key goals of a spectrum allocation algorithm in open spectrum systems are spectrum utilization and fairness. Specific combinations of these two goals form different utility functions that can be customized for each type of network application. In this section, we define a theoretical model to represent the general allocation problem, and describe three utility functions that trade off spectrum utilization and fairness. We then show a reduction from the optimal allocation problem to a variant of a graph-coloring problem.

A. Allocation Model and Utility Functions

In our model, we assume that environmental conditions such as user location, available spectrum are static during the time it takes to perform spectrum assignment. This corresponds to a slow varying spectrum environment where users quickly adapt to environmental changes by re-performing network-wide spectrum allocation. Therefore, we focus on a model for a fixed topology.

We assume a network of N secondary users indexed from 0 to $N-1$ competing for M spectrum channels indexed 0 to $M-1$. Each secondary user can be a transmission link or a broadcast access point. The channel availability and rewards for each secondary user can be calculated based on the location and channel usage of nearby primary users. We define the key components of our model as follows:

Channel availability: $L = \{l_{n,m} | l_{n,m} \in \{0,1\}\}_{N\times M}$ is a N by M binary matrix representing the channel availability: $l_{n,m} = 1$ if and only if channel m is available at user n. Using the example in Section II, if $d_S(n,m) < d_{min}$ then $l_{n,m} = 0$, otherwise $l_{n,m} = 1$.

Channel reward: $B = \{b_{n,m}\}_{N\times M}$, a N by M matrix representing the channel reward: $b_{n,m}$ represents the maximum bandwidth / throughput that can be acquired (assuming no interference from neighbors) by user n using channel m. Using the example in Section II, the reward can be the coverage of a secondary user using a channel:

$$b_{n,m} = d_S(n,m)^2,\ d_{min} \le d_S(n,m) \le d_{max}, \tag{1}$$

or the capacity using a channel (assuming the signal to noise ratio (SNR) is a function of $d_S(n,m)$):

$$b_{n,m} = log(1 + f(d_S(n,m))),\ d_{min} \le d_S(n,m) \le d_{max}. \tag{2}$$

Obviously, $b_{n,m} = 0$ if $l_{n,m} = 0$.

Interference constraint: Let $C = \{c_{n,k,m} | c_{n,k,m} \in \{0,1\}\}_{N\times N\times M}$, a N by N by M matrix, represents the interference constraints among secondary users. If $c_{n,k,m} = 1$, users n and k would interfere with each other if they use channel m simultaneously. The constraint depends on channel availability, *i.e.*, $c_{n,k,m} \le l_{n,m} \times\ l_{k,m}$ and $c_{n,n,m} = 1 - l_{n,m}$. In this paper, we use a binary geometry model where two users conflict if they are located within certain distance of each other. In particular, $c_{n,k,m} = 1$ if $Dist(n,k) \le d_S(n,m) + d_S(k,m)$. Again, this constraint is channel specific: two users might be constrained on one channel but not another. A detailed pseudo code for generating channel availability, channel award and interference constraint is shown in Appendix I.

This model provides an approximation to the effects of interference in real wireless systems. It captures the way interference is manifested in wireless environments without delving into complex detection and decoding algorithms and protocols. We are currently investigating the impact of non-binary interference metric on the proposed approach.

Conflict Free Channel Assignment: $A = \{a_{n,m} | a_{n,m} \in \{0,1\}, a_{n,m} \le l_{n,m}\}_{N\times M}$ is a N by M binary matrix that represents the assignment: $a_{n,m} = 1$ if channel m is assigned to user n. A *conflict free* assignment needs to satisfy all the interference constraints defined by C, that is,

$$a_{n,m} + a_{k,m} \leq 1, \;\; if \;\; c_{n,k,m} = 1, \forall\, n, k < N, m < M. \quad (3)$$

Let $\Lambda(L, C)_{N,M}$ denote the set of conflict free spectrum assignments for a given set of N users and M spectrum bands and constraints C.

Radio Interface Limit: C_{max} represents the maximum number of channels that can be assigned to a secondary user. The assignment at each user n needs to satisfy $\sum_{m=0}^{M-1} a_{n,m} \leq C_{\max}$.

User Reward: $\Re = \{\beta_n = \sum_{m=0}^{M-1} a_{n,m} \cdot b_{n,m}\}_{N\times 1}$ represents the reward vector that each user gets for a given channel assignment.

Network Utilization: The channel allocation is to maximize network utilization $U(\Re)$.

Given the model above, we can define the spectrum assignment problem by the following optimization function:

$$A^* = \underset{A \in \Lambda(L,C)_{N,M}}{arg\max} \; U(\Re). \quad (4)$$

We can obtain utility functions for specific application types using sophisticated subjective surveys. An alternative is to design utility functions based on traffic patterns and fairness inside the network. We consider and address utility in terms of single-hop flows, since they are the simplest format in wireless transmissions.

- *Max-Sum-Reward*: This maximizes the total spectrum utilization in the system regardless of fairness. The optimization problem is expressed as:

$$U_{sum} = \sum_{n=0}^{N-1} \beta_n = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} a_{n,m} \cdot b_{n,m}. \quad (5)$$

- *Max-Min-Reward*: This maximizes the spectrum utilization at the bottleneck user, or the user with the least allotted spectrum. The optimization problem is expressed as:

$$U_{min} = \min_{0 \leq n < N} \beta_n = \min_{0 \leq n < N} \sum_{m=0}^{M-1} a_{n,m} \cdot b_{n,m}. \quad (6)$$

  Roughly, Max-Min-Reward driven allocation gives the most poorly treated user (i.e. the user who receives the lowest reward) the largest possible share, while not wasting any network resources. This is the simplest notion of fairness.

- *Max-Proportional-Fair*: Consistent with prior work [9], [13], [16], [22], we consider and address fairness for single-hop flows. the corresponding fairness-driven utility optimization problem is expressed as:

$$\begin{aligned} U_{fair} &= \sum_{n=0}^{N-1} log(\beta_n) \\ &= \sum_{n=0}^{N-1} log(\sum_{m=0}^{M-1} a_{n,m} \cdot b_{n,m}). \end{aligned} \quad (7)$$

  The essence of proportional fair is that if for any other feasible assignment A' and the associated $\beta_n^{'}$, the aggregate of proportional changes in user reward is zero or negative: *i.e.*

$$\sum_{n=0}^{N-1} \frac{\beta_n^{'} - \beta_n}{\beta_n} \leq 0.$$

  To make it comparable to U_{min} and U_{sum}, we modify the fairness utility to

$$U_{fair} = \left(\prod_{n=0}^{N-1} \beta_n \right)^{\frac{1}{N}} = \left(\prod_{n=0}^{N-1} \sum_{m=0}^{M-1} a_{n,m} \cdot b_{n,m} \right)^{\frac{1}{N}}. \quad (8)$$

Note that under the same assignment, $\frac{1}{N} U_{sum} \geq U_{fair} \geq U_{min}$.

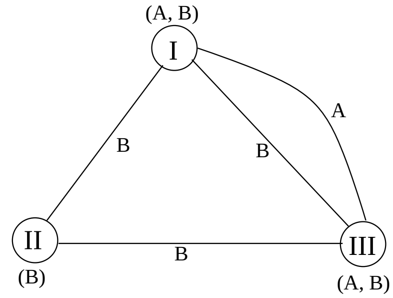


Fig. 2. An example CSGC graph for Figure 1

B. Color-Sensitive Graph Coloring

Our approach to solving this complex optimization problem is to reduce it to a variant of the graph coloring problem by mapping spectrum channels into colors, and assigning them to users (vertices in a graph). Past work has demonstrated the effectiveness of using conflict graphs to model interference [10], [19], [28]. Our work extends the model to a multi-color conflict graph by taking in to account the impact of primary users on secondary users' interference condition.

We define a bidirectional graph $G = (V, L, E)$, where V is a set of vertices denoting the users that share the spectrum, L is the available spectrum or the color list at each vertex, defined in section III-A, and E is a set of undirected edges between vertices representing interference between any two vertices. For any two vertices u,$v \in V$, a m-colored edge exists between u and v if $c_{u,v,m} = 1$. The set of edges depend on the interference constraint C (see section III-A), which is determined by the spectrum usage of nearby primary users and the transmit power of user u and v on channel m.

The spectrum allocation problem is equivalent to coloring each vertex using a number of colors from its color list to maximize system utility. The coloring scheme is constrained by that if a m colored edge exists between any two vertices, they cannot simultaneously use color m. This is a variant of the traditional graph coloring problem. In the traditional problem, graphs are colorless, colors have the same reward, and two connected vertices only have one colorless edge; in our problem, vertices can be connected via multiple colored edges. We call this problem *color-sensitive graph coloring* (CSGC).

Figure 2 illustrates the reduced CSGC graph that corresponds to the network from Figure 1. Channel A is available to secondary user I and III, so that in the corresponding CSGC, vertex I and III have A on their color list. Since the transmission areas of I and III on channel A overlap, they can conflict on channel A, and there is a color A edge between I and III. Channel B is available for three users and they all conflict with each other. Hence, B is on each vertex's color list and a color B edge exists between any two vertices. Thus we can use a conflict graph G to model the network setup of each deployment of primary and secondary users, reducing spectrum allocation to a graph coloring problem. We note that CSGC only optimizes color assignment for a fixed topology. If the topology changes (*e.g.* due to user movement), the graph coloring algorithm needs to be repeated.

IV. Spectrum Allocation Algorithms

The optimal coloring problem is known to be NP-hard [7]. In this section, we apply existing graph coloring solutions to present heuristic based approaches that produce good approximations for our problem.

