Valuate Outsourcing Contracts from Vendors' Perspective: A Real Options Approach

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ABSTRACT

To date, most applications of real options theory (ROT) in outsourcing literature are modeled from the clients' side. Little attention has been paid to vendors' options in outsourcing. In this article, we study outsourcing from the vendors' perspective by analyzing vendors' value of waiting. The contribution of our research to the literature lies in our analysis of a model that compensates for vendors' loss of option to wait since they have to exercise outsourcing contracts at clients' given timing. Such a compensation-oriented model yields new insights about the vendors' valuation of outsourcing opportunities, and offers important practical guidance to vendors' decision making.

*Subject Areas: **Mathematical Models, Outsourcing Contracts, and Real Options Theory.***

INTRODUCTION

There are two sides to an outsourcing contract: the client (contract-granting firm) and the vendor (contract-receiving firm). When the vendor undertakes the outsourcing contract, it must incur an amount of initial investment costs. For example, sometimes the vendor has to update or expand capacity to facilitate this contract, and sometimes the vendor has to take over the client's former relevant facilities and employees. Even though an outsourcing contract brings a relatively certain demand to the vendor, it does not necessarily bring solid profits to the vendor. More or less, an outsourcing contract puts the vendor under financial and/or operational risks,

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which mainly come from three aspects: competitive bidding process, uncertainty of costs, and pressure of shorter contract-duration.

Outsourcing deals are often secured as the result of sealed-bid auctions, which bring price pressures to potential vendors. Based on prices and other criteria, the client invites a short list of vendors to an intense round of negotiations. Vendors invited to tender outsourcing deals are typically required to respond within a short period of time, and often based on less than ideal tender information. Specifically, a vendor never knows its competitors' bidding prices. Selection of a "winner" is based on a number of criteria and, not surprisingly, price is a critical component (Li & Kouvelis, 1999). To beat its opponents, a *vendor* may provide its bid calculated with marginal costs, leaving a thin boundary upon which it can make profits. Indeed, the vendor who wins the outsourcing contract may even sign a deal as a "loss leader" (Kern, Willcocks, & van Heck, 2002).

However, the vendor's operating costs usually are not constant over the contract duration due to the difficulty of defining a clear baseline, fast changing technology, varying economies of scale, and establishing the learning curve associated with cooperation between vendor and client (Chopra & Sodhi, 2004). For instance, UK's Inland Revenue Department outsourced its data center operations to EDS under a 10-year outsourcing agreement in 1993. EDS incurred losses over several years from running the data center. Profitability eventually emerged in the late 1990s from the economies of scale achieved by pooling this data center's operations with those of the Department of Social Security, a contract EDS also managed (Kern et al., 2002).

Currently there is a trend toward shorter duration outsourcing contracts (Johnson, 2000; McDougall, 2005). Clients use short-term contracts to recover faster from mistakes (e.g., selecting a wrong vendor or poorly defining the baseline), to motivate vendors' performance through the incentive of renewal, and to ensure that the agreed price is not out of step with the market. Under a short-duration contract, the vendor takes the risk in hoping that it can recover its initial investment costs by getting the renewal after the current contract expires (Lacity & Willcocks, 1998). However, an unspoken assumption in this process is that there is a successful client–vendor relationship (Webb & Laborde, 2005). Unfortunately, in many cases the client's inability to define the baseline requirements clearly, together with the subsequent expectations that additional or undocumented services would be provided by the vendor without additional costs, causes client–vendor relationships to deteriorate (Barthelemy, 2003). If the vendor tries to please its client by suffering uncovered additional costs, it still faces the risk that the client may turn to other more competitive vendors when this unprofitable contract expires.

Thus far current research in outsourcing mainly focuses on the clients' side (Levina & Ross, 2003). Little research investigates the vendors' decision-making (i.e., how should vendors valuate clients' outsourcing offers). Facing the aforementioned pressures, the need for a generic blueprint of dos and don'ts for vendors can hardly be overemphasized. To study this important unanswered question, the remainder of this research is structured as follows. In the next section, we review why the real options theory (ROT) provides a better way to valuate an investment opportunity under uncertainty, and point out the neglected vendors' options in outsourcing research. Then we develop the basic model of vendors' decision making, find out vendors' investment thresholds ("strike prices") through the standard

net present value approach and the real options approach respectively, and reveal why the consideration of vendors' options can reduce the risk of becoming the "loss leader." To that end, we investigate the relationships between vendors' decision-making thresholds and three parameters, the effects of learning on renewal, and the effects of competition among vendors. Finally, we conclude this research and provide managerial implications.

LITERATURE REVIEW

Current outsourcing literature has been exploring the reasoning behind clients' outsourcing decisions. As recently reported (Chalos & Sung, 1998; Li & Kouvelis, 1999; Barthelemy, 2003), the vendors' cost advantages provide the strongest motivation for clients to seek sourcing arrangements. The decisive criteria for winning outsourcing contract bids tend to be cost savings (Davis & Applegate, 1995; Ang & Straub, 1998). A survey by Deloitte Consulting Group reveals that 83% of respondents mentioned cost savings as the primary factor for outsourcing (Deloitte, 2005).

A potential problem that arises when clients primarily aim at cost minimization is that vendors might have to undercut prices in order to get the contract and as a consequence will operate at a loss (Van Tulder & Mol, 2002). More often than not, the exact value and service requirements of an outsourcing contract cannot be clearly defined (Reyniers & Tapiero, 1995). The difficulty in such bidding circumstances misleads vendors to valuate the profitability of outsourcing contracts (Ang & Straub, 1998; Gopal, Sivaramakrishnan, Krishana, & Mukhopadhyay, 2003). It is not uncommon that *vendors* make unrealistic bidding promises to ensure they win outsourcing contracts, but subsequently discover that they are unable to recover their tendering and operational costs in the near future. Kern et al. (2002) name this situation the "winner's curse," as the winner of an outsourcing bid systematically bids above the actual value of the contract and thereby systematically incurs losses. Lacity and Willcocks (1998) find 21 out of 85 outsourcing deals were in the winner's curse mode through empirical research.

Thus far the current literature does not provide practical guidance to vendors for outsourcing contract valuation, helping them avoid the risk of the winner's curse. In a study on vendors' decision making, Jeffery and Leliveld (2004) find that most vendors are using the standard net-present-value (NPV) to valuate their outsourcing contracts. If the NPV is positive the project is worthwhile and should be pursued; if it is negative the project should be turned down; if the NPV is zero it does not matter to the vendor whether the project is accepted or rejected. For example, Dayanand and Padman (2001) use the standard NPV approach (i.e., NPV $\geq$ 0) to study the outsourcing contract's progress payments problem.

Theoretical critiques of the standard NPV approach have brought up the issue that without considering the dynamics in operation conditions, the standard NPV analysis on investor's decision making may miss the additional value of managerial flexibility (i.e., an option to exercise the investment opportunity immediately or hold the investment for a while) (Bowman & Hurry, 1993; Luehrman, 1998; McGrath, Ferrier, & Mendelow, 2004). Actually, a vendor can ignore a client's outsourcing project now, but may invest in other external or internal projects in the future. This freedom makes investment timing an important instrument,

which the standard NPV neglects, to optimize the vendor's decision making. Dixit and Pindyck (1994) apply the ROT to discuss optimal investment timing in the framework of irreversibility and uncertainty, and point out the parallels between an investment opportunity and a "call option." A call option gives an investor the right to acquire an asset of uncertain future value. If conditions favorable to investing arise, the investor can exercise the option by taking the "strike price."

