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VALIDITY OF THE INTERFERENCE MODEL FOR π N SCATTERING *)

Charles B. Chiu
CERN - Geneva

and

A. V. Stirling
C.E.N. - Saclay

ABSTRACT

It is shown in an extensive analysis of the πN scattering data between 2 and 6 GeV/c that the interference model does not account correctly for some features of the data. Therefore some modifications of the model would be needed, if it is further applied to analyze all the forward and backward data in the intermediate energy region.

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E R R A T A

Page 4, line 7 : should read "... and t-s for the invariant amplitude..."

line 8 : the table on the left should be

s \ u	$\frac{3}{2}$	$\frac{1}{2}$
$\frac{3}{2}$	1/3	2/3
$\frac{1}{2}$	4/3	-1/3

Page 5, Eq. (II.4)

$$a_\ell^{\pm} = e^{i\delta_\ell^{\pm}} \sin \delta_\ell^{\pm} = \frac{x_\ell^{\pm}}{\epsilon - i}$$

Page 15, lines 13, 14 :

$\alpha(u) = \ldots\ldots\ldots$

$\alpha(u) = \ldots\ldots\ldots$

Page 25, line 5 : "where $\nu = (s-u)/(4M)$."

Eq. (A.I.4)

$$F^{(+)} \xrightarrow[\text{large } \nu]{} F_R^{(+)}(\nu) = \sum_i \beta_i \left[\nu^{\alpha_i} + (-\nu)^{\alpha_i} \right] \csc \pi\alpha_i (-2M)$$

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Page 26, Eq. (A.I.5)

$$(-\nu)^{\alpha_i} = |\nu|^{\alpha_i} e^{-i\pi\alpha_i} \quad \text{for } \nu > 0$$

$$\nu^{\alpha_i} = |\nu|^{\alpha_i} e^{i\pi\alpha_i} \quad \text{for } \nu < 0$$

line 4 : "... β is equal to $\gamma/2M$ up to α factors..."

last equation:

$$\int_{-\infty}^{0} \frac{Im\left[\nu'^{\alpha_i} + (-\nu')^{\alpha_i}\right]\nu'}{\nu' - \nu}\, d\nu' = -\int_{0}^{\infty} \frac{Im\left[\nu'^{\alpha_i} + (-\nu')^{\alpha_i}\right]\nu' d\nu'}{\nu' - \nu}$$

Page 27, Eq. (A.I.8)

$$\cdots + \frac{4M}{\pi} \int_0^\infty \nu' \left(q'\sigma' - \sum_i \beta_i \nu'^{\alpha_i} \right) d\nu'$$

line 4

$$\cdots = \int_0^\infty \nu' f(\nu')\, d\nu'$$

Page 28, Eq. (A.II.4)

$$\frac{Im\, F_R^{\,i}}{q} = -\frac{2}{\pi} P \int_0^\infty d\nu'\, Re\left(\frac{F_R^{\,i}}{q'}\right) \frac{\nu'}{\nu'^2 - \nu^2}$$

I. INTRODUCTION

The Regge pole model accounts for many features of the data at high energy $s^{\frac{1}{2}}$ and small momentum transfer [1] $(-t)^{\frac{1}{2}}$, but the extension of this model to lower energies meets with some difficulties. Theoretically, the lower the energy, the more difficult it becomes to justify the asymptotic assumptions and approximations used for the derivation of the Regge amplitudes, and experimentally the data reveal noticeable resonance effects. These effects are illustrated in Figs. 1 and 2, with the difference of the π^+p and π^-p total cross-sections [2] and the backward π^-p differential cross-section [3] (d.c.s.), respectively, which cannot be fitted with the crossed channel Regge pole contribution alone.

For intermediate energies, the interference model which adds the direct channel resonance amplitude to the Regge pole amplitude has received some attention [3],[4],[5], in particular for the determination of the spin and parity of the πN resonances.

Let us briefly recall the development of the interference model. The origin of the interference prescription can be traced back to the early analyses of total cross-sections, where one was naturally led to superimpose resonance contributions to a smooth background. In this way, Höhler et al. [4] studied the $\sigma^{(\pm)}$ data in the intermediate energy region using the extrapolation of Regge amplitude as a background. They also extracted from the same data the $I=\frac{1}{2}$ and $\frac{3}{2}$ πN resonance contributions [4]. One of the next proposals for studying the higher πN resonances was given by Heinz and Ross [4]. They pointed out that it is particularly interesting to look for interference effects near the backward direction. The Michigan group [3] observed striking interference effects in the π^-p backward d.c.s. These data were analyzed successfully with the interference model by Barger and Cline [5]. Stimulated by this result, other pieces of data have been fitted in the same way by Barger and Olsson [5], by Baacke and Yvert [4], by Grannis et al. [4], and by many others [4]. From these analyses, the higher πN resonances have been tentatively assigned as recurrences on Regge trajectories.

Our original goal was to make a self-consistent fit to the πN data mainly in the region between 2-6 GeV/c (near the forward and backward directions) using this interference model. We varied the resonance parameters and we used adjustable Regge amplitudes constrained by the high energy data, to achieve maximum flexibility in the fit. But in some cases the data could not be fitted. So the purpose of this paper is to make a systematic survey on πN data, in order to see what are the cases where fits can or cannot be achieved with this model. We also review results obtained by other authors using the interference model.

The plan of this paper is as follows. The definitions and the formalism of this model are given in Section II. The data concerned are [6]: a) the $\pi^{\pm}p$ total cross-sections; b) the $\pi^{\pm}p$ backward d.c.s.; c) the $\pi^{\pm}p$ d.c.s., polarization and charge exchange (c.ex.) d.c.s. near the forward direction. They are respectively analyzed in Sections III, IV and V. A summary of our main results and conclusions is presented in Section VI.

The recent work by Dolen et al. [4] came to our attention after we had finished our data analysis. As they have shown that dispersion relations help to understand some difficulties of this model, part of their evaluations is repeated in the Appendices.

II. PION-NUCLEON SCATTERING AMPLITUDES

1. General formalism [7)]

Define the invariant amplitude F, such that d.c.s. in the c.m. system is written as :

$$\frac{d\sigma}{d\Omega} = \sum_{spin} \left| \frac{F}{8\pi W} \right|^2 \qquad \text{(II.1)}$$

where $F = 8\pi W \chi_f^+ [f_1 + f_2(\sigma\cdot\hat{k}_f)(\sigma\cdot\hat{k}_i)] \chi_i$, $\hat{k}_i$ and $\hat{k}_f$ are unit vectors denoting the direction of the incoming and the outgoing pion in the c.m. system, and W is the total c.m. energy. In terms of the A and B amplitudes, one has :

$$f_1 = \frac{E+M}{8\pi W}\left[A + (W-M)B\right]$$

$$f_2 = \frac{E-M}{8\pi W}\left[-A + (W+M)B\right] \qquad \text{(II.2)}$$

where E is the nucleon energy in the c.m. system. The partial wave expansions for f_1 and f_2 are given by :

$$f_1 = \frac{1}{k}\sum_\ell \left[a_\ell^+ P'_{\ell+1}(x) - a_\ell^- P'_{\ell-1}(x)\right]$$

$$f_2 = \frac{1}{k}\sum_\ell \left[a_\ell^- - a_\ell^+\right] P'_\ell(x) \qquad \text{(II.3)}$$

where k is the c.m. momentum, $a_\ell^\pm$ are the partial wave amplitudes with the ± signs denoting the states with $J = \ell \pm \frac{1}{2}$ and $x = \hat{k}_f \cdot \hat{k}_i$.

Thus far we have suppressed the isotopic spin indices. The $\pi^{\pm}p$ elastic scattering amplitudes and the c.ex. amplitude are related to the $I=\frac{3}{2}$ and $\frac{1}{2}$ amplitudes by the matrix :

	$\frac{3}{2}$	$\frac{1}{2}$
π^+p	1	0
π^-p	1/3	2/3
c.ex.	$\sqrt{2}/3$	$-\sqrt{2}/3$

For example, $f(\pi^-p)=\frac{1}{3}f(\frac{3}{2})+\frac{2}{3}f(\frac{1}{2})$. Tables below are interpreted in a similar manner. Also define $F^{(\pm)}=\frac{1}{2}(F^{\pi^-p}\pm F^{\pi^+p})$. The isotopic crossing matrices between s–u and s–t for the invariant amplitude F are :

u \ s	$\frac{3}{2}$	$\frac{1}{2}$
$\frac{3}{2}$	1/3	4/3
$\frac{1}{2}$	2/3	−1/3

t \ s	$\frac{3}{2}$	$\frac{1}{2}$
0	2/3	1/3
1	−1/3	1/3

2. The amplitudes f_s, f_t and f_u

Define f_s to be the contribution of the direct channel resonances to the scattering amplitude, or loosely speaking, the direct channel resonance amplitude; similarly, f_t the t channel exchange amplitude, and f_u the u channel exchange amplitude. The function f_s is dominated by the contribution of the nearby resonances. The amplitudes f_t and f_u are relatively large near the forward and the backward directions, respectively, and are the dominating contributions to the scattering amplitude in these regions at high energy. We will assume the Regge pole dominance in f_t and f_u and, for the sake of discussing the interference model, also assume the asymptotic forms for f_t and f_u can be extended to the lower energy region. The detailed forms for f_s, f_t and f_u will be discussed below. The assumption of the interference model is that the amplitude can be approximated by the sum of these f_s, f_t and f_u contributions. Generally f_u is small near the forward direction and f_t is small near the backward direction. For the forward amplitude we use (f_s+f_t) and for the backward (f_s+f_u).

a. The s channel resonance contribution f_s

Near a resonance, the contribution of the resonance is assumed to be approximated by the Breit-Wigner formula. In particular, the resonating partial wave amplitude can be written as :

$$a_\ell^\pm = \frac{e^{i\delta_\ell^\pm}\sin\delta_\ell^\pm}{2i} = \frac{x_\ell^\pm}{\epsilon - i} \qquad \text{(II.4)}$$

where

$x = \Gamma_e/\Gamma$, Γ_e = elastic width of the resonance;

Γ = width of the resonance;

$\epsilon = \dfrac{s_0 - s}{\sqrt{s_0}\,\Gamma}$, $\sqrt{s_0}$ = mass of the resonance.

In the energy region far away from the resonance, if x is constant, this formula gives a $1/s$ energy dependence for the real part of the amplitude $a_\ell^\pm$. In πN scattering, with this formula there are cases where a distant resonance could contribute substantially to the full scattering amplitude. For example, in the π^+p d.c.s. at 8 GeV/c near the backward dip region, the contribution from all the known low energy resonances is about ten times larger than the data (20% of the amplitude comes from the P_{33} resonance which is about 90 half-widths away).

Since the Breit-Wigner formula was derived to approximate a resonance amplitude near the position of the resonance [8], this far-reaching extension of the tails of the resonance amplitudes cannot be taken seriously. In order to evaluate how much the results obtained with this model depend on the shape of the resonance amplitude, we have used an exponential cut-off factor b(s), defined as follows :

$$b(s) = 1 \qquad \text{for} \quad s \leq s_o$$

$$b(s) = \exp\left[-c(s-s_o)\right] \quad \text{for} \quad s > s_o \qquad \text{(II.5)}$$

$$\text{with} \quad c \approx 0.5$$

so that

$$a_\ell^{\pm} = \frac{x_\ell^{\pm}}{\epsilon - i}\, b(s)$$

In this way we could estimate, by turning on and off this cut-off factor, the contribution of the distant low energy resonances (e.g., P_{33}) to the full scattering amplitude in the region of the higher resonances [e.g., Δ (2450)].

b. The t channel exchange contribution f_t

The t channel exchange amplitude is assumed to be dominated by the exchange of P, P' trajectories in the I = 0 state and the ρ trajectory in the I = 1 state. The asymptotic helicity non-flip and flip Regge amplitudes A' and B at total lab. pion energy E_L can be written as [9] :

$$A_i' = \alpha_i(\alpha_i+1)\,\gamma_i(t)\,E_L^{\alpha_i}\,\xi_+ \qquad \text{for } P \text{ and } P'$$

$$A_\rho' = (\alpha_\rho+1)\,\gamma_\rho(t)\,E_L^{\alpha_\rho}\,\xi_- \qquad \text{for } \rho$$

and (II.6)

$$B_i = \alpha_i^2(\alpha_i+1)\,\bar{\gamma}_i(t)\,E_L^{\alpha_i-1}\,\xi_+ \qquad \text{for } P \text{ and } P'$$

$$B_\rho = \alpha_\rho(\alpha_\rho+1)\,\bar{\gamma}_\rho(t)\,E_L^{\alpha_\rho-1}\,\xi_- \qquad \text{for } \rho$$

