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Validity of Linearized Unsteady Euler Equations with Shock Capturing

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By examining the nature of shock movement, this article first shows that the unsteady lift due to the movement of a shock is linear with the magnitude of the shock movement. The argument that is presented holds true regardless of the shock structure, which is determined by the level of viscosity. This proof is the basis for showing that the linear perturbation equations can be used to determine not only the unsteadiness of the flowfield away from the shock but also the effect of the shock movement as well. Shock capturing is a computational technique that in effect adds a large amount of viscosity in the shock region, smearing shocks over several cells. After proving that the analytical linear viscous equations can be used to represent the flowfield, computational aspects relating to shock capturing are examined. These arguments provide the basis for using the linearized unsteady Euler equations with shock capturing as a viable computational technique. The strength of this new technique is the reduced computational effort required to find the effect of an unsteady perturbation on the flowfield.

Introduction

UNSTEADY flow calculations are of great importance to the aeronautical gas turbine industry because of the need to predict the onset of flutter, as well as the magnitude of forced response loading and oscillations. In recent years, many methods have been developed to solve the unsteady, nonlinear, two-dimensional and three-dimensional Euler or Navier-Stokes equations, but these methods are in general too expensive for everyday industrial use. Therefore, industry continues to use two-dimensional, linearized potential methods that require significantly less computational effort for most unsteady calculations.[1,2] These methods are limited, however, in their suitability for flows with moderate shocks and three-dimensional vorticity. Accordingly, there is a strong interest in developing a new class of methods, linearized Euler methods, in which the steady and unsteady flows can be both three-dimensional and vortical and the only assumption is that the level of unsteadiness is small.[3-7]

A two-dimensional linearized Euler method was developed by Hall and Crawley[3] and Hall,[8] who showed the usefulness of the linear assumption up to surprisingly large levels of unsteadiness. This work also demonstrated a limited capacity for shock fitting. This is an important subject because a significant fraction of the unsteady lift in compressor or fan flutter comes from the shock motion.

It is believed, however, that shock fitting in three dimensions will prove to be an intractable problem, and so an alternative shock-capturing approach has been developed. Although Whitehead[1] and Verdon and Casper[2] have used linearized shock capturing with the potential equations, their work lacks any mathematically rigorous justification. The purpose of this paper is to prove the validity of using shock capturing in perturbation methods. This proof is based on the linearization of the standard, unsteady, nonlinear shock-capturing Euler methods and will be used to define a new perturbation scheme.

Analysis of the Navier-Stokes Equations

There are discretization and computational issues associated with modeling an unsteady transonic flowfield, but before considering these issues a better understanding of the analytical problem must be gained. In particular, the interest is in shock motion. Since viscosity plays an important role in the detailed characteristics of a shock, the analysis will start with the Navier-Stokes equations. The first flowfield that will be examined is a constant area duct since this represents the simplest transonic flowfield. Next, a variable area duct, which has a gradient in the flow upstream and downstream of the shock, will be studied to more realistically represent two- and three-dimensional flowfields.

Shock Motion in a Constant Area Duct

A moving shock in a constant area duct is usually referred to experimentally as a shock tube and is often used to probe the internal structure of a shock. Here it will be used to isolate and explore the effect of viscosity on a moving shock. An understanding of the analytical solution to the shock tube problem will point to features of the flowfield that must be modeled computationally.

Since the effect of viscosity is of interest, the one-dimensional Navier-Stokes equations will be considered:

$$\frac{\partial U}{\partial t}+\frac{\partial F_E}{\partial x}+\frac{\partial F_V}{\partial x}=0 \tag{1}$$

where

$$U=\begin{bmatrix}\rho\\ \rho u\\ \rho E\end{bmatrix},\quad F_E=\begin{bmatrix}\rho u\\ \rho u^2+p\\ \rho u H\end{bmatrix},\quad F_V=\begin{bmatrix}0\\ -\tau_{xx}\\ u\tau_{xx}-q_x\end{bmatrix} \tag{2}$$

In the previous expression for the viscous flux F_V, τ_{xx} is the viscous stress with viscosity μ and q_x is the heat conduction term with conductivity κ,

$$\tau_{xx}=\mu\frac{\partial u}{\partial x}$$

$$q_x=-\kappa\frac{\partial T}{\partial x}$$

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