In outsourcing literature, the application of ROT is booming. For example, Johnstone (2002) treats outsourcing as a call option for the public sector and use the cost of purchasing as the strike price. If the in-house operating cost is higher than the purchasing cost in the open market, this public sector should outsource its former in-house activities. Nembhard, Shi, and Mehmet (2003) address the same issue: the bottom-line cost (strike price) associated with an outsourcing decision (option). Alvarez and Stenbacka (2006) use the level of market uncertainty as the strike price, by which to decide an organization's production mode—partial or complete outsourcing (option). In the outsourcing process, there is no doubt that clients play an active role by deciding whether to outsource, when to outsource, and how much/many to outsource. Therefore it is a straightforward study to look at outsourcing as an option of clients. In the current literature of outsourcing, all ROT-related studies treat outsourcing as an option in clients' hands.

Actually, vendors also have two embedded options in the outsourcing process: (i) whether to accept a client's outsourcing offer; (ii) if accepted, when to exercise this contract. While the first option is understandable, the second one is not obvious. In fact, the exercise timing of an outsourcing contract usually is out of the vendor's control. It is rare that a client would hold an outsourcing opportunity to meet a vendor's optimal exercising moment. If a vendor signs an outsourcing contract, it must exercise this contract at the given time. From the ROT perspective, however, the given exercise timing forces the vendor to give up the option to wait. Because losing the option to wait could expose the investor under a potential loss of money (Luehrman, 1998; Zhu & Weyant, 2003), it is a commonly agreed principle in financial economics that no investment should take place unless its net benefits at least compensate for the loss of "value of waiting" (Dixit & Pindyck, 1994; McGrath, 1997; Huchzermeier & Loch, 2001). Conventionally, investment is seen as acceptable when the NPV exceeds zero. The variation of this standard NPV approach dictated by ROT is that any action that reduces managerial flexibility is acceptable only if its NPV exceeds the financial value of the option given up. In order words, NPV larger than zero is not the critical point of investment, but NPV larger than the value of the lost option is (Pindyck, 1988; Smith & Nau, 1995; Tiwana, Keil, & Fichman, 2006). Since the vendor who has to exercise an outsourcing contract at the given time is no longer hedged by its option to wait, the vendor must make up this lost option value in the bid. Thus far, little attention has been paid to vendors' options in the existing outsourcing literature.

THE MODEL

No vendor wants to lose a bid due to being way out of step with competitors' bidding prices, but similarly, no vendor wants to win a bid only to discover that a satisfactory return is not possible. As a result, there is significant desire to establish

a decision-making model to figure out the vendor's decision-making criteria—the lowest bidding price, the highest operating cost, and the shortest contract duration.

The Basic Setting

To win an outsourcing contract, a vendor's bidding price P must be competitive. If the vendor wins, it will invest I to implement the outsourcing contract and incur an operating cost $C(t)$. Here, $C(t)$ evolves over time as a general Brownian motion (GBM), which is the continuous-time formulation of the random walk. This is the standard setting in ROT (Dixit, 1989) and also a good first approximation for uncertainties (Kamien & Li, 1990; Ingersoll & Ross, 1992; Abel & Eberly, 1994; Dixit & Pindyck, 1994; Murto, 1997). Specifically,

$$dC/C = \mu\, dt + \sigma\, dB_t, \tag{1}$$

where dB_t denotes a standard GBM process; μ implies the shift rate of expected future change; σ describes the uncertainty rate of such a process. We assume $\mu < 0$, that is, after taking over an outsourcing contract, the vendor's expected cost keeps decreasing. The source of cost reduction is the outsourcing firm's access to economies of scale, more information, and the unique know-how or learning curve that the vendor has been establishing in the practice (Anderson & Weitz, 1986; Roodhooft & Warlop, 1999).

The Strike Prices

We represent the time when the vendor exercises the contract as t_0. As a consequence, such an outsourcing contract's net present value NPV over the contract duration D is

$$
\begin{aligned}
NPV_{t_0} &= E\left[\int_{t_0}^{t_0+D} (P - C_t)e^{-\rho(t-t_0)}dt\right] - I \\
&= P\frac{1 - e^{-\rho D}}{\rho} - C_{t_0}\frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} - I. \tag{2}
\end{aligned}
$$

Here, ρ is the discount rate of the vendor. Because we assume that the vendor is risk neutral, this discount rate does not include a term proportional to the uncertainty of the outsourcing contract. Instead, it can be interpreted as the cost of capital the vendor or the vendor's industry faces (i.e., it depends positively on the real interest rate and the industry-specific risk rate).

For each possible bidding price P, there is a particular highest operational cost C_{NPV} for the vendor by letting equation (2) = 0:

$$C_{NPV} = \left(P\frac{1 - e^{-\rho D}}{\rho} - I\right)\frac{\rho - \mu}{1 - e^{-(\rho-\mu)D}}. \tag{3}$$

If $C_{t_0} \leq C_{NPV}$, the vendor can sign this outsourcing contract with this particular price P; if $C_{t_0} > C_{NPV}$, the vendor should not accept this contract with this P.

Similarly, given the operational cost C_{t_0} at t_0, the vendor's lowest bidding price P_{NPV} can be obtained by this standard NPV approach, $P_{NPV} = \{P : C_{NPV}(P) = C_{t_0}\}$. Hence,

$$P_{NPV} = \left(C_{t_0} \frac{1 - e^{-(\rho - \mu)D}}{\rho - \mu} + I \right) \frac{\rho}{1 - e^{-\rho D}}. \tag{4}$$

According to ROT, it is helpful to consider that before undertaking an outsourcing contract, a vendor has the option to wait—and thus not to wait. As a result, the dynamic *NPV* (= standard *NPV* + value of option to wait) more accurately reflects the value of an investment opportunity than the standard *NPV* does (Benaroch, 2002; Daily & Kotlikoff, 2006). The dynamic *NPV* at t_0, the time before the vendor exercises the contract, can be described by the standard real options expression:

$$F(t_0) = \max_{T \geq t_0} E_{t_0} \left[(NPV_T)^+ e^{-\rho(T - t_0)} \right]$$

$$= \max_{T \geq t_0} E_{t_0} \left[\left(P \frac{1 - e^{-\rho D}}{\rho} - C_T \frac{1 - e^{-(\rho - \mu)D}}{\rho - \mu} - I \right)^+ e^{-\rho(T - t_0)} \right], \tag{5}$$

where $X^+ = \max(X, 0)$. This reflects the essence of an option: the value of option to wait can never make things worse but can possibly make them better. Because, by definition, there is no obligation to exercise an option, the value of option to wait is always nonnegative. For example, if the standard *NPV* is negative at t_0, the vendor will select to wait rather than to exercise, that is, the value of option to wait is larger than zero. The vendor can maximize its dynamic *NPV* at t_0 by selecting the optimal investment time *T* in the future.