Here

$$\xi_{\pm} = - \frac{e^{-i\pi\alpha} \pm 1}{\sin \pi\alpha}$$

(with + for P and P', and − for ρ). The A' amplitude is related to the amplitudes A and B through the relation [10] :

$$A' = A + B \frac{E_L + \frac{t}{4M}}{1 - \frac{t}{4M^2}} \qquad (II.7)$$

The α factors in Eq. (II.6) for the P and P' amplitudes are parametrized according to Chew's ghost killing mechanism [11]. An alternative parametrization for the α factors for P and P' has also been suggested which is the no-compensation mechanism [12], where :

$$\begin{aligned} A_i' &= \alpha_i^2 (\alpha_i + 1)^2 \gamma_i (t) E_L^{\alpha_i} \xi_+ \\ B_i &= \alpha_i^2 (\alpha_i + 1) \bar{\gamma}_i (t) E_L^{\alpha_i - 1} \xi_+ \end{aligned} \qquad (II.8)$$

For $\pi^{\pm}p$ elastic and $\pi^{-}p$ c.ex. amplitudes, one has :

$$A'^{\pi^{\pm}p} = A_P' + A_{P'}' \mp A_\rho' \ , \qquad A'^{cex} = -\sqrt{2}\, A_\rho' \qquad (II.9)$$

Similar expressions can also be written for the B amplitudes.

c. <u>The u channel exchange contribution f_u</u>

We assume for πN backward scattering, the exchange of the Δ and nucleon trajectories is dominating and the exchange of N_γ trajectory is relatively weak as compared to the nucleon trajectory and is neglected. The contribution due to the exchange of these trajectories can be written as [13] :

$$f^{\pi^\pm p}(\sqrt{s},u) = f_1^{\pm}(\sqrt{s},u) - \cos\theta\, f_1^{\pm}(-\sqrt{s},u) + i\sin\theta\, \sigma\cdot\hat{n}\, f_1^{\pm}(-\sqrt{s},u)$$

where θ is the scattering angle in the c.m. system, $\hat{n} = ((\hat{k}_i \times \hat{k}_f)/|\hat{k}_i \times \hat{k}_f|)$ and

$$f_1^{\pm}(\sqrt{s},u) \xrightarrow[u \text{ fixed}]{s\to\infty} \sum_{i=N,\Delta} \left[D(\sqrt{s},\sqrt{u})\, F_i^{\pm}(s,\sqrt{u}) + D(\sqrt{s},-\sqrt{u})\, F_i^{\pm}(s,-\sqrt{u}) \right]$$

with

$$D(\sqrt{s},\sqrt{u}) = \frac{\left[(\sqrt{s}+M)^2-\mu^2\right]\left[\sqrt{u}-\sqrt{s}+2M\right]}{4s\sqrt{u}}$$

$$F_i^{\pm}(s,\sqrt{u}) = C_i^{\pm}\, \beta_i(\sqrt{u})\, \xi_i(\sqrt{u}) \left(\frac{s}{s_0}\right)^{\alpha_i(\sqrt{u})-\frac{1}{2}} \qquad \text{(II.10)}$$

and

$$\xi_i(\sqrt{u}) = -\frac{1 \pm \exp\left[-i\pi\left(\alpha_i(\sqrt{u})-\frac{1}{2}\right)\right]}{\sin\pi\left(\alpha_i(\sqrt{u})-\frac{1}{2}\right)}$$

where μ is the pion mass, $\pm$ denotes $\pi^{\pm}p$ as in Eq. (II.9). The symbols $C_i^{\pm}$ stand for the isotopic spin crossing coefficients. They are given explicitly in Section II.1.

3. The amplitude parameters

In our analysis, the f_s parameters used are essentially those given in Tables I and II [14]. The f_t parameters are comparable to the two sets of values given in Ref. 12). The f_u parameters are similar to those used in Ref. 13). To obtain maximum flexibility, in our fit the parameters for all these three amplitudes were varied, and the forward and backward data were analyzed separately.

III. ANALYSIS OF THE πN TOTAL CROSS-SECTIONS

The contribution of the amplitudes f_s and f_t to the sum and the difference of the π^+p and π^-p total cross-sections is given by :

$$\sigma^{(\pm)} = \frac{1}{2}\left[\sigma^{\pi^-p} \pm \sigma^{\pi^+p}\right] = \frac{4\pi}{k}\, Im\, f_s^{(\pm)} + \frac{1}{P_L}\, Im\, A'^{(\pm)}$$

$$\sigma^{(\pm)} = \frac{4\pi}{k}\, Im \begin{cases} \frac{2}{3} f_s^{3/2} + \frac{1}{3} f_s^{1/2} \\ -\frac{1}{3} f_s^{3/2} + \frac{1}{3} f_s^{1/2} \end{cases} + \frac{1}{P_L}\, Im \begin{cases} A'_P + A'_{P'} \\ A'_\rho \end{cases} \qquad \text{(III.1)}$$

1. Low energy data

It has been shown by Barger and Olsson that a fit to $\sigma^{(-)}$ data with the interference model can be obtained down to 700 MeV/c [5]. We first investigate the application of this model for $\sigma^{(\pm)}$ at these low energies. The experimental data for $\sigma^{(+)}$ are shown on Fig. 3a, where Höhler et al. [15] have compared the extrapolated (P+P') Regge pole contribution $R^{(+)}$ to the background $B^{(+)}$ estimated from Roper's phase shift analysis. The ratio $B^{(+)}/R^{(+)}$ is presented on Fig. 3b. We have analyzed the $\sigma^{(-)}$ cross-section in a similar way. The extrapolated Regge pole contribution $R^{(-)}$ of the ρ and the non-resonance background $B^{(-)}$ calculated from the phase shift solution of Bareyre et al. [16] are illustrated in Fig. 1. The ratio $B^{(-)}/R^{(-)}$ presented on Fig. 3b is similar to $B^{(+)}/R^{(+)}$. Since the magnitude of B and R relative to the full amplitude is so different in the "(+)" and "(-)" cases, we shall treat them separately.

In the "(+)" case, below 1 GeV/c the extrapolated Regge amplitudes in the t channel deviate substantially in phase and magnitude from the non-resonating background amplitude given by the phase shift solution. This deviation could start around 2 GeV/c. Furthermore, in this case, all the resonance amplitudes add constructively to the background amplitude :

$$\sigma^{(+)} = \frac{4\pi}{k}\left[Im\, f_{Regge} + Im\, f_{res} \right] , \qquad Im\, f_{res} \geqslant 0 \qquad \text{(III.2)}$$

From Fig. 3a one sees that the $\sigma^{(+)}$ data oscillate about the (P+P') contributions extrapolated below 2 GeV. It is certainly not possible to fit these data by simply adding positive contributions from the resonances to the Regge background $R^{(+)}$. So the lower energy limit for the application of this model in this case should be around 2 GeV/c.

In the "(−)" case, below 1 GeV/c, the extrapolated Regge amplitudes also deviate substantially in phase and magnitude from the non-resonating background amplitude given by the phase shift solutions [16]. However, one should note that below 2 GeV/c the resonance contributions are large compared to the B and R contributions and cancellations appear between the contributions of $I = \frac{1}{2}$ and $I = \frac{3}{2}$ resonances. So a wide range of relatively different background can be used to fit the $\sigma^{(-)}$ data with a slight adjustment of the resonance parameters.

The phase shift analyses [16], which give a complete description of all the pion-nucleon data up to 2 GeV/c have shown that it is unlikely that a Breit-Wigner plus a background amplitude obtained by fitting the $\sigma^{(-)}$ data alone can be extended to fit the strenuous constraints coming from all the other data at these low energies.

2. Dispersion relations

Dispersion relations help one to understand intuitively [17] the behaviour of $\sigma^{(+)}$ and $\sigma^{(-)}$ presented in Figs. 1 and 2, where the $\sigma^{(+)}$ data oscillate about the (P+P') contribution and the $\sigma^{(-)}$ data about the ρ contribution. It has been shown above that this behaviour of $\sigma^{(+)}$ makes it impossible to obtain a fit to the data. From the Regge asymptotic assumption and the dispersion relation, it can be shown (see Appendix A) that at $t = 0$ one has the following relation :

$$\frac{f^2\mu^2}{2M} + \frac{1}{\pi}\int_0^\infty \nu' \left[\sigma^{(+)} q' - \sum_i \beta_i \nu'^{\alpha_i} \right] d\nu' = 0 \qquad \text{(III.3)}$$

where $\sum_i$ denotes the summation over all $I = 0$ t channel trajectories (for $\alpha_i \geqslant -2$) with the corresponding residue functions β_i; $f^2/4\pi \approx 0.08$. The symbols ν' and q' are the pion lab. energy and the lab. momentum, respectively. This relation shows that the asymptotic amplitude represented by the sum $\sum_i \beta_i \nu'^{\alpha_i}$ should describe the average behaviour of the amplitude over the entire energy region. In particular, in the resonance region, one expects the term $\sigma^{(+)} q'$ to oscillate around this average $\sum_i \beta_i \nu'^{\alpha_i}$. Now, if one assumes that (P+P') contribution does not deviate significantly from $\sum_i \beta_i \nu'^{\alpha_i}$, then $\sigma^{(+)} q'$ should oscillate about this (P+P') contribution. This is essentially what one observes in Fig. 2 below 2 GeV/c. An analogous expression can be written down involving a $\sigma^{(-)}$ term [17], where one finds the total cross-section $\sigma^{(-)}$ also oscillates around the corresponding contribution from the asymptotic term. Again if ρ is dominating, $\sigma^{(-)}$ is expected to oscillate about the ρ contribution. This is also what one observes (see Fig. 1).

In summary, for the $\sigma^{(+)}$ case, the resonance contribution adds constructively and the dispersion relation helps to understand intuitively why the data cannot be fitted. For the $\sigma^{(-)}$ case, the resonance contribution has opposite sign and a fit has been achieved [5].

Similar problems occur in the other non-zero degree data when the resonance contributions have the same sign. The connection between the non-cancellation of the resonance contributions and the difficulty in fitting the data was first clearly pointed out by Dolen et al. [17] in their work on finite energy sum rule. For completeness we shall repeat some of their arguments in Appendix B, incorporating our results from this Section and from Sections IV and V.

3. High energy data

Above 2.5 GeV/c, we have obtained fits to the $\sigma^{(\pm)}$ data. However, the resonance contributions are relatively small in the $\sigma^{(+)}$ case, and they have opposite sign for $I=\frac{1}{2}$ and $I=\frac{3}{2}$ in the $\sigma^{(-)}$ case. The interference effects are small. These fits only indicate that the interference model is compatible with the data.

IV. ANALYSIS OF THE $\pi^{\pm}p$ BACKWARD SCATTERING DATA

1. The $\pi^- p$ d.c.s. at 180°

As the resonance amplitude f_s contributes with either sign to the $\pi^- p$ amplitude at 180° depending on the parity of the state involved, the analysis of the structure observed in the backward data (Fig. 2) with the interference model seemed very promising [3)-5)]. But some of the difficulties encountered with this model are the following :

a) The fitted parameters of the scattering amplitudes are model dependent. For instance the magnitude of the contribution from the low energy resonances, under various assumptions for the expression of the Breit-Wigner resonant amplitude, is illustrated on Fig. 4. The continuous line shows the fit obtained with the resonance parameters listed in Table I; the dashed curve shows the d.c.s. calculated with the same parameters, except for the cut-off factor b(s) defined in (II.5) applied to Δ (1236), Δ (1924), N_γ (1512) and N_α (1688) resonant amplitudes. As the contribution of these low energy resonances is large and depends in such a crucial way on the expression used for the tail of the Breit-Wigner resonant amplitude (many half-widths from the resonance energy), the study of the higher resonances, e.g., Δ(2840) is extremely difficult.

Moreover, we have obtained satisfactory fits of the same data, using the preceding cut-off factor b(s), simply by varying the elasticity parameters within the experimental errors. As these resonance parameters are not well known [4)-6)], this gives rise to some ambiguities.

b) Even within a given model for the resonance amplitude the parameters of the π N resonances obtained from the fit of the 180° $\pi^- p$ data are still ambiguous. We obtained many different solutions both with and without cut-off factor b(s). This ambiguity is mainly due to the fact that the interference effects between nearby resonances having opposite parities are so large that it is difficult to isolate the interference effect between one resonant amplitude f_s and the background amplitude f_u. Also the number of conjectured resonances is not known with certainty. For instance, we have obtained equally satisfactory fits to the data with the two different sets of resonance parameters listed in Tables I and II.

c) The phase of the background amplitude f_u is not crucial. The u channel exchange Regge amplitudes which we used were extrapolated from previous high energy analysis [13]. To test the sensitivity of the fits on the phase of the background amplitude f_u, two sets of α_Δ trajectories [constrained to go through $\Delta(1238)$ and $\Delta(1924)$] have been used :

$$(u) = 0.1 - 0.02\sqrt{u} + 0.93\ u$$

$$(u) = -0.4 + 0.7\ \sqrt{u} + 0.7\ u$$

The phase difference between the corresponding amplitudes f_u at 180° in the energy region of interest is about 30°, but within the experimental errors we obtained equally good fits both with and without cut-off factor $b(s)$.

d) The magnitude of the Regge background amplitude is not crucial. Recently Dikmen [18] was able to fit the same data by using the direct channel resonance amplitudes alone (with no cut-off factors on the resonance tails), neglecting the Regge background amplitude.

