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Valuation of Index Options.

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VALUATION OF INDEX OPTIONS

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ABSTRACT

This dissertation examines the valuation of index options. The first chapter analyzes the value of early exercise for an index option, specifically the Standard and Poor's 100 Index (OEX) option. The value is found by estimating European put-call parity and comparing it to the price difference between an American call and put. Zivney (1991) estimated this value for closing prices. The value of early exercise is re-estimated by using bid-ask prices and time-series/cross-sectional data analysis, effectively mitigating the non-synchronous data problem and heteroskedasticity, which may be severe in Zivney's study. The results from using intraday bid-ask prices are compared to last bid-ask and transaction prices. The value of early exercise is added to the Black-Scholes European option model value, and the combination is found to price index options better. The *average* estimate of early exercise is about 4.1 percent for calls in-the-money and 10.87 percent for puts in-the-money.

The second chapter of the dissertation looks at various index proxies. This analysis seeks to discern how arbitragers are capturing arbitrage profits and thereby keeping the options near some equilibrium price. The index is proxied with other index options, the Standard and Poor's 500 Index futures, and small mimicking portfolios of stocks. Arbitrage profits are examined. The benchmark prices are generated with the Black-Scholes option pricing model, the Black-Scholes model plus the value of early exercise found above, the binomial option pricing model, and put-call parity. The mimicking portfolios of stocks produce the poorest profits. This is due to the deviation between the portfolio and the actual index increasing with time.

The final section examines price discovery. Using a technique relying on vector error correction models, it is found that most price discovery is found in the cash OEX index, with a smaller portion occurring in other options. Virtually no price discovery is found in the SPX futures.

INTRODUCTION

This dissertation analyzes the problems and some potential solutions inherent in the valuation of index options. The findings should be relevant to market participants who are attempting to achieve their investment objectives as effectively as possible and to researchers who are concerned with security design and market efficiency.

Stock index options were initially offered in the early 1980's as a response to the risk-management needs of different groups of investors, especially portfolio traders. As the name implies, a stock index option is an option whose underlying security is an index. The index itself is not a traded security, but the stocks comprising it are. The most popular option in terms of trading volume is written on the Standard and Poor's (S&P) 100 Index and began trading on March 11, 1983. Harvey and Whaley (1992b) illustrate its growth in popularity by noting that the level of trading volume had reached over 400,000 contracts per day by 1986. The proper valuation of index options, specifically the S&P 100 Index option, is a complex task because of their diverse features and is the subject of this dissertation.

The first chapter of my dissertation concerns the early exercise feature of the S&P 100 Index (OEX) option. This feature gives the holder the right to exercise the option any time prior to expiration. Various researchers (for instance, Merton (1973)) have written that the only reason a call option should be exercised early is if the dividend becomes large enough. So, investors are continually weighing the opportunity cost of holding onto the option position and receiving a risk-free return against exercising and receiving the dividend. However, even if the dividend feature of an index option could be properly modelled, Diz and Finucane (1993) and French and Maberly (1992) show that there are other more important factors affecting the early exercise decision.

Diz and Finucane (1993) studied the early exercise patterns of OEX options. Of the options that were exercised in their sample, 30 percent of the calls and 38 percent of the puts were exercised prior to maturity. Of these early call exercises, about 7 percent can be attributed to the dividend. Twenty-two percent are explained by the wildcard feature[1], and 48 percent are explained by the size of the bid-ask spread. Nineteen percent are exercised because of unknown reasons, and the rest are exercised because of a combination of reasons. Therefore, a model that incorporates only dividends as an impetus for early exercise is misspecified.

One problem, therefore, is to find an effective pricing mechanism for American index options. The route taken here is to estimate the value of the early exercise feature directly. Then, it will be added to the Black and Scholes [BS] (1973) European option pricing model to produce a reasonable value for an American option. In other words, since the only difference between a European and an American option is the early exercise feature, a European option estimate plus the value of early exercise should give a reasonable estimate of the value of an American option.

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[1]Since the stocks making up the OEX cease trading at 3:00 P.M. Central Standard Time and the OEX options continue trading until 3:15 P.M., there is a 15 minute window in which the option holder will exercise a call (put) if he believes the implied index price has dropped (risen) to some level below (above) the actual 3:00 P.M. index price. This "wildcard" option has the same payoff pattern as a put. See Fleming and Whaley (1994) or Chapter 4 of this dissertation for a more detailed explanation.

Various researchers have estimated the value of early exercise. Brenner and Galai (1986) estimated this value for puts using implied interest rates from put-call parity and found it to be from 0.9 to 3.5 percent. Blomeyer and Johnson (1988) looked at the difference in put values between the Geske and Johnson (1984) American option pricing model and the BS model and found the value to be about four percent. Zivney (1991) used put-call parity and the closing prices of the OEX option to find the value for calls to be 3.5 percent and for puts to be 10 percent. This dissertation will overcome some limitations of Zivney's paper and otherwise extend it. This particular valuation method has the advantage that it is contingent upon neither the level of dividends nor the assumptions of a particular model.

This study improves upon Zivney (1991) by using intraday bid-ask prices (prices at which investors can actually trade at that moment in time given their information) for OEX options for 1986 thus mitigating much of the nonsynchronous data problem, correcting for autocorrelation and heteroskedasticity where needed, incorporating dividend data, and using a more realistic proxy for the risk-free rate of interest. An average estimate of early exercise for calls at-the-money is found to be 4.1 percent, and for puts at-the-money it is 10.87 percent. For comparison, closing transaction and bid-ask quotes are also examined. It is further found that adding the value of early exercise to the BS model gives a better estimate of market values than the Black-Scholes model does alone.

The second section of my dissertation investigates the underlying security of an index option and arbitrage relationships. The underlying security for an index option is the index itself. Although the various indices were not traded during the time period of this study, Standard and Poor's Depository Receipts, representing the S&P 500 index stocks, are currently available on the American Stock Exchange. This security may one day become an attractive to include in arbitrage portfolios. However, the individual stocks comprising the various indices have always been available to arbitragers. This feature has implications for valuation purposes, because most of the extant option pricing models are based on arbitrage relationships. The issue is that since the index is not traded, if prices drift from equilibrium values, there are no direct arbitrage forces to pressure them back towards equilibrium.

It has been suggested that a portfolio of the actual stocks making up the index could be used. For instance, if an index call became overvalued relative to the put, a riskless arbitrage portfolio could be formed by writing the overvalued call, buying the put, and buying the portfolio of stocks. All of this would have to be done simultaneously to ensure the risk-free status. It would be difficult, but not completely impossible, to purchase all 100 stocks in the OEX simultaneously. But, if the put is overvalued, then the arbitrage portfolio would require a long call, short put, and short positions in all 100 equities simultaneously. The up-tick rule on the New York Stock exchange would prevent the formation of this portfolio. The up-tick rule requires investors to wait until the last price change of a security is positive before initiating a short sale. Therefore, the arbitrage forces for index options may not be as strong as they are for equity options.

The purpose of this section of the dissertation is to analyze the mechanisms arbitragers use to profit from index option mispricing and force prices back towards equilibrium. Evnine and Rudd (1985) suggest that traders construct proxy portfolios of several highly liquid stocks that closely mimic the index. They also suggest, along with Fleming and Whaley (1994), that some arbitragers use the S&P 500 futures to value OEX options. However, whether a proxy portfolio or S&P 500 futures are used, pricing errors will develop relative to the OEX. Arbitrage, therefore, becomes risky. Another alternative is the formation of a synthetic index using index options.

This study compares arbitrage profits using the synthetic index, S&P 500 futures, and index proxy portfolios. Each proxy index is examined relative to put-call parity, the Black-Scholes model, the Black-Sholes model plus the estimated value of early exercise, and the binomial option pricing model.

Finally, the third chapter of the dissertation will look at price discovery in the index option market. The analysis here will seek to determine where relevant pricing information first enters the options market. The problem is that, while the option is explicitly written on the Standard and Poor's 100 Index, investors might be receiving pricing information from other sources. Already mentioned is the S&P 500 futures market. Along with it, the actual OEX cash index and the index implied from put-call parity will be examined.

The methodology used will rely on a system of equations made up of current and lagged values of the indices called a vector autoregression. The innovations from the system will then be incorporated into a vector moving average representation of the same system. This makes it easier to discern where price discovery is taking place.

CHAPTER 1
THE VALUE OF EARLY EXERCISE
IN STANDARD AND POOR'S 100 INDEX OPTIONS

1.1 Literature Review

European options cannot be exercised prior to maturity. However, American options give the holder the right to exercise any time up to and including maturity. Merton (1973a and 1973b) provided the conditions for rational early exercise of American options and showed that the early exercise feature of the American option makes it at least as valuable as its European counterpart. The importance of the early exercise feature as a determinant of American options prices was demonstrated by Whaley (1982) and Geske and Roll (1984). Both studies empirically examined option pricing models that incorporated a positive probability of early exercise. They found that these option pricing models valued American style options more accurately than European option pricing models.

Investors, also, seem to value the early exercise feature, judging by the percentage of options that are exercised early. According to French and Maberly (1992), of the Standard and Poor's 100 Index (OEX) options that were exercised between April, 1983 and March, 1990, approximately 28 percent of the calls were exercised early. For the period from April, 1983 until December, 1988, Diz and Finucane (1993) find that 30% of the exercised OEX calls and 38% of the exercised OEX puts are exercised early.