The maximum value of the investment opportunity from any initial state *C* and any initial time *t* is described by the so-called value function that is defined by

$$F(C_t, t) = \max_{u(s), s \in (t, \bar{T})} E_t \left[F(C_{\bar{T}}, \bar{T}) e^{-\rho(\bar{T} - t)} \right.$$

$$\left. - \int_t^{\bar{T}} u(s) I(s) e^{-\rho(s - t)} ds + \int_t^{\bar{T}} (P - C_s) \xi(s) e^{-\rho(s - t)} ds \right], \tag{6}$$

where $\bar{T}$ is an arbitrary time in the future. $u(t) = \{0, 1\}$ and $\xi(t) = \{0, 1\}$ are the investment control function and the indication function, respectively. When $u(t) = 1$, the vendor accepts the client's offer and decides to invest $I(t)\, dt \equiv I$ in the outsourcing contract; when $\xi(t) = 1$, the vendor is really implementing the contract. The value of the investment opportunity is the discounted expectation of the value at future time $\bar{T}$ and the discounted value for the intermediate cash flows.

Solving equation (6) yields (see Appendix A)

$$F(t_0) = \begin{cases} P \dfrac{1 - e^{-\rho D}}{\rho} - C_{t_0} \dfrac{1 - e^{-(\rho - \mu)D}}{\rho - \mu} - I, & C_{t_0} \leq C_{ROT} \\ A_2 C_{t_0}^{\beta_2}, & C_{t_0} > C_{ROT} \end{cases} \tag{7}$$

$$C_{ROT} = \frac{\beta_2}{\beta_2 - 1} \left(P \frac{1 - e^{-\rho D}}{\rho} - I \right) \frac{\rho - \mu}{1 - e^{-(\rho - \mu)D}}, \tag{8}$$

where

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} < 0$$

and

$$A_2 = C_{ROT}^{(1-\beta_2)} \frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} \frac{-1}{\beta_2}.$$

When $C_{t_0} \le C_{ROT}$, the top half of equation (7) is just as same as equation (2) (i.e., the dynamic NPV = standard NPV). It means that if the vendor's cost is lower than C_{ROT}, the value of option to wait becomes zero and the option to undertake the contract is immediately valuable (in *ROT* term it is "in the money"); when $C_{t_0} > C_{ROT}$, the option to undertake the contract is of no use (in *ROT* term it is "out of the money"). Thus, C_{ROT} is the threshold of the vendor's decision-making criterion (in *ROT* term it is "strike price").

From equations (7) and (8), we can obtain the vendor's lowest bidding price through the *ROT* approach, $P_{ROT} = \{P : C_{ROT}(P) = C_{t_0}\}$. Hence,

$$P_{ROT} = \left(\frac{\beta_2 - 1}{\beta_2} C_{t_0} \frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} + I\right) \frac{\rho}{1 - e^{-\rho D}}. \tag{9}$$

Facing an outsourcing contract that must be exercised at t_0, if the vendor does not consider the value of option to wait, its investment thresholds are C_{NPV} and P_{NPV}; if the vendor compensates for the loss of option to wait, its investment thresholds become C_{ROT} and P_{ROT}. Comparing equations (3) and (8) and equations (4) and (9), we obtain

$$\frac{C_{ROT}}{C_{NPV}} = \frac{\beta_2}{\beta_2 - 1} < 1, \tag{10}$$

$$\frac{P_{ROT}}{P_{NPV}} = 1 + \frac{\dfrac{-1}{\beta_2} C_{t_0} \dfrac{1 - e^{-(\rho-\mu)D}}{\rho - \mu}}{C_{t_0} \dfrac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} + I} = 1 + \frac{-1}{\beta_2} \frac{1}{1 + \dfrac{I}{C_{t_0}} \dfrac{\rho - \mu}{1 - e^{-(\rho-\mu)D}}} > 1. \tag{11}$$

These results reveal that the standard *NPV* approach provides more aggressive criteria to the vendor's decision making than the *ROT* approach does, because the likelihood of $C_{t_0} < C_{NPV}$ is larger than that of $C_{t_0} < C_{ROT}$, and the likelihood of $P > P_{NPV}$ is larger than that of $P > P_{ROT}$. The larger likelihood of passing the threshold means the vendor is more likely to exercise the outsourcing contract. Therefore, the standard *NPV* approach may increase the vendor's risk of "winner's curse."

The Shortest Contract Duration

To understand the risk of an investment, the investor should also understand the investment's payback period—the contract duration (Wambach, 2000). Because contract duration is most likely to be an endogenous variable in outsourcing (e.g.,

Figure 1: The shortest contract durations under the NPV and ROT approaches.
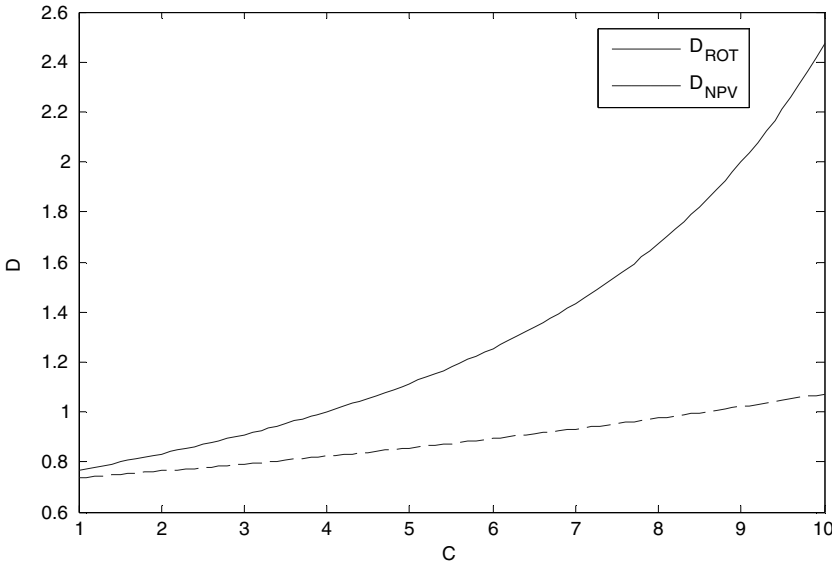
a client usually predetermines the contract's duration), a vendor must figure out whether its investment costs and operating costs could be covered over this given duration.

According to the standard *NPV* approach, D_{NPV}, the shortest contract duration the vendor needs to cover its costs under a bidding price P, is defined as $D_{NPV} = \{D^+ : C_{NPV}(D) = C_{t_0}\}$. According to the *ROT* approach, the vendor's shortest contract duration D_{ROT} under a bidding price P is defined as $D_{ROT} = \{D^+ : C_{ROT}(D) = C_{t_0}\} = \{D^+ : C_{NPV}(D) = C_{t_0} \frac{\beta_2 - 1}{\beta_2}\}$. Since $\partial C_{NPV}/\partial D > 0$ (See Appendix B), $(\beta_2 - 1)/\beta_2 > 1$, thus we have $D_{NPV} \leq D_{ROT}$.

At each state C, the relative D_{NPV} and D_{ROT} can be solved numerically. Figure 1 shows that at the same operating cost level, the standard *NPV* approach tends to require shorter contract duration to cover vendors' investment I and operating cost $C(t)$ than the *ROT* approach does. Again, the standard *NPV* approach valuates the outsourcing contract opportunity more aggressively than the *ROT* approach does.