A. Approximation via Labeling

In [19], the author proposes *progressive minimum neighbor first* (PMNF) as a sequential heuristic solution to graph coloring for generalized channel assignment. He shows that the worst case performance of PMNF significantly outperforms other heuristic approaches. The algorithm assigns each vertex a unique label, colors the vertex with the highest label with the lowest indexed color without violating the constraints. The algorithm removes the colored vertex and associated edges from the graph, and repeats until all the vertices are colored. In PMNF, the objective is to minimize the total colors required to color each vertex, hence the basic idea of the algorithm is to color the “most difficult” vertices first. This way the vertices are labelled proportional to the size of their neighborhood.

We apply a similar approach to our problem, we need to consider conflict constraints in addition to different color lists and color rewards at each vertex. The colors are assigned iteratively, as shown in Figure 3. A vertex is ”saturated” if its channel assignment has reached $C_{\max}$. In each stage, the algorithm labels all the non-saturated vertices with a non-empty color list according to some policy-defined labeling rule. We define each labeling rules later in this section. The algorithm picks the vertex with the highest valued label and assigns the color associated with the label to the vertex. The algorithm then deletes the color from the vertex's color list, and also from the color lists of the constrained neighbors. It also deletes all the edges from the colored vertex in the color graph, so the interference constraint of a vertex keeps on changing as other vertices are processed, and the labels of the colored vertex and its neighbor vertices are modified according to the new graph. The algorithm enters the next stage until every vertex's color list becomes empty or every vertex saturates.

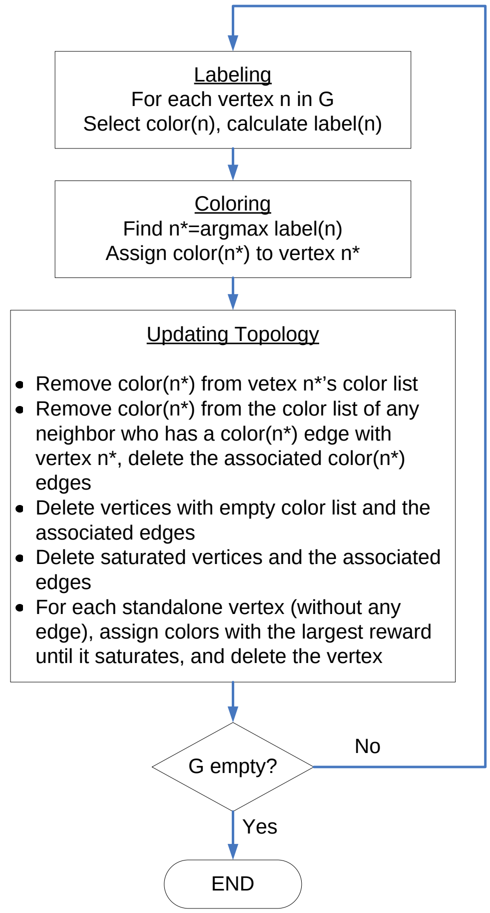


Fig. 3. Flow Chart of Coloring

Note that our graph coloring problem wants to maximize utility while the conventional graph coloring problem [24], [19] wants to minimize the number of colors used. While the labeling rule in our approach is different from PMNF, the intuition is similar. We choose to color the “most valuable” vertices first, *i.e.* the vertices that contribute to the system utility the most. In particular, it can be shown that when $C_{max} = M$, the problem of maximizing sum reward is equivalent to a combination of maximum weighted independent set (WIS) problem on each color. In [21], the authors show that a greedy approach is a tight $\frac{1}{\Delta(G)}$-approximation of WIS, where $\triangle(G)$ is the maximum degree of the graph. The authors propose an iterative approach to repeatedly select the vertex u from the graph G with maximal $\frac{b(u)}{d_G(u)+1}$, where $b(u)$ is the bandwidth and $d_G(u)$ is the degree of u in G. The algorithm then deletes u and its associated edges from the graph G. Here $\frac{b(u)}{d_G(u)+1}$ approximates the contribution of u to sum reward in its local neighborhood.

B. Centralized vs. Distributed Network Architectures

The algorithm we describe above assumes a central allocation server with knowledge about all users in the system. In this section, we discuss the challenges facing the centralized version of the algorithm, and describe a distributed version of the algorithm.

a) Centralized Architecture: In a centralized architecture, a central spectrum server makes decisions on channel assignment. The server collects location, power, spectrum and interference information from both primary and secondary users, and runs the assignment algorithm to distribute channels among secondary users. It then broadcasts the assignments on a predefined channel. Secondary users listen to the broadcast and communicate using their assigned channels.

While a central server can optimize across network-wide information, there are two serious limitations to this approach. First, this scheme requires a communication path between the spectrum server and all secondary users, *i.e.* all users need interference-free access to a pre-assigned dedicated control channel, possibly in a licensed band. In addition, as networks grow in density, a pre-defined control channel will limit the bandwidth available for control messages. Second, the server processing complexity will scale at least polynomially with the number of devices. Any central spectrum server will quickly become a computational bottleneck.

b) Distributed Architecture: As an alternative to the central spectrum server, secondary users can use a distributed algorithm to determine its own spectrum assignment. It must use only locally available information. Gathering and disseminating information to a large neighborhood not only incurs high delay, but also limits the scalability of the network.

The distributed algorithm works as follows. Each secondary user detects the presence of primary users to determine its own channel availability and transmission constraints. It then coordinates with nearby neighbors to determine channel assignments in an iterative fashion. In each iteration, each user labels itself according to one of the policy-driven labeling rules described in Section IV-C, and broadcasts the label to its neighborhood[4]. After collecting all the labels from its neighbors, the secondary user with the maximum label within its neighborhood selects the associated channel and broadcasts its selection. The neighbors who conflict with this user on this channel remove the channel from their respective available lists. After collecting assignment information from surrounding neighbors, each secondary user updates its list of available channels and recalculates its label. The process is repeated until each user's available channel list is empty or users are satisfied. Through these iterative broadcasts, this algorithm allows cooperation beyond a node's immediate neighbors, producing effects similar to global optimization through cooperative local actions distributed throughout the system.

TABLE I
SUMMARY OF LABELING RULES

Utility \ Rule Type	Collaborative	Non-collaborative
Max Sum Reward	CSUM	NSUM
Max Min Reward	CMIN	NMIN
Max Prop. Fair	CFAIR	NFAIR

C. Labeling Rules

We have described both centralized and distributed allocation algorithms based on iteratively coloring nodes using label values. In this section, we define a number of labeling rules that correspond to each of the utility functions described in Section III-A for both centralized and distributed algorithms. For distributed algorithms, we use collaborative rules that consider the impact of interference on neighbors when labeling. Table IV-C summarizes how the proposed rules correspond to utility functions and centralized or distributed approaches.

For each vertex n, its m color-specific degree, $D_{n,m}$, is the number of conflict edges it shares with its neighbors for color m. This is the number of neighbors who cannot simultaneously use m with n, *i.e.*:

$$D_{n,m} = \sum_{k=0,k\neq n}^{N-1} c(n,k,m) \cdot l_{n,m} \cdot l_{k,m}. \tag{9}$$

$D_{n,m}$ is a good measure of the impact to neighbors when a color is assigned to a vertex. Now we describe the relevant labeling values organized by the utility function they optimize for.

Max Sum Reward:

Collaborative-Max-Sum-Reward (CSUM): This rule aims to maximize the sum reward defined in (5). When a vertex n is assigned with a color m, its contribution to the sum reward in a local neighborhood can be computed as $b_{n,m}/(D_{n,m}+1)$ since some of its neighbors cannot use this color. We propose to label the vertex n according to

$$\begin{aligned} label_n &= \max_{m\in\ell_n} b_{n,m}/(D_{n,m}+1), \\ color_n &= arg \max_{m\in\ell_n} b_{n,m}/(D_{n,m}+1) \end{aligned} \tag{10}$$

where ℓ_n represents the color list available at vertex n at this assignment stage. This rule considers the tradeoff between spectrum utilization (in terms of selecting the color with the largest reward) and interference to neighbors (in terms of degree). This rule is collaborative, since it takes into account the impact to neighbors.

Non-collaborative-Max-Sum-Reward (NSUM): This rule aims to improve the sum of reward without considering the impact of interference to neighbors. The vertex with the maximum reward will be colored, *i.e.* a vertex n is labeled with

$$\begin{aligned} label_n &= \max_{m\in\ell_n} b_{n,m}, \\ color_n &= arg \max_{m\in\ell_n} b_{n,m}. \end{aligned} \tag{11}$$

[4]This requires a coordination scheme so that secondary users can communicate with each other without interfering primary users. A detailed study on this subject can be found in [26].

When colors are homogeneous, this corresponds to a random labeling. Compared to CSUM, this rule is relatively selfish. It is non-collaborative, since each vertex only considers its own reward and ignores impact on the overall system.

Max Min Reward:

Collaborative-Max-Min-Reward (CMIN): This rule tries to distribute colors uniformly among vertices to improve the minimum reward that a vertex can get, while considering interference to neighbors. This rule tries to solve Max-Min optimization as defined in (6). In each stage, a vertex n is labeled according to

$$\begin{aligned} label_n &= -\sum_{m=0}^{N-1} a_{n,m} \cdot b_{n,m}, \\ color_n &= arg \max_{m \in \ell_n} b_{n,m}/(D_{n,m}+1). \end{aligned} \tag{12}$$

where $a_{n,m}$ represents the reward obtained at n before this assignment stage. Note that unlike $CSUM$ and $NSUM$, the label depends on the reward obtained in previous stages. In each stage, the vertex with the minimum accumulated reward will be colored with the color that maximizes utilization while considering interference. If two vertices have the same label, then the vertex with larger $\max_{m \in \ell_n} b_{n,m}/(D_{n,m}+1)$ value gets a higher label.