The question remains, however, as to the value investors place on the early exercise feature. Brenner and Galai (1986) examined implied interest rates from put-call parity for American equity options. They compared the rate from deep-out-of-the-money puts (options that should not be rationally exercised) to rates from other exercise prices. The difference was what they called the value of early exercise. It ranged from 0.9% to 3.5%. Blomeyer and Johnson (1988) compared the accuracy of the Geske and Johnson (1984) American put valuation model with the Black-Scholes put valuation model. The difference between the model values could be looked at as the value of early exercise. It was about 4% for puts in-the-money. However, since this estimate is based on model values, any mis-specification can interfere with an accurate measurement of early exercise. Zivney approximated the market value of the early exercise feature of American options and based his study on the 1985 closing prices of the Chicago Board Options Exchange's (CBOE) Standard and Poor's 100 Index option. He found the value to be about 3.5% for calls and 10% for puts.

Zivney defined this value to be the price difference between an American call and put and the identical European options. More specifically, he examined the deviation from European put-call parity. That is, using his notation, the value of early exercise is:

$$A = |(C_A - P_A) - (C_E - P_E)'| \tag{1}$$

where $(C_A - P_A)$ is the difference between an American call and put on the same security with the same strike price and expiration date trading at the same moment, and $(C_E - P_E)'$ represents the left hand side of the European put-call parity relationship:

$$C_E - P_E = S - D_T - Xe^{-rT}, \tag{2}$$

where C is the price of the call, P is the price of the put, S is the price of the underlying security, D_T is the discounted value of dividends between the current date and expiration, X is the strike price, r is the risk-free rate of interest, and T is the time to expiration.

Zivney used the put-call parity relationship rather than European and American option pricing models to develop the early exercise value. Stoll (1969) showed that this relationship suggests that, if the price of a European call is known, the price of the put with the same exercise price and time to maturity on the same underlying security can be calculated. Because put-call parity is built on the concept of a replicating portfolio, testing can be done on actual market prices, and, therefore, there is no need for a joint test of a model to determine which is superior.

Zivney also observed that this measure of early exercise value behaved like an option. It increased with increases in the time to expiration, the risk-free interest rate, and the index price. The value of early exercise decreased with increases in the exercise price. He proposed that adding this observed value to the theoretical price of an option obtained from the Black-Scholes European option model would result in an "appropriate" value of an American option.

While Zivney's assertion that his measurement is actually the value of early exercise is reasonable, there are several weaknesses in his paper. First, Zivney used the closing prices of the Standard and Poor's 100 Index and the associated options for the year 1985. There are problems with this, including the problem of non-synchronous data. Zivney found 3635 pairs of options matching for put-call parity. By definition, they would have to trade simultaneously. However, using the data described in Section 1.3 below, it was found that the average of the absolute value of the differences in the times of the last call and put trades was about 57 minutes. The average of the absolute value of the differences in the times between the last call trade and the three o'clock index close was about 40.31 minutes, and the same measure for puts was approximately 42.18 minutes. Bodurtha and Courtadon (1986) demonstrate that the non-simultaneity of spot and option prices in the currency option market produces a significant number of put-call parity boundary violations.

Also, French and Maberly (1992) have examined the problem of using the last trade of the day involving the OEX option. The idiosyncrasy in question is the so-called wildcard feature. The wildcard occurs during the period after the stock market closes, 3:00 P.M., and when the OEX stops trading, 3:15 P.M. Central Standard Time. The settlement price of the option exercise is based upon the 3:00 P.M. index closing price. The option holder can exercise the option at any point between those two times, and he may be inclined to do so if he believes that the implied index is priced differently from its closing price. For instance, some type of bad news can occur after the stock market closes that may affect stock prices, or the investor may observe lower stock prices on other exchanges. In this instance, it may be optimal to exercise an in-the-money call early. Since it would not be optimal to exercise before the close, closing prices are affected by the wildcard feature. Therefore, closing prices of the OEX should not be used to determine the value of early exercise on this index option. So, Zivney might be measuring the value of the wildcard option, the value of early exercise, or a combination of both.

Second, in the last half hour of trading, market-makers are making adjustments to meet margin requirements. This influences both the price and the volume of options.

Along the same lines, Evnine and Rudd (1985) study the efficiency of the OEX market using intraday bid-ask quotes and find the market to be less than completely efficient. Evnine and Rudd cite an earlier version (Evnine and Rudd, 1983) of their study in which they used closing prices and found different results. So, the time factor does seem to matter.

In the first half hour of trading, each option series is being opened, and overnight limit orders are being filled. These prices may not be representative of a "typical" option. This study solves the beginning- and end-of-day problems by creating a five minute bid-ask price series by choosing the latest bid-ask quote in each five minute interval between the hours of 9:00 A.M. and 2:45 P.M.[2] If there is no change in the quote for a particular interval, the latest prevailing quote is used.[3] This effectively eliminates any bias arising from the opening or the closing, and the non-synchronous problem is also mitigated by the use of bid-ask data. These prices indicate what an investor could trade at that particular moment, given the information he has on the price of the index at that moment.

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[2]For the time period of this study, OEX options were traded on the CBOE from 8:30 A.M. until 3:15 P.M. Central Standard Time each weekday.

[3]A current bid or ask quote is good for ten option contracts.

Third, Zivney incorporated interest rates implied from the parity relationship. Previous studies on the implied interest rate from American options using put-call parity, such as those by Brenner and Galai (1986) and Frankfurter and Leung (1991), have found negative interest rates. Brenner and Galai (1986) found that, if early exercise were doubtful, option positions could be a substitute for riskless investments. Conversely, they discovered that a strong possibility of early exercise causes the interest rate implied by put-call parity to stray from market risk-free rates. Zivney does not mention whether he did or did not find negative rates or, if he did, what relevance they had. However, whether he found them or not, the rate implied from American options using European put-call parity will be downwardly biased. And, in general, the rate implied from index options, whether American or European, will also be downwardly biased. In Zivney's study, because a single implied interest rate was calculated for each maturity everyday, and this rate was assigned to various levels of strike prices, there was a possibility that a single negative rate would be applied to many options which otherwise might have had positive rates.

Using my call bid and put ask prices, the number of options with negative rates did turn out to be very large. With no filters on the data, 42.64% of the options were assigned negative interest rates when the discounted dividends were left out. With the dividends included, the percentage dropped to 7.88.

There are two explanations for the downwardly biased interest rates. First, although a European put-call relationship is being assumed, the actual put and call prices are based on American options. The put-call parity relationship for an American option is founded on an inequality rather than a strict equality. In fact, according to Brenner and Galai (1986), the implied rate is defined by the upper boundary of the American relationship as

$$C_A - P_A \leq S - D_T - Xe^{-rT} \tag{3}$$

So, the interest rate must be downwardly biased to force this into a European style relationship. In other words, to make the above relationship into an equality, a lower interest rate must be used. So, for this reason, the implied rate from an American option would be inappropriate.

Second, the implied risk-free rate should be downwardly biased when considering an index option. Two types of arbitrage situations can result. There can be a situation in which the call is relatively more expensive than the put. In this case, an investor will write the call, buy the put and buy the index to make arbitrage profits. This ensures that any mispricing along these lines will not last long. Even though it would be difficult to buy all 100 of the stocks comprising the S&P 100 Index simultaneously, it would not be impossible. Also, according to Evnine and Rudd (1985), investors can construct proxy portfolios consisting of many fewer stocks.

However, if the put is relatively more expensive than the call, arbitrage is generally not possible. An investor would have to write the put, buy the call, and short the index. The problem is that an index is made up of a portfolio of stocks. It would be almost impossible to find a situation in which every stock in the index could be shorted simultaneously; i.e. every stock's previous trade would have to have been on an up-tick. Even if a proxy portfolio could be found consisting of only a handful of stocks, the investor would encounter the same problem.

So, in the first arbitrage situation, the price of the put is being driven up, and the price of the call is being forced down. In the second situation, arbitrage is not possible, so the price of the put can become relatively much larger than the price of the call. From Equation (4), it can be seen that this situation can lead to the numerator of the fraction being, sometimes, greater than the denominator when looking at transaction prices; i.e., the natural log of a number greater than one is positive and, combined with the negative sign in front, can lead to negative interest rates.

$$r' = -(\text{Ln} ((S - D_T - C + P)/ X))/T. \tag{4}$$

Since only the lending side of the process is being examined, this situation would not affect this study, but Zivney's interest rates may have been affected. The problem remains, however, that the rate in this study would be biased downwardly if the rate implied from American options is employed.

To further demonstrate that there may have been a problem with Zivney's implied rates, his footnote number four regresses his implied rates on the time to maturity and the moneyness of the options. He finds a statistically significant negative relationship between the interest rate and the time to maturity. However, looking at market rates during 1985, the yield curve is upward sloping.

Fourth, as Stoll (1969) found in his data, there is both heteroskedasticity and autocorrelation in the market data. Zivney makes no mention of this or of any corrections he may have made, and, therefore, his conclusions may be questionable; i.e., his estimators may be unbiased but inefficient. So, any tests using these estimators cannot be relied upon.

And fifth, Zivney ends his paper with the hypothesis that the value of early exercise plus the Black-Scholes (1973) option value might produce a "better" estimate of an American option value. The Black-Scholes (1973) model has certain empirically documented biases in it. For instance, it prices options well at-the-money, but becomes less precise as the option gets deeper in- or out-of-the-money. If the value of early exercise can partially account for these biases, a more accurate method for valuing options will have been found. He gives no empirical evidence that this combination would price American options more accurately, but, logically, this makes sense. This possibility is tested.

The next section of this paper describes the data and methodology used to improve upon Zivney's original work and test his hypothesis that the value of early exercise plus the option value calculated from the Black-Scholes (1973) model will produce a "better" value for an American option. Both intraday and end of day prices are used for comparison and contrast.