ANALYSIS AND DISCUSSIONS

The Thresholds of Decision Making

Analyzing vendors' decision-making thresholds, C_{ROT} and P_{ROT}, can provide a useful lens to explore the insights into the vendors' value of waiting (W). If a vendor's operating cost C (or bidding price P) is lower (or higher) than the threshold C_{ROT} (or P_{ROT}), the vendor's value of waiting W becomes zero (i.e., the vendor should act rather than wait); otherwise $W > 0$, the vendor should do more waiting and less investing.

Equations (8) and (9) show that three exogenous parameters of an outsourcing contract, ρ, μ, and σ, exercise influence upon the vendors' decision-making thresholds C_{ROT} and P_{ROT}. The partial derivatives $\partial C_{ROT}/\partial\rho$, $\partial C_{ROT}/\partial\mu$, and $\partial C_{ROT}/\partial\sigma$ are

$$\frac{\partial C_{ROT}}{\partial\rho} = \frac{2}{(\beta_2-1)^2\sigma^2\sqrt{(\mu/\sigma^2-1/2)^2+2\rho/\sigma^2}}\frac{\rho-\mu}{1-e^{(\rho-\mu)D}}\left(P\frac{1-e^{-\rho D}}{\rho}-I\right)$$
$$+\frac{\beta_2}{\beta_2-1}\left[\frac{(1-\rho^{-(\rho-\mu)D})-(\rho-\mu)D\rho^{-(\rho-\mu)}D}{(1-e^{-(\rho-\mu)D})^2}\right.$$
$$\left.\times\left(P\frac{1-e^{-\rho D}}{\rho}-I+\frac{\rho-\mu}{1-e^{-(\rho-\mu)D}}\left(P\frac{\rho De^{-\rho D}-(1-e^{-\rho D})}{\rho^2}\right)\right)\right],$$

$$\frac{\partial C_{ROT}}{\partial\mu} = \left[\frac{\sqrt{(\mu/\sigma^2-1/2)^2+2\rho/\sigma^2}+(\mu/\sigma^2-1/2)}{\sigma^2(\beta_2-1)^2\sqrt{(\mu/\sigma^2-1/2)^2+2\rho/\sigma^2}}\frac{\rho-\mu}{1-e^{-(\rho-\mu)D}}\right.$$
$$\left.+\frac{\beta_2}{\beta_2-1}\frac{(\rho-\mu)De^{-(\rho-\mu)D}-(1-e^{-(\rho-\mu)D})}{(1-\rho^{-(\rho-\mu)D})^2}\right]\left(P\frac{1-e^{-\rho D}}{\rho}-I\right),$$

$$\frac{\partial C_{ROT}}{\partial\sigma} = \frac{\rho-\mu}{1-e^{-(\rho-\mu)D}}\left(P\frac{1-e^{-\rho D}}{\rho}-I\right)\frac{-1}{(\beta_2-1)^2}\frac{\partial\beta_2}{\partial\sigma} < 0$$

(See Appendix C for proof).

Among the three partial derivatives, only $\partial C_{ROT}/\partial\sigma$ has an obvious analytical result: $\partial C_{ROT}/\partial\sigma < 0$. The same results occur to the partial derivatives of P_{ROT}. To save space, we omit the details.

Such a difficulty of obtaining analytical results is typical in the ROT literature, because influential factors are always compounded together to impact the strike price. Therefore, we apply the mainstream methodology in the discipline of ROT— numerical simulation—to determine how the values ρ, μ, and σ affect the vendor's exercise-or-wait thresholds C_{ROT} and P_{ROT} (See Dixit and Pindyck [1994] for similar numerical analysis). During the calculation, ρ varies from .04 to .2, σ varies from .1 to .6, and μ varies from 0 to .2, respectively. In Figures 2–4, each time we hold one parameter in order to study the relation between the strike price (C_{ROT} or P_{ROT}) and other two parameters. Note that the lower the C_{ROT} or the higher the P_{ROT} is, the higher the waiting value W is. These figures clearly reveal that C_{ROT} is positively proportional to ρ and μ but negatively proportional to σ and P_{ROT} is negatively proportional to ρ and μ but positively proportional to σ.
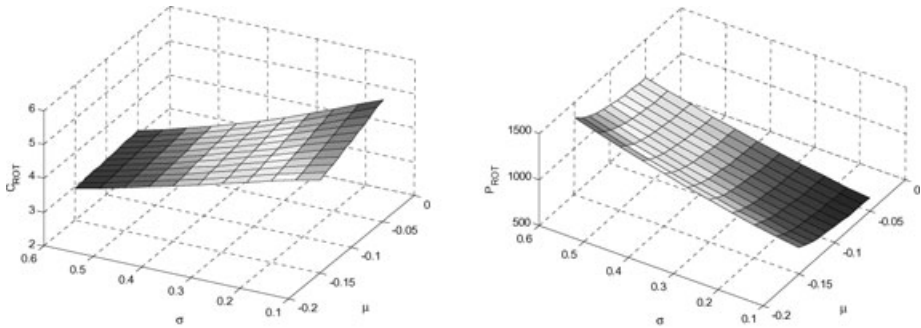
By definition, ρ is the capital depreciation that the vendor has to take. Due to the positive proportional relation between C_{ROT} and ρ (the negative proportional relation between P_{ROT} and ρ), an increasing ρ will lead an increasing C_{ROT} (decreasing P_{ROT}), so that the likelihood of $C_{t_0} \leq C_{ROT}$ ($P \geq P_{ROT}$) will increase and the waiting value W will decrease (i.e., the vendor prefers to exercise the outsourcing contract opportunity immediately to avoid a higher investment cost in the future); a decreasing ρ will lead a decreasing C_{ROT} (increasing P_{ROT}), so that the likelihood of $C_{t_0} \leq C_{ROT}$ ($P \geq P_{ROT}$) will decrease and W will increase (i.e., the vendor hesitates to exercise this outsourcing contract opportunity in order

Figure 2: The decision-making thresholds versus σ and ρ (μ is fixed)[a].
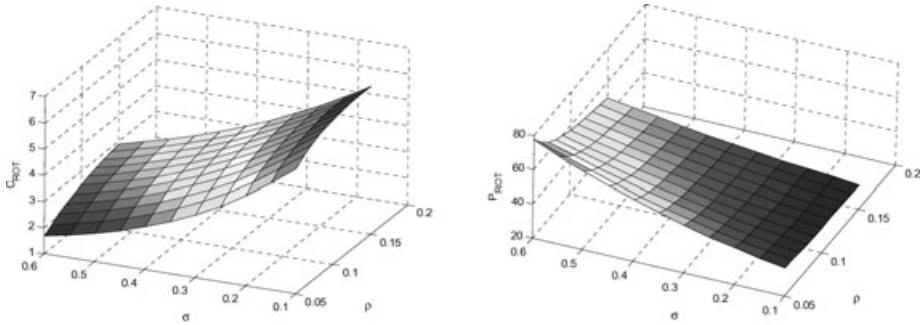
[a] The value of waiting is positively proportional to σ, but negatively proportional to ρ.

