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Validity of fracture mechanics concepts applied to wood by finite element calculation

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Summary. Since Porter (1964), some authors have applied fracture mechanics concepts to wood by assuming that the material is orthotropic and elastic. In order to verify these assumptions, the objective of this paper is to compare experimental results and finite elements computation.

Symbols

a	Crack length
b	Specimen thickness
C	Compliance
E	Elastic modulus
G_{Ic}	Critical strain energy release rate for mode 1 so-called toughness
G	Shear modulus
K_{Ic}	Critical stress intensity factor for mode 1
P	Load
S^*	$(\text{Apparent elastic modulus})^{-1}$
S_{ij}	Elastic compliance tensor
w	Specimen width
λ	Crack lips opening
ε	Strain
σ	Stress
τ	Shear stress
ν	Poisson's ratio

Introduction

Owing to its particular structure, wood is a very anisotropic, inhomogeneous and non-linear elastic material. In order to estimate some characteristic fracture mechanics parameters, it is necessary to make assumptions for the simplification of calculations. In fact, if it is easy to determine the strain energy release rate for mode 1, G_I, with a compliance method for instance, the determination of the stress intensity factor K_I with the aid of the relation

$$G_I = R \cdot K_I^2$$

where R is a proportionality coefficient, is not obvious. This relation is valid for orthotropy, plane strain conditions and linear elastic behavior (Sih et al. 1965). The purpose of this work is to compare experimental measurements of fracture toughness of wood with theoretical calculations by a finite elements method and an analytical method, and, by this way, to test the validity of the assumptions.

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Experimental

Strain energy release rate determination

The strain energy release rate G_{1c} is measured with the compliance method. G_{1c} is the available energy for crack propagation. Briefly, this method consists in measuring the variation of the compliance of a cracked specimen with the length of the crack. G_{1c} is then given by:

$$G_{1c} = \frac{P_c^2}{2\,b} \cdot \frac{\partial C}{\partial a}. \qquad (1)$$

Details on this method can be found in basic books on fracture mechanics (Knott 1973).

Tests were performed on a European hardwood, Beech (Fagus sylvatica) at temperature and humidity conditions of the laboratory. The moisture content of the samples is about 10%. The sampling orientation is denoted TL, that is to say, the crack lies in the RL plane, the load is applied in the T direction and the crack propagates in the L direction (Fig. 1). Two types of specimens were used, single edge notch tensile (SENT) and double cantilever beam (DCB), with the dimensions given in Fig. 1. The specimens were set on a tensile testing machine and loaded at a crosshead speed of 1 mm/mn. The load versus crack lips opening curves ($P - \lambda$) were recorded (Figs. 2 and 3). Several methods for the determination of the load corresponding to the beginning of the propagation of the crack have been used (Triboulot 1981) and they have shown that this load corresponds to the point where the $P - \lambda$ curve is no longer linear. The compliance C is the reverse of the slope of the $P - \lambda$ curve and C is plotted versus a/w. The experimental points are fitted by an exponential curve $C = A \exp(B.\, a/w)$ which is easily derived. G_{1c} is calculated by (1). The results obtained on both type of specimens are reported on Fig. 4.