1.2 Method of Analysis

The general technique applied here is initially very similar to the method used by Zivney. The logic of using put-call parity to value the early exercise feature is as follows. American options can be exercised early. If investors believe that the early exercise feature has value, there will be a difference in price between American and European options. This value can be found by comparing the difference between

American puts and calls and European puts and calls. Therefore, since the American call and put bid-ask prices can be observed and assigned to the same relationship as in the left hand side of equation 2, $(C_A - P_A)$, and the only difference between American and European options is the possibility of early exercise, then the difference between $(C_A - P_A)$ and the right hand side of equation 2 can be called the value of early exercise.

As mentioned, this study makes use of bid-ask prices. In the put-call parity relationship, borrowing and lending relationships can be developed using bid-ask prices. Assuming there are no profitable riskless arbitrage opportunities available, the investor can lend through the following relationship:

$$P + S - C = Xe^{-rt} \qquad (5)$$

That is, he can buy a put, buy the index (the stocks comprising the index or a reasonable proxy portfolio), and write the call. Of course, since all of this must be done simultaneously, the investor will be a price taker; so, in this case, the put will be the put ask price and the call will be the call bid.

An investor can borrow Xe^{-rt} dollars by buying a call, shorting the index, and writing the put. That is,

$$-P - S + C = -Xe^{-rt} \qquad (6)$$

Therefore, he will buy the call at the ask and sell the put at the bid. In fact, he must short the index (stock) at the bid also, but that will not be examined here, because the S&P 100 index is not a traded security, although the stocks that make it up are. Therefore, because of the immense difficulty that would occur in trying to short all 100 stocks in the index at the same moment, it will be assumed that the investor is in a lending situation. The put ask and call bid prices will be employed, unless specifically mentioned to the contrary.

1.3 Data

The 1986 option and index data of the S&P 100 Index employed in this study were obtained from the resorted format of the Berkeley Options Data Base tapes. The data is re-sorted by date, time, and stock symbol. Although Zivney used 1985 data for his study (he does not mention using the Berkeley option tapes), the Berkeley option tapes are unreliable for the S&P 100 data prior to 1986 and S&P 500 values through 1986. Specifically, the index values were mis-coded for various time periods before 1986. Sheikh (1991) found errors in the consolidated format. The same type of errors are in the resorted format. Bid-ask price data was gathered for the entire year of 1986 at five minute intervals throughout each day, as described previously. In addition, the last bid-ask and transaction prices are collected each day.

3,755,834 OEX bid-ask records were found for puts and calls on the tapes for 1986. The figures for the bid-ask prices are found in rounded form on the tapes. For instance, instead of a bid price of 1262.5 cents, this would be recorded as 1263 cents. Since prior research has indicated that price discreetness is important (see Chan, Chung, and Johnson (1993), for example), the prices were corrected for this study. In other words, using the figures above as an example, 1263 was corrected to 1262.5.

The bid-ask data was filtered in several stages. First, although Zivney used closing prices, this study uses the entire day's worth of data, except for the first and last half hour. Sheikh (1991) pointed out that overnight limit orders make up the bulk of the trades in the first half hour of trading. In the last fifteen minutes to half hour, market makers are cleaning up their books to meet margin requirements. Also, there is the wildcard option problem. This brought the total number of records down to 3,289,513. The data was next paired by puts and calls with the same strike price and maturity date in five minute intervals.

Second, any set of prices in which the index value was greater (less) than the value of the strike price plus (minus) the call (put) in-the-money was deleted, because it would indicate non-synchronous problems and would not meet put-call parity. Table A1 shows that this filter removed roughly 3.1 percent of the five minute interval series.

According to Whaley (1982), the CBOE does not allow new positions to be established in options whose premia is below fifty cents. Therefore, all pairs where either the bid or the ask is below fifty cents are deleted.

Finally, all observations which occurred in the expiration week were deleted. Several authors, including Day and Lewis (1988), attribute higher volatility to expiration week. The main data set (607,420 call bid and put ask prices or 303,710 matches for put-call parity) consisted of all pairs of puts and calls that matched as to date, maturity, and strike in five minute intervals for 1986. The index value was the value at the five minute mark. This effectively removes the non-synchronous problem. For instance, the amount of time it takes from the moment a trade takes place until the time it gets stamped is no longer a problem. What are left are the prices at which traders can make transactions given their information on the index value. The other two data sets consist of final bid-ask or transaction prices. The last index price is the closing index price each day. These two sets of final prices may be strongly affected by non-synchroneity, as the final price for one option in a pair may occur several hours earlier than the other.

The dividend data came from Harvey and Whaley (1992). Harvey and Whaley (1992) demonstrate that dividends on the S&P 100 Index are highly cyclical and, while on average are small compared to the value of the index, at various times they are large enough for calls to exhibit a positive probability for early exercise. Their dividend series consists of daily cash dividends on the S&P 100 Index from 1982 through 1988. This paper employs 1986 data along with the first four months of 1987. For each option, all daily cash dividends paid between the current date and the maturity date of the option were discounted back from the date of the dividend payment to the current date, as in the formula,

$$D_T = \sum_{i=1}^{N} D_i e^{-rt_i} \tag{7}$$

where r is the risk-free rate of interest, D_i is the ith dividend paid between the current date and maturity, and t_i is the period from the current date until the ith dividend is paid. N is the number of daily dividends paid between the current date and maturity date. The normal procedure is to discount dividends from the ex-dividend date to the option trade date. However, this is an index consisting of a portfolio of 100 stocks of large companies. Therefore, the actual day of the dividend payment was used, as Brenner and Galai (1986) did for individual stocks and Sheikh (1991) did for the S&P 100. This method assumes that dividend payments are known beforehand. But, Brenner and Galai point out that, with very large companies, this is a reasonable assumption. Figlewski

(1984) makes the assertion that, for highly diversified portfolios such as this, uncertainty about the dividend stream is insignificant. Black (1975) suggests netting out the dividends from stock prices for equity options. Corrado and Miller (1993) state that this method should also be used with index options. Harvey and Whaley (1992) concur with netting out the present value of expected dividends from the index value. That is done here.

The interest rate used to discount the dividends was the T-bill risk-free rate, as Harvey and Whaley (1992) did. For each day of the option's life, the T-bill rate on that day was used in the model. The T-bill that matured just following the expiration of the option was chosen. It was employed rather than the T-bill that matured two days before the option's expiration date for several reasons. First, since that T-bill matured two days before the option, there would be two days when there would be no rate at which to discount the dividend. Second, the *Wall Street Journal*, from which the rates were obtained, listed the rates only until several days before the T-bill matured. For instance, The T-bill maturing on January 16, 1986 (January option maturity was on the 18th) was quoted until January 14th. Therefore, the T-bill that matured the soonest after the option's expiration was chosen.

Given that a lending relationship is examined, the t-bill rate is appropriate. This is the minimum rate an investor would accept to construct this portfolio. It would be unreasonable to assume that an investor would take on a risk-free investment that returns less than the risk-free t-bill rate.

T-bill rates, discounted dividends, observed index values, strike prices, and times to maturity were substituted into Equation (2) to find the value for European put-call parity. Some simple statistics on the data can be found in Table A2.

1.4 Econometric Models

If the early exercise feature gives the holder of the option a right that is valued, then it should have movements that have been theorized for options. This hypothesis is tested with the following model, which is known in the econometric literature as the Hsiao (1975) random coefficients model [RCM]:

$$A_{it} = \beta_{0it} + \beta_{1it}f(S_{it} - X_{it}) + \beta_{2it}g(T_{it}) + \beta_{3it}h(r_{it}) + \beta_{4it}i(\sigma_{it}) + u_{it} \tag{8}$$

where A_{it} is the estimated value of early exercise, $f(S_{it} - X_{it})$ is the degree the option is in the money, $g(T_{it})$ is the number of days until maturity, $h(r_{it})$ is the risk-free rate of interest, and $i(\sigma_{it})$ is a measure of volatility based on the standard deviation of returns of the index over the past sixty days.

If A is an option, it should increase in value with the increases in moneyness, days to maturity, and the risk-free rate of interest. Because of the manner in which the estimate of early exercise is calculated, separate tests are made on put-call parity pairs where the call is in-the-money and where the put is in-the-money. Therefore, when the call is in-the-money, the signs of the coefficients should be positive. When the put is in-the-money, however, it should be remembered that A will be negative. So, the sign of the moneyness coefficient should be positive, and the rest should be negative. Zivney found the theorized signs. Furthermore, these movements should be statistically significant if investors value this right.

Because of the form of the data, we argue that a representation of the data generation process based on the Hsiao (1975) random coefficients model, equation 8, would be appropriate. This model recognizes that the coefficients may vary over cross-sections and time. Ordinary least squares is inefficient under these conditions. For instance, from equation (8), the change in the index value (S) is the same for each *A*, but the strike price (X) is or can be different. Therefore, a change in S will affect different *A*s differently. The average effect over all individuals will show up in the coefficient, but the individual reactions will go into the error.

This formulation indexes all coefficients and variables across individuals and time. It is based on the idea that, whether we have data measured in five minute intervals or at the close of trading, each measure of *A* may respond differently in each time period and for each individual to changes in the independent variables. The possibility that there are different individuals or different numbers of individuals in each time period means that there is very likely that there is heteroskedasticity present. For instance, *A* derived from an option pair where the call is deeply in the money is going to respond differently to a change in the risk-free rate of interest in each time period from when *A* is derived from a pair of options where the call is near-the-money.