Barger and Cline [5] have first obtained a fit to these data. Their work, together with that of Dikmen [18] and our results, indicates that there are many different solutions both with and without cut-off factors on the tails of the Breit-Wigner amplitudes. Therefore, it does not seem possible to obtain a conclusive description of the 180° $\pi^- p$ data with the interference model.

2. Determination of the resonance spin and parity assignments

The 180° $\pi^- p$ data have been used to determine the parity of conjectured resonances. All the work discussed above, together with that of Kormanyos et al. [3], is effectively consistent with the same parity assignments. One should stress the difficulties which arise from the fact that the number of conjectured resonances is not known. Recently, there have been suggestions for the existence of many more pion-nucleon resonances from phase shift solutions [16], which indicates that the situation is clearly more complicated than what one had anticipated. Around the 2.2 GeV mass region the phase shift analyses suggest the following resonances [16] :

	M(MeV)	x	Γ(MeV)
$P31^+$	1934	0.30	339
$D35^-$	1954	0.15	311
$F35^+$	1913	0.16	350
$F37^+$	1946	0.39	221
$H3,11^+$	2400	0.18	340
$P13^+$	1863	0.15	296
$F17^+$	1983	0.13	225
$D13^-$	2057	0.26	293
$G17^-$	2265	0.35	298
$H19^+$	≃ 2450	≃ 0.40	350

instead of $N_\gamma(2190)$, $N_\alpha(2200)$, $\Delta(1920)$ and $\Delta(2450)$ with the parameters listes in Tables I and II.

If there are indeed so many different resonances close to each other which give non-negligible contribution to the d.c.s., the interference model is incapable of disentangling the individual contribution and specifying the spin parity of each of these resonances from the consideration of 180° data alone. Our analysis of the data was finished before these new resonances were proposed and we considered only the resonances listed in Tables I and II.

3. The backward π^+p differential cross-section

Some of the backward π^+p differential cross-section data [6] are shown on Fig. 5. The backward dip observed at high energy has been explained by a zero in the amplitude f_u resulting from the N trajectory passing through $-\frac{1}{2}$ [13]. The curves on Fig. 5 are the fitted cross-sections obtained with the interference model from the extrapolated high energy Regge amplitudes [13] and the resonance parameters listed in Table II (with cut-off factors applied to all the resonances). Some significant defects, which cannot be eliminated with an adjustment of the resonance and the background amplitude parameters, can be noticed near 180° : the fitted curves are consistently larger than the data.

The recent more accurate π^+p backward data of Ashmore et al. [19] confirm the existence of the backward dip at high energy and therefore the interpretation in terms of N_α and Δ trajectory exchange [13]. It also confirms the extrapolation of the Regge amplitudes near the backward direction below 6 GeV/c [20] which we have used in the interference model.

It is interesting to remark that the most sensitive place to test the validity of the interference model is precisely the π^+p backward scattering for the following reasons : the background Regge amplitude f_u extrapolated from the high energy data is comparatively well defined, and the known π^+p resonances have the same parity, so that there are no interference effects between nearby resonances of opposite parity which would give a confusing picture like the preceding π^-p backward scattering analysis; furthermore, all these parity (+) resonances add constructively at 180° with the Regge amplitude.

We found the magnitude of the extrapolated Regge amplitudes essentially saturate the backward π^+p d.c.s. near 180°. The sum of the Regge and the resonance amplitudes gives a too large contribution in comparison with the data. Since the Regge and the resonance contributions add constructively, similar to the case of the $A'^{(+)}$ amplitude discussed in Section III, it strongly suggests double counting effects.

The magnitude of the π^+p backward data near 180^o has been explained by Dobrowolski et al., Minami, and Baker et al. [21] using the resonance amplitudes alone. So the magnitude is explained qualitatively either by the extrapolated "bona fide" Regge amplitude, or by the resonance amplitude, but not by the sum of the two [22),23)].

V. ANALYSIS OF THE FORWARD $\pi^{\pm}p$ AND CHARGE EXCHANGE DATA

The interference effects produced by the direct channel resonances are illustrated on a plot of the $\pi^{+}p$ differential cross-section data at constant momentum transfer as a function of the incident pion lab. momentum between 2 and ~6 GeV/c, (see Fig. 6), where the t channel Regge amplitude alone would give a smooth contribution. Similar effects have been observed on the c.ex. data [24], and sharp variations have also been noticed on the exponential slope of the $\pi^{-}p$ diffraction peak [25].

Previous analyses of the $\pi^{\pm}p$ polarization [26] and the c.ex. cross-section data [6),24] have been reported. In the present work, we have searched for a simultaneous fit of the combined $\pi^{\pm}p$ and c.ex. d.c.s. and $\pi^{\pm}p$ polarization data in the momentum transfer region $0 \leqslant (-t) \leqslant 1.5\ (\mathrm{GeV/c})^2$ together with the total cross-sections in an attempt to resolve the ambiguities on the πN resonances mentioned in the analysis of the backward scattering data (Section IV).

1. The $\pi^{\pm}p$ differential cross-section

It has been shown [12] with the parametrization of Eq. (II.6) that the amplitude f_t alone does not give a satisfactory fit to the dip and secondary maximum for the $\pi^{\pm}p$ d.c.s. in the energy interval between 2 and 5 GeV/c. When we added f_s to f_t, the fit was not improved. On the other hand, with f_t alone parametrized according to Eq. (II.8) one can obtain a qualitative fit to the same data [12]. We had anticipated that by taking into account the effect of the direct channel resonances, a quantitative fit would be achieved. But in all our trials we found too large interference effects, especially near the dip and the secondary maximum region.

It is particularly instructive to study in detail the $\pi^{+}p$ scattering with the interference model in the neighbourhood of $\Delta(2450)$, where the behaviour of the amplitude f_t is essentially constrained by the data at higher energy, and the amplitude f_s is relatively well defined. We found that the non-flip amplitudes in f_s and f_t have approximately the same phase near the dip and near the secondary maximum, thus giving rise to the too large interference effects mentioned previously. The failure of this model in the π^+p case is particularly significant and strongly suggests double counting effects.

2. The $\pi^{\pm}p$ polarization data

We have obtained fits to the π^-p polarization data above 2.0 GeV/c similar to that given by Grannis et al. [26]. In their work they have also reported great difficulties in fitting the π^+p polarization data [6]. In the neighbourhood of the Δ (2450), they obtained large negative polarization near $t=-0.6$ $(\text{GeV/c})^2$, which is not present in the experimental data. Our search with different parametrizations for the P and P' exchange amplitudes and with a range of values for the J, M, Γ and x parameters of the resonances (listed in Tables I and II) was also unsuccessful. This difficulty is present at all energies of the available data (from 2 to 4 GeV/c). It can be correlated with the fact that the $I=\frac{3}{2}$ resonances have the same $(J-\ell)$ assignment. They add constructively and give a large contribution to the B amplitude [23]. In the π^-p case, where the data can be fitted, one notes that $I=\frac{1}{2}$ and $\frac{3}{2}$ resonance contributions to the B amplitude have opposite sign.

3. The charge exchange differential cross-sections

The c.ex. data discussed here [6] below 2.5 GeV/c are from Carroll et al., and above 2.5 GeV/c from Yvert et al.. We have not obtained satisfactory fits for the data around 2 GeV/c. These data have been fitted by Carroll et al. [4] and, more recently, by Yokosawa [4]. C.ex. data are sensitive to the $(J-\ell)$ resonance assignment. These two analyses give opposite signs for the $(J-\ell)$ assignment of N(2190).

Carroll et al. obtain $J^P = \frac{7}{2}^+$ while Yokosawa gives $\frac{7}{2}^-$. Yokosawa also shows that the data cannot be fitted with the usual $A'\rho$ amplitude [as given in Refs. 9),12), and in Eq. (II.6)], where there is a zero in the $A'\rho$ near $t = -0.2$ (GeV/c)^2. So the c.ex. data around 2 GeV/c are not well understood with the simple interference model considered. This could be linked to the fact that the resonance amplitude is comparable to the Regge background and that of the full amplitude. Also, the resonance amplitude used might need to be modified, see Refs. 17),23) and also Appendix B.

For the c.ex. data above 2.5 GeV/c, a phenomenological analysis has been reported in Ref. 24). We have included these data in our search. Satisfactory fit to these data was readily achieved. We note that the resonance contribution f_s is relatively small and the amplitude f_t is not known to sufficient accuracy (in particular, we have neglected possible low-lying trajectories, e.g., the ρ' trajectory), compared to the magnitude of the resonance contributions, so different solutions can be obtained for the amplitudes f_s and f_t with slight adjustment of their parameters. Furthermore, the total number of resonances in this region is poorly known. Thus, for example, the difference between Tables I and II for the higher resonances cannot be resolved by fitting these data. So one can only conclude that the interference model is compatible with these data.

The $(J-\ell)$ resonance assignment for $\Delta(2450)$ obtained by Baacke et al. [24] from the analysis of the charge data does not depend on these uncertainties.

VI. SUMMARY

From the preceding review of the work of Höhler et al.[4),15)], Barger et al.[5)], Baacke and Yvert[24)], Grannis et al.[26)], and many others mentioned in Refs. 3),4),21), together with the work of Dikmen[18)] and our contribution, the following conclusions have been reached.

The interference model is compatible between 2 and 6 GeV/c with the $\pi^{\pm}p$ total cross-section, the forward $\pi^{-}p$ polarization, c.ex. d.c.s., and the $\pi^{-}p$ 180° d.c.s.. In all these cases, there are either large cancellations between nearby resonance amplitudes or the resonance contributions are much smaller than the Regge background.

The interference model does not give a unique quantitative description of the $\pi^{-}p$ d.c.s. data at 180°, for which many different solutions can be obtained, both with and without the cut-off factors applied to the resonance amplitudes.

The interference model does not work for the $\pi^{+}p$ backward d.c.s., for which it predicts too large cross-sections near 180°. It does not explain the forward $\pi^{\pm}p$ d.c.s. for which it predicts too large interference effects; and, finally, it does not account correctly for the $\pi^{+}p$ polarization data near $t = -0.6\ (\mathrm{GeV/c})^2$. In most of these cases, the resonance contributions are large and have the same sign. Dispersion relations help one to understand intuitively the origin of these difficulties.

Recently Dolen et al.[17)] have clearly pointed out, in their work on finite energy sum rules, the connection between the non-cancellation of the resonance contributions and the difficulty in fitting the data. They have shown that dispersion relations help one to understand intuitively the origin of these difficulties. They have independently reached similar phenomenological conclusions on this model (Appendix B).

From all these considerations, the interference model which directly adds "background + resonance" contributions appears to be inadequate. In other words, it is unnatural to identify in all cases the extrapolation of the Regge amplitude with the so-called non-resonating background.

Should one further explore the possibility of using "background + resonance" amplitudes to fit various data, the usual prescription would have to be modified :

1. if the extrapolation of some leading asymptotic term(s) is used for the background contribution, the resonance contribution would have to oscillate about this background (see Section III.2 and Appendix B). This could be achieved, for instance, with the following form of the resonance contribution $f_{res} \simeq x(\epsilon+i)/(\epsilon-i)$.
2. On the other hand, if one wants to keep the usual Breit-Wigner resonance amplitude, the simple asymptotic term(s) would have to approach zero in the low energy region where the resonance contributions are dominating.
3. For both cases, the influence of far away resonances should not play a significant role in fitting the data.
4. Lastly, should there be more resonances present in the intermediate energy region, as can be conjectured from the latest phase shift analyses below 2 GeV [16], they should be included in the f_{res} amplitude.

Further experimental data would certainly help to find out if these suggestions can improve the applicability of the model in the intermediate energy region.