Figure 3: The decisionmaking thresholds versus with σ and μ (ρ is fixed)[a].
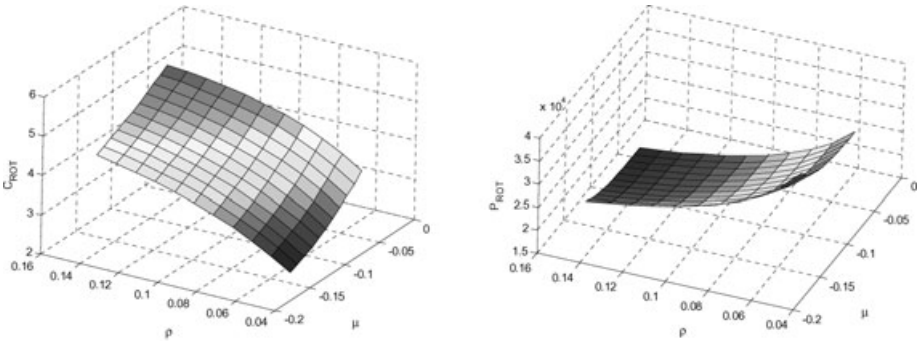

[a] The value of waiting is positively proportional to σ, but negatively proportional to μ.

to enjoy a lower investment cost in the future). The managerial relevance is that given that other conditions are stable, the vendor's decision making is more aggressive when the capital cost increases.

Based on the assumption $\mu < 0$, μ measures the decreasing rate of the vendor's operational cost. A smaller μ implies a more significant cost reducing process. Because of the positive proportional relation between C_{ROT} and μ (the negative proportional relation between P_{ROT} and μ), an increasing μ will lead an increasing C_{ROT} (decreasing P_{ROT}), so that the likelihood of $C_{t_0} \leq C_{ROT}$ ($P \geq P_{ROT}$) will increase and W will decrease (i.e., if the effect of cost reduction in the future is not significant, the vendor prefers to exercise this outsourcing contract opportunity now); a decreasing μ will lead a decreasing C_{ROT} (increasing P_{ROT}), so that the likelihood of $C_{t_0} \leq C_{ROT}$ ($P \geq P_{ROT}$) will decrease and W will increase (i.e., if the effect of cost reduction is significant in the future, the vendor hesitates to exercise this outsourcing contract opportunity too soon). The managerial relevance is that given that other conditions are stable, the vendor's decision making is more aggressive when its cost is not expected to decrease.

Figure 4: The decisionmaking thresholds versus with ρ and μ (σ is fixed)[a].

[a] The value of waiting is negatively proportional to μ and ρ.

Because σ describes the uncertainty of the vendor's operational cost during the implementation of an outsourcing contract, a larger σ means that the effect of cost reduction is highly uncertain. As the proportional relation between C_{ROT} and σ is negative, (the proportional relation between P_{ROT} and σ is positive), an increasing σ will lead a decreasing C_{ROT} (increasing P_{ROT}), so that the likelihood of $C_{t_0} \leq C_{ROT}$ ($P \geq P_{ROT}$) will decrease and W will increase (i.e., if the effect of cost reduction is fuzzy, the vendor hesitates to invest in this outsourcing contract); a decreasing σ will lead an increasing C_{ROT} (decreasing P_{ROT}), so that the likelihood of $C_{t_0} \leq C_{ROT}$ ($P \geq P_{ROT}$) will increase and W will decrease (i.e., if the effect of cost reduction is certain, the vendor will prefer to exercise this outsourcing contract opportunity soon). The managerial relevance is that given that other conditions are stable, the vendor's decision making is more aggressive when the effect of cost reduction is certain.

The Ratio of D_{ROT}/D_{NPV}

Based on the discussion regarding the shortest contract durations in the last section, it is clear that D_{ROT} is more conservative than D_{NPV}, because the ROT approach considers the vendor's lost option to wait. To gain further insight, we need to investigate the relationships between the ratio of D_{ROT}/D_{NPV} and ρ, μ, and σ. Again, it is impossible to analytically solve the derivatives of $\partial(D_{ROT}/D_{NPV})/\partial\rho$, $\partial(D_{ROT}/D_{NPV})/\partial\mu$, and $\partial(D_{ROT}/D_{NPV})/\partial\sigma$. So we use the simulation approach to obtain the relations between this ratio and ρ, μ, and σ (see Figure 5).

From Figure 5a, for the given μ, when σ is low, D_{ROT}/D_{NPV} is close to 1 and robust to σ and ρ; when σ is high, D_{ROT}/D_{NPV} becomes sensitive to σ (positively proportional) and ρ (negatively proportional). Since a lower level of uncertainty or a higher capital cost reduces the value of waiting, D_{ROT} and D_{NPV} are close and respond to the capital cost change at the same pace; a higher level of uncertainty or a lower capital cost increases the value of waiting and the difference between D_{ROT} and D_{NPV} becomes significant.

Figure 5: The ratio D_{ROT}/D_{NPV} versus ρ, μ, and σ.
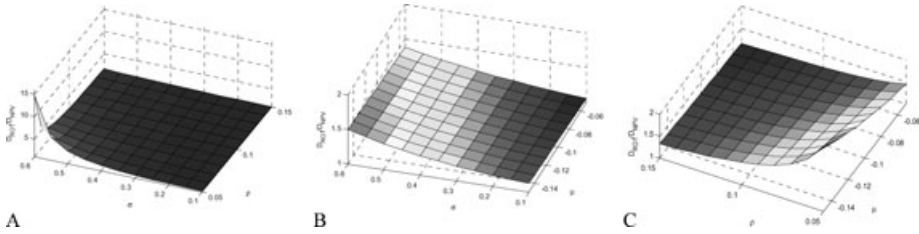
From Figure 5b, for the given ρ, D_{ROT}/D_{NPV} is positively proportional to σ and negatively proportional to μ. Since a lower level of uncertainty or a slight cost reduction reduces the value of waiting, D_{ROT} and D_{NPV} are close; a higher level of uncertainty or a significant cost reduction increases the value of waiting, the difference between D_{ROT} and D_{NPV} becomes significant.

From Figure 5c, for the given σ, D_{ROT}/D_{NPV} is negatively proportional to both ρ and μ. Since a lower capital cost or a slight cost reduction reduces the value of waiting, D_{ROT} and D_{NPV} are close; a higher capital cost or a significant cost reduction increases the value of waiting, thus the difference between D_{ROT} and D_{NPV} becomes significant.