The variance of the ordinary least squares (OLS) estimator is biased in this case, because the first assumption of the classical linear regression model is violated. This assumption requires the parameters of the model to remain constant. This means that the test statistics will be unreliable due to inefficiency. This results in OLS estimators that are unbiased but inefficient. However, the estimated generalized least squares estimator variance is unbiased, because it accounts for the heteroskedasticity produced by the random coefficients. The estimated generalized least squares estimator is asymptotically consistent and more efficient than the OLS estimator. The model we propose produces an efficient estimator.

In the Hsiao model, the β_{kit}'s are random variables, with means $\bar{\beta}_k$, the population average response coefficient. Therefore, each coefficient can be thought of as being an average estimate plus a component specific to time and a component specific to an individual:

$$\beta_{kit} = \bar{\beta}_k + v_i + e_t. \tag{9}$$

where

$$v_i \sim N(0,\sigma_v^2),$$

$$e_t \sim N(0,\sigma_e^2),$$

$$\varepsilon_{it} \sim N(0,\sigma_\varepsilon^2),$$

$$\mathrm{cov}(v_i,v_j) = 0,$$

$$\mathrm{cov}(e_t,e_s) = 0,$$

$$\mathrm{cov}(v_i,e_t) = 0.$$

The u_{it} random errors can be broken down into three parts,

$$u_{it} = \sum_{i=1}^{k} v_i x_{it} + \sum_{1}^{k} e_t x_{it} + \epsilon_{it}. \tag{10}$$

and are distributed $N(0,\sigma_u)$ where

$$\sigma^2_{u_{it}} = \sigma^2_u \sum x^2_{it} + \sigma^2_u \sum x^2_{it} + \sigma^2_e$$

Thus $\bar{\beta}_I$ is the average intercept. v_i is the deviation from the mean for the ith individual and is constant over time. e_t, on the other hand, varies over time but is constant over individuals.

Zivney did not recognize the time series/cross-sectional nature of his data and estimated equation (8) as an ordinary least squares model where the parameters are considered to be constant. This formulation assumes that $v_i = e_0 = 0$. Also, the heteroskedasticity in ε is ignored:

$$A = \beta_0 + \beta_1 f(S - X) + \beta_2 g(T) + \beta_3 h(r) + \varepsilon. \tag{11}$$

Several potential problems lie in using the Hsiao procedure with this data set. It assumes each individual estimated value of early exercise will react to changes in the independent variables in the same way over time. It also assumes that all of the estimated values of early exercise react in the same way to some factor in a given time period. For instance, it would assume, if the put-call parity pairs of options in one cross-section included one pair at-the-money, one pair where the call is five dollars in-the-money, and one pair where the put is five dollars in-the-money, that these would react in the same way in a time period to some influence. Since the tests are run separately for puts in-the-money and calls in-the-money, the reactions of the estimated values to changes in the independent variables should be similar. A cross-section of calls in the money will react in the same way to a change in the interest rate whether they are near-the-money or far in-the-money. Only the degree of the reaction will be different. Over time, the story is much the same. The reaction would change to the greatest degree when, for instance, a call goes from in- to out-of-the-money. Since the tests control for this, there should be little difficulty in interpretation.

In the random coefficients model we estimate, the time series component will be represented by the usual five minute intervals for the intraday data and days for the end-of-day data. The cross sections are define by the degree of moneyness. The reasoning is as follows.

The method of estimating early exercise depends on the difference between the values of a call and a put, one being in-the-money and the other out-of-the-money. Since it would be irrational for an investor to exercise the option that is out-of-the-money, it is logical to believe that the early exercise value connected to that option is nil. However, it should not be zero. We can assume this because of an arbitrage argument: if the value of early exercise were zero except when the option went into-the-money, an investor could buy the option the instant before it became in-the-money (assuming this could be predicted), hold it until it went into-the-money an instant later, and then sell it after it jumped in price due to the early exercise value. This is contrary to observation and theory. Therefore, out-of-the money options must have a positive value of early exercise due to the probability that they will go into-the-money. Using Zivney's method of finding this value for near-the-money options can only give a muddy picture of the true value for options that are near-the-money.[4] For this reason, the cross-sections are defined as to the degree of moneyness. The cross-sections are

[4]OLS estimates were obtained on the data both where one of the options was at least five percent in-the-money and where both options were within five percent of being at-the-money. The signs of the coefficients were correct in the first case but not in the second.

delineated at five percentage point intervals in the degree of moneyness. For 1986, the degree of moneyness ranged from being in-the-money by 15.55 percent for calls and in-the-money by 13.18 percent for puts.

Also, it should be noted that none of the data sets here or in Zivney is a homogeneous time series. For instance, in the intraday data set, each five minute interval may contain a different number of observations. In addition, even if there are the same number of observations in each interval, the quotes in one interval may be based on a completely different set of options from the previous interval. This means that the concept of stationarity is not an issue here.

The estimation method is maximum likelihood (ML). This means that the sample estimator, $\bar{b}$, is consistent, it is asymptotically normal, and asymptotically efficient. Judge, et. al. (1985) show that the estimators have the same form as estimated generalized least squares (EGLS) estimators, but the unknown disturbance covariance matrix here is based on maximizing a log-likelihood function rather than a least squares estimate. This is important, because even though both the ML and the EGLS estimators are asymptotically consistent, the EGLS estimators are not necessarily as efficient as the ML estimators.[5]

Finally, we want to determine whether or not the addition of the early exercise estimate to the European Black-Scholes value can give a better approximation to the

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[5]SAS's PROC MIXED is used to estimate the model. A GROUP effect is specified to allow the coefficients to vary.

market value of options than the Black-Scholes model alone. The theorized relationship is:[6]

$$|BS_{option} - Actual_{option}| \geq |BS_{option} + A - Actual_{option}| \quad (12)$$

where the Black-Scholes values are given by

$$call = S^{*}N(d_1) - Xe^{-rT}N(d_2)$$

and

$$put = Xe^{-rT}N(-d_2) - S^{*}N(-d_1)$$

where

$$d_1 = \frac{\ln(S^{*}/X) + rT}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}. \quad (13)$$

Black-Scholes option prices are formed with S^* representing the market determined index value less the discounted dividends, X is the contractually specified strike price, the time to maturity is T, the T-bill rate of interest proxies for the risk-free rate, r, and σ is the historical volatility mentioned above. $N(d_1)$ and $N(d_2)$ are cumulative probabilities for unit normal variables.

The intuition behind this test is based on the theorized relationship between American and European options. Theory tells us that an American option is worth at least as much as its European counterpart. Furthermore, if investors value the right to

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[6]This test was suggested by Mitchell Stern at the 1994 Eastern Finance Association Meeting.

exercise early, then the American option will be greater than the otherwise equivalent European option. So, if the relationship in equation 12 holds, we can infer that investors value the early exercise feature, and its addition to the Black-Scholes model value gives us a better estimate of market-determined American options than the Black-Scholes values alone.

The random coefficients model is also used here. The reasons are similar to those given above. Each Black-Scholes value and each market determined price can vary over time along with the estimates of early exercise. In addition, for any time period, there may be different options and different numbers of options.

1.5 Empirical Results

The estimated value of early exercise is calculated as the difference between the left-hand side of equation 3 and the right-hand side of equation 2. For those instances involving calls in-the-money with a positive value of A and puts in-the-money with a negative value of A, the average values of early exercise are 4.1% and 10.87% respectively for the 5 minute data. The last quote prices finds a higher estimated value of early exercise for calls at 6.88%, and the figure for puts is 8.76%. The last trade data shows the value to be 4.57% for calls and 9.45% for puts.

To test the hypothesis that Zivney used an incorrect testing procedure, tests similar to his are conducted using intraday bid-ask data, end-of-day bid-ask data, and end-of-day transaction data for 1986. The positive value of early exercise from calls in-the-money was regressed on the moneyness, time to maturity, and the risk-free rate of interest from the corresponding call. Ordinary least squares regressions are first executed. A quick examination of the Durbin-Watson statistic reveals that it is less than one on several occasions and does not come close to 2 in the remainder of the tests. So, corrections are made for autocorrelation by using the Prais-Winsten transformation. Regressions are executed for values from puts in-the-money that have a negative value of early exercise. The purpose of this is to compare the reaction of the value of early exercise to theorized movements of options. The results are in Tables 1, 1a, and 1b.

In Table 1, all matches where the calls that should be exercised early, according to the dividend rule, have been left out. For comparison, Table 1a leaves them in. This means that it is less likely that the value of early exercise will show up in Table 1, and there is more of a possibility of it appearing 1a. As can be seen from Tables 1, 1a, and 1b, the coefficients and the degrees of significance change depending on the data set being examined. In addition, the coefficients are not what theory would predict in all cases. We would expect that if any one of the data sets would produce the hypothesized coefficients, the five minute data would produce them, because it is made up of synchronous data. However, not all coefficients are as hypothsized. For instance, the coefficient on the interest rate variable for the intraday data in Table 1b is a good example. Therefore, a test to determine whether the coefficients are changing is conducted.

Judge, Griffiths, Hill, Lütkepohl, and Lee (1985) suggest the Breusch-Pagan test for heteroskedasticity induced by the random coefficients. This is a Lagrange multiplier test that is asymptotically distributed $\chi^2_{(K-1)}$. It tests for the error variance being related to more than one variable by taking the residuals from the original estimated model, squaring them, and regressing them on the variables that are thought to be causing the variance. These could be all of the variables in the original model. The regression sum of squares from this second regression is divided by two times the square of the sum of the squared errors divided by the number of observations from the first regression to form the test statistic. The intuition is that if the independent variables in the second model explain the dependent variable(the squared errors from the first model), then the test statistic will be large and the null hypothesis that the coefficients of the second model are zero will be rejected. The results in Tables 1, 1a, and 1bclearly indicate a rejection of the null. This leads us to conclude that heteroskedasticity is present and may be caused by random coefficients. In addition, using equation (11) and OLS as Zivney did, the data is examined quarter by quarter. Each quarter's model reveals coefficients different from the next and different degrees of significance, along with some signs that are not as hypothesized.