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APPENDIX A

DISPERSION RELATIONS

I. THE DISPERSION RELATION FOR $\mathrm{Im}\,F^{(+)}$

For the forward scattering amplitude $F^{(+)}$, one can formally write the unsubtracted dispersion relation [7]:

$$\mathrm{Re}\,F^{(+)}(\nu) = -\frac{g^2\mu^2}{2M}\left(\frac{1}{\nu_B-\nu} + \frac{1}{\nu_B+\nu}\right) + \frac{P}{\pi}\int_{-\infty}^{-\mu}\frac{\mathrm{Im}\,F^{(+)}(\nu')}{\nu'-\nu}d\nu' + \frac{P}{\pi}\int_{\mu}^{\infty}\frac{\mathrm{Im}\,F^{(+)}(\nu')}{\nu'-\nu}d\nu' \qquad (A.I.1)$$

where $\nu = (s-u)/4M$). At $t=0$, ν is the incident pion lab. energy, $\nu_B = -(\mu^2/2M)$, q and k are the lab. and c.m. pion momentum, $g^2 = (4M^2f^2/\mu^2)$ and $(f^2/4\pi) \approx 0.08$. From Eqs. (III.1) and (II.1), one has

$$\sigma^{(+)} = \frac{4\pi}{k}\,\mathrm{Im}\,f^{(+)} = \frac{1}{2k\sqrt{s}}\,\mathrm{Im}\,F = \frac{1}{2Mq}\,\mathrm{Im}\,F^{(+)} \qquad (A.I.2)$$

Also, $F^{(+)}$ has the crossing symmetric property [7]

$$\mathrm{Im}\,F^{(+)}(-\nu+i\epsilon) = -\mathrm{Im}\,F^{(+)}(\nu+i\epsilon) \qquad (A.I.3)$$

Assume that for sufficiently large ν, $F^{(+)}$ can be expanded in a Regge asymptotic series

$$F^{(+)} \xrightarrow[\text{large }\nu]{} F_R^{(+)}(\nu) = \sum_i \beta_i\left[\nu^{\alpha_i} + (-\nu)^{\alpha_i}\right]\csc\pi\alpha_i \qquad (A.I.4)$$

with the phase convention defined such that :

and

$$(-\nu)^{\alpha_i} = |\nu|^{\alpha_i} e^{i\pi\alpha_i} \quad \text{for } \nu > 0$$
$$\nu^{\alpha_i} = |\nu|^{\alpha_i} e^{-i\pi\alpha_i} \quad \text{for } \nu < 0 \qquad (A.I.5)$$

The residue function β is equal to $2M\nu^{-1}q\gamma$ up to α factors, and γ is given in Eqs. (II.6) and (II.8). Now define a function $H(\nu) = [F^{(+)}(\nu) - F_{R'}^{(+)}(\nu)]\,\nu$, where $F_{R'}^{(+)}(\nu)$ contains a sum of Regge pole terms down to $\alpha_i = -2$. So, by construction,

$$H(\nu) \xrightarrow[\nu \to \infty]{} \frac{1}{\nu^{1+\epsilon}} \quad \text{with } \epsilon > 0 \qquad (A.I.6)$$

One can write an unsubtracted dispersion relation for H analogous to Eq. (A.I.1), namely

$$\mathrm{Re}\,[\nu(F^{(+)} - F_{R'}^{(+)})] = -2Mf^2\left(\frac{\nu_B}{\nu_B - \nu} - \frac{\nu_B}{\nu_B + \nu}\right) + \frac{P}{\pi}\int_0^\infty 2M\left(q'\sigma(\nu') - \sum_i \beta_i \nu'^{\alpha_i}\right)\frac{2\nu\nu' d\nu'}{\nu'^2 - \nu^2} \qquad (A.I.7)$$

where one has used the following identities obtained from (A.I.3) and (A.I.5) with some algebra

$$\int_{-\infty}^{-\mu} \frac{\mathrm{Im}F^{(+)}(\nu')\nu'}{\nu' - \nu} d\nu' = -\int_{\mu}^{\infty} \frac{\mathrm{Im}F^{(+)}(\nu')\nu'}{\nu' + \nu} d\nu'$$

and

$$\int_{-\infty}^{0} \frac{\nu'^{\alpha_i}\nu'}{\nu' - \nu} d\nu' = -\int_{0}^{\infty} \frac{\nu'^{\alpha_i}\nu'}{\nu' + \nu} d\nu'$$

Therefore, from Eq. (A.I.6)

$$\nu \, \mathrm{Re}\, H(\nu) \underset{\nu\to\infty}{\longrightarrow} 0 = 4Mf^2\nu_B + \frac{4M}{\pi}\int_0^\infty \left(q'\sigma' - \sum_i \beta_i \nu'^{\alpha_i}\right) d\nu' \qquad \text{(A.I.8)}$$

where one has used the following identity for a convergent integral

$$\lim_{\nu\to\infty} \nu P\int_0^\infty \frac{f(\nu')\nu' d\nu'}{\nu - \nu'} = \int_0^\infty f(\nu')\, d\nu'$$

Since $\nu_B = -(\mu^2/2M)$, from Eq. (A.I.8), one arrives at Eq. (III.3).

II. THE DISPERSION RELATION FOR Re $F^{(+)}$

Following the work of Gilbert [27], the dispersion relation for $F^{(+)}$ on the right-hand cut reads

$$\mathrm{Im}\left[\frac{F^{(+)}(\nu)}{q}\right] = -\frac{P}{\pi}\int_{-\infty}^{-\mu} \mathrm{Re}\left(\frac{F^{(+)}(\nu')}{q'}\right)\frac{d\nu'}{(\nu'-\nu)} - \frac{P}{\pi}\int_{\mu}^{\infty} \mathrm{Re}\left(\frac{F^{(+)}(\nu')}{q'}\right)\frac{d\nu'}{(\nu'-\nu)} + \frac{g^2\mu^2}{M\sqrt{\mu^2-\nu_B^2}}\left[\frac{1}{\nu_B-\nu}+\frac{1}{\nu_B+\nu}\right] \qquad \text{(A.II.1)}$$

So we have

$$\mathrm{Im}\left(\frac{F^{(+)}(\nu)}{q}\right) = -\frac{2}{\pi} P\int_{S_0}^{\infty} \mathrm{Re}\left(\frac{F^{(+)}(\nu')}{q'}\right)\frac{\nu' d\nu'}{(\nu'^2-\nu^2)} \qquad \text{(A.II.2)}$$

where, for simplicity, we include the pole term in the integral. Analogously to Eq. (A.I.4), we assume [28]

$$F^{(+)} \xrightarrow[\text{large } \nu]{} \sum_i \beta_i \left[\nu^{\alpha_i} + (-\nu)^{\alpha_i} \right] \frac{q}{\nu} \csc \pi\alpha_i \equiv \sum_i F_R^i \qquad \text{(A.II.3)}$$

with each Regge term, satisfies the relation :

$$\frac{\mathrm{Im}\, F_R^i}{q} = -\frac{2}{\pi} P \int_0^\infty d\nu' \, \mathrm{Re}\left(\frac{F_R^i}{q'}\right) \frac{1}{\nu'^2 - \nu^2} \qquad \text{(A.II.4)}$$

Define $F_{R'}^{(+)}$ to be the sum of Regge contributions including all the terms with $\alpha_i \geqslant -1$. So

$$\nu^2 \, \mathrm{Im}\left[\frac{F^{(+)}(\nu)}{q} - \frac{F_{R'}^{(+)}(\nu)}{q} \right] \xrightarrow[\nu \to \infty]{} 0 \qquad \text{(A.II.5)}$$

This implies

$$\int_{S_0}^\infty \mathrm{Re}\left[\frac{F^{(+)}(\nu')}{q'} \right] \nu' d\nu' - \int_0^\infty \mathrm{Re}\left[\frac{F_{R'}^{(+)}(\nu')}{q'} \right] \nu' d\nu' \doteq 0 \qquad \text{(A.II.6)}$$

So the real part of the asymptotic amplitude $F_{R'}^{(+)}$ describes the average behaviour of the real part of the amplitude.

APPENDIX B

CLASSIFICATION OF THE DATA

Our phenomenological fit of the data was completed prior to the work of Dolen, Horn and Schmid (D.H.S.) on finite energy sum rules [17]. They defined the conditions under which the interference model is expected to work or not to work. They have independently made a survey of some of the πN data similar to that considered in this work. We shall repeat for completeness their evaluation in connection with the results presented in Sections III, IV and V.

I. D.H.S. PRESCRIPTION ON THE INTERFERENCE MODEL

They pointed out that the correct prescription for the amplitude in an interference model should be :

$$f = f_{Regge} + f_{res} - \langle f_{res} \rangle$$

where $\langle f_{res} \rangle$ is the average behaviour of the resonance contribution, instead of the usual prescription $f = f_{Reg.} + f_{res}$. If the cases where : $\langle f_{res} \rangle \ll (f_{Reg.} + f_{res})$, the usual prescription is approximately valid. In particular, the average $\langle f_{res} \rangle$ can be negligible in some cases if f^i_{res}'s have opposite sign. On the other hand, when all f^i_{res}'s add constructively, the average will always be different from zero. We shall consider these two cases separately.

CASE I : The resonant amplitudes add constructively

In the energy intervals listed in column 3 below, there are difficulties in fitting the data, especially in those regions where the "relevant resonance amplitudes" add constructively and give rise to large interference effect. The energy intervals where the interference effects are small and fits have been achieved are mentioned in column 4.

Data	Relevant resonance amplitude	No fit	Fit
$\sigma^{(+)}$	$\mathrm{Im}\,A'^{(+)}$	$P \lesssim 2$ GeV/c	$P \gtrsim 2$ GeV/c
π^+p d.c.s. near 180^0	$\mathrm{Im}(f_1-f_2)\,\pi^+p$	$2 \lesssim P \lesssim 4$	$P \gtrsim 4$
forward $\pi^{\pm}p$ d.c.s.	$\mathrm{Im}\,A'\,\pi^{\pm}p$	$2 \lesssim P \lesssim 4$	$P \gtrsim 4$
forward c.ex. d.c.s.	$\mathrm{Im}\,B^{(-)}$	$P \sim 2$	$P \gtrsim 2.5$
forward π^+p polarization	$\mathrm{Im}\,B\,\pi^+p$	$2 \lesssim P \lesssim 4$	---

In the latter case where fits have been obtained the Regge amplitudes f_{Regge} are not determined accurately enough as compared to the magnitude of the resonance contribution and we can only conclude that the interference model is compatible with the data at this stage.

CASE II: The resonant amplitudes have opposite sign

We have indicated for the data listed below the energy region where the resonance interference effects are large in column 3 and where they are small in column 4. There are large cancellations in the resonance amplitudes especially in the "relevant resonant amplitudes" indicated in column 2.

Data	Relevant resonant amplitude	Fit	
		large interference effects	small interference effects
$\sigma^{(-)}$	Im $B^{(-)}$	$P \lesssim 3.5$ GeV/c	$3.5 \lesssim P \lesssim 6$
$\pi^- p$ 180°	Im A' $\pi^- p$	$2 \lesssim P \lesssim 4$	$4.0 \lesssim P \lesssim 6$
$\pi^- p$ polarization	Im B $\pi^+ p$	$2 \lesssim P \lesssim 4$	

In all these cases the data have been fitted. However, in the energy region where the interference effects are small (column 4), the same remark indicated in case I applies.

II. FURTHER COMMENTS

One sees that in all the cases where the interference model does not work, the prescription given by D.H.S. is in agreement with the results either analyzed or reviewed in this paper. In the cases where there are cancellations in the resonance amplitude, fits to the data can be achieved. However, even for these cases one should be aware of some deficiencies of this model as discussed in the text :

a) It is difficult to extract the total number of resonances from the data without going through a phase shift analysis. If there are many more resonances than what one originally anticipated and if they give non-negligible contributions, the interference prescription would be incapable of disentangling the effects of the individual resonances. [Incidently, in the latest phase shift analysis results, there are one or more resonances suggested in all the partial wave amplitudes up to G waves [16].] By the same token, should there be for instance $I=\frac{3}{2}$ parity minus and $(J-\ell)=-\frac{1}{2}$ resonances near Δ (2450), the situation for the π^+p polarization and π^+p d.c.s. near 180° would have to be re-examined. The interpretation of the c.ex. data near 2 GeV/c would also be further complicated if there are more resonances present.

b) The tails of the Breit-Wigner amplitude in some cases play an important role in these fits.

c) There are uncertainties associated with the Regge amplitudes, in particular the total number of trajectories which one should include in the analysis is not known. This difficulty becomes more serious in the low energy region.

APPENDIX C

LIST OF THE DATA ANALYZED BETWEEN 2-6 GeV/c

I. THE $\pi^{\pm}p$ TOTAL CROSS-SECTION DATA

a) A. Citron, W. Galbraith, T.F. Kycia, B.A. Leontić, R.H. Phillips, A. Rousset and P.H. Sharp, Phys.Rev. 144, 1101 (1966).

Data	P_L GeV/c	Number of data points	Ref.
π^+p	2.11 - 6.84 GeV/c	45	a)
π^-p	2.52 - 6.94 GeV/c	41	a)

II. THE $\pi^{\pm}p$ DIFFERENTIAL CROSS-SECTION

a) C.T. Coffin, N. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger and D. Williams, Phys.Rev. 159, 1169 (1967).

b) R.J. Esterling, R.E. Hill, N.E. Booth, S. Suwa and A. Yokosawa, University of Chicago report, EFINS 66-29 (1966).