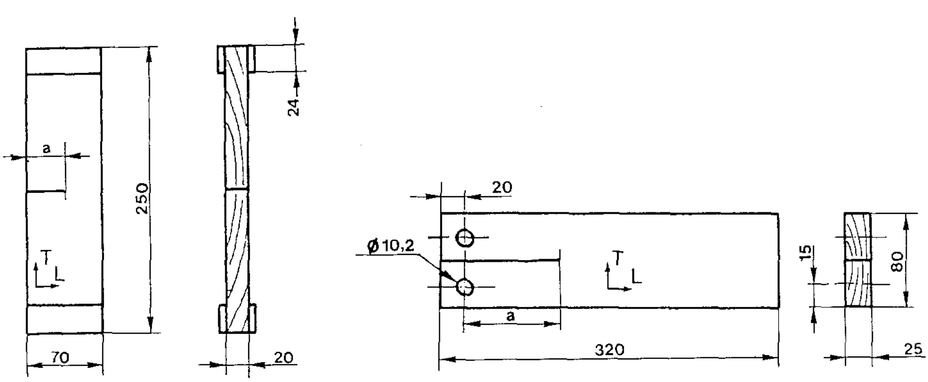

Fig. 1. Design of specimens (a) SENT; (b) DCB

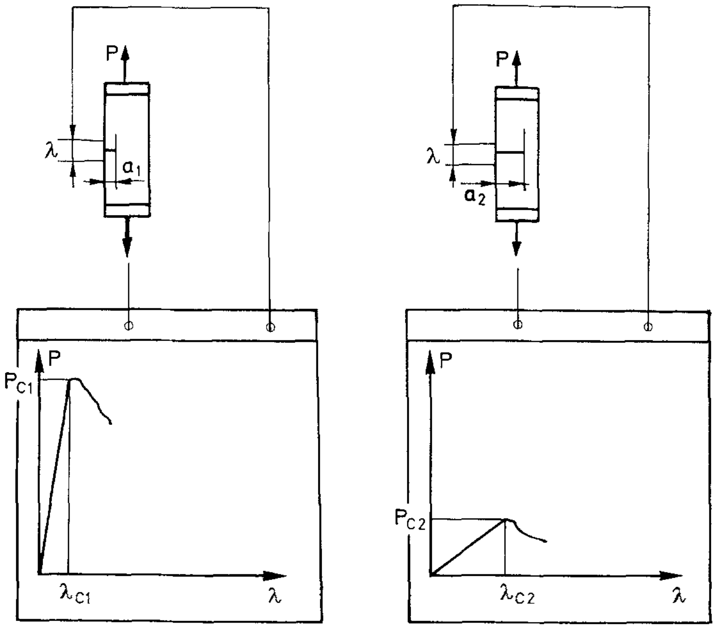

Fig. 2. Schematic representation of experimental device and typical recording for SENT specimens

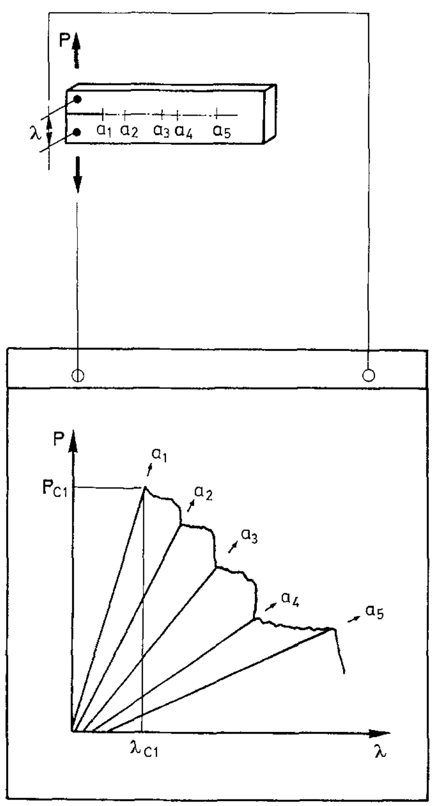

Fig. 3. Typical recording for DCB specimens

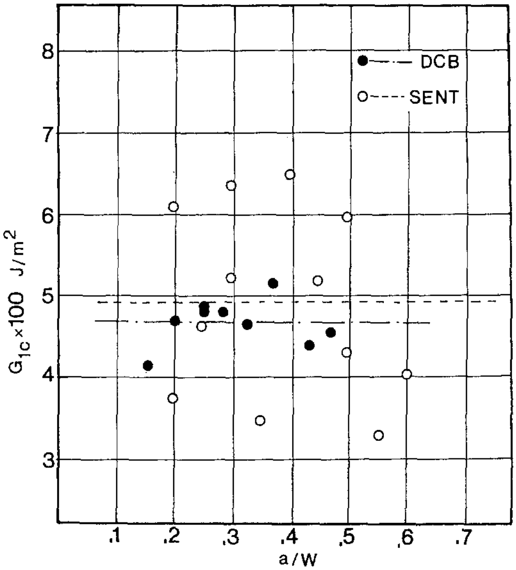

Fig. 4. Results obtained on (a) SENT specimens; (b) DCB specimens

Stress intensity factor calculation

Sih et al. (1965) have shown that in an orthotropic cracked body, the critical stress intensity factor K_{Ic} is related to G_{Ic} by:

$$G_{Ic} = S^* \cdot K_{Ic}^2 \tag{2}$$

where S^* is the reverse of an apparent elastic modulus

$$S^* = \left[\frac{S_{11} S_{22}}{2}\right]^{1/2} \left[\left(\frac{S_{22}}{S_{11}}\right)^{1/2} + \left(\frac{2 S_{12} + S_{66}}{2 S_{11}}\right)\right]^{1/2} \tag{3}$$

with $S_{11} = \frac{1}{E_L}$, $S_{22} = \frac{1}{E_T}$, $S_{12} = \frac{\nu_{TL}}{E_T}$, $S_{66} = \frac{1}{G_{LT}}$.

The elastic compliance tensor is experimentally determined according to a recently published method (Seichepine 1980). The values are given in Table 1.

Using these values it is possible to determine K_{Ic} with (2). The results are given in Table 2.