Table 1. Generalized Least Squares Estimates Using Prais-Winsten Two-step Estimators for: $A = b_0 + b_1 f(S - X) + b_2 g(T) + b_3 h(r) + \varepsilon$
Where Calls are in-the-money and $A > 0$
All Data Are Included Except Where $D_T > X(1 - e^{-rT})$

	b_0	b_1	b_2	b_3	BP Test
Last Trade Data 652 Observations	0.153 (0.425)	0.022 (7.203)**	0.004 (3.553)**	2.800 (0.466)	78.135
Last Quote Data 851 Observations	0.6411 (3.696)**	0.02733 (10.833)**	0.00123 (1.552)	-7.191 (-2.570)**	108.602
5 minute data 38356 Observations	-0.26402 (-6.680)**	0.01407 (31.841)**	0.00025 (2.562)**	11.2454 (17.332)**	11971.220

Numbers in parenthesis are t-values. The degree of significance is for a one-tailed test.
** Significant at the .01 level * Significant at the .05 level
BP Test is the Breusch-Pagan Test

Table 1a. Generalized Least Squares Estimates Using Prais-Winsten Two-step Estimators for: $A = b_0 + b_1 f(S - X) + b_2 g(T) + b_3 h(r) + \varepsilon$
Where Calls Are in-the-money and A > 0
All Data Are Included

	b_0	**b_1**	**b_2**	**b_3**	**BP Test**
Last Trade Data 765 Observations	0.245 (0.789)	0.022 (7.943)**	0.004 (3.827)**	1.172 (0.223)	96.677
Last Quote Data 1996 Observations	0.8437 (13.542)**	0.01688 (7.621)**	0.0132 (21.712)**	-18.671 (-21.401)**	41.569
5 minute data 41415 Observations	-0.16833 (-4.698)**	0.01378 (32.298)**	-0.00008 (-0.993)	10.0199 (16.731)**	14832.310

Numbers in parenthesis are t-values. The degree of significance is for a one-tailed test.
** Significant at the .01 level * Significant at the .05 level
BP Test is the Breusch-Pagan Test

Table 1b. Generalized Least Squares Estimates Using Prais-Winsten Two-step Estimators for: $A = b_0 + b_1 f(S - X) + b_2 g(T) + b_3 h(r) + \varepsilon$
Where Puts Are in-the-money and A < 0

	b_0	**b_1**	**b_2**	**b_3**	**BP Test**
Last Trade Data 87 Observations	2.252 (3.831)**	0.059 (5.318)**	-0.004 (-2.071)*	-25.846 (-3.082)**	129.273
Last Quote Data 236 Observations	2.3548 (6.018)**	0.0439 (7.120)**	-0.0134 (-10.950)**	-27.746 (-4.656)**	208.966
All 117532 Observations	-0.7755 (-27.499)**	0.0578 (177.508)**	-0.0094 (-158.264)**	11.041 (22.366)**	14622.400

Numbers in parenthesis are t-values. The degree of significance is for a one-tailed test.
** Significant at the .01 level * Significant at the .05 level
BP Test is the Breusch-Pagan Test

As mentioned, heteroskedasticity is present in the random coefficients model. To test this, a likelihood ratio test is performed between the null model, which does not consider heteroskedasticity, and one that explicitly models the heterogeneity of the cross-sections.[7] The likelihood ratio test simply tests whether there is a significant difference between the maximized likelihood functions of the null and the restricted models. The test is $\lambda_{LR} = -2[L(\Psi)-L(\Psi_0)]$ where $L(\Psi)$ is the likelihood function of the null model and $L(\Psi_0)$ is the same when explicitly modelling heterogeneity. This is asymptotically distributed as a chi-square variable with $(K-1)$ degrees of freedom. In all cases, the null model is rejected in favor of the one modelling the heteroskedasticity.

A variable for volatility is included in the models estimated in Tables 2, 2a, and 2b, since this is one of the variables hypothesized to cause movements in options. The results in Table 2a for the situation where calls are in-the-money indicate that the signs of the coefficients are as hypothesized except for the time variable. The reason we get a negative coefficient can be explained. First, Fleming and Whaley (1995) find that the wildcard option increases in value as a proportion of the index option with decreases in time to maturity. They explain that as the time value of the option decreases, the wildcard value becomes more significant. The same should be happening here. In fact, Harvey and Whaley (1992) find that the number of options exercised early increases with decreases in the time to maturity. Therefore, the early exercise feature becomes more valuable to investors as maturity approaches. This is what is found here. de Roon and Veld (1995) do a similar study on DAX index options using OLS and find that their coefficient on the time to maturity variable is also negative.

[7]Using SAS's MIXED procedure, each cross-section, represented by five percentage point increments of moneyness, is modeled with its own set of covariance parameters.

The moneyness coefficient is significant at the 5% level and of the correct sign. As an option goes deeper in-the-money, the time value decreases as a proportion of the option value. The value of the option approaches its intrinsic value, max(0, S-X), or the exercise value. Therefore, the closer the option comes to its exercise value, the greater the value of early exercise.

Table 2. Random Coefficient Model Estimates of
$A_{it} = \beta_{0it} + \beta_{1it}f(S_{it} - X_{it}) + \beta_{2it}g(T_{it}) + \beta_{3it}h(r_{it}) + \beta_{4it}i(\sigma_{it}) + u_{it}$
For Calls in-the-money and A > 0
All Data Are Included Except Where $D_T > X(1 - e^{-rT})$

	$\bar{b}_0$	$\bar{b}_1$	$\bar{b}_2$	$\bar{b}_3$	$\bar{b}_4$
Last Trade Data Observations	0.8652 (1.07)	0.0204 (4.70)**	0.0043 (4.57)**	-2.3785 (-0.47)	-3.4179 (-0.80)
Last Quote Data Observations	1.0245 (2.33)**	0.0217 (6.76)**	-0.0006 (-0.77)	-8.0459 (-3.40)**	-1.3181 (-0.52)
5 minute data	-1.0407 (-14.67)**	0.0013 (1.19)	-0.0043 (-31.70)**	22.5911 (43.24)**	3.8484 (11.01)**

Numbers in parenthesis are asymptotic t-values. The degree of significance is for a one-tailed test. ** Significant at the .01 level * Significant at the .05 level

Table 2a. Random Coefficient Model Estimates of
$A_{it} = \beta_{0it} + \beta_{1it}f(S_{it} - X_{it}) + \beta_{2it}g(T_{it}) + \beta_{3it}h(r_{it}) + \beta_{4it}i(\sigma_{it}) + u_{it}$
For Calls in-the-money and A > 0
All Data Are Included

	$\bar{b}_0$	$\bar{b}_1$	$\bar{b}_2$	$\bar{b}_3$	$\bar{b}^4$
Last Trade Data Observations	1.0109 (1.33)	0.0194 (4.84)**	0.0041 (4.71)**	-2.8854 (-0.61)	-3.9125 (-0.99)
Last Quote Data Observations	3.3733 (8.64)**	0.0206 (6.82)**	0.0098 (16.89)**	-15.6470 (-17.77)**	-16.8481 (-7.03)**
5 minute data	-0.8925 (-12.99)**	0.0019 (1.83)*	-0.0041 (-30.34)**	20.4776 (40.84)**	3.5691 (10.68)**

Numbers in parenthesis are asymptotic t-values. The degree of significance is for a one-tailed test. ** Significant at the .01 level * Significant at the .05 level

Table 2b. Random Coefficient Model Estimates of
$A_{it} = \beta_{0it} + \beta_{1it}f(S_{it} - X_{it}) + \beta_{2it}g(T_{it}) + \beta_{3it}h(r_{it}) + \beta_{4it}i(\sigma_{it}) + u_{it}$
For Puts in-the-money and A < 0

	$\bar{b}_0$	$\bar{b}_1$	$\bar{b}_2$	$\bar{b}_3$	$\bar{b}_4$
Last Trade Data Observations	2.5229 (3.83)**	0.0382 (7.74)**	-0.0092 (-12.90)**	-4.8633 (-1.05)	-15.9696 (-5.15)**
Last Quote Data Observations	0.4126 (0.31)	0.0077 (0.41)	-0.0123 (-4.50)**	-7.5883 (-1.60)	-4.2958 (-0.57)
5 minute data	1.6947 (36.03)**	0.0378 (57.98)**	-0.0119 (-135.9)**	-0.1709 (-0.47)	-13.188 (-69.17)**

Numbers in parenthesis are asymptotic t-values. The degree of significance is for a one-tailed test. ** Significant at the .01 level * Significant at the .05 level

The sign on the interest rate coefficient is correct also. Since a call has a value worth at least max(0, $S\text{-}Xe^{-rt}$), a higher interest rate will increase its value due to the lower discounted strike price. Finally, the volatility is positively related to the value of early exercise. The greater the volatility, the greater the probability the underlying security will increase in value, hence a higher call price. For the case where the calls are in-the-money but some have been exercised, we obtain similar results.

The estimates on puts-in-the-money also produce the correct signs. Similar reasoning as for calls can be made here to explain the signs. However, two points should be kept in mind. First, the estimate of *A* is negative for puts in-the-money. And, second, although the effect of movements in the riskless rate of return has the opposite effect on puts, the effect on the probability of early exercise is positive. So, because we have a negative *A*, the coefficient on the risk-free rate is and should be negative. This differs from Table 1b where the coefficient on the interest rate variable was positive for the intraday data. However, here it is not significant. This is consistent with prior findings that, of the variables that influence options, the risk-free rate of interest has the smallest effect. This leads us to conclude that the right to exercise early has movements like traded options and should be considered as an option.