Data	P_L GeV/c	$t(GeV/c)^2$	Number of data points	Ref.
π^+p	2.3	$0 \geq t \geq -1.5$	28	a)
	2.5	"	27	"
	2.7	"	27	"
	3.0	"	17	"
	3.5	"	16	"
	3.7	"	16	"
	4.0	"	17	"

Data	P_L GeV/c	$t(GeV/c)^2$	Number of data points	Ref.
$\pi^- p$	2.5	$0 \geqslant t \geqslant -1.5$	27	a)
	3.0	"	15	
	3.5	"	23	
	4.0	"	19	
	5.0	"	15	
	6.0	"	14	
	2.07	$0 \geqslant t \geqslant -1.5$	21	b)
	2.27	"	17	
	2.50	"	15	

III. $\pi^{\pm} p$ BACKWARD DIFFERENTIAL CROSS-SECTION

a) S. Kormanyos, A.D. Krisch, J.R. O'Fallon, K. Kuddic and L.G. Ratner, Phys.Rev.Letters 16, 709 (1966).

b) H. Brody, R. Lanza, R. Marshall, J. Niederer, W. Selove, M. Sochet and R. Van Berg, Phys.Rev.Letters 16, 828 (1966).

c) W.F. Baker, P.J. Carlson, V. Chabaud, A. Lundby, E.G. Michaelis, J. Banaigs, J. Berger, C. Bonnel, J. Duflo, L. Goldzahl and F. Plouin, Phys.Letters 23, 605 (1966); 25, 361 (1967).

d) T. Dobrowolski, B.N. Guskov, M.F. Likhachev, A.L. Lubimov, Y.A. Matulenko, V.S. Stavinsky and A.S. Vovenko, Phys.Letters 24, 203 (1967).

e) W.R. Frisken, A.L. Read, M. Ruderman, A.D. Krisch, J. Orear, R. Rubinstein, D.B. Scarl and D.H. White, Phys.Rev.Letters 15, 313 (1965).

f) C.T. Coffin, N. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger and D. Williams, Phys.Rev.Letters 15, 838 (1965); 17, 458 (1966).

Data	P_L GeV/c	$u(GeV/c)^2$	Number of data points	Ref.
$\pi^+ p$	2.06 - 4.70	$\sim 180^0$	13	d)
	2.3		7	
	2.5		7	
	2.7	$0.10 \gtrsim u \gtrsim -1.0$	7	f)
	3.0		6	
	3.5		5	
	4.0		6	
	2.85	$0.11 \gtrsim u \gtrsim 0$	9	
	3.30	$0.09 \gtrsim u \gtrsim 0$	6	c)
	3.55	$0.08 \gtrsim u \gtrsim 0$	10	
	4.0	$\sim 180^0$	2	e)
	4.4	$-0.14 \gtrsim u \gtrsim -0.8$	4	b)
	6.1	$-0.16 \gtrsim u \gtrsim -1.3$	6	
$\pi^- p$	2.0 ~ 5.3	$\sim 180^0$	42	a)
	3.30	$0.08 \gtrsim u \gtrsim 0$	4	c)
	3.55	$0.09 \gtrsim u \gtrsim 0$	10	c)
	4.0	$\sim 180^0$	2	e)
	4.2	$-0.14 \gtrsim u \gtrsim -0.8$	4	b)
	6.2	$-0.16 \gtrsim u \gtrsim -1.4$	6	

IV. $\pi^{\pm}p$ POLARIZATION

a) O. Chamberlain, M.J. Hansroul, C.H. Johnson, P.D. Grannis, L.E. Holloway, L. Valentin, P.R. Robrish and H.M. Steiner, Phys.Rev.Letters 18, 975 (1966).

b) S. Suwa, A. Yokosawa, N.E. Booth, R.J. Esterling and R.E. Hill, Phys.Rev. Letters 15, 560 (1965), and EFINS 66-29, report, University of Chicago, Argonne National Laboratory.

Data	P GeV/c	t $(GeV/c)^2$	Number of data points	Ref.
π^+p	1.988	$0 \gtrsim t \gtrsim -1.5$	17	a)
	2.535	$0 \gtrsim t \gtrsim -1.5$	12	
	3.260	$0 \gtrsim t \gtrsim -0.5$	6	
	3.747	$0 \gtrsim t \gtrsim -0.5$	7	
π^-p	1.988	$0 \gtrsim t \gtrsim -1.5$	16	a)
	2.535	"	11	
	2.912	"	10	
	3.260	"	8	
	2.07	"	19	b)
	2.27	"	17	
	2.50	"	15	

V. CHARGE EXCHANGE DIFFERENTIAL CROSS-SECTION

a) A.S. Caroll, I.F. Corbett, C.J.S. Damerell, N. Middlemas, D. Newton, A.B. Clegg and W.S.C. Williams, Phys.Rev.Letters 16, 288 (1966).

b) M. Yvert, private communication, Saclay-Orsay collaboration, to be published.

P GeV/c	$t (GeV/c)^2$	Number of data points	Ref.
2.070	$0 \gtrsim t \gtrsim -1.5$	19	a)
2.265	"	17	
2.460	"	16	
2.55	$0 \gtrsim t \gtrsim -0.9$	16	b)
2.76	$0 \gtrsim t \gtrsim -1.1$	17	
2.92	$0 \gtrsim t \gtrsim -1.1$	17	
3.07	$0 \gtrsim t \gtrsim -1.3$	18	
3.22	$0 \gtrsim t \gtrsim -1.3$	18	
3.38	$0 \gtrsim t \gtrsim -1.6$	19	
3.52	"	"	
3.66	"	"	
3.86	"	"	
4.06	"	"	
4.25	"	"	
4.45	"	"	
4.64	"	"	
4.82	"	"	
5.01	"	"	
5.21	"	"	
5.38	"	"	
5.55	"	"	
5.74	"	"	

TABLE CAPTIONS

TABLE I The πN resonance parameters tabulated by Barger et al.[5]. We refer to this reference for further details. In our work, only those resonances with their elasticity parameters x_I's explicitly given in the column labelled "$\pi^- p \to p \pi^-$ at 180^o" were used. Our x_I values are comparable to these tabulated values.

TABLE II Suggested assignment of πN resonances compatible with the daughter trajectory scheme. [See Ref. 14).] The resonance parameters x_I, M and Γ listed are typical average values obtained from the separate analyses of the forward and the backward data.

REMARK : In both of these tables, the "conjectured" πN resonances have been indicated with an asterisk (*). Latest phase shift analyses, as reviewed by Lovelace[16], have indicated the existence of many more resonances than indicated in these Tables.

Y = +1 Fermion Resonances

	Resonance (mass in MeV)	Spin-Parity (J^P) = established	Width (BeV)	Elasticity (x_I) Determinations			
				$\pi^- p \to p\pi^-$ at 180°	$\pi^- p \to \pi^0 n$ at 0°	Total Cross Sections	Phase Shifts
REGGE RECURRENCES	$\Delta_\delta(1236)$	$3/2^+$	0.12	1.0	1.0	1.0	1.0
	$\Delta_\delta(1924)$	$7/2^+$	0.17	0.35-0.50*	0.49	0.33-0.41	0.50
	$\Delta_\delta(2450)^*$	$11/2^+$	0.28	0.12	0.12	0.11	...
	$\Delta_\delta(2840)^*$	$15/2^+$	0.40	0.05	0.03	0.03	...
	$\Delta_\delta(3220)^*$	$19/2^+$	0.44	0.02	0.003	0.006	...
	$N_\gamma(1512)$	$3/2^-$	0.12	0.60	0.77	0.76	0.50-0.71
	$N_\gamma(2210)$	$7/2^-$	0.24	0.20	0.25	0.15-0.25	...
	$N_\gamma(2640)$	$11/2^-$	0.40	0.05	0.08	0.07	...
	$N_\gamma(3020)$	$15/2^-$	0.40	0.015	0.011	0.007	..
	$N_\gamma(3350)$	$19/2^-$	0.10	0.003-0.01	0.003	...	...
	$N_\alpha(938)$	$1/2^+$	...	...	...	...	...
	$N_\alpha(1688)$	$5/2^+$	0.10	0.60	1.04	0.80	0.66
	$N_\alpha(2220)^*$	$9/2^+$	0.20	0.05	...	...	...
	$N_\alpha(2610)^*$	$13/2^+$	0.30	0.025	...	...	
	$N_\alpha(2970)^*$	$17/2^+$	...	...	...	...	
UNCLASSIFIED STATES	$N_\alpha(\sim 1470)$	$1/2^+ (P_{11})$	0.20				0.60-0.70
	$N_\beta(\sim 1560)$	$1/2^- (S_{11})$	0.28				0.40
	$N_\beta(\sim 1650)$	$5/2^- (D_{15})$	0.13	0.20			0.20-0.40
	$\Delta_\beta(\sim 1690)$	$1/2^- (S_{31})$	0.15-0.23	0.30			0.25-0.44
	$N_\beta(\sim 1715)^*$	$1/2^- (S_{11})$					0.90

TABLE I

Resonance	Mass (MeV)	Spin-Parity (J^P)	Elasticity x	Width Γ	$(J \pm \frac{1}{2})$	Signature ξ
N_α	1688	$3 - \frac{1}{2} = \frac{5}{2}^+$	0.7	0.10	+	+
	2200*	$5 - \frac{1}{2} = \frac{9}{2}^+$	0.07	0.24	+	+
N_β	1650	$2 + \frac{1}{2} = \frac{5}{2}^-$	0.33	0.10	-	+
	2640*	$4 + \frac{1}{2} = \frac{9}{2}^-$	0.06	0.42	-	+
N_γ	1512	$2 - \frac{1}{2} = \frac{3}{2}^-$	0.70	0.12	+	-
	2190	$4 - \frac{1}{2} = \frac{7}{2}^-$	0.20	0.24	+	-
	3020*	$6 - \frac{1}{2} = \frac{11}{2}^-$	0.015	0.40	+	-
Δ	1238	$1 + \frac{1}{2} = \frac{3}{2}^+$	1.00	0.12	-	-
	1924	$3 + \frac{1}{2} = \frac{7}{2}^+$	0.40	0.17	-	-
	2450*	$5 + \frac{1}{2} = \frac{11}{2}^+$	0.12	0.28	-	-
	2840*	$7 + \frac{1}{2} = \frac{15}{2}^+$	0.05	0.40	-	-

TABLE II

REFERENCES AND FOOTNOTES

1) L. Van Hove, Rapporteur's talk, XIIIth International Conference on High Energy Physics, Berkeley (1966). We note that the charge exchange polarization at high energy is different from zero, which cannot be accounted for by the ρ trajectory exchange alone. Various suggestions have been made to explain these polarization data. We are also aware that the sum of the π^+p and π^-p polarization data is positive at 6 GeV/c and negative at 12 GeV/c at $t=-0.2(\text{GeV/c})^2$ [e.g., G. Höhler et al., Karlsruhe preprint (1967)]. This effect cannot be explained within the framework of the P, P' and ρ models; however, the present experimental errors are quite large.

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Also many others.

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11) G.F. Chew, Phys.Rev.Letters 16, 60 (1966).

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14) The assignment given in Table II is in agreement with the daughter trajectory scheme proposed by Freedman and Wang [D.Z. Freedman and J.M. Wang, Phys.Rev.Letters 17, 569 (1966)]. With this scheme, the existing πN resonances can be fitted into two families, one for $I = \frac{3}{2}$ and one for $I = \frac{1}{2}$, with the MacDowell symmetry connecting the even and odd parity states [S.W. MacDowell, Phys.Rev. 116, 774 (1959)]. The family for $I = \frac{1}{2}$ is illustrated in Fig. 8. For $I = \frac{3}{2}$, the $S_{31}(\sim 1690)$ resonance would lie on the daughter trajectory of the Δ.

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C. Lovelace, Rapporteur's talk at the 1967 Heidelberg Conference.
Note-The uncertainty in the non-resonance background $B^{(-)}$ is $\lesssim$ 15%. This error takes into account the difference between different phase shift solutions and the difference between various possible ways in extracting the non-resonance background from a phase shift solution.

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22) Barger and Cline have fitted the same data using the interference model [5)]. However, their Regge amplitude includes only the small Δ trajectory contribution. It certainly cannot account for the magnitude of the π^+p backward cross-sections at 8 GeV/c.

23) If there were more resonances, especially $I = \frac{3}{2}$ with opposite $(J-\ell)$ assignment, resonances present in this energy region, giving non-negligible contributions, the situation would be more complicated. We found that the model with resonance assignments given in Tables I and II is inadequate.

24) J. Baacke and M. Yvert, Report submitted to the XIIIth International Conference on High Energy Physics, Berkeley (1966), see Ref. 4).

25) L.D. Jacobs, Thesis, report UCRL-16877 (1966), see Ref. 6).

26) P. Grannis, H. Steiner and L. Valentin, report UCRL-16750 (1966).

27) W. Gilbert, Phys.Rev. 108, 1078 (1957).

28) A similar asymptotic behaviour has been used by :
M. Olsson, Phys.Rev.Letters 19, 550 (1967).

FIGURE CAPTIONS

Figure 1 Im $kf^{(-)}$ plotted v. P_{inc}, the incident pion lab. energy and v. $W_{c.m.}$ the total energy in the centre-of-mass system. The symbol k is the c.m. momentum, $f^{(-)}$ is the forward scattering amplitude, and $kf^{(-)}$ is a dimensionless quantity. Curve (I) is $(k^2/4\pi)B^{(-)}$ obtained from phase shift solution with the dashed line indicating its smooth extrapolation, and curve (II) is $(k^2/4\pi)R^{(-)}$. The resonances are labelled corresponding to the assignment of Table II. [For definitions of $B^{(-)}$ and $R^{(-)}$, see Section III.]

Figure 2 The backward $\pi^- p$ d.c.s. v. P, the incident pion lab. momentum in GeV/c. The solid points are from Kormanyos et al. of Ref. 3), and the triangular point is from Frisken et al. of Ref. 6). The resonances labelled are according to the assignment of Table II.