Validity of the assumptions

Plane strain assumption

A rigorous plane strain condition in the specimens tested implies that the strain in the transverse direction R is equal to zero. In real cases, this cannot be achieved because the free surfaces are necessarily in plane stress conditions. The validity of

Table 1. Elastic compliances in Pa^{-1}

S_{11}	S_{22}	S_{12}	S_{66}	S^*
$7.10 \cdot 10^{-11}$	$1.16 \cdot 10^{-9}$	$-2.80 \cdot 10^{-11}$	$5.10 \cdot 10^{-10}$	$5.45 \cdot 10^{-10}$

Table 2. Critical strain energy release rate and critical stress intensity factor

Specimen	G_{1c} J/m^2	S^* m^2/N	K_{1c} $MPa\sqrt{m}$
SENT	491	$5.45 \cdot 10^{-10}$	0.95
DCB	475	$5.45 \cdot 10^{-10}$	0.93

a plane strain assumption is then tested by the ratio of the volume of material in plane stress condition over the volume of material in plane strain condition. In the case of our experiments, the strain of the specimen across the thickness near the crack tip has been recorded during the loading (Triboulot 1981). An elastic calculation has shown that the plane strain conditions are satisfied over about 65% of the total thickness of the specimen. The plane strain assumption is considered as being valid.

Orthotropy and elastic behavior

These assumptions have been tested by a finite elements calculation on the SENT specimen. The mesh had 156 triangular elements and 99 nodes (Fig. 5). An elastic and orthotropic behavior law of the material was introduced. At the crack tip there were eight special singular elements to take into account the singularity of the stress field. According to Zienkiewicz (1971):

$$[\sigma] = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [D] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{4}$$

where [D] is the stiffness matrix defined by:

$$[D] = \frac{E_T}{(1+\nu_{LT})(1-\nu_{LT}-2n\,\nu_{TL}^2)} \cdot \begin{bmatrix} n(1-n\,\nu_{TL}^2) & n\,\nu_{TL}(1+\nu_{LT}) & 0 \\ n\,\nu_{LT}(1+\nu_{LT}) & (1-\nu_{LT}^2) & 0 \\ 0 & 0 & m(1+\nu_{LT})(1-\nu_{LT}-2n\,\nu_{TL}^2) \end{bmatrix}$$

and:

$$n = \frac{E_L}{E_T} = \frac{\nu_{LT}}{\nu_{TL}}; \qquad m = \frac{G_{LT}}{E_T}$$

and from the data of Table 1:

D (1, 1) = 17044 MPa
D (2, 1) = D (1, 2) = 575 MPa
D (2, 2) = 889 MPa
D (3, 3) = 1961 MPa
D (1, 3) = D (2, 3) = D (3, 1) = D (3, 2) = 0 .

The load is concentrated on node 98, and the rotation is free around this node, according to the experimental device. The results of the computations are the displacements of the nodes, and it is then possible to draw a computed load-crack lips opening curve. This curve is in good agreement with the experimental one (Fig. 6). It is also possible to obtain a computed stress intensity factor from the stresses of element 8 (Fig. 5) (Triboulot 1981). This value (0.91 MPa $\sqrt{\text{m}}$) agrees well with the experimental values of Table 2.

It can be concluded from these calculations that the orthotropic and linear elastic behavior model is valid.

Comparison with an analytical method

From the data of the eight singular elements around the crack tip, it is possible to obtain the stresses distribution. This can be also obtained from the analytical expressions given by Sih et al. (1965) who have used the Westergaard's (1939)