Non-collaborative-Max-Min-Reward (NMIN): This rule is a non-collaborative version of CMIN where the impact of interference is not considered in the vertex labeling and coloring, *i.e.*

$$\begin{aligned} label_n &= -\sum_{m=0}^{N-1} a_{n,m} \cdot b_{n,m}, \\ color_n &= arg \max_{m \in \ell_n} b_{n,m}. \end{aligned} \tag{13}$$

In each stage, the vertex with the minimum accumulated reward will be colored with the color that has the largest reward. If two vertices have the same label, then the vertex with larger $\max_{m \in l_n} b_{n,m}$ is assigned with a higher label.

Max Proportional Fair:

Collaborative-Max-Proportional-Fair (CFAIR): This rule aims to achieve a specific fairness among vertices, corresponding to (8). It is well known that proportional fair scheduling assigns resource to the user with the highest $r_n/\hat{R}_n$, where r_n represents the reward generated by using a time slot and $\hat{R}_n$ is the average reward that the user n has received in the past [17], [3]. The concept of proportional fair scheduling is applied to this problem by viewing colors as time slots. In each stage, each vertex n is labeled according to

$$\begin{aligned} label_n &= \frac{\max_{m \in \ell_n} b_{n,m}/(D_{n,m}+1)}{\sum_{m=0}^{M-1} a_{n,m} \cdot b_{n,m}}, \\ color_n &= arg \max_{m \in \ell_n} b_{n,m}/(D_{n,m}+1). \end{aligned} \tag{14}$$

where $label_n$ represents the ratio of the maximum *interference-weighted* reward from using a color and the accumulated reward in past stages. This rule is in general different from the traditional proportional fair rule as it captures the difference in the impact of interference generated by a color assignment.

Non-collaborative-Max-Proportional-Fair (NFAIR): This is a non-collaborative version of the CFAIR rule. Each vertex n is labeled according to

$$\begin{aligned} label_n &= \frac{\max_{m \in \ell_n} b_{n,m}}{\sum_{m=0}^{M-1} a_{n,m} \cdot b_{n,m}}, \\ color_n &= arg \max_{m \in \ell_n} b_{n,m}. \end{aligned} \tag{15}$$

When all the channels have uniformed bandwidth, *i.e.* $b_{n,m} = 1$, this rule becomes *NMIN* rule.

V. Simulation Results and Discussions

In this section, we conduct experimental simulations to quantify the performance of open spectrum systems, and validate the proposed spectrum allocation algorithms. We start by examining the appropriateness of the labeling rules designed for different utility functions. We then compare the performance of collaborative and non-collaborative approaches to a baseline approach, and study the impact of system settings on utility performance. We also compare the performance of centralized and distributed implementations and their respective associated complexity.

We conduct our simulations under the assumption of a noiseless, immobile radio network. We randomly place a number of primary and secondary users in a given area (10x10). Each primary user randomly selects one channel to utilize from a pool of channels (*e.g.* 10 channels). For simplicity, we assume that primary users have uniform protection ranges, *i.e.* $D_P = const$. Given the location and channel selection of primary users, each secondary user n adjusts its transmit power (and hence interference range) on each channel m, *i.e.* $d_S(n,m)$ to avoid interference with primary users. Channel availability, reward and interference constraints are derived according to section III. By default, we assume that there are 10 channels, 20 primary users and 10 secondary users. We set $C_{\max} = 10$, $D_P = 2$, $d_{min} = 1$ and $d_{max} = 4$. Each deployment of primary and secondary users produces a topology and a colored conflict graph. We study the statistical performance of spectrum allocation in terms of the average system utility over 500 deployments.

We modify the definition of two utility functions to facilitate the simulations. We use mean reward instead of sum reward in the following simulations, *i.e.*:

$$U_{mean} = \frac{1}{N} \sum_{n=0}^{N-1} \beta_n, \tag{16}$$

so that all three utilities are within the same scale. In addition, the fairness based utility defined in (8) becomes 0 if there exists a secondary user without any channels assigned, *i.e.* a starved user. For a better illustration of the performance at non-starved users, we modify (8) into:

$$U = \left(\prod_{n=0}^{N-1} (\beta_n + 1e-4) \right)^{\frac{1}{N}}. \tag{17}$$

by assuming a baseline reward of $1e{-}4$ at each secondary user. Overall, the results are indexed *Mean Reward*, *Min Reward* and *Fairness* respectively.

A. Labeling Rules and Utilities

We start by examining the relationship between the proposed rules and the utility functions. For this purpose, we use the centralized implementation and the default setting defined above. Figure 4 illustrates the system utilities corresponding to each of the 40 topologies chosen randomly. The results confirm that each proposed collaborative rule outperforms others in optimizing the respective targeted utility function. From their definitions, we see that the CFAIR rule is a combination of CSUM and CMIN rules. Hence, CFAIR's performance is in between that of CSUM and CMIN in both *Mean Reward* and *Min Reward*. In terms of *Min Reward* and *Fairness*, CMIN and CFAIR have similar performance while CSUM performs poorly. This is because both utilities are critically limited by the "poor" users. Those users are located in crowded areas and near primary users, and hence have many edges and small color list in the corresponding conflict graph. CMIN and CFAIR rules grant priority to these users by taking into account the accumulated reward in the labeling metric. As the priority is mainly determined by the accumulated reward, these two rules perform similarly. The same conclusion applies to non-collaborative rules, and we omit those results because of space constraints.

B. Collaborative vs. Non-collaborative Rules

In this section, we compare the performance of collaborative and non-collaborative rules. We also introduce a baseline random labeling approach which assigns a random label between 0 and 1 and selects a color randomly from the color list. For easy notation, we will use CA, NCA, and RAND to represent collaborative, non-collaborative and random rules. It is well-known that the performance of graph coloring depends heavily on the topology of the conflict graph. Hence, only through comprehensive evaluations under different network settings can we thoroughly understand the problem. Next, we present simulation results evaluating the impact of four system parameters: the number of primary users, the number of secondary users, the maximum transmission power d_{max} of secondary users and the number of channels.

1) Impact of the Number of Primary Users: We start by quantifying the performance of labeling rules under different configurations of primary user deployment. Note that the configuration of primary users determines channel availability, reward and interference constraints seen by secondary users. In the simulated system, increasing the number of primary users or increasing the protection range d_P would both expand the primary protection area, and force affected secondary users to reduce their power and thus d_S. The impact is two-fold. First, the number of available channels, and channel reward at secondary users are reduced, degrading spectrum utilization. Second, the interference among secondary users decreases, improving the possibility of spectrum reuse by multiple secondary users. The final impact on system utility depends on the tradeoff between the two, which in turn depends on the settings of channel reward and interference constraints. Figure 5 shows that in the current setting, increasing the number of primary users would degrade all three utilities. Similar trends can be obtained by increasing d_P, and those results are omitted due to space constraints.

Compared to CA and NCA, RAND rule performs poorly in term of all three utilities. This is because both CA and NCA rules take into account certain property of *spectrum heterogeneity* by approximating the contribution of a channel assignment to system utility. Overall, results in Figure 5 shows that CA and NCA rules outperform RAND rule by $30{-}50\%$ in terms of *Mean Reward*, $2-14$-fold in terms of *Min Reward* and $2-4$-fold in terms of *Fairness*. Improvements in both *Mean Reward* and *Fairness* are much more significant since they depend heavily on the "poor" user's performance. Using random labeling, a "poor" user's available channels diminish quickly due to its small available channel list and large number of interference constraints. This limits the system utility.

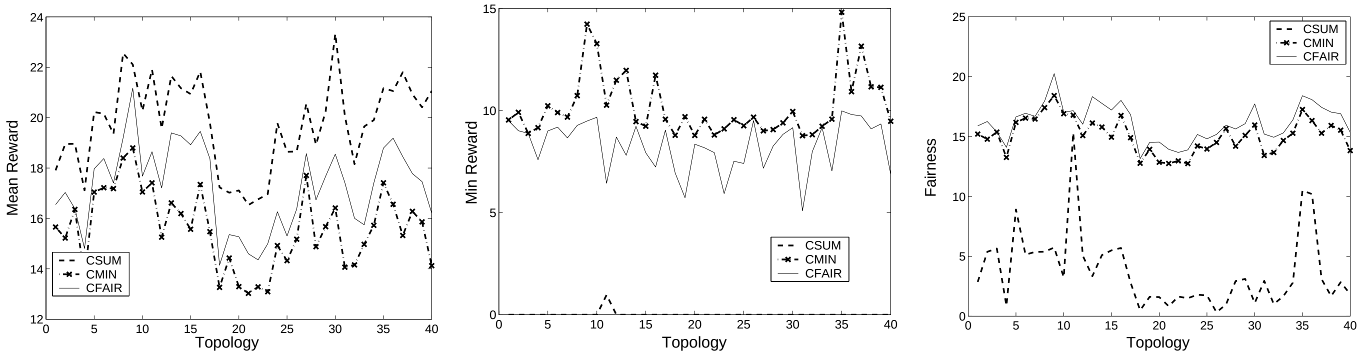
Compared to NCA rules, CA rules not only consider the reward obtained for each individual user, but also the consequence of interference and its impact on overall system utility. The label provides a more accurate characterization of the user's contribution to system utility. Figure 5 shows that CA leads to an improvement of $5{-}30\%$ in *Mean Reward*, $15{-}80\%$ in *Min Reward* and $15-40\%$ in *Fairness*.

2) Impact of the Number of Secondary Users: Next, we examine the performance of different rules under different configurations of secondary user deployment. We start by varying the number of secondary users in the area, *i.e.* user density. Increasing density clearly creates additional interference constraints, thus increasing the vertex degree in the conflict graph. Hence, Figure 6 shows that all three utilities degrade as the number of secondary users increases. In addition, the performance difference among CA, NCA and RAND rules is similar to that in Figure 5.

3) Impact of the Number of Channels: We now examine how system utility scales with the number of channels. Figure 7 quantifies the performance of different rules as the number of channels changes. We see that in general all three utilities scale linearly with the number of channels (at least when the number of channels exceeds 10). We also observe that the scale depends on the number of secondary users.