Also included in Tables 2, 2a, and 2b are the estimates using the closing data. The coefficients and levels of significance in many cases are not what theory predicts. However, one of the goals of this paper is to show that the use of nonsynchronous data can cause problems. These estimates demonstrate that problems arise from using end-of-day data even when a model has been found that gives consistent and asymptotically efficient estimators.

The results in Tables 3 and 4 show that the value of early exercise is a significant contributor to the Black-Scholes model in pricing American options. The significant and positive intercepts are what theory predicts. The signs of the coefficients are also as expected. However, the same statistics over the three series are very different. This is further evidence that nonsynchronous can affect the results.

Table 3. Hsiao Random Coefficient Model Estimates of
$[\text{Abs(BS Call - Actual)}]_{it} = \beta_{0it} + \beta_{1it}[\text{Abs(BS Call + A - Actual)}] + u_{it}$

	Avg.Abs(BS Call - Actual)	**Avg. Abs(BS Call + A - Actual)**	**Estimates**
Last Trade	1.297	0.527	$0.675 + 1.131\bar{b}$
		t-values:	11.13** 18.30**
Last Quotes	1.444	0.437	$0.915 + 0.857\bar{b}$
		t-values:	22.81** 25.71**
5-min. data	1.364	0.799	$0.341 + 1.027\bar{b}$
		t-values:	20.96** 1198.3**

** Significant at the .01 level   * Significant at the .05 level

Table 4. Hsiao Random Coefficient Model Estimates of
$[\text{Abs(BS Put - Actual)}]_{it} = \beta_{0it} + \beta_{1it}[\text{Abs(BS Put + A - Actual)}] + u_{it}$

	Avg.Abs(BS Put - Actual)	**Avg. Abs(BS Put + A - Actual)**	**Estimates**
Last Trade	1.320	0.455	1.165 + 0.853$\bar{b}$
		t-values:	17.43** 13.72**
Last Quotes	1.446	0.519	1.016 + 0.506$\bar{b}$
		t-values:	8.26** 4.61**
5-min. data	1.524	0.502	1.571 + 0.998$\bar{b}$
		t-values:	41.83** 1015.0**

** Significant at the .01 level * Significant at the .05 level

The average prices in Tables 6 and 7 bear closer examination. For calls, the average last prices are several dollars higher than the intraday prices. This is indicative of the heavier volume of trading of options that are closer to the money. The last price series give equal weight to each option series, so a far in-the-money option gets the same weight on the average as an at-the-money option. The intraday data has more trades in near-the-money options, hence the lower average price.

Table 5. Average Values of Calls and
the Estimated Value of Early Exercise

	Avg. A	**Avg. Call**	**Avg. B-S Call**
Last Trade	0.776	16.99	15.70
Last Quotes	1.150	16.72	15.33
5-min. data	0.568	13.69	12.32

Table 6. Average Values of Puts and the Estimated Value of Early Exercise

	Avg. A	Avg. Put	Avg. B-S Put
Last Trade	-1.104	11.68	10.37
Last Quotes	-1.227	14.01	12.67
5-min. data	-1.299	11.95	10.43

1.6 Chapter 1 Summary

Zivney's paper showed that there is a value to early exercise. It is also found in this analysis. The average values found here are different from those that Zivney found.

He asserted and found that the value had movements that would be expected from options. His results may have been obtained by chance, however. Both his choice of ordinary least squares as an estimation technique and his selection of data consisting of end-of-day observations are shown to produce spurious results. A model that explicitly incorporates cross-sectional and time series variability and gives efficient estimators is used here. The movements that have been attributed to options in response to movements in certain variables were found when using this technique. However, they are not found when using ordinary least squares as Zivney did. Also, when the value of early exercise is added to the Black Scholes option pricing model, a more accurate pricing mechanism results. The most striking results show that not only do the tests depend on the type of data (intraday or end-of-day), but also on the time series and cross-sectional variability.

CHAPTER 2
MARKET EFFICIENCY AND RISK-FREE ARBITRAGE OPPORTUNITIES IN THE STANDARD AND POOR'S 100 INDEX OPTION MARKET

2.1 Literature Review

This section of the dissertation examines arbitrage opportunities in the OEX market. Arbitrage opportunities occur in markets where investors can find strategies on an unlimited scale, in theory, in which they can earn positive returns with zero net investment. Arbitrage opportunities do not occur, however, in markets where prices fully and instantaneously reflect all available relevant information (i.e., the market is efficient). Therefore, an investigation of the extent of arbitrage opportunities in a market is, at the same time, an investigation of the efficiency of a market.

Arbitrage opportunities may develop in actual capital markets if various frictions (taxes, short sale restrictions, non-divisibility of securities, costly information, etc.) hinder the instantaneous flow of information to prices. However, the presence of these frictions transforms riskless arbitrage into risky situations. For instance, the time it takes to recognize an arbitrage opportunity, enter a market order[9] to profit from the event, and have the order filled might be enough time for prices to change and the opportunity to pass. This is called immediacy risk. However, even if investors are risk averse, some will have a competitive advantage over others with respect to dealing with these frictions and still be able to identify and profit from potential arbitrage opportunities.

[9]A limit order would also be risky, because there is the chance the order would not be filled.

An example would be the cost and time advantages a market maker has over a private individual investor.

An arbitrage opportunity may occur in an option market if an option's market price deviates from a model value. As long as the pricing model is correctly specified, that option is said to be mispriced. Arbitragers identify the mispricing and form portfolios to earn arbitrage profits. The manner in which the portfolios are formed force prices back to equilibrium.

However, these arbitrage opportunities differ according to the market. Mispricings of equity options are generally thought not to last long, because arbitragers can easily and quickly enter the stock and options markets and earn risk-free profits. This action drives prices back towards equilibrium levels and implies a relatively high degree of efficiency in these markets.

Due to various idiosyncracies, however, forces constraining arbitrage opportunities may not be as strong in the index options market. The problems revolve around the apparent difficulties in forming arbitrage portfolios. For instance, index options are written on a non-traded index, so arbitrage portfolios would need to include either a short or long position in the securities making up the index in their exact proportions. The index that is the subject of this paper, the Standard and Poor's 100 Index, consists of one hundred different stocks. Therefore, the option is written on a non-traded portfolio of 100 stocks. An arbitrage trade that could take advantage of and correct a mispricing would be difficult to establish due to the difficulty in instantaneously forming the portfolio of 100 stocks mentioned above. In the case where the arbitrage situation requires the investor to go long the underlying security, he would have to purchase one hundred different securities simultaneously. This would be difficult but not impossible. However, the immediacy risk would be very high, and prices would have to deviate substantially from equilibrium to entice investors into the market, i.e., a high risk situation should promise the arbitrager a high expected return. If the situation required shorting the index, the arbitrager would be required to short one hundred stocks simultaneously on an up-tick. This is so improbable as to be impossible. This is one example of a friction that can prevent index options from fully and instantaneously reflecting all available information.

The same argument can be extended to a Black-Scholes world. Black and Scholes (1973) demonstrated that riskless returns could be earned by continuously rebalancing a portfolio of calls and the underlying stock, as long as there are no large price movements in the equity. If investors can continuously rebalance portfolios in the exact proportions required by the Black-Scholes (1973) model, mispricings will not last long, and the market will gain efficiency. The difficulty in this method lies in four areas. First, it is not possible to continuously rebalance portfolios in practice.

Second, the correct ratio between options and shares or options and futures which would allow the investor to capture risk-free returns may indicate the necessity to go short or long a fractional share of the underlying security.[9] For instance, Figlewski (1989) points out that if the market value of the basket of stocks on which index options

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[9]Cox and Rubinstein (1985) show that rebalancings should be carried out through the underlying security and not the option. This prevents the fluctuating call price from affecting the profit. For instance, adjusting the hedge through the option may require the investor to purchase an overpriced option or write an underpriced one.

are written is $500,000, and a Standard and Poor's 500 future with a value of $140,000 is used as the proxy for the OEX in the hedge, then only when the hedge ratio is 0.28, 0.56, or 0.84 can a riskless hedge be carried out.

If the investor is using stocks and options, then a portfolio mimicking the index would have to be formed. Using an argument similar to Harvey and Whaley (1992b), if the market value of the stocks comprising the index is $42,000 and if IBM makes up about 4% of the value, then the investor's portfolio must contain $1680 of IBM. If the price of IBM is $63, then the investor would have to purchase 26.67 shares for the hedge portfolio, clearly an impossibility. The problem is exacerbated, because this forces the investor to purchase an odd lot of shares. The odd lot transaction costs are significantly greater than trading in round lots. Thus, the riskless rate will not be obtained.

Third, the transaction costs of continuous or repeated rebalancing would be prohibitive. Because these large portfolio trades put a strain on the distribution system and on specialists' capital positions, according to Gastineau (1991), transactions costs may increase and significantly affect the cost of the hedge. Phillips and Smith (1980) studied the costs involved in these types of trades and determined that they could be large enough to eradicate most arbitrage profits.

And, fourth, there is still the problem of trading large numbers of stocks simultaneously. As mentioned above, it would be almost impossible to go short one hundred stocks simultaneously. So, whether the arbitrage strategy involves periodically rebalancing a portfolio or a quick index arbitrage trade, various frictions may hamper the efficiency of the index options market.