Influence of Learning on Renewal

Any outsourcing contract will expire after its duration D. If a vendor considers the renewal of an outsourcing contract during the bidding stage, such a long-term strategy will directly impact the vendor's bidding. For example, with a strategic intent to hold the contract permanently (always get the renewal), the vendor may offer a low bidding price, believing that it can recoup the investment and broaden margins later. Consequently, facing an outsourcing contract opportunity, a key decision the vendor has to make is whether to valuate this opportunity over a short-term period (for only one-period D) or a long-term period (for multiple-period Ds). To analyze the two different bidding strategies, we introduce two scenarios as follows:

Case I: the vendor is keen to gain the renewal. While the vendor is actually bidding for the outsourcing contract over $(T, T + D)$, its bidding strategy extends to the next contract duration $(T + D, T + 2D)$. As a result, the vendor considers a long-term bidding strategy over $(T, T + 2D)$:

$$V_I = E_T \left[\int_T^{T+2D} (P - C_t)e^{-\rho(t-T)}\, dt \right]. \tag{12}$$

Case II: the vendor's bidding strategy only covers one period $(T, T + D)$. As a result, under this short-term bidding strategy, the renewal over the next period $(T + D, T + 2D)$ is an option—renew or not—in the future rather than a scheduled event in Case I.

$$V_{II} = E_T \left[\int_T^{T+D} (P - C_t)e^{-\rho(t-T)}dt + \left(\int_{T+D}^{T+2D} (P - C_t)e^{-\rho(t-(T+D))}dt \right)^+ \right].$$

(13)

From equations (12) and (13), we get (see Appendix D):

$$\Delta V = V_{II} - V_I \geq 0,$$

(14)

$$\partial \Delta V / \partial \sigma \geq 0.$$

(15)

Equation (14) reveals that Case II brings a higher value to the vendor than Case I does. The managerial relevance is that when facing an outsourcing contract opportunity, the vendor should bid this opportunity by focusing on a single period and treat the renewal as an option instead of an obligation. The higher value of Case II comes from more information or knowledge and options the vendor possesses. During the first period $(T, T + D)$, the vendor usually establishes the learning-curve for the client's special requirements or demands, so that it can use the learning from the first period to revamp itself more knowledgeable of the operational cost over the second period $(T + D, T + 2D)$. The higher the future uncertainty is, the higher the value of Case II is (see equation (15)). In Case II, based on its new knowledge, the vendor can more wisely decide to accept the client's renewal invitation or withdraw from the client's outsourcing business at the end of the first period. In Case I, however, the vendor does not have such an option, because it has to follow its original strategy to carry on the outsourcing contract over the second period.

Effects of Competition

In different industries vendors have different levels of market power. For certain industries where vendors do not have enough opportunities to invest later, waiting for the next opportunity probably means a dramatic reduction in profits and market shares or even going bankrupt. To understand how vendors should valuate the value of option to wait in different industries, it is important to investigate the risk of waiting or abandoning the current offer.

We model the risk of losing the investment opportunity by assuming the contract's cash flow or potential cash flow may be lost following a Poisson process (or a jump to bankruptcy) with probability $\phi \, dt$ over the next instant, independent of $C(t)$ and other variables in the model. Let $\zeta(t)$ be an indicator function indicating whether a loss of opportunity has occurred. $\zeta(t)$ begins as one and becomes zero when a loss occurs. Hence from equation (6), the value of investment opportunity becomes

$$F(C_t, t) = \max_{u(s), s \in (t, \bar{T})} E_t \left[F(C_{\bar{T}}, \bar{T})e^{-\rho(\bar{T}-t)} \right.$$

$$\left. - \int_t^{\bar{T}} u(s)I(s)e^{-\rho(s-t)}ds + \int_t^{\bar{T}} (P - C_s)\xi(s)\zeta(s)e^{-\rho(s-t)} \, ds \right].$$

(16)

Due to the independence of the loss process, for all s, $t \leq s \leq \bar{T}$, we have

$$E_t[\zeta(s)|C(s)] = e^{-\phi(s-t)}. \tag{17}$$

Based on this fact, we can replace $\zeta(s)$ with its expectation. Such process behaves as a depreciation rate in the valuation and gets involved by an adjustment to the discount rate, $\hat{\rho} = \rho + \phi$ (Berk, Green, & Naik, 2004). When $\phi = 0$, the vendor has strong market power and no competition threats, hence this case reduces to the basic model. When ϕ is large, there is high probability of losing an investment opportunity due to competition among vendors, future cash flow will have less present value, hence such a case has high discount rate.

We now look at the impact of ϕ on the vendor's value of waiting. Notice that

$$\frac{\partial C_{ROT}}{\partial \phi} = \frac{\partial C_{ROT}}{\partial \hat{\rho}} \frac{\partial \hat{\rho}}{\partial \phi} = \frac{\partial C_{ROT}}{\partial \hat{\rho}} \text{ and } \frac{\partial P_{ROT}}{\partial \phi} = \frac{\partial P_{ROT}}{\partial \hat{\rho}} \frac{\partial \hat{\rho}}{\partial \phi} = \frac{\partial P_{ROT}}{\partial \hat{\rho}}.$$

Due to the aforementioned positive proportional relation between C_{ROT} and $\hat{\rho}$ (the negative proportional relation between P_{ROT} and $\hat{\rho}$), an increasing ϕ will lead an increasing C_{ROT} (decreasing P_{ROT}), so that the likelihood of $C_{t_0} \leq C_{ROT}$ ($P \geq P_{ROT}$) will increase and the waiting value W will decrease (i.e., the vendor prefers to exercise the outsourcing contract opportunity immediately in a highly competitive industry); a decreasing ϕ will lead a decreasing C_{ROT} (increasing P_{ROT}), so that the likelihood of $C_{t_0} \leq C_{ROT}$ ($P \geq P_{ROT}$) will decrease and W will increase (i.e., the vendor is not hurried to exercise an outsourcing contract opportunity in a less competitive industry). The managerial relevance is that the vendor's decision making is more aggressive in a highly competitive industry.

MANAGERIAL IMPLICATIONS AND CONCLUSIONS

The outsourcing literature has addressed the vendor's problem of valuating outsourcing contracts from the standpoint of NPV criterion. This perspective ignores the vendor's managerial flexibility. That is, vendors have two embedded options in the outsourcing process: (i) whether to accept the client's outsourcing offer; (ii) if accepted, when to exercise this contract. This fact motivates our article and addresses the void in the literature.

In this article , by employing a real-options framework, we valuate outsourcing from the vendor's perspective in a setting characterized by the vendor's lost option to wait. By compensating for this lost option value, we use the "strike price" concept from ROT to provide the criteria (P_{ROT} and C_{ROT}) for the vendor's decision making. In particular, the vendors' lowest bidding price P_{ROT} (the highest operating cost C_{ROT}) resulting from the ROT approach is higher (lower) than its counterpart $P_{NPV}(C_{NPV})$ from the standard *NPV* approach, in which the vendor's options are neglected. This conservative threshold may protect vendors from the risk of the so-called winner's curse. Given this setup, we concurrently examine the relationships between vendors' waiting value (W) and influential factors (ρ, μ, and σ). We also analyze vendors' learning effects within the context of outsourcing contract renewal. The result also reflects the value of vendors' option: the vendors' benefits from treating renewal as an option are higher than an otherwise identical

outsourcing contract that treats renewal as an obligation. At last, we investigate the influence of competition on vendor decision making. Our vendors-oriented approach is novel to the literature of outsourcing, and offers important practical implications for vendors as follows:

First, valuate an outsourcing contract opportunity not only in standard terms of its expected profit and variance, but also based on vendor's readiness. If a vendor is not ready to undertake an outsourcing contract at the given exercising time, its bidding price must compensate for this hurriedness. The vendor has the flexibility to choose whether and when to invest, despite the fact that the client usually does not provide the vendor the timing flexibility in the outsourcing contract. In reality, however, the vendor tends to ignore the opportunity cost of the investment, that is, the option value to wait, under the pressure of losing the contract opportunity.