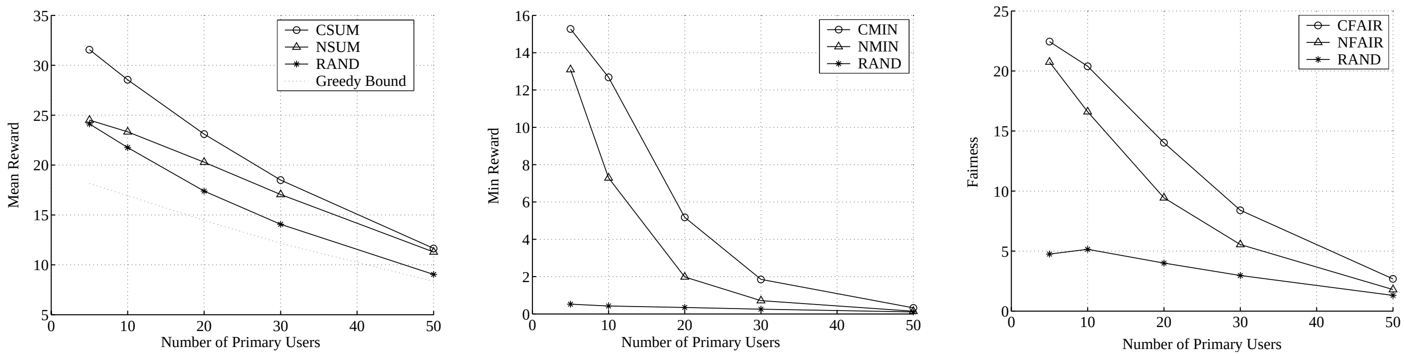
4) The Impact of d_{max}: We then study the impact of varying the value of d_{max}. Raising d_{max} allows secondary users to transmit at higher d_S, which leads to improved spectrum utilization for secondary users who are distant from primary users. However, since there is no power control among secondary users [5], this also leads to additional interference constraints and reduced possibility of spectrum sharing. Hence, there exists a tradeoff between improving spectrum utilization and degrading spectrum sharing. Figure 8 illustrates

[5]Secondary users can adjust their transmission power and thus d_S accordingly to avoid interference among themselves. This is the conventional power control problem. In this paper, we assume that secondary users only use power control to avoid interfering primary users. Research on combining power control and spectrum allocation to further improve system utility will be included in another study.



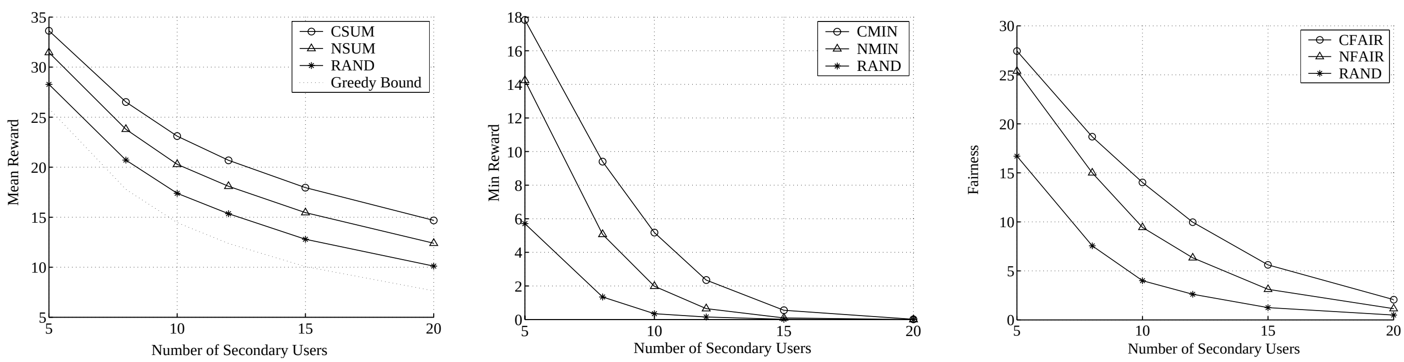
(a)*Mean Reward* (b)*Min Reward* (c)*Fairness*

Fig. 4. System Utilities under different labeling rules for various topologies.



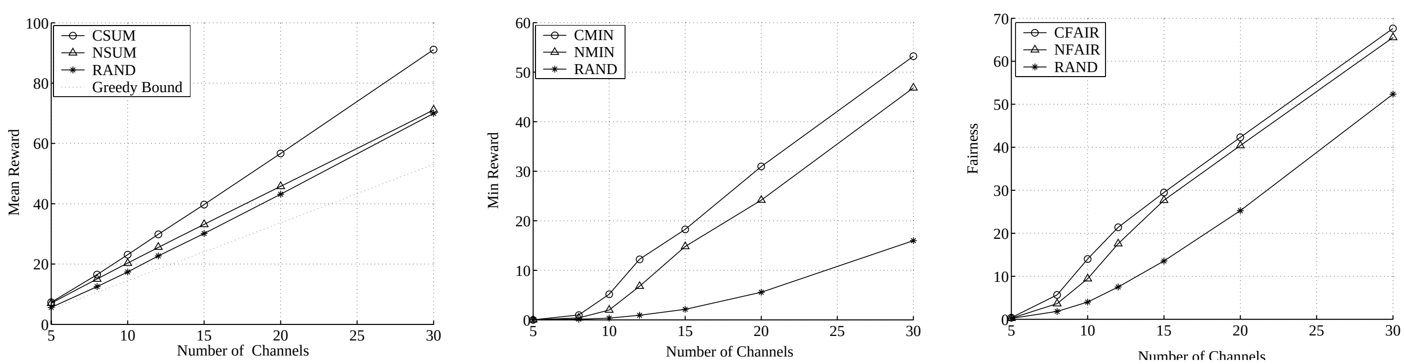
(a) *Mean Reward* (b) *Min Reward* (c)*Fairness*

Fig. 5. Spectrum allocation performance with varying number of primary users.



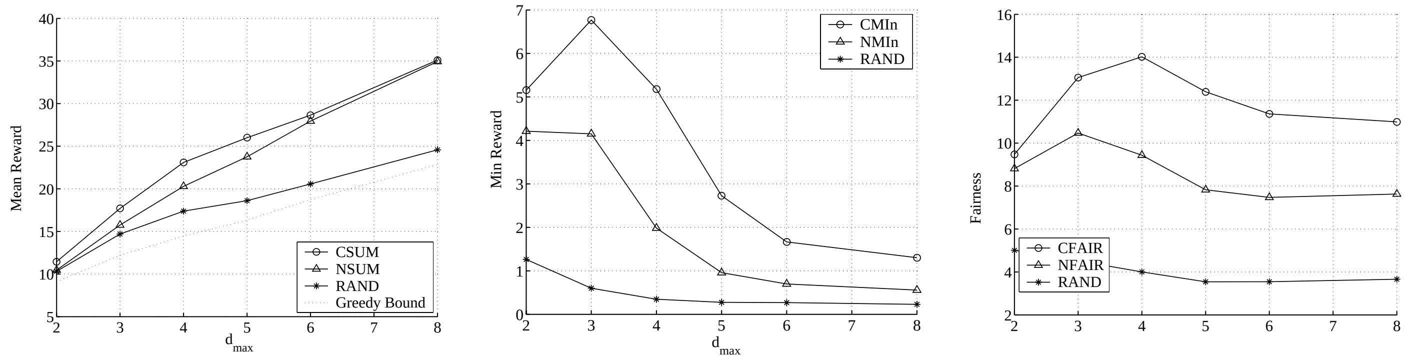
(a) *Mean Reward* (b)*Min Reward* (c)*Fairness*

Fig. 6. Spectrum allocation performance with varying secondary users.



(a) *Mean Reward* (b) *Min Reward* (c)*Fairness*

Fig. 7. Spectrum allocation performance with varying channels.

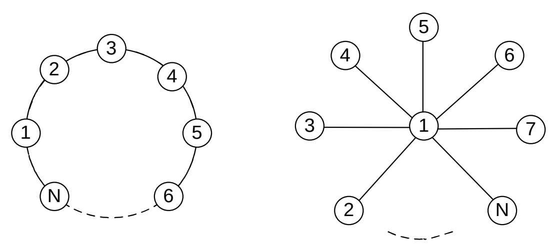


(a)*Mean Reward* (b)*Min Reward* (c) *Fairness*

Fig. 8. Spectrum allocation performance with varying maximal transmission ranges for secondary users.