Figure 3

a. The solid curve is the experimental cross-section v. T_π, the incident pion kinetic energy in GeV. The dashed curve is $B^{(+)}$ and the dotted-dashed curve is $R^{(+)}$. This figure has been obtained by Höhler et al. A similar figure is shown in Ref. 15).

b. The ratio B/R plotted v. P_{inc}, the incident pion lab. momentum in GeV/c, for both the (+) and the (−) cases, Ref. 16). The solid curve is an empirical fit to the points.

Figure 4 The backward $\pi^- p$ d.c.s. v. P, the incident pion lab. momentum in GeV/c. The solid points are from Kormanyos et al. [Ref. 3)] and the triangular point is from Frisken et al. [Ref. 6)]. The resonances labelled are according to the assignment of Table II. Curve I - a typical fit to the data with the resonance parameters given in Table I. Curve II - is the d.c.s. calculated with the same parameters except for the cut-off factor $t(s)$ defined in (II.5) applied to $\Delta(1236)$, $\Delta(1924)$, $N_\gamma(1512)$ and $N_\alpha(1688)$ resonance contributions.

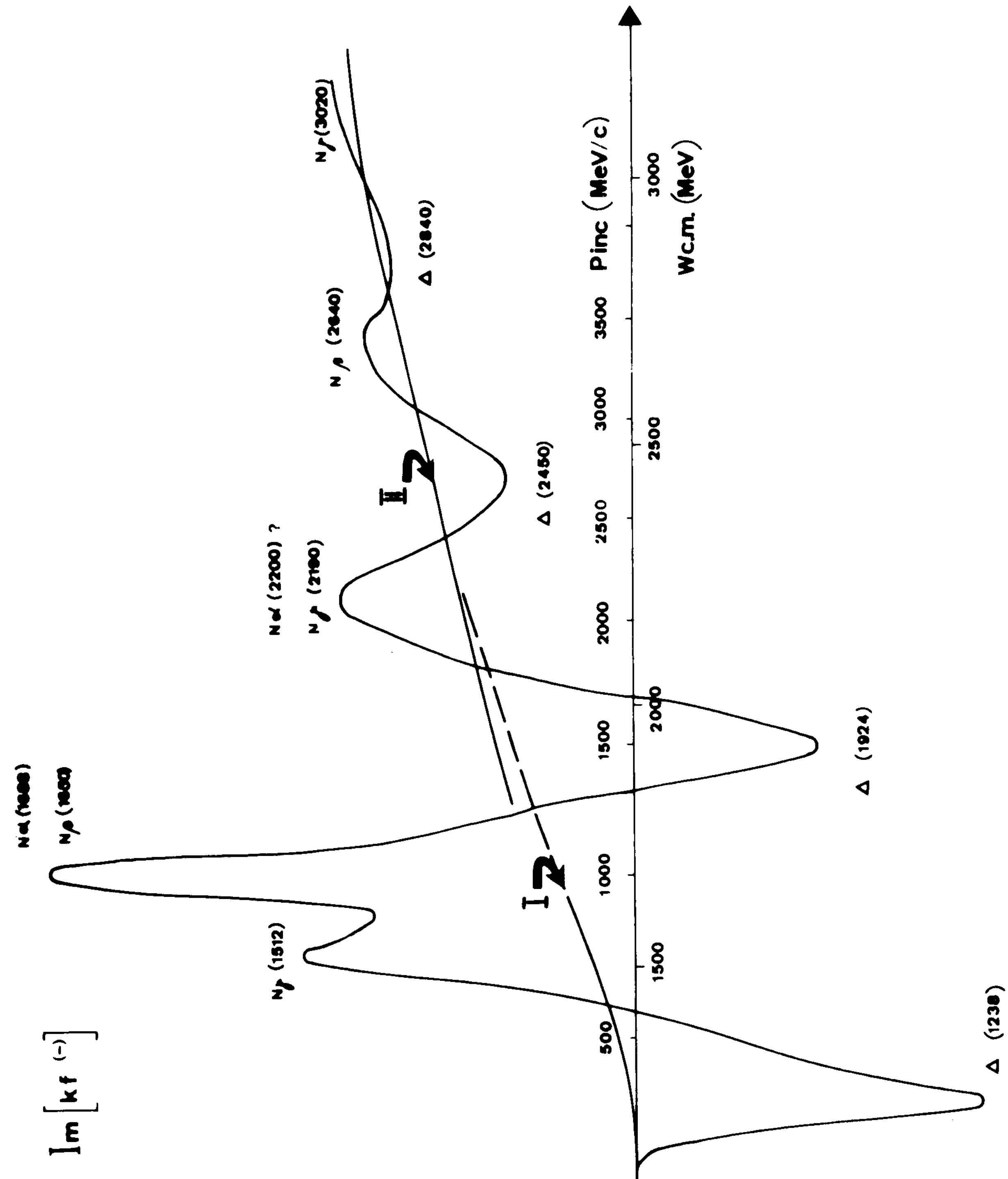
Figure 5 The π^+p d.c.s. near the backward direction v. $\cos\theta^*$, where θ^* is the c.m. scattering angle. The data points shown are from Ref. 6). The open circles joined by the solid lines are a sample fit.

Figure 6 The d.c.s. for π^+p elastic scattering at various t values v. P, the incident pion momentum in GeV/c. The data points below 5 GeV/c are from Coffin et al. [6], joined by the dashed curves. The data points beyond 5 GeV/c are from Foley et al. [Phys.Rev.Letters 11, 425 (1963)], joined by the dashed curves, and from Harting et al. [Nuovo Cimento 38, 60 (1965)], joined by the solid lines. (Lines are hand drawn to guide the eyes.)

Figure 7 The π^+p polarization plotted v. t at incident pion lab. momentum 2.54 GeV/c. The points are from Chamberlain et al. [6]. The solid curve is a typical fit given by the interference model.

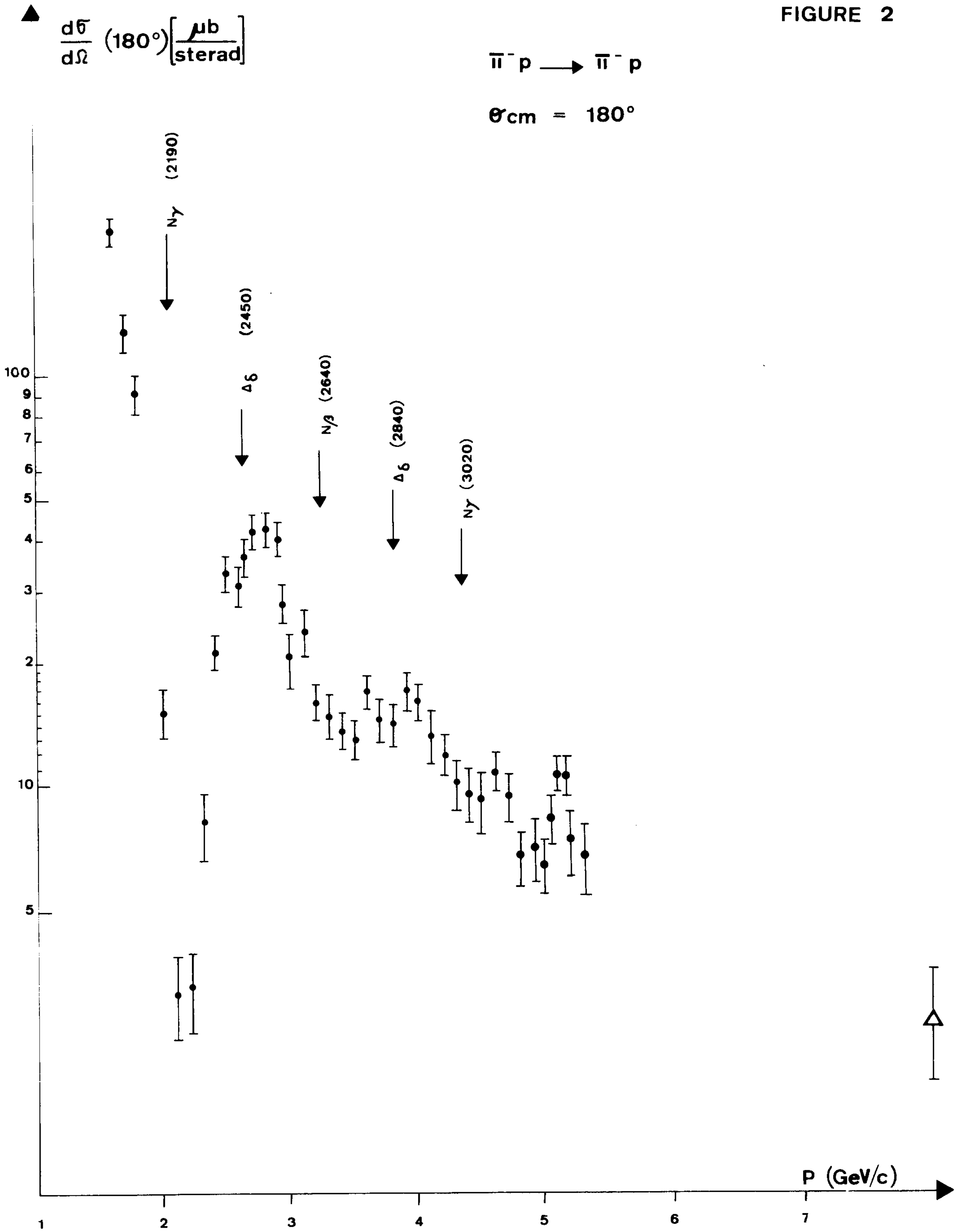
Figure 8 The suggested scheme for the assignment of the quantum numbers for the $I=\frac{1}{2}$ resonances. We group all the resonances shown into one large Regge family comprising the "parent" and the "daughter" and the "grand-daughter" trajectories. The trajectories with opposite parities are connected by MacDowell symmetry [14]. (See also Table II.) Many more new resonances have been proposed by the latest phase shift analyses, reviewed by Lovelace [16].

FIGURE 1



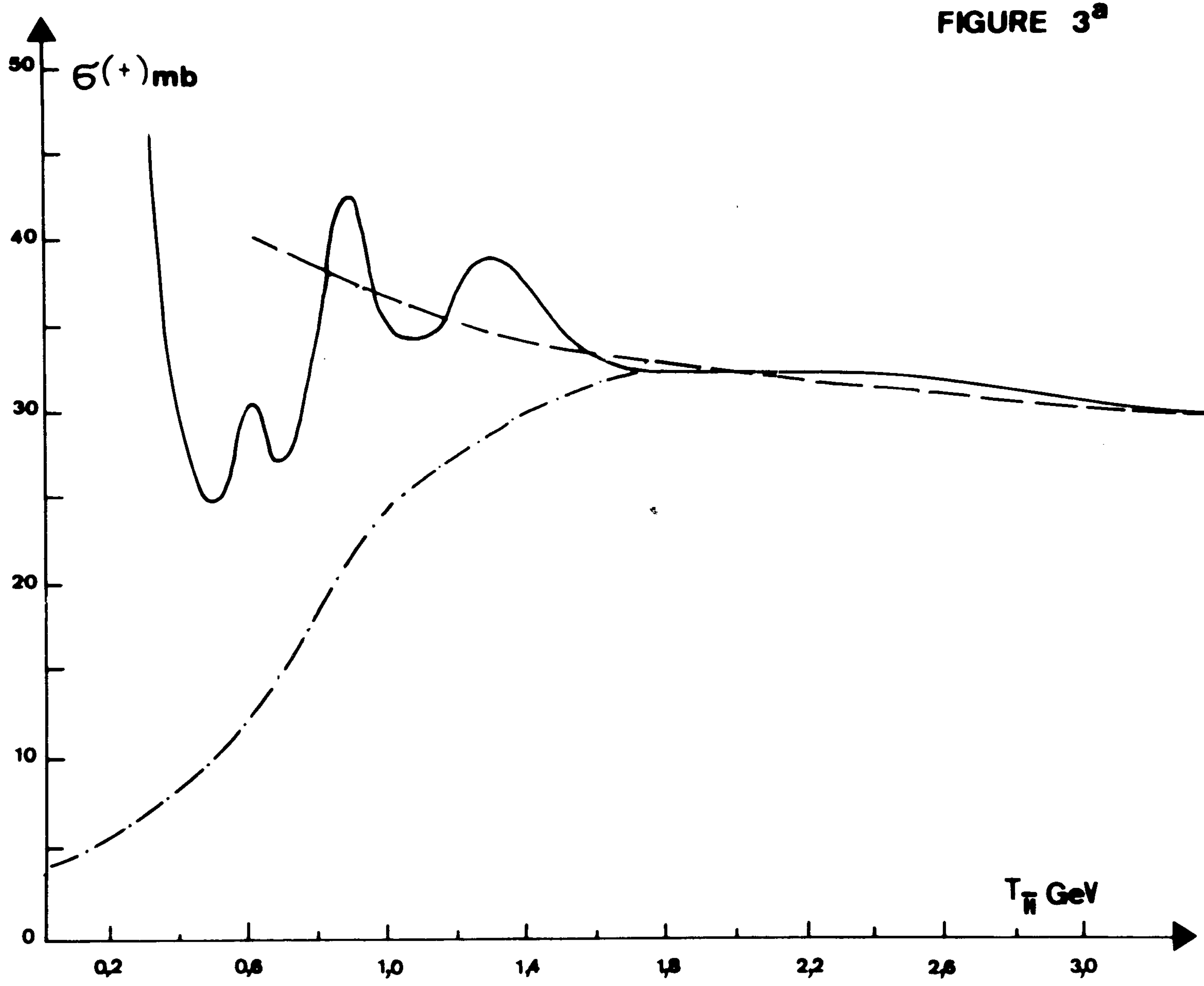
SIS/K 0436

FIGURE 2



$\frac{d\sigma}{d\Omega}$ (180°) $\left[\frac{\mu b}{sterad}\right]$
$\pi^- p \longrightarrow \pi^- p$
θ_{cm} = 180°
Nγ (2190)
Δδ (2450)
Nβ (2640)
Δδ (2840)
Nγ (3020)
100
9
8
7
6
5
4
3
2
10
5
P (GeV/c)
1
2
3
4
5
6
7