An important aspect of the arguments above centers on the question of how rapidly traders can execute transactions involving large numbers of different securities. Trading entire portfolios of stocks can be done as a form of program trading. Currently, program trading can mean portfolio trading, index arbitrage, computerized trading (a computer program generates buy and sell orders directly with no human interference), and portfolio insurance. Rubinstein (1989, p.29) defines program trading for the purposes of his paper "as the simultaneous entry, but separate execution, of orders in stocks in proportion to their relative representation in major indexes." *The Wall Street Journal* (for instance, see the July 8, 1994 issue, p. C6) defines the expression in its weekly segment, "Program Trading," as the "simultaneous purchase or sale of at least 15 different stocks with a total value of $1 million or more." The common denominator among the definitions is that program trading entails the buying or selling of portfolios of stocks rather than individual stocks. This reduces some of the non-simultaneous problem and, hence, lessens the risk of the trader missing the arbitrage profit, if there is one.

In fact, Stoll (1988) contends that most program or portfolio trading is executed in small units through the New York Stock Exchange Designated Order Turnaround system (DOT). DOT is an order-entry system constructed to send small orders to the appropriate specialist. The SuperDot system has followed and was able to handle market orders of up to 2099 shares in 1986.[10] Therefore, the portfolio trade is broken up among many specialists and is executed, usually, as a market order and may not capture the prices required to earn risk-free profits. In addition, each trading firm must maintain a separate line to the trading floor for portfolio trades. This separate line allows exchange officials to inhibit program trading and give retail trades an opportunity to be executed in case program trades completely dominate the market. This feature has never been used, but it adds another dimension of risk for the index arbitrager.

However, the problem remains that it is difficult, if not impossible, to simultaneously trade 100 stocks in the exact proportions as the index would require. So, while pressures can develop to push the index and index option prices away from equilibrium, the arbitrage forces which should keep index option prices in equilibrium seem to be weaker than in the case of individual equity options. This means that the flow of information between markets is impeded, and efficiency is hindered. The purpose of this study will be to address the issue of whether or not the Standard and Poor's 100 Index option is efficiently priced and, if it is not, whether there is some mechanism that can drive the prices toward equilibrium.

One mechanism that is examined is an arbitrage portfolio of a call, a put, a discount bond, and a synthetic index, rather than a synthetic option, as in portfolio insurance. The synthetic index would be constructed from another set of puts, calls, and discount bond whose payoff structure matches the payoff of the index. For instance, if the mispricing requires a short position in the underlying security, this position could

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[10] In September, 1989, the SuperDot system was upgraded to handle post-opening orders of up to 30,099 shares.

be mimicked by writing a call, buying a put, and borrowing an amount equivalent to the discounted strike price. Rebalancing would not be required to earn a riskless return. It would be necessary, though, that the options making up the synthetic index be priced correctly or better (i.e., the purchased option could be underpriced or the written option could be overpriced).

This is the so-called box-spread. It is used here because it does not rely on any particular model and its assumptions. To identify the mispriced options for use in forming arbitrage portfolios, put-call parity boundaries, the binomial option pricing model, the Black-Scholes option pricing model, and the Black-Scholes option pricing model plus the estimated value of early exercise will be used as benchmarks.

Also examined will be portfolios using S&P 500 futures and subsets of the OEX to mimic the S&P 100 index. While the literature states that these portfolios are used by actual traders, there are no studies relating portfolios of OEX options and SPX futures or OEX options and subsets of the index to arbitrage portfolios. Arbitrage portfolios will be constructed with the relevant costs taken into account. Profits, if any, will be determined.

Recently, a new security has been introduced. These are Standard and Poor's Depository Receipts (SPDRs). They are unit investment trusts which contain the SPX basket of stocks and are traded on the American Stock Exchange. Although they were not traded during the time period of this study, it could be argued that they can be used as part of an arbitrage portfolio with the OEX options. However, in conversations with an OEX option trader, these were not mentioned, although there is no reason they could not be used. The trader did acknowledge that other options and the SPX futures were commonly used to form the arbitrage portfolios though.[11]

The next section examines how previous researchers have seen the problem. The section following that will outline the methodology. Subsequently, the data description and results are provided.

Previous studies have examined the efficiency of the Standard and Poor's 100 Index option market of the Chicago Board Options Exchange (CBOE). For instance, in an early study of the efficiency of the OEX market, Evnine and Rudd (1985) test the OEX and the Major Market Index options against arbitrage boundaries, put-call parity, and binomial option pricing model theoretical values. They show that the index options are often mispriced and attribute the mispricings to the difficulty in forming arbitrage portfolios.

While Evnine and Rudd (1985) use intraday bid-ask data, Chance (1988) employs closing bid-ask prices of the OEX and finds that violations of boundary conditions are rare. On closer examination of the two studies' only common test, however, both find violations of the lower boundary condition (C_a - I + K $\geq$ 0 for Evnine and Rudd and $C_a + Ke^{-rT} - Ie^{-DT}$ for Chance) in about 2.5% of their data. The use of intraday data in the Evnine and Rudd study and end of the day data by Chance could be the cause of the difference in their overall conclusions about the efficiency of the OEX market. In fact, Evnine and Rudd mention that they obtained different results

[11]The SPDR's volume is still low relative to the SPX futures. Therefore, arbitragers would more likely use the SPX futures for price discovery and forming arbitrage portfolios. Also, the futures would reflect any changes in the composition of the SPX index, while a unit investment trust does not change its composition except for liquidating various holdings.

when using closing prices in an earlier study (Evnine and Rudd, 1983). Sheikh (1991) discovers that the market values of OEX options are different from values produced by the Black-Scholes (1973) option pricing model by using a methodology very similar to Rubinstein (1985). Diz and Finucane (1993) find inefficiencies in the OEX market relative to early exercise decisions. They find that investors sometimes exercise early even when the value of this decision is less than the value they could have received if the option were sold. The major problem, as mentioned above, is that mispricings cannot be arbitraged away easily with an index option due to the non-tradeable status of the underlying security. In fact, the studies mentioned above use actual index values in their analyses even though such a security does not exist. Various authors have suggested proxies for the index.

Evnine and Rudd (1985) suggest that traders construct proxy portfolios consisting of several highly liquid stocks appropriately weighted to closely match the index. Also mentioned is that futures on the S&P 500 Index (SPX) are sometimes used to proxy for the S&P 100 Index. Fleming and Whaley (1994), also, assert that traders use the S&P 500 futures returns to value OEX options. However, the use of these or any other proxies introduces risk from the imperfect correlation between the index proxy and the S&P 100. This risk prevents the formation of *riskless* arbitrage portfolios and permits the existence of pricing errors.

To illustrate, using the analysis from Evnine and Rudd (1985), if C_t and I_t are the price of the call and index at time t, then an investor can attempt to profit from overpricing of the call by forming a riskless hedge portfolio consisting of a long position in the index and a written call. The investor would have to use a proxy for the index, P_t, such that $P_t = I_t$ for all t. In reality, this condition can only hold for a moment, so the relationship becomes, at some time T, $P_T = I_T + \varepsilon_T$, where ε_T is the size of the tracking error. If the hedge ratio is Δ, then the investor can look forward to a profit over the time period $[t, T]$ of:

$$(\Delta P_T - C_T) - (\Delta P_t - C_t)$$

$$= (\Delta I_T + \Delta \epsilon_T - [\hat{C}_T + u_T]) - (\Delta I_t - [\hat{C}_t + u_t])$$

$$= [(\Delta I_T - \hat{C}_T) - (\Delta I_t - \hat{C}_t)] + \Delta \epsilon_T - u_T + u_t \qquad (14)$$

where $\hat{C}$ is the option's theoretical price, and the pricing error between the market value and the theoretical value of the call is $u_t = C_t - \hat{C}_T$. The term in brackets after the second equality represents the risk-free return on an arbitrage between a correctly priced option and the index. u_t is the mispricing that investors initially identify and try to take advantage of through arbitrage. u_T is the mispricing when the hedge is unwound. Evnine and Rudd assume it can be neglected if the hedge is continuously rebalanced. $\Delta\varepsilon_T$ is the remaining risk and may not disappear. Therefore, risk-free arbitrage may not be possible, and pricing errors may remain.

On the other hand, Chen and Johnson (1985) demonstrate that the underlying security does not have to be part of the hedge portfolio. Chen and Johnson (1985) assert that a hedge can be constructed with only options. If a mispricing requires a hedge of a short position of *n1* units of option *c1* and a long position of one share of stock, for instance, investors may also be able to find a hedge on the same stock of going long *n2* units of option *c2* and shorting one share of the stock. The hedge ratio becomes *-n1/n2*, and the stock is netted out. The strategy is to adjust the hedge by buying the underpriced option and selling the overpriced one. Thus, an option-option hedge is created, and the underlying security need not be held. Continuous rebalancing is still required, and fractional units of options are still assumed to be available for risk-free hedges, assumptions that do not hold in real options markets.

The next section will demonstrate how risk-free arbitrage portfolios can be constructed using instruments that are available to market participants. Included will be a portfolio comprised of only options and a discount bond. This will curtail the need for periodic rebalancing.

2.2 Method of Analysis

The empirical problems with arbitrage involving an index option are that the portfolios require periodic rebalancing, trading in fractional units, and the simultaneous trading of a large number of securities. This adds to the cost and requires constant supervision by the investor. Several alternatives will be examined here which might mitigate these problems. The alternatives include a synthetic index composed of options and a risk-free bond, the S&P 500 futures contract, and small portfolios of equities to mimic the index.

2.2.1 Synthetic Index

A method to test for efficiency without the need for continuous portfolio rebalancing would be to construct a synthetic index with options and a discount bond, and use this portfolio in conjunction with the mispriced option to produce a risk-free arbitrage portfolio. Of course, this presumes that the entire market is not out of equilibrium at the same time. That is, the options used to construct the synthetic index should be, at least, properly priced.