Second, wisely exploit the sense of proportion to judge the exercise timing by valuating the value of wait. Higher σ, higher uncertainty, makes the option to wait more valuable. Now the vendor should do more waiting. Higher μ, lower cost decreasing rate, increases the opportunity cost of keeping the option alive. Now the vendor should do more investing but less waiting. Higher ρ or ϕ, higher capital cost or higher competition, erodes the value of waiting. Now the vendor should do more investing.

Finally, treat the contract renewal as an option instead of an obligation during the bid process. A vendor should not simply pursue long-term outsourcing opportunities, but pay attention to establish its dynamic learning curve under uncertainties. The attempt to lock in the client first and hope to recoup broaden margins later would lead to the notorious "winner's curse."

There are two major limitations to our model. First, the assumption of risk-neutral vendors may be unrealistic, especially for small vendors. For example, vendors may become risk-seeking, when they are short of business due to recession, are a new entrant into the market, have become less powerful competitively, or want to lock out competitors. Another limitation of the model is that it is only appropriate to the fixed-price outsourcing contract. While this kind of contract is still the mainstream in outsourcing, flexible-pricing contracts are emerging, such as cost plus, market pricing, fixed fee adjusted by volume fluctuation, and benefit sharing. In order to incorporate these features into the model, a major change in the setup is anticipated, and we hope that further research will follow this vendor-oriented avenue to incorporate these aspects of outsourcing opportunity valuating. [Received: November 2006. Accepted: January 2008.]

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APPENDIX A

The maximum value of the investment opportunity from any initial state C and any initial time t is described by the so-called value function that is defined by equation (6).

Before time T when the investment happens, the investment opportunity, $F(C)$, produces no cash flow, the return is just the capital appreciation. Hence as shown in Dixit and Pindyck (1994), in the continuation region (not optimal to invest) the Bellman equation is

$$\rho F \, dt = E(dF). \tag{A.1}$$

Equation (A.1) tells that the expected rate of capital appreciation, $E(dF)$ equals the expected return for the investment opportunity, $\rho F \, dt$.

Using Ito's Lemma, the Bellman equation becomes

$$\frac{1}{2}\sigma^2 C^2 F''(C) + \mu C F'(C) - \rho F = 0. \tag{A.2}$$

In addition, $F(C)$ should satisfy the following boundary conditions:

$$F(\infty) = 0 \tag{A.3}$$

$$F(C^*) = P\frac{1 - e^{-\rho D}}{\rho} - C^*\frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} - I \tag{A.4}$$

$$F'(C^*) = \left(P\frac{1 - e^{-\rho D}}{\rho} - C^*\frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} - I \right)'. \tag{A.5}$$

Condition (A.3) arises since the option to invest will be of zero value when $C = \infty$. Conditions (A.4) and (A.5) are smooth pasting and value matching conditions coming from optimality. Equations (7) and (8) follow from (A.2)–(A.5). To find $F(C)$, we need to solve equation (A.2) subject to the boundary functions (A.3)–(A.5). In the continuation region (not optimal to invest), the general solution for equation (A.2) must take the form $F(C) = A_1 C^{\beta_1} + A_2 C^{\beta_2}$, where A_1 and A_2 are constants to be determined, and $\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}} > 1$, $\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}} < 0$. To satisfy the condition (A.3), we must have $A_1 = 0$, so the solution must have the form $F(C) = A_2 C^{\beta_2}$. The values of A_2, C_{ROT} will be solved from the conditions (A.4)–(A.5).

The derivation of the above Bellman equations relies on a number of standard technical conditions. In particular, the following sufficient technical conditions (Duffie, 2001, Appendix E) are satisfied:

- (i) all of ρ, g, h, μ, σ, and F are continuous. In this article, $g(C) = 0$, $h(C, t) = 0$.
- (ii) F satisfies a polynomial growth condition in C, that is, for some positive numbers M and γ, $|F(C, t)| \leq M(1 + \|C\|^\gamma)$, $(C, t) \in R^N \times [0, \infty)$
- (iii) g, h satisfy a polynomial growth condition in C or are each nonnegative.
- (iv) ρ is nonnegative; and
- (v) μ and σ satisfy growth and Lipschitz conditions in C.

APPENDIX B

To show that $\frac{\partial C_{NPV}}{\partial D} > 0$, it remains to show that $\frac{\partial \frac{1-e^{-\rho D}}{1-e^{-(\rho-\mu)D}}}{\partial D} > 0$, and $\frac{\partial \frac{-1}{1-e^{-(\rho-\mu)D}}}{\partial D} > 0$.
First,

$$\frac{\partial \frac{1-e^{-\rho D}}{1-e^{-(\rho-\mu)D}}}{\partial D} = \frac{\rho(-\mu)}{(1-e^{-(\rho-\mu)D})^2} \left(\frac{1-e^{-\mu D}}{\mu} - \frac{1-e^{-\rho D}}{\rho} \right) > 0,$$

where $f(x) = \frac{1-e^{-xD}}{x}$ is decreasing function of x, since $f'(x) = \frac{e^{-xD}(1+xD)-1}{\mu} \leq 0$. Note that $h(x) = e^{-xD}(1+xD) - 1 \leq 0$, since $h(0) = 0$,

$$h'(x) = -D^2 x e^{-xD} \begin{cases} < 0 & x > 0 \\ > 0 & x < 0. \end{cases}$$

Second,

$$\frac{\partial \frac{-1}{1-e^{-(\rho-\mu)D}}}{\partial D} = \frac{(\rho-\mu)e^{-(\rho-\mu)D}}{(1-e^{-(\rho-\mu)D})^2} > 0.$$

APPENDIX C

It remains to show that

$$\frac{\partial \beta_2}{\partial \sigma} = \left(\frac{2\mu}{\sigma^3} - \frac{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right) \frac{-2\mu}{\sigma^3} + \frac{-2\rho}{\sigma^3}}{\sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}} \right)$$

$$= \frac{2\mu \left[\sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} + \left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right) + \frac{\rho}{\mu} \right]}{\sigma^3 \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}} > 0,$$

which follows from

$$\sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} + \left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right) + \frac{\rho}{\mu} < 0,$$

because

$$\left(\frac{\mu}{\sigma^2} - \frac{1}{2} + \frac{\rho}{\mu}\right)^2 = \left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2} + \frac{\rho}{\mu}\left(\frac{\rho}{\mu} - 1\right)$$

$$> \left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}, \quad \text{and} \quad \frac{\mu}{\sigma^2} - \frac{1}{2} + \frac{\rho}{\mu} < 0.$$

APPENDIX D

Case I: $(T, T + 2D)$

$$V_I = E_T\left[\int_T^{T+2D} (P - C)e^{-\rho(t-T)}dt\right]$$

$$= P\frac{1 - e^{-\rho \cdot 2D}}{\rho} - C\frac{1 - e^{-(\rho-\mu)\cdot 2D}}{\rho}. \tag{D1}$$

Case II: $(T, T + D), (T + D, T + 2D)$

$$\text{At } T + D, \ V_{T+D} = \int_{T+D}^{T+2D} (P - C)e^{-\rho(t-(T+D))}dt$$

$$= P\frac{1 - e^{-\rho D}}{\rho} - C_{T+D}\frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu}$$

$$V_{II} = E_T\left[\int_T^{T+D} (P - C)e^{-\rho(t-T)}dt + V_{T+D}^+\right]$$

$$= P\frac{1 - e^{-\rho D}}{\rho} - C_T\frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} + E_T\left[V_{T+D}^+\right]. \tag{D2}$$

It remains to solve $E_T[V_{T+D}^+]$.