FIGURE 3[a]



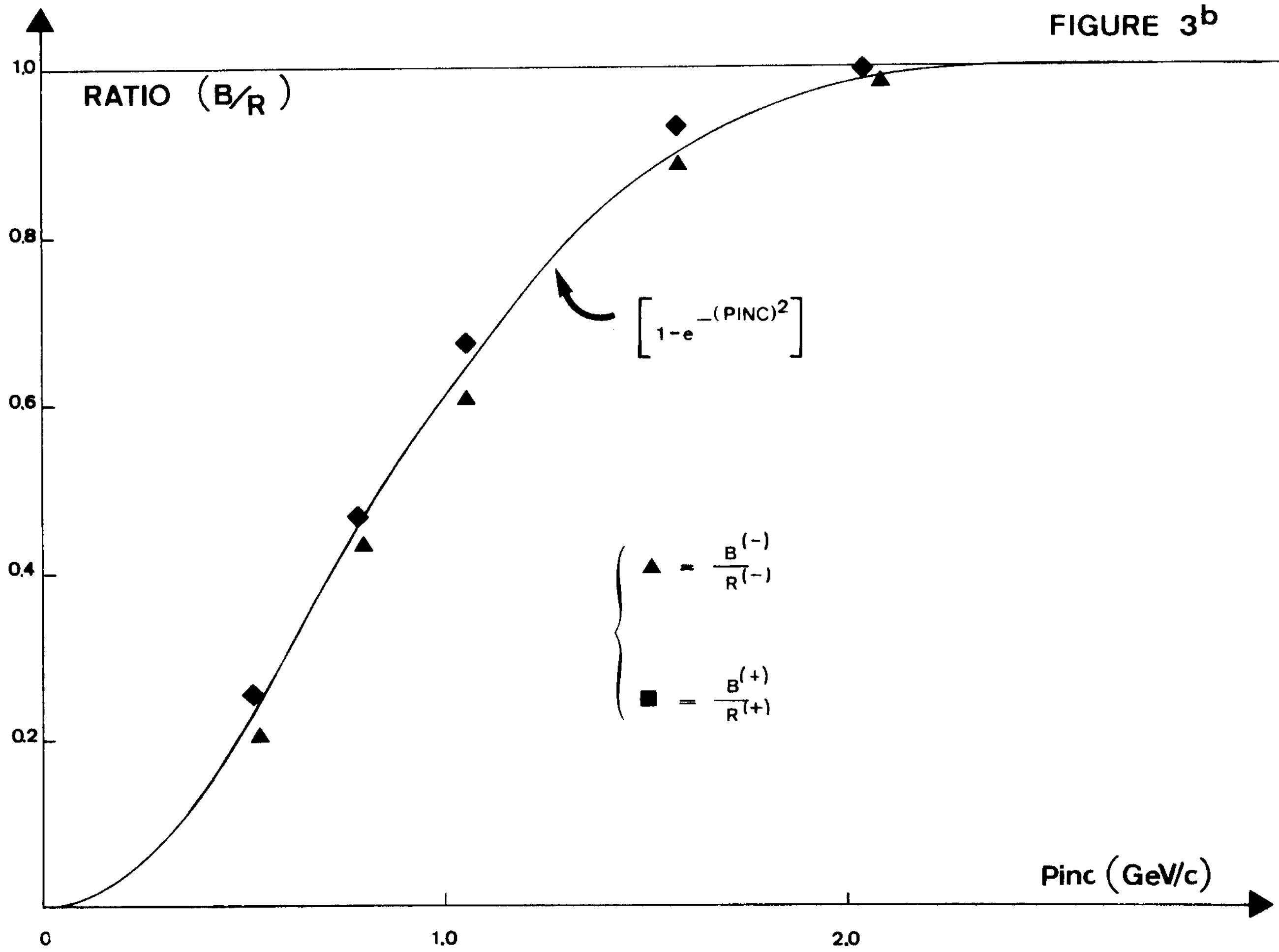
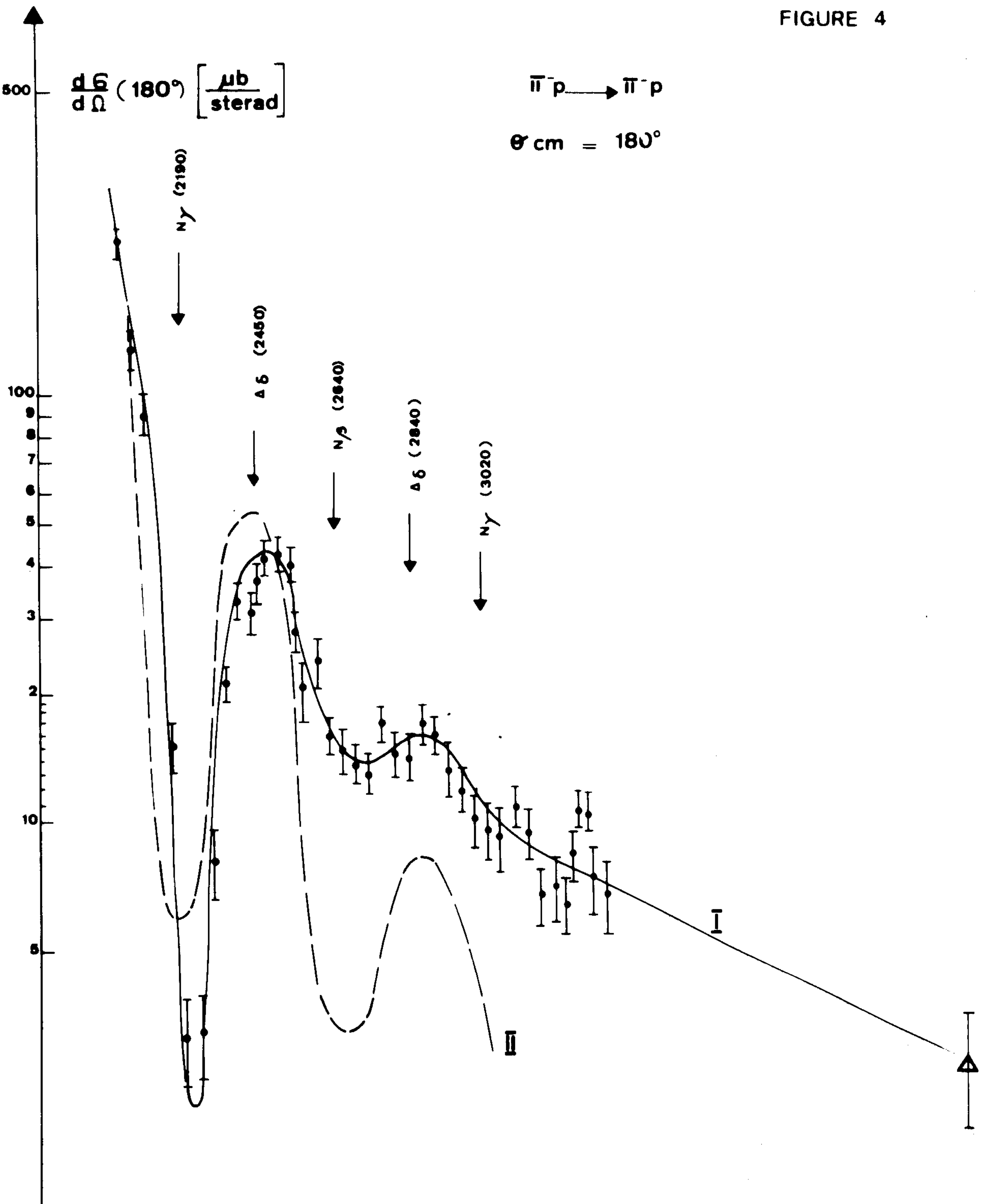
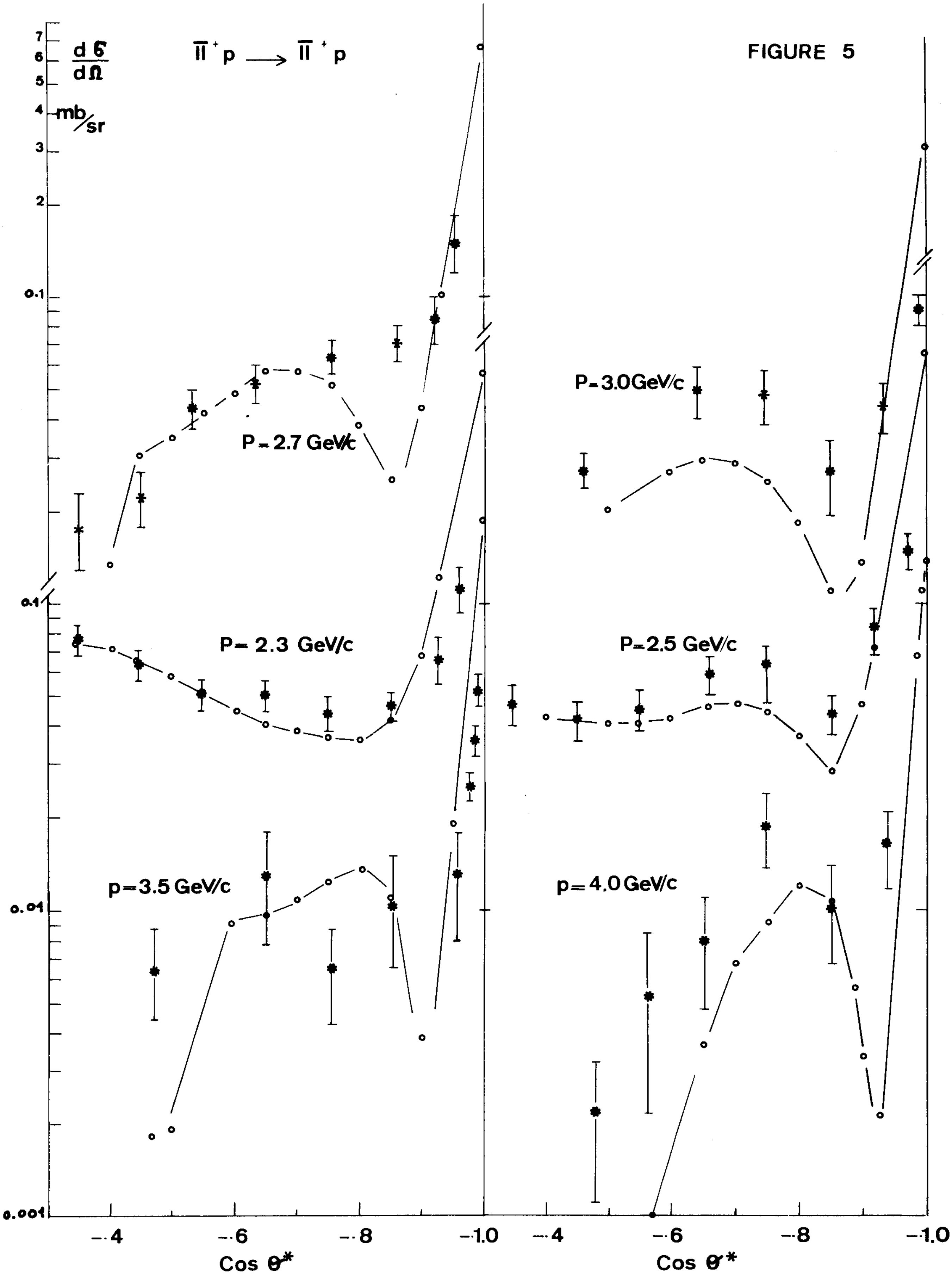


FIGURE 3b
RATIO (B/R)
1.0
0.8
0.6
0.4
0.2
0
1.0
2.0
Pinc (GeV/c)
[1-e−(PINC)2]
▲ = B(−)/R(−)
■ = B(+)/R(+)