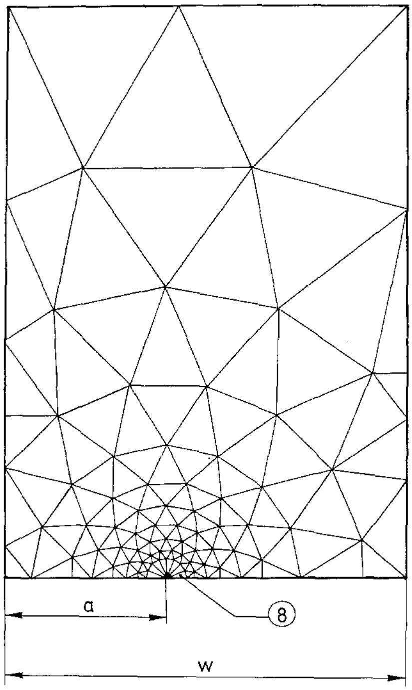

Fig. 5. Mesh used for finite elements calculations

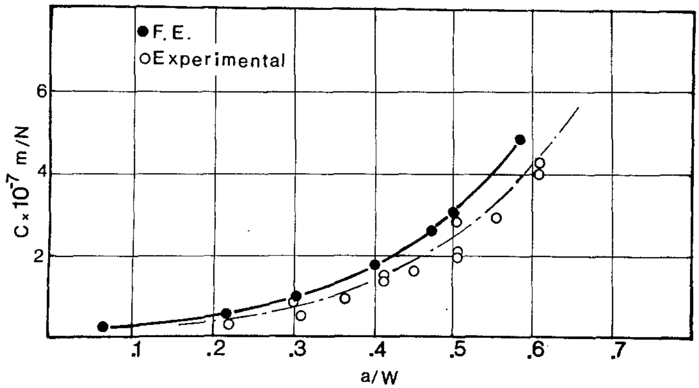

Fig. 6. Compliance curve. Experimental and finite elements results

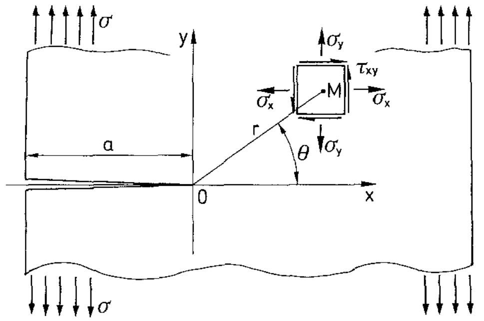

Fig. 7. Coordinate system and stress notation at crack tip

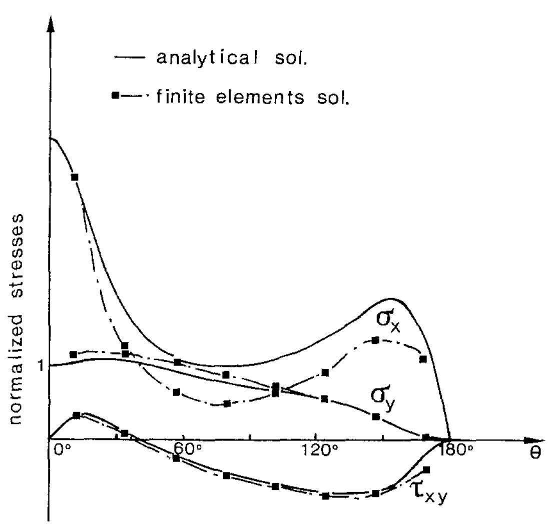

Fig. 8. Comparison between analytical and finite elements calculations for stresses around crack tip

method. The stresses σ_x, σ_y and τ_{xy} (Fig. 7) are obtained as a function of $K_I/\sqrt{2\pi r}$, θ, and of the elastic compliances. The normalized stresses $\sigma_x\sqrt{2\pi r}/K_I$, $\sigma_y\sqrt{2\pi r}/K_I$ and $\tau_{xy}\sqrt{2\pi r}/K_I$ are plotted versus θ and compared with the finite elements solution. The agreement is quite good and confirms the validity of the orthotropic and linear elastic behavior assumption (Fig. 8).

Conclusion

Although many values for critical strain energy release rate and critical stress intensity factors can be found in the literature, their credibility is not obvious, since the assumptions for plane strain, elastic behavior and orthotropy have not been discussed.

For the experimental conditions described in this paper, it is possible to obtain reliable values for the fracture toughness of the species under study. These values can be used by structural engineers. But it must be borne in mind that wood exhibits a high natural variability and this can only be taken into account by a statistical analysis giving a range of variation and a mean value for the fracture toughness data. A more refined model for wood must be elaborated to take into account the fibrous nature of its structure, its viscoelastic behavior and the strong influence of humidity. Moreover, if the sampling orientation is LT instead of TL, the crack propagation occurs perpendicular to the original crack plane RT in the L direction, and this phenomenon is not yet completely understood (Triboulot 1981).

References

Knott, J. F. 1973: Linear elastic fracture mechanics. In: Fundamentals of fracture mechanics: 94–113. London: Butterworth

Porter, A. W. 1964: On the mechanics of fracture in wood. For. Prod. J. 14: 325–331

Seichepine, J. L. 1980: Mise au point d'une méthode expérimentale destinée à l'identification de la matrice des complaisances élastiques de solides anisotropes. Application au bois. Thesis, Nancy, I.N.P.L., France

Sih, G. C.; Paris, P. C.; Irwin, G. R. 1965: On cracks in rectilinearly anisotropic bodies. Int. J. Fracture 1: 189–203

Triboulot, P. 1981: Application de la mécanique de la rupture aux bois massifs. Thesis, University of Technology of Compiègne, France

Westergaard, H. M. 1939: Trans. ASME. Ser. E. J. Appl. Mech. 61: 49–53

Zienkiewicz, O. C. 1971: The finite element method in engineering science; 52–55. London: McGraw-Hill

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