TABLE II
COMPARISON TO GLOBAL OPTIMUM - RANDOM TOPOLOGIES

Relative Difference (%)	Sum Reward	Min Reward	Fairness
CA	0.08	35	20
NCA	0.25	44	28
RAND	15	76	65



(a) (b)

Fig. 11. Fixed topology (a) Ring (b) Star

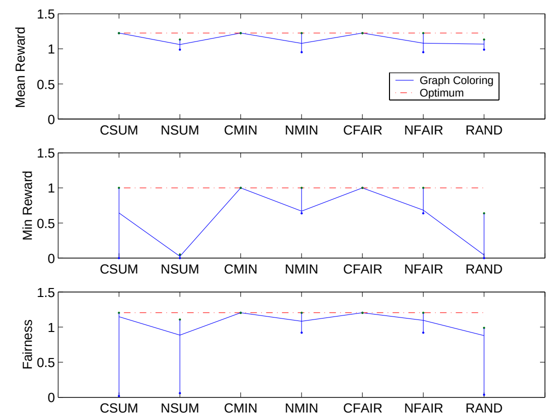


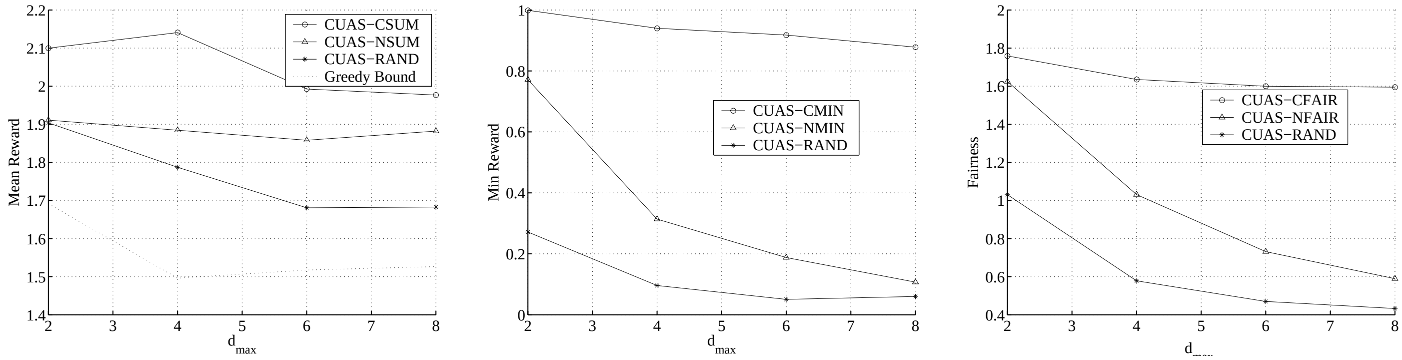
Fig. 12. Comparison to the optimal solution using 18 node ring topology

the system utilities where d_{max} varies from 2 to 8. We see that system utilities are quite sensitive to variations in d_{max}. In particular, *Mean Reward* increases with d_{max}, and *Min Reward* and *Fairness* reach the maximum for $d_{max} = 3$ and 4 and converge after d_{max} exceeds 5. Hence, we should adjust d_{max} to optimize system utility, or equivalently, invoke power control to adjust d_{max} at each secondary user.

Note that the above results are obtained by assuming that $b_{n,m} = D_S(n,m)^2$. We also examine the impact of d_{max} where $b_{n,m}$ is computed differently. Figure 9 illustrates the system utilities where the channel reward is defined by $b_{n,m} = log(1 + D_S(n,m)^2)$. In this case, the gain from improving spectrum utilization becomes less significant. In this case, CA rules that consider interference to neighbors in labeling are less sensitive to variations in d_{max} comparing to NCA and RAND. Figure 10 illustrates the utility performance where $d_{max} = d_{min}$ and $b_{n,m} = d_{max}^2$. This represents the case where users transmit at a fixed power, hence get homogeneous reward by using each channel. Note that even without reward heterogeneity, CA rules perform significantly better than NCA and RAND rules. Results show that the system performance is sensitive to d_{max}, and a proper setting of d_{max} is essential for good system performance. We also observe that *Mean Reward* in general reaches its maximum value at a higher d_{max} compared to the other utilities. This can be explained as follows. Increasing d_{max} could help "rich" users who are located in a sparse area, but degrade the performance of "poor" users who are within close distance with each other. As *Mean Reward* depends heavily on "rich" users, the impact of increasing d_{max} remains positive until these "rich" users become "poor" users as d_{max} increases.

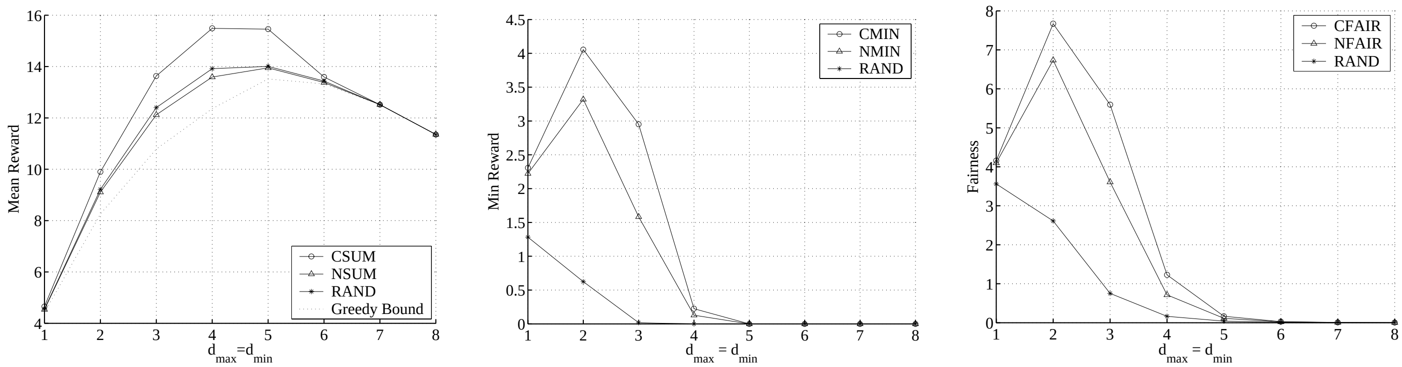
C. Comparing to the Optimal Solution

We now compare the system utility derived from the proposed heuristic based approaches to the optimal value. We use exhaustive search to find the channel assignment that maximizes each system utility. Given the complexity of the exhaustive search scales exponentially with the number of nodes, we use simple topologies (see Fig. 11) with limited number of nodes and channels. Topo I and II are two extreme topologies: a star topology with one vertex interferes with the rest and a ring topology with uniformed interference condition. Fig 13 and 12 summarize the results for 18 node ring topology and 10 node star topology, assuming 3 channels with throughput 1, 0.81 and 0.64, respectively. There is randomness in the graph coloring assignment (if two vertices have the same label, the algorithm randomly picks one vertex). Hence, results are represented as mean with 90% confidence interval. We observe that the proposed collaborative rule based approaches



(a) *Mean Reward* (b)*Min Reward* (c)*Fairness*

Fig. 9. Spectrum allocation performance with varying maximal transmission ranges for secondary users where $b_{n,m} = log(1 + D_S(n, m)^2)$.



(a) *Mean Reward* (b) *Min Reward* (c)*Fairness*

Fig. 10. Spectrum allocation performance with varying maximal transmission ranges for secondary users where $d_{max} = d_{min}$.

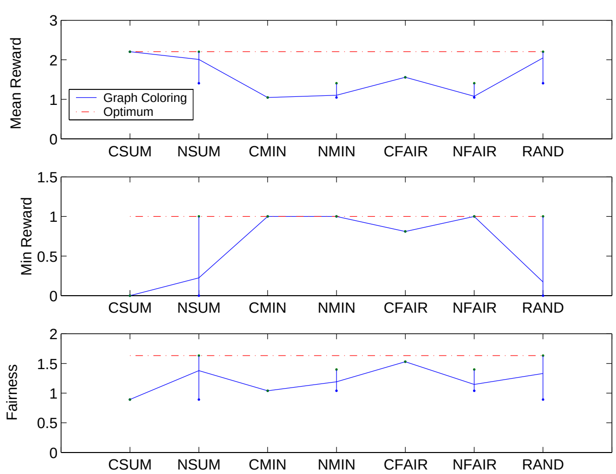


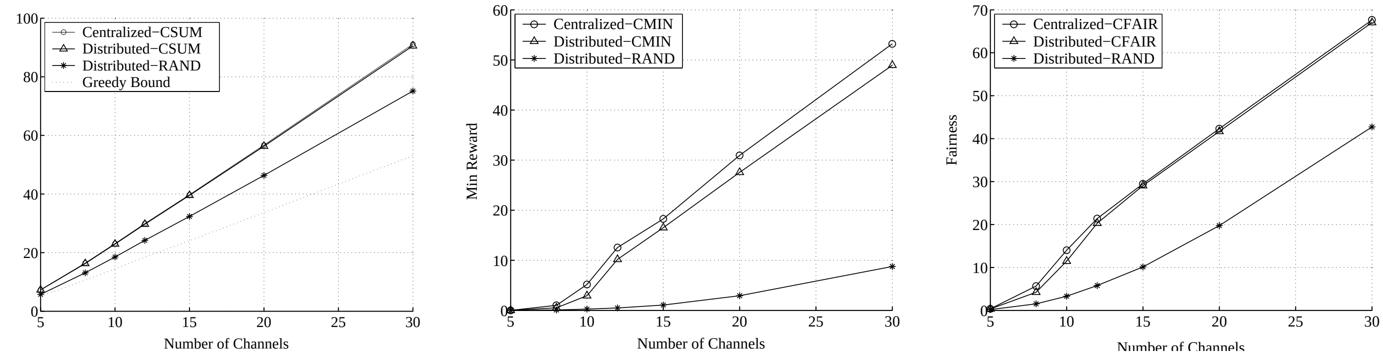
Fig. 13. Comparison to the optimal solution using 10 node star topology

achieve similar performance compared to the global optimal. For star topology, the performance under fairness utility is slightly worse.

We also consider a set of small random topologies assuming 5 secondary users, 10 primary users and 5 channels. The topologies are formed by randomly deploying primary and secondary users following the procedure in Appendix I. We set $D_P = 2$, $d_{min} = 1$ and $d_{max} = 4$. For a clear illustration, we introduce another performance metric:*Relative difference*. This measures the difference of system utility provided by the proposed graph coloring approach and the global optimum. If the utility obtained through graph coloring using a particular rule x is $T(n)$ and the global optimum is T_{opt}, the relative difference is $1-T(n)/T_{opt}$. When $T_{opt} = 0$, the relative difference is 0. Table V-B.4 summarizes the *Relative difference* for different system utilities averaged over 100 random topologies. Similar to the above, CA and NCA refer to the collaborative and non-collaborative rules under different system utilities. We see that there is still visible difference between the proposed approach and the global optimum, particularly for min reward and fairness. Overall, CA provides the best approximation compared to NCA and RAND.