FIGURE 4

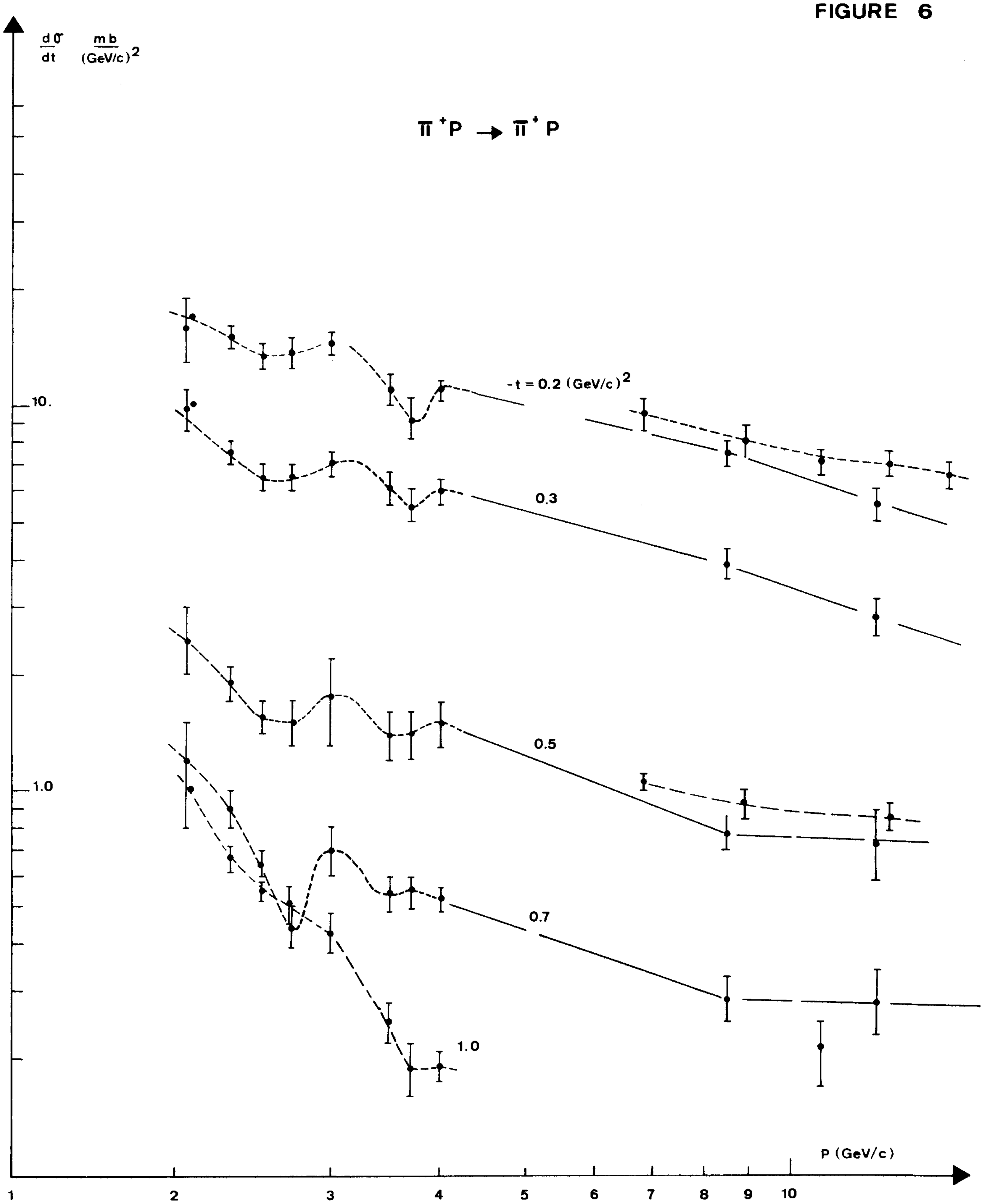


500
100
9
8
7
6
5
4
3
2
10
5
1
dσ/dΩ (180°) [μb/sterad]
π⁻p → π⁻p
θ cm = 180°
Nγ (2190)
Δδ (2450)
Nβ (2640)
Δδ (2840)
Nγ (3020)
I
II
P (GeV/c)
1
2
3
4
5
6
7



$\frac{d\sigma}{d\Omega}$ mb/sr
$\pi^+ p \longrightarrow \pi^+ p$
FIGURE 5
P = 2.7 GeV/c
P = 2.3 GeV/c
p = 3.5 GeV/c
P = 3.0 GeV/c
P = 2.5 GeV/c
p = 4.0 GeV/c
7
6
5
4
3
2
0.1
0.01
0.001
-.4
-.6
-.8
-1.0
Cos θ*

FIGURE 6



dσ/dt mb/(GeV/c)^2
π⁺P → π⁺P
10.
1.0
-t = 0.2 (GeV/c)^2
0.3
0.5
0.7
1.0
P (GeV/c)
1
2
3
4
5
6
7
8
9
10

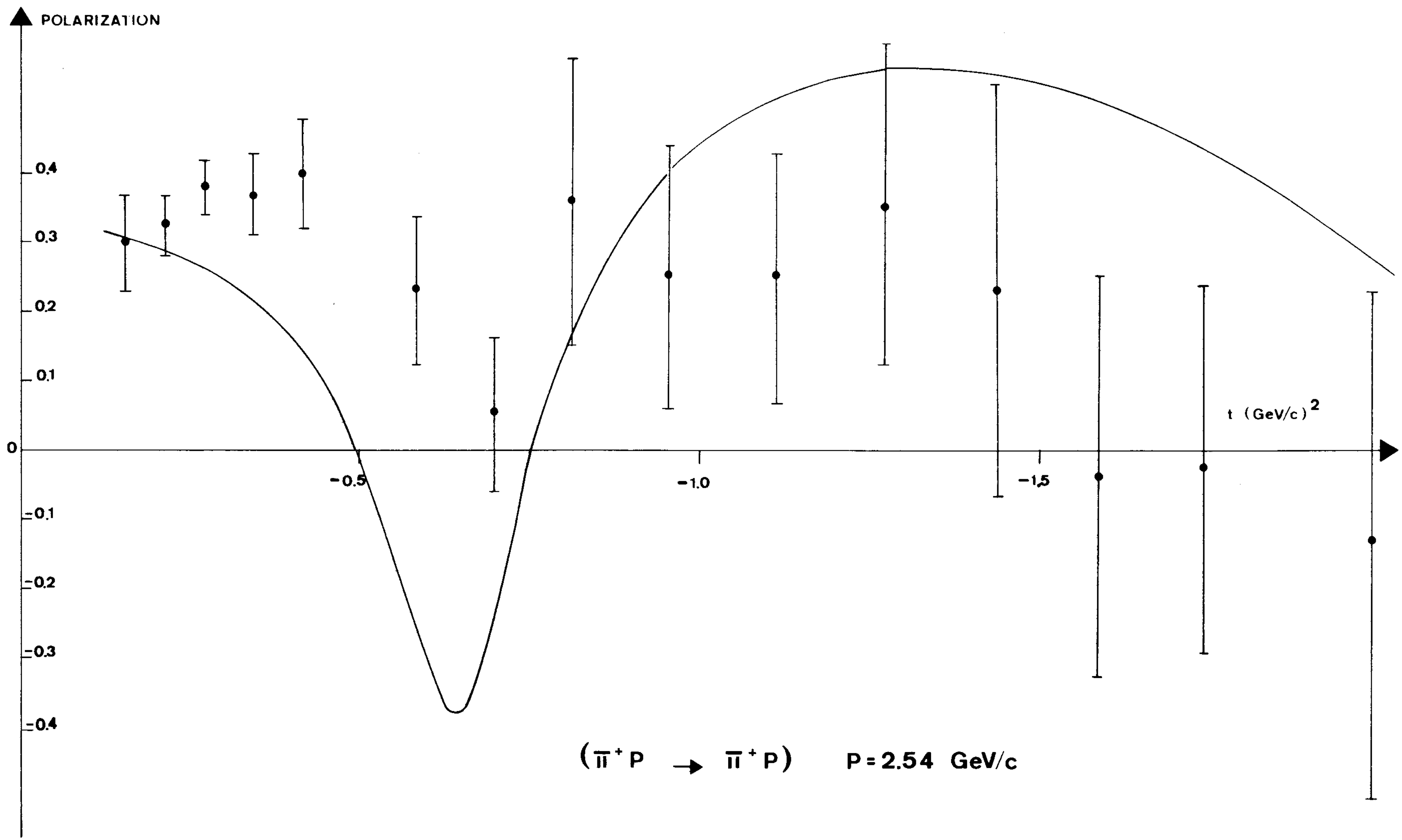
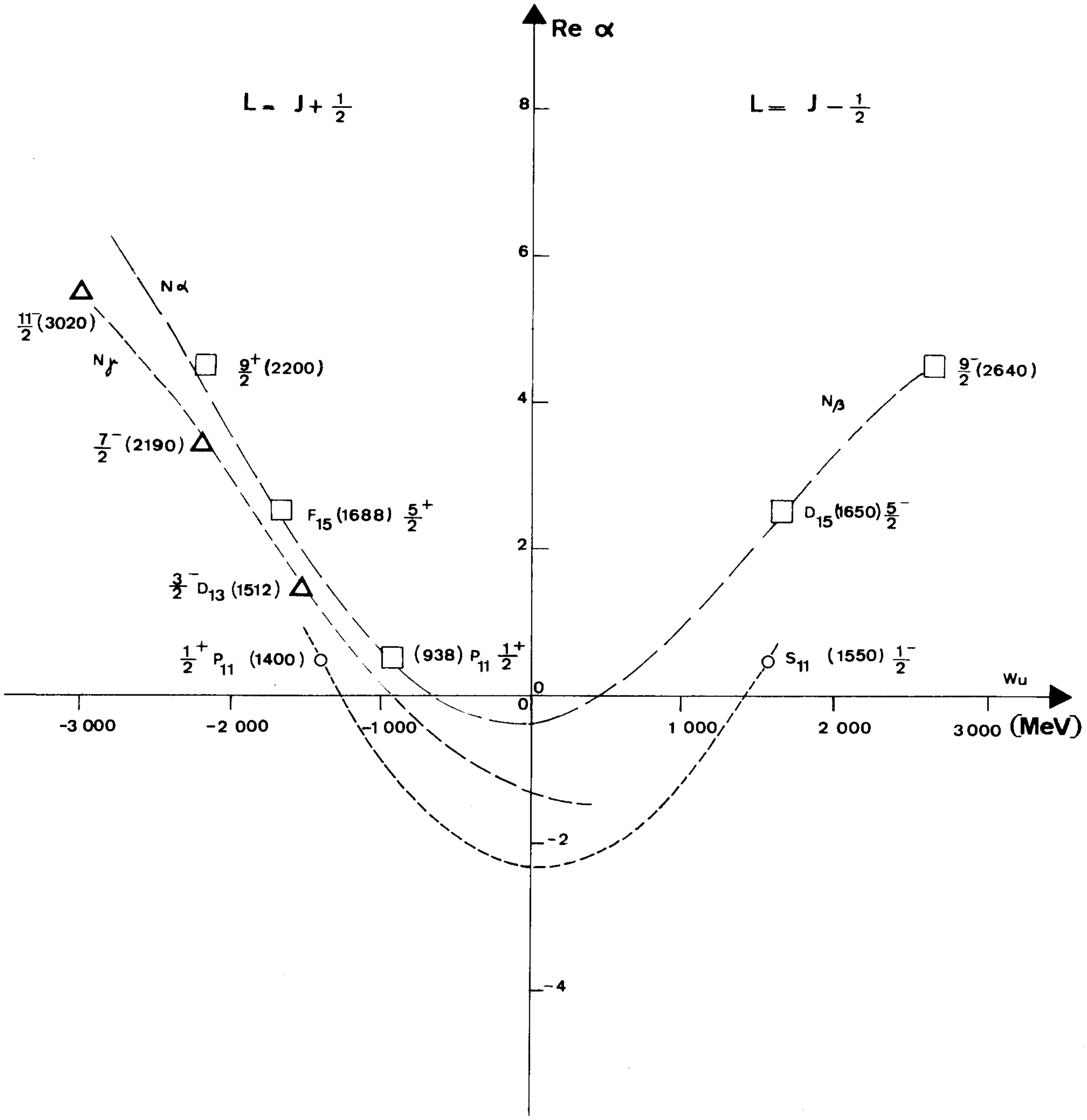

POLARIZATION
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-1.0
-1,5
t (GeV/c)²
(π⁺P → π⁺P) P = 2.54 GeV/c

FIGURE 7

FIGURE 8



Re α
L = J + 1/2
L = J − 1/2
8
6
4
2
0
-2
-4
Nα
Nγ
Nβ
11/2 (3020)
9/2+ (2200)
9/2− (2640)
7/2− (2190)
F15 (1688) 5/2+
D15 (1650) 5/2−
3/2− D13 (1512)
1/2+ P11 (1400)
(938) P11 1/2+
S11 (1550) 1/2−
Wu
-3 000
-2 000
-1 000
1 000
2 000
3000 (MeV)