Let $x_t = \frac{\ln C_{t+T} - \ln C_T - (\mu - \frac{1}{2}\sigma^2)t}{\sigma}$. Then

$$E_T\left[V_{T+D}^+\right] = E_T\left[\left(P\frac{1 - e^{-\rho D}}{\rho} - C_{T+D}\frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu}\right)^+ \Big| C_T\right]$$

$$= \int_{-\infty}^{x^*} \left(P\frac{1 - e^{-\rho D}}{\rho} - C_{T+D}\frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu}\right)\frac{1}{\sqrt{2\pi D}}e^{-\frac{x^2}{2D}}\,dx$$

where

$$x^* = \left\{x : P\frac{1 - e^{-\rho D}}{\rho} - C_{T+D}(x)\frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} = 0\right\}.$$

At $t = D$,

$$
\begin{aligned}
x^* &= \frac{\ln C^*_{t+T} - \ln C_T - \left(\mu - \frac{1}{2}\sigma^2\right) t}{\sigma} \\
&= \frac{\ln\left(P \dfrac{1 - e^{-\rho D}}{\rho} \dfrac{\rho - \mu}{1 - e^{-(\rho - \mu)D}}\right) - \ln C_T - \left(\mu - \frac{1}{2}\sigma^2\right) D}{\sigma} \\
&= \frac{\ln \dfrac{P}{C_T} + \ln \dfrac{\rho - \mu}{\rho} + \ln \dfrac{1 - e^{-\rho D}}{1 - e^{-(\rho - \mu)D}} + \left(\frac{1}{2}\sigma^2 - \mu\right) D}{\sigma}
\end{aligned}
$$

Notice $x_0 = 0$, $dx_t = dB_t$, hence, $X_t \sim N(0, t)$.

$$
P \frac{1 - e^{-\rho D}}{\rho} \int_{-\infty}^{x^*} \frac{1}{\sqrt{2\pi D}} e^{-\frac{x^2}{2D}} dx = P \frac{1 - e^{-\rho D}}{\rho} \phi\left(\frac{x^*}{\sqrt{D}}\right),
$$

where $\phi(.)$ is the cumulative Normal distribution function.

Since $C_{T+D} = C_T e^{(\mu - \frac{1}{2}\sigma^2)D + x_D \sigma}$,

$$
\begin{aligned}
\int_{-\infty}^{x^*} C_{T+d} \frac{1}{\sqrt{2\pi D}} e^{-\frac{x^2}{2D}} dx &= \int_{-\infty}^{x^*} C_T e^{(\mu - \frac{1}{2}\sigma^2)D + x\sigma - \frac{x^2}{2D}} \frac{1}{\sqrt{2\pi D}} dx \\
&= \int_{-\infty}^{x^*} C_T e^{(\mu - \frac{1}{2}\sigma^2)D + \frac{\sigma^2}{2} D} \frac{1}{\sqrt{2\pi D}} e^{-\frac{(x - \sigma D)^2}{2D}} dx \\
&= C_T e^{\mu D} \phi\left(\frac{x^* - \sigma D}{\sqrt{D}}\right).
\end{aligned}
$$

Now we have

$$
E_T\left[V^+_{T+D}\right] = P \frac{1 - e^{-\rho D}}{\rho} \phi\left(\frac{x^*}{\sqrt{D}}\right) - \frac{1 - e^{-(\rho - \mu)D}}{\rho - \mu} C_T e^{\mu D} \phi\left(\frac{x^* - \sigma D}{\sqrt{D}}\right).
$$

Consequently,

$$
\begin{aligned}
V_{II} &= P \frac{1 - e^{-\rho D}}{\rho} \left(1 + \phi\left(\frac{x^*}{\sqrt{D}}\right)\right) \\
&\quad - C \frac{1 - e^{-(\rho - \mu)D}}{\rho - \mu} \left(1 + \phi\left(\frac{x^* - \sigma D}{\sqrt{D}}\right) e^{\mu D}\right).
\end{aligned}
\tag{D3}
$$

From Equations (D1) and (D3), we have

$$\Delta V = V_{II} - V_I$$

$$= E_T \left[\left(P \frac{1 - e^{-\rho D}}{\rho} - C_{T+D} \frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} \right)^+ \bigg| C_T \right]$$

$$- E_T \left[\left(P \frac{1 - e^{-\rho D}}{\rho} - C_{T+D} \frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} \right) \bigg| C_T \right]$$

$$= E_T \left[\left(P \frac{1 - e^{-\rho D}}{\rho} - C_{T+D} \frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} \right)^- \bigg| C_T \right] \geq 0$$

where $x^- = (-x)^+$.

$$E_T \left[\left(P \frac{1 - e^{-\rho D}}{\rho} - C_{T+D} \frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} \right)^- \bigg| C_T \right]$$

$$= - \int_{x^*}^{+\infty} \left(P \frac{1 - e^{-\rho D}}{\rho} - C_{T+D} \frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} \right) \frac{1}{\sqrt{2\pi D}} e^{-\frac{x^2}{2D}} dx$$

$$= - \left\{ P \frac{1 - e^{-\rho D}}{\rho} \left(1 - \phi \left(\frac{x^*}{\sqrt{D}} \right) \right) - C \frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} \left(1 - \phi \left(\frac{x^* - \sigma D}{\sqrt{D}} \right) \right) \right\}.$$

$$\frac{\partial \Delta V(\sigma)}{\partial \sigma} = -P \frac{1 - e^{-\rho D}}{\rho} (-1) \frac{1}{\sqrt{2\pi}} e^{-\frac{x^{*2}}{2D}} \frac{\partial x^*/\sqrt{D}}{\partial \sigma}$$

$$+ C \frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} (-1) \frac{1}{\sqrt{2\pi}} e^{-\frac{(x^*-\sigma D)^2}{2D} + \mu D} \frac{\partial (x^* - \sigma D)/\sqrt{D}}{\partial \sigma}.$$

Since

$$x^* = \frac{\ln \dfrac{P}{C} + \ln \dfrac{\rho - \mu}{\rho} + \ln \dfrac{1 - e^{-\upsilon D}}{1 - e^{-(\upsilon-\mu)D}} + \left(\dfrac{1}{2}\sigma^2 - \mu \right) D}{\sigma},$$

we obtain

$$\frac{\partial \Delta V(\sigma)}{\partial \sigma} = P \frac{1 - e^{-\rho D}}{\rho} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^{*2}}{2D}} (1 - 1) \frac{\partial x^*}{\partial \sigma} \frac{1}{\sqrt{D}}$$

$$- C \frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x^*-\sigma D)^2}{2D} + \mu D} (-\sqrt{D})$$

$$= C \frac{1 - e^{-(\rho-\mu)D}}{\rho - \mu} \sqrt{D} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x^*-\sigma D)^2}{2D} + \mu D} \geq 0.$$

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