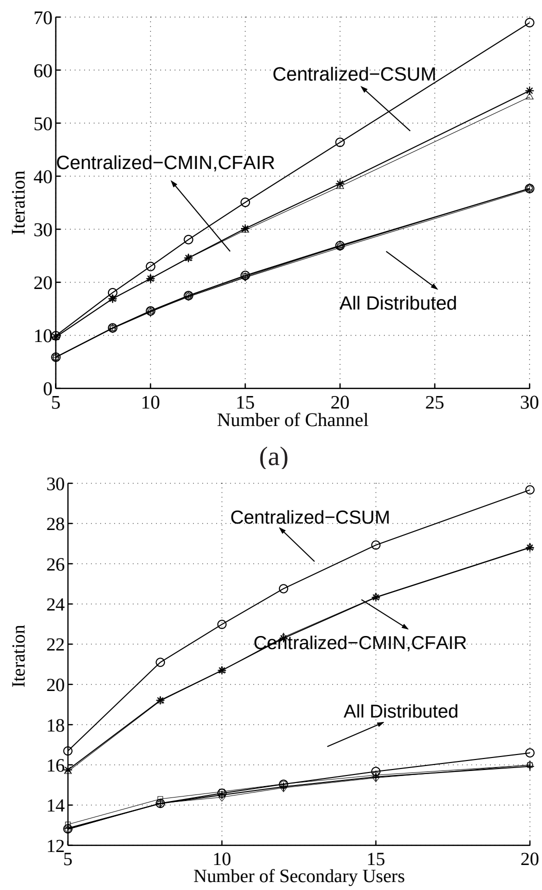
D. Centralized vs. Distributed Implementation

In this section, we compare the performance of centralized and distributed implementations. Figure 14 compares the performance of collaborative rules as the number of channels varies. We also include a distributed implementation of "RAND" rule. We observe that the centralized and distributed implementations of collaborative rules perform similarly and significantly outperform the distributed implementation of RAND rule. There is a visible difference between two implementations in terms of *Min Reward*. This is because *Min Reward* represents the worst user performance in the system and thus requires system-wide optimization. Centralized implementation is designed to maximize the performance



(a) *Mean Reward* (b) *Min Reward* (c)*Fairness*

Fig. 14. Spectrum allocation performance using centralized and distributed algorithms.



(a)

(b)

Fig. 15. Number of labeling stages (a) regarding the number of channels (b) regarding the number of secondary users

of the "poorest" user within the network, while distributed implementation aims to maximize the performance of the "poorest" user in local neighborhood.

We also examine the complexity of two implementations. The major difference between two implementations is that during each coloring stage, centralized implementation selects one user while distributed implementation selects multiple users. Hence the number of labeling/coloring stage required for distributed implementations is much less than that of centralized implementation. Figure 15 compares the number of labeling stages in both implementations. We see that distributed implementation cuts the number of stages by almost half. For centralized implementation, the number of stages equals to $\sum_{n=0}^{N-1}\sum_{m=0}^{M-1} a_{n,m}$. Hence, the number of stages required for *Mean Reward* is much higher than that of *Min Reward* and *Fairness*. As expected, the number of stages scales linearly with both the number of channels and the number of secondary users.

VI. Theoretical Lower Bound

When $C_{max} = M$, seeking channel assignment to maximize sum reward is equivalent to finding the maximum weighted independent set problem. The work in [21] shows that a greedy approach that selects to color the vertex with maximum $\frac{b_n}{(D_n+1)}$ outputs an independent set of weight at least $\sum_{n\in V}\frac{b_n}{(D_n+1)}$. In this section, we conduct theoretical analysis on the lower bound of sum reward using the CSum rule and a centralized implementation, under the C_{max} constraint.

For each vertex n, we sort the channel list by the CSum label $b_{n,m}/(D_{n,m}+1)$ in decreasing order. Define $\pi(n,K)$ as the collection of up to K highest labeled channels at n. Define coloring bound

$$GB(K) = \sum_{n=0}^{N-1} \sum_{m\in\pi(n,\ K)} \frac{b_{n,m}}{D_{n,m}+1}. \tag{18}$$

In particular, if $K = M$, $\pi(n,K) = \{0, ..., M-1\}$, and

$$GB(M) = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} \frac{b_{n,m}}{D_{n,m}+1} \tag{19}$$

and if $K = 1$, *i.e.* every user can only use one channel, $\pi(n, C_{max}) = arg\max_{0\le m\le M-1} b_{n,m}/(D_{n,m}+1)$, and

$$GB(1) = \sum_{n=0}^{N-1} \max_{0\le m\le M-1} \frac{b_{n,m}}{D_{n,m}+1} \tag{20}$$

Let

$$S(C_{max}) \triangleq \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} a_{n,m}\cdot b_{n,m}, \tag{21}$$

represent the sum reward obtained using the CSum labeling rule, which depends on the choice of C_{max}. Since $\forall n$, $\sum_{m=0}^{M-1} a_{n,m} \le C_{\max}$.

Theorem 1: Using centralized implementation and CSum rules, the sum reward is bounded.

$$S(C_{\max}) \geq GB(C_{\max}). \tag{22}$$

Theorem 1 expands the lower bounds derived in [21] regarding weighted independent set problem (WIS) into the proposed color-sensitive graph coloring problem with the constraint C_{max}. When $C_{max} = M$, the problem can be reduced into finding the maximum WIS on each color graph. A color m graph is derived from graph G by removing color m from all the color lists, and removing color m edges. The proof is straightforward following the work in [21]. For general choice of C_{max}, we need to jointly consider the color assignment for all the color graphs, and the results in [21] are not directly applicable. The detailed proof is shown in the Appendix II

Corollary 1: The same lower bounds can be obtained using distributed implementations.

Distributed and centralized implementations differ in terms of the choice of vertex to be colored in each stage. In distributed implementations, more than one vertex can be colored in each stage, but each chosen vertex is associated with the highest labeling in its neighborhood. Hence, the selected user's contribution to the sum throughput can still compensate for the throughput loss at its conflicting neighbors. A detailed proof is included in Appendix III. The derived lower bounds are shown in Figure 5 to 14.

Theorem 2: Using centralized implementation and CSum rules, the *performance ratio* of the proposed approach to the optimal solution $\rho = inf_G \frac{S_G(C_{max})}{\alpha_G(C_{max})}$, where $S_G(C_{max})$ is the sum reward obtained using the CSum labeling rule on graph G and $\alpha_G(C_{max})$ is the optimal sum reward. When $C_{max} = M$, the *performance ratio* is bounded by

$$\frac{1}{\min_m \max_n D_{n,m}} \geq \rho \geq \frac{1}{\max_m \max_n D_{n,m}}. \tag{23}$$

The proof is straightforward following the results in [21].

VII. Related Work

Extensive research exists on the general problem of channel allocation. Both analytical framework and practical strategies have been proposed. Analytical frameworks in [16], [9] address fairness for single-hop flows, and derive an estimate of the rate at each flow to achieve Max-Min fairness. However, there is no guarantee that a feasible scheme exists to achieve the rate.

Practical strategies have been proposed for sharing a single channel. Contention based schemes invoke a random access protocol like ALOHA and CSMA, where users contend in time to share a common channel [13], [9], [16]. While this scheme provides fairness and utilization on a single channel system probabilistically, its application to a multi-channel system requires each user to know how many and which channel(s) to access. Another approach, conflict free time slot scheduling, provides guaranteed channel usage by reserving time slots for each flow. Solutions in [20], [1], [19] assign exactly one time slot to each flow. This approach can be used in multi-channel systems if each user uses only one channel. Another solution [22] allows users to use multiple slots/channels to achieve Max-Min-fair, but does not consider interference from neighbor transmissions.

Multi-channel assignment strategies were developed mostly for cellular networks. The work in [11] provides solutions to assign frequency bands among base stations to minimize call blocking probability for voice traffic. There is no notion of fairness as the traffic determines the number of channels each base station should use. Distributed channel assignment for OFDM based systems has been studied in [8] but only for fully-connected network, where all the flows interfere with each other.

While most existing approaches allocate channels according to a fixed user demand, *i.e.* call requests or one channel per user, our goal is to optimize spectrum utilization across the entire network while taking fairness into account. In addition, we consider the issue of *spectrum heterogeneity*, where users perceive different channel availability and different channel interference constraints as a function of time and their location. For Sum Reward based utility, and unlimited channel access *i.e.* $C_{\max} = M$, the optimization is exactly a Weighted Independent Set (WIS) problem [21]. However, we generalize the optimization to $C_{\max} \leq M$ and derive the theoretical lower bound. We consider a general multi-hop network topology, while most work on OFDM based channel allocation are based on fully-connected single hop wireless networks.

VIII. Conclusion and On-going Work

In this paper, we define a general model and utility functions for optimizing utilization and fairness in spectrum allocation for open spectrum systems. By reducing the optimal allocation to one of *Color-Sensitive Graph Coloring (CSGC)*, we show that it is an NP-hard problem. While taking into account spectrum heterogeneity, we describe a set of approximation algorithms for both centralized and distributed approaches to spectrum allocation. Our experimental results show that not only can our algorithms drastically improve network performance by reducing interference, but our distributed algorithm provides benefits comparable to the centralized approach while drastically reducing computation complexity.

While we propose several computationally efficient distributed allocation algorithms in this paper, we assumed a static network environment and focused on optimizing a snapshot of the network. If we consider a dynamic network, network-wide spectrum allocation becomes a more complex problem. The algorithm needs to recompute allocations as the topology changes. We develop an adaptive approach that adapts to topology variations through local optimizations [4]. To reduce communication overhead, we develop a rule based spectrum management scheme where users observe local interference patterns and act independently according to preset spectrum rules [27]. We are also examining the impact of changing spectrum availability and bandwidth distributions on our algorithms.

Acknowledgments

The authors would like to acknowledge the anonymous reviewers for their insightful suggestions.

APPENDIX I
PSEUDO CODE FOR MODELING NETWORK CONFLICT GRAPH

Deploy K primary users: each primary user k ($1 \le k \le K$) locates in x_k, and uses channel y_k.
Deploy N secondary users: each secondary user n ($1 \le n \le N$) locates in ϕ_n.

for $n = 1$ to N **do**
  $D_{SE}(n,m) = \min(d_{max},\ \min_{k=1..K, y_k=m}\{DIST(\phi_n, x_k) - D_{PR}\})$
  if $D_{SE}(n,m) > d_{min}$ **then**
    $B_{n,m} = D_{SE}(n,m)^2$, $l_{n,m} = 1$
  else
    $B_{n,m} = l_{n,m} = 0$
  end if
end for
for $n = 1$ to $N-1$ **do**
  for $i = n+1$ to N **do**
    for $m = 1$ to M **do**
      if $D_{SE}(n,m) + D_{SE}(i,m) \ge DIST(\phi_n, \phi_i)$ **then**
        $c(n,i,m) = c(i,n,m) = 1$
      else
        $c(n,i,m) = c(i,n,m) = 0$
      end if
    end for
  end for
end for

APPENDIX II
PROOF OF *Theorem* 1

In this section, we provide the proof of *Theorem* 1. We start with the following denotations.

- $\mathcal{S}^{(i)} = \{(n_i, m_i)\}$: The chosen vertex and the associate color, *i.e.* vertex-color pair at the ith coloring stage;
- $\mathcal{A}_n^{(i)} = \{(n,m) | (n,m) \in \mathcal{S}^{(k)}, k \le i\}$: The list of color assignment at vertex n before the ith stage.
- $l_{n,m}^{(i)}$: The availability of m after the ith coloring stage. (After each assignment, a set of colors are removed (or disabled) from some vertices.)
- $\mathcal{F}^{(i)} = \{(n,m) | l_{n,m}^{(i)} = 1\}$:The set of available vertex-color pairs after ith stage.
- $u_{n,m}^{(i)}$: An indicator of the disabled vertex-color pair due to the ith coloring, *i.e.* $u_{n,m}^{(i)} = 1$ only if $l_{n,m}^{(i-1)} = 1$ and $l_{n,m}^{(i)} = 0$.
- $\mathcal{U}^{(i)} = \{(n,m) | \mathbf{u}_{n,m}^{(i)} = 1\}$: The set of disabled vertex-color pair due to ith coloring.
- $D_{n,m}^{(i)}$: Vertex n's degree on color m after ith coloring, *i.e.* $D_{n,m}^{(i)} = \sum_{k=0,k\ne n}^{N-1} c_(n,k,m) \cdot l_{n,m}^{(i)} \cdot l_{k,m}^{(i)}$. Let $D_{n,m} = D_{n,m}^{0}$.
- $N_m^{(i)}(n) = \{k | c_{n,k,m=1}, l_{k,m}^{(i)} = 1, k < N, k \ne n\}$: The set of m colored neighborhood of vertex n after ith coloring. $D_{n,m}^{(i)} = |N_m^i(n)|$.
- $\mathcal{M}(K)$: The set of K preferred vertex-color pairs in the system *i.e.*

$$\mathcal{M}(K) = \{(n,m) | m \in \pi(n,K), n < N\}. \tag{24}$$

The following lemmas will be used in the proof.

Lemma 1:
$\mathcal{F}^{(i)}$, $\mathcal{U}^{(i)}$ and $\mathcal{S}^{(i)}$ are related by

$$\begin{aligned}
\mathcal{F}^{(i-1)} &= \mathcal{F}^{(i)} \cup \mathcal{U}^{(i)} \cup \mathcal{S}^{(i)}, \\
\mathcal{F}^{(i)} \cap \mathcal{U}^{(i)} &= \mathcal{F}^{(i)} \cap \mathcal{S}^{(i)} = \mathcal{U}^{(i)} \cap \mathcal{S}^{(i)} = \emptyset \\
\mathcal{F}^{(0)} &= \Big(\bigcup_i \mathcal{U}^{(i)}\Big) \cup \Big(\bigcup_i S^{(i)}\Big).
\end{aligned} \tag{25}$$

The proof is trivial and thus omitted.

Lemma 2:

$$D_{n,m}^{(i)} \le D_{n,m}^{(j)} \le D_{n,m}^{(0)}, \quad \forall\ i \ge j \ge\ 0. \tag{26}$$

Proof: Since $l_{n,m}^{(i)} \le l_{n,m}^{(i-1)}$, then it is obvious that the vertex degree $D_{n,m}^{(i)}$ is a non-increasing function of i. ■

Lemma 3:
Using CSum rule based coloring scheme, the color assignment at ith coloring stage is (n_i, m_i), and

$$(n_i, m_i) = \underset{(n,m)\in\mathcal{F}^{(i-1)}}{\arg\max} \frac{b_{n,m}}{D_{n,m}^{(i-1)} + 1}. \tag{27}$$

It satisfies,

$$\begin{gathered}
\forall i > 0,\ \forall (n,m) \in \mathcal{F}^{(i-1)} \\
\frac{b_{n_i,m_i}}{(D_{n_i,m_i}^{(i-1)} + 1)} \ge \frac{b_{n,m}}{(D_{n,m}^{(i-1)} + 1)} \ge \frac{b_{n,m}}{(D_{n,m}^{(0)} + 1)}.
\end{gathered} \tag{28}$$

Proof: The proof is trivial by combining (25),(26) and (27). ■

To prove Theorem 1, we start by analyzing $\mathcal{M}(K)$. Since $\mathcal{M}(K) \subseteq \mathcal{F}^{(0)}$, we can divide it into two groups, *i.e.*

$$\mathcal{M}(K) = \underbrace{\Big(\bigcup_i \mathcal{U}^{(i)} \cap \mathcal{M}(K)\Big)}_{\text{joint with disabled vertex-color pair}} \cup \underbrace{\Big(\bigcup_i \mathcal{S}^{(i)} \cap \mathcal{M}(K)\Big)}_{\text{joint with selected vertex-color pair}}. \tag{29}$$

The first group represents the vertex-color pair in $\mathcal{M}(K)$ which is selected at each coloring stage, and the second group represents the vertex-color pair in $\mathcal{M}(K)$ that is discarded at each coloring stage (in topology updating). Next, we analyze the vertex-color pairs of two groups separately.

For a vertex-color pair (n,m) chosen at ith coloring stage, it is obvious that the following lemma holds.

Lemma 4:

$$\begin{aligned}
& \sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{S}^{(i)}} \frac{b_{n,m}}{D_{n,m}^{(0)} + 1} \\
\le & \sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{S}^{(i)}} \frac{b_{n_i,m_i}}{D_{n_i,m_i}^{(i-1)} + 1} \quad (by\ Lemma\ 3) \\
= & \ |\mathcal{M}(K) \cap \mathcal{S}^{(i)}| \frac{b_{n_i,m_i}}{D_{n_i,m_i}^{(i-1)} + 1} \\
= & \ (1 - |\mathcal{S}^{(i)} \backslash \mathcal{M}(K)|) \frac{b_{n_i,m_i}}{D_{n_i,m_i}^{(i)} + 1}
\end{aligned} \tag{30}$$

For a vertex-color pair (n, m) disabled at ith stage, the following lemma holds.

Lemma 5:

$$\begin{aligned}
&\sum_i \sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{U}^{(i)}} \frac{b_{n,m}}{D_{n,m}^{(0)}+1} \\
\leq\ & \sum_i \frac{b_{n_i,m_i}}{D_{n_i,m_i}^{(i-1)}+1}(D_{n_i,m_i}^{(i-1)} + |\mathcal{S}^{(i)}\backslash\mathcal{M}(K)|). \quad (31)
\end{aligned}$$

Proof: There are two reasons to disable a vertex-color pair.

- At each coloring stage, after a vertex is colored, the color will be deleted from the neighbors of the vertex to avoid future conflict. Let
$$\mathcal{U}_{\mathcal{N}}^{(i)} = \bigcup_{n\in N_{m_i}^{(i-1)}(n_i)} (n, m_i)$$
represent the set of vertex-color pairs disabled at the ith coloring stage to prevent future conflict. These vertices are the neighbors of the selected vertex n_i, who share a m_i colored edge with n_i. Obviously the size of $\mathcal{U}_{\mathcal{N}}^{(i)}$ is the number of conflicting neighbors of n_i who has m_i available, *i.e.* $|\mathcal{U}_{\mathcal{N}}^{(i)}| = D_{n_i,m_i}^{(i-1)}$.
- At each coloring stage, after a vertex is colored, the assignment of the vertex might reach the maximum constraints $K = C_{max}$. The vertex and its color list will be deleted. Let
$$\mathcal{U}_{\mathcal{S}}^{(i)} = \bigcup_{m\neq m_i, l_{n_i,m}^{(i)}=1, |\mathcal{A}_{n_i}^{(i)}|=K} (n_i, m)$$
represent the vertex-color pairs that are disabled because vertex n_i's assignment reaches K.

Obviously

$$\mathcal{U}^{(i)} = \mathcal{U}_{\mathcal{N}}^{(i)} \cup \mathcal{U}_{\mathcal{S}}^{(i)},\ \mathcal{U}_{\mathcal{N}}^{(i)} \cap \mathcal{U}_{\mathcal{S}}^{(i)} = \emptyset \quad (32)$$

For each $(n_i, m) \in \mathcal{M}(K) \cap \mathcal{U}_{\mathcal{N}}^{(i)}$, since it hasn't been chosen by the labeling rule, we have

$$\begin{aligned}
&\sum_i \sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{U}^{\mathcal{N}(i)}} \frac{b_{n_i,m}}{D_{n_i,m}^{(0)}+1} \\
\leq\ & |\mathcal{U}_{\mathcal{N}}^{(i)}| \frac{b_{n_i,m_j}}{D_{n_i,m_j}^{(j-1)}+1} = D_{n_i,m_i}^{(i-1)} \frac{b_{n_i,m_j}}{D_{n_i,m_j}^{(j-1)}+1}. \quad (33)
\end{aligned}$$

For each $(n_i, m) \in \mathcal{M}(K) \cap \mathcal{U}_{\mathcal{S}}^{(i)}$, since $|\mathcal{A}_{n_i}^{(i)}| = K$, then

$$K = |\mathcal{A}_{n_i}^{(i)}| = |\mathcal{M}(K) \cap \mathcal{A}_{n_i}^{(i)}| + |\mathcal{A}_{n_i}^{(i)}\backslash\mathcal{M}(K)| \quad (34)$$

From the definition of $\pi(n_i, K)$, we have

$$\begin{aligned}
K &= |\pi(n_i, K)| = |\pi(n_i, K) \cap \mathcal{A}_{n_i}^{(i)}| + |\pi(n_i, K)\backslash\mathcal{A}_{n_i}^{(i)}| \\
&= |\mathcal{M}(K) \cap \mathcal{A}_{n_i}^{(i)}| + |\mathcal{M}(K) \cap \mathcal{U}_{\mathcal{S}}^{(i)}| \\
&\quad + \sum_{j<i} |\pi(n_i, K) \cap \mathcal{U}_{\mathcal{N}}^{(j)}|. \quad (35)
\end{aligned}$$

Combining (34) and (35), we get

$$\begin{aligned}
&|\mathcal{M}(K) \cap \mathcal{U}_{\mathcal{S}}^{(i)}| \\
=\ & \delta(|\mathcal{A}_{n_i}^{(i)}| = K)(|\mathcal{A}_{n_i}^{(i)}\backslash\mathcal{M}(K)| \\
& -\sum_{j<i} |\pi(n_i, K) \cap \mathcal{U}_{\mathcal{N}}^{(j)}|) \\
\leq\ & \delta(|\mathcal{A}_{n_i}^{(i)}| = K)|\mathcal{A}_{n_i}^{(i)}\backslash\mathcal{M}(K)|. \quad (36)
\end{aligned}$$

For each $(n_i, m) \in \mathcal{M}(K) \cap \mathcal{U}_{\mathcal{S}}^{(i)}$, since it hasn't been chosen by the labeling rule, the following holds:

$$\begin{aligned}
\frac{b_{n_i,m}}{D_{n_i,m}^{(0)}+1} &\leq \frac{b_{n_i,m_j}}{D_{n_i,m_j}^{(j-1)}+1},\ \forall j \leq i,\ (n_i, m_j) \in \mathcal{A}_{n_i}^{j} \\
&\leq \min_{j\leq i, (n_i,m_j)\in\mathcal{A}_{n_i}^{j}} \frac{b_{n_i,m_j}}{D_{n_i,m_j}^{(j-1)}+1}. \\
&\triangleq \xi(n_i, i). \quad (37)
\end{aligned}$$

Hence, we can derive

$$\begin{aligned}
&\sum_i \sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{U}_{\mathcal{S}}^{(i)}} \frac{b_{n,m}}{D_{n,m}^{(0)}+1} \\
\leq\ & \sum_i \delta(|\mathcal{A}_{n_i}^{(i)}| = x)|\mathcal{M}(K) \cap \mathcal{U}_{\mathcal{S}}^{(i)}|\xi(n_i, i) \\
\leq\ & \sum_i \delta(|\mathcal{A}_{n_i}^{(i)}| = x)|\mathcal{A}_{n_i}^{(i)}\backslash\mathcal{M}(K)|\xi(n_i, i) \\
\leq\ & \sum_i \frac{b_{n_i,m_i}}{D_{n_i,m_i}^{(i-1)}+1}|\mathcal{S}^{(i)}\backslash\mathcal{M}(K)|. \quad (38)
\end{aligned}$$

■

Combining *lemma 4* and *5* we have

$$\begin{aligned}
&GB(K) \\
=\ & \sum_{n=1}^{N-1} \sum_{m\in\pi(n,K)} \frac{b_{n,m}}{D_{n,m}^{(0)}+1} \\
=\ & \sum_{(n,m)\in\mathcal{M}(K)} \frac{b_{n,m}}{D_{n,m}^{(0)}+1} \\
=\ & \sum_i \left(\sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{S}^{(i)}} \frac{b_{n,m}}{D_{n,m}^{(0)}+1} \right) \\
& + \sum_i \left(\sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{U}_{\mathcal{N}}^{(i)}} \frac{b_{n,m}}{D_{n,m}^{(0)}+1} \right) \\
& + \sum_i \left(\sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{U}_{\mathcal{S}}^{(i)}} \frac{b_{n,m}}{D_{n,m}^{(0)}+1} \right) \\
\leq\ & \sum_i \frac{b_{n_i,m_i}}{D_{n_i,m_i}^{(i-1)}+1}(1 - |\mathcal{S}^{(i)}\backslash\mathcal{M}(K)|) \\
& + \sum_i \frac{b_{n_i,m_i}}{D_{n_i,m_i}^{(i-1)}+1} \left(|\mathcal{U}_{\mathcal{N}}^{(i)}| + |\mathcal{S}^{(i)}\backslash\mathcal{M}(K)| \right) \\
=\ & \sum_i \left(\frac{b_{n_i,m_i}}{D_{n_i,m_i}^{(i-1)}+1} + D_{n_i,m_i}^{(i-1)} \cdot \frac{b_{n_i,m_i}}{D_{n_i,m_i}^{(i-1)}+1} \right) \\
=\ & \sum_i b_{n_i,m_i}. \quad (39)
\end{aligned}$$

This completes the proof of **Theorem 1**.

In the special case of $x = M$ ($x \geq M$), that is, each vertex can use as many colors as possible, $\mathcal{M}(K)$ is equal to $\mathcal{F}^{(0)}$, $\mathcal{U}_{\mathcal{S}}^{(i)} = \emptyset$, and then (39) can be rewritten as

$$\begin{aligned} & GB(M) \\ = & \sum_{n<N}\sum_{m<M} \frac{b_{n,m}}{D_{n,m}+1} \\ = & \sum_i \left(\sum_{(n,m)\in\mathcal{S}^{(i)}} \frac{b_{n,m}}{D_{n,m}+1} + \sum_{(n,m)\in\mathcal{U}^{(i)}} \frac{b_{n,m}}{D_{n,m}+1} \right) \\ \leq & \sum_i \left(\frac{b_{n_i,m_i}}{D^{(i-1)}_{n_i,m_i}+1} + \sum_{n\in N^{(i-1)}_{m_i}(n_i)} \frac{b_{n,m_i}}{D^{(i-1)}_{n,m_i}+1} \right) \\ = & \sum_i \left(\frac{b_{n_i,m_i}}{(D^{(i-1)}_{n_i,m_i}+1)} + D^{(i-1)}_{n_i,m_i} \cdot \frac{b_{n_i,m_i}}{(D^{(i-1)}_{n_i,m_i}+1)} \right) \\ = & \sum_i b_{n_i,m_i} \end{aligned} \tag{40}$$

Actually, (40) obtains the generalization of Weighted Independent Set Problem in Color-Sensitive Graph Coloring Problem in [21].

Appendix III
Proof of *Corollary 1*

In a similar way, we can expand the above results to the distributed cases, as *Corollary 1*. The difference between the centralized and distributed implementations is the vertex choice, that is, more than one vertex may be chosen at one coloring stage in distributed implementation, and $\mathcal{S}^{(i)}$ probably consists of multi pairs, $\mathcal{S}^{(i)} = \{(n_{i_1}, m_{i_2}), (n_{i_2}, m_{i_2}), \cdots\}$. Let $\mathcal{U}^{(i)}_{(n_i,m_i)}$ represent the individually disabled pair set by (n_i, m_i), obviously $\mathcal{U}^{(i)} = \underset{(n_i,m_i)}{\cup} \mathcal{U}^{(i)}_{(n_i,m_i)}$. Since multi pairs may be chosen at one stage, their individual disabled sets may be overlapped, therefore,

$$\begin{aligned} & \sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{U}^{(i)}} \frac{b_{n,m}}{D^{(0)}_{n,m}+1} \\ \leq & \sum_{(n_i,m_i)\in\mathcal{S}^{(i)}} \left(\sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{U}^{(i)}_{(n_i,m_i)}} \frac{b_{n,m}}{D^{(0)}_{n,m}+1} \right) \end{aligned} \tag{41}$$

Hence,

$$\begin{aligned} & GB(x) \\ = & \sum_i \left(\sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{S}^{(i)}} \frac{b_{n,m}}{D^{(0)}_{n,m}+1} \right) \\ & + \sum_i \left(\sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{U}^{(i)}} \frac{b_{n,m}}{D^{(0)}_{n,m}+1} \right) \\ \leq & \sum_i \sum_{(n_i,m_i)\in\mathcal{S}^{(i)}} \sum_{(n,m)\in\mathcal{M}(K)\cap(n_i,m_i)} \frac{b_{n,m}}{D^{(0)}_{n,m}+1} \\ + & \sum_i \sum_{(n_i,m_i)\in\mathcal{S}^{(i)}} \sum_{(n,m)\in\mathcal{M}(K)\cap\mathcal{U}^{(i)}_{(n_i,m_i)}} \frac{b_{n,m}}{D^{(0)}_{n,m}+1} \end{aligned} \tag{42}$$

Using similar approach as in proving (39), (42) can be expanded as

$$\begin{aligned} & GB(x) \\ \leq & \sum_i \sum_{(n_i,m_i)\in\mathcal{S}^{(i)}} \left(\frac{b_{n_i,m_i}}{D^{(i-1)}_{n_i,m_i}+1} + D^{(i-1)}_{n_i,m_i} \frac{b_{n_i,m_i}}{D^{(i-1)}_{n_i,m_i}+1} \right) \\ = & \sum_i b_{n_i,m_i}. \end{aligned} \tag{43}$$

□

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