The main difficulty with an index option lies in the fact that the put can become overvalued relative to the call. Then an arbitrageur would write the put, buy the call, lend an amount equal to the discounted strike price, and short the index to generate riskless returns. It is the last input that causes problems. However, an investor could write a call, buy a put, and borrow an amount equal to the discounted strike price to produce a payoff pattern identical with the index and remain consistent with put-call parity.

Because we assume something cannot be created from nothing, the initial cost should be positive:

$$C_B - C_{WI} + P_{BI} - P_W - K_2 e^{-rt} + K_1 e^{-rt} > 0 \tag{15}$$

where P_W is the price of the written put, C_B is the price of the purchased call, K_2e^{-rt} is the amount borrowed as part of the index proxy portfolio, K_1e^{-rt} is the amount loaned at the strike price of the mispriced option, and C_{WI} and P_{BI} are the respective prices of the written call and purchased put comprising the index. If there are no frictions in the market, this would be a strict equality, and the ending payoffs would be zero. In other words, an investment that has zero cost has a zero return. This is illustrated in Table 7 where the index proxy portfolio is formed at a higher strike price. However, the initial cost may empirically be found to be non-positive due to mispricing, and the ending payoffs to be positive.

Table 7 indicates that there are three possible states of the world at maturity. The price of the index can be below the lower strike price, it can be above the higher strike price, or it can be between the two strike prices. No matter which state occurs, if put-call parity holds, the value of the portfolio will be zero at maturity. Therefore, there is no uncertainty, no cost, and no payoff.

Table 7. Final Payoffs for an Option Arbitrage With a Written Call, Purchased Put, and an Amount Borrowed Proxying for the Shorted Underlying Security

Ending Possibilities:		$S \leq K_1$	$K_1 < S < K_2$	$S \geq K_2$
K_1	P_W	$S - K_1$	0	0
	C_B	0	$S - K_1$	$S - K_1$
	$-K_1e^{-rt}$	K_1	K_1	K_1
K_2	C_{W1}	0	0	$K_2 - S$
	P_{B1}	$K_2 - S$	$K_2 - S$	0
	K_2e^{-rt}	$-K_2$	$-K_2$	$-K_2$
Ending Payoffs:		0	0	0

S = end of period price of underlying security, K_1 = strike price of mispriced option, K_2 = strike price of index proxy which is assumed to be greater than K_1, P_W = written put, C_B = purchased call, C_{W1} = written call, and P_{B1} = purchased put.

There are three benefits to forming the arbitrage portfolio this way. First, it does not have to be continually rebalanced. Investors know the initial cost and the minimum ending payoff. There are no additional costs from continual rebalancing or monitoring. The investor would enter into the relationship only if the ending payoff is positive or the initial cost is negative. Second, this will help force the mispriced options back to an equilibrium level. The price of an overpriced put will be forced down, and the price of an underpriced call will be forced up. And, third, there is no problem with a hedge ratio demanding fractional share or option positions to be undertaken.

An argument against this portfolio is that the written put or the written call may be exercised before maturity. However, this will not affect the risk of the situation. This can be shown whether the arbitrager receives notice of the exercise immediately or not. For instance, consider the written call at the higher strike price. It will not be exercised unless $S > K_2$. If this condition occurs, and the arbitrager is notified immediately of the exercise, the writer will have to provide the difference between the index value and the strike price. Instead, however, the investor can exercise the call he holds at the lower strike price to offset this transaction. The payoff will be $-(S - K_2) + (S - K_1)$ or $K_2 - K_1$. The puts at this point are out-of-the-money and would not be exercised. However, if the price of the index subsequently dropped below K_1, and if the put at the lower strike price were exercised, then the payoff would be $-(K_1 - S) + (K_2 - S) = K_2 - K_1$, again.

Unfortunately, the actual trader on the CBOE is notified of the exercise the next day, not immediately. To account for this, the arbitrager would assess the probability of early exercise each day in the wildcard period. If the price of $C_W < S - K_2$, then he assumes his option will be exercised. He, therefore, exercises his call at the lower strike price early. He receives $S - K_1$ for his exercise and purchases an offsetting option at the higher strike price to close out that position. This will provide an additional profit, because the price of the new call purchased will be less than the amount above K_2 the arbitrager receives upon exercising his option. If the cost of the new call is $(S - K_2)$ minus e, where $e > 0$, then his payoff will be,

$$-((S - K_2) - e) + (S - K_1) = K_2 - K_1 + e > K_2 - K_1. \tag{16}$$

Since the initial criteria for setting up this arbitrage portfolio was that it must be risk-free and that $K_2 - K_1$ must be greater than or equal to the risk-free return, the arbitrager earns an even greater profit. A similar argument can be made for the put being exercised early.

2.2.2 Futures

Fleming and Whaley (1994) assert that the return stream from the S&P 500 future is more correlated with the OEX than the S&P 100 Index is with the OEX. In fact, they state, "traders use the S&P 500 futures rather than the reported S&P 100 index level to value OEX options" (Fleming and Whaley, 1994, p.230). A simple correlation analysis is made of the OEX cash index, the cash SPX index, and the SPX futures. Three series are formed representing 17,368 observations of each at five minute intervals. The OEX data is from the Berkeley Options tapes, and the SPX data comes from the Tick Data Corporation. These series are used in the third part of this dissertation, except that an adjustment is made to the futures data at that point. The returns of the respective instruments are also analyzed here. As can be seen in Table 8, the OEX, SPX, and SPX future are all highly correlated. However, in Table 9 it can be seen that there is less of a linear relationship between the returns. But, they are still highly significant. Therefore, an analysis incorporating the S&P 500 future rather than the call-put-bond index proxy is executed. Arbitrage profits are identified and recorded.

Table 8. Correlation Analysis of the OEX, SPX, and SPX Future

	Cash OEX	Cash SPX	SPX Future
Cash OEX	1.0000 (0.0000)	0.98853 (0.0001)	0.98859 (0.0001)
Cash SPX		1.0000 (0.0000)	0.99789 (0.0001)
SPX Future			1.0000 (0.0000)

Numbers in parentheses are P-values.

Table 9. Correlation Analysis of the OEX, SPX, and SPX Futures Returns

	Cash OEX Returns	Cash SPX Returns	SPX Futures Returns
Cash OEX Returns	1.0000 (0.0000)	0.86343 (0.0001)	0.58580 (0.0001)
Cash SPX Returns		1.0000 (0.0000)	0.62728 (0.0001)
SPX Futures Returns			1.0000 (0.0000)

Numbers in parentheses are P-values.

The problem is in determining the adjustment to the SPX futures price to use in pricing the OEX options, since the S&P 100 and S&P 500 Indexes are not perfectly correlated. We can construct a simulated S&P 100 Index in the following manner by regressing the OEX cash index on the discounted SPX future as follows:

$$OEX_t = \alpha + \beta\left(\frac{SPXF_t}{(1 + r - d)^T}\right) + e_t \qquad (17)$$

This gives us the predicted value of the S&P 100 index, which can now be used in put-call parity or other pricing relationships. In fact, the regression is run each day. Those parameters from the model are then used with the following day's SPX futures to price the OEX options.

2.2.3 Stock Portfolios

Evnine and Rudd (1985) suggest using portfolios of several stocks to proxy for the S&P 100 cash index. Proxy portfolios can be selected by minimizing the return errors between various subsets of the OEX and the total OEX.

There are several criteria in finding a mimicking portfolio for the OEX. First, as few stocks as possible should be chosen. This reduces costs as well as alleviating the difficulty in purchasing or shorting one hundred stocks simultaneously. And second, the difference between the returns from the mimicking portfolio and the index portfolio should be minimized. Since the goal is to find the *optimal* portfolio subject to several criteria, a genetic algorithm[12] was chosen to generate the portfolio.

A genetic algorithm is a global optimization technique. It relies on the laws of probability, as well as a preference for better performing solutions, to search over a

[12]For a description of genetic algorithms and applications in finance, see Grace (1994).

population of potential solutions to a problem simultaneously. The potential solutions are modeled as binary strings for computational efficiency. Most other techniques search over one potential solution at a time. In this case, the search is for a set of weights ranging between zero and one for one hundred different stocks, given the above criteria. In using the genetic algorithm to select the optimal set of stocks to mimic the OEX, a population of potential solutions composed of 400 binary strings, each 2000 digits long is generated. The 2000 binary digits decodes into 100 weights. Two thousand five hundred generations or iterations are run.

Short sale restrictions are taken into account by shorting only on an uptick. When the stocks are bought or sold, I use the appropriate bid-ask quotes.

2.2.4 Transactions Costs

Any study of market efficiency that does not take into account transactions costs can be misleading. Where it may appear that arbitrage profits are available, including a realistic set of costs may cause those positive profits to disappear.

To estimate the relevant costs, Phillips and Smith (1986) separately examine the cost structure of market makers on the floor of the options exchange and members of brokerage firms who specialize in arbitrage trades involving stocks and options. Although the latter group is in the most favorable position cost-wise to trade a portfolio of options and stocks judging by their Table 1, the market maker would be able to capture mispriced options more rapidly. Unless otherwise specified, the term arbitrager will refer to either type of trader.

Phillips and Smith (1980) identify explicit and implicit costs. Explicit costs would entail items such as commissions and SEC transactions fees. Implicit costs are found in the bid-ask spread. For instance, the market maker will charge a higher price to sell his inventory (the ask price) than to buy (the bid price) more inventory. Since bid-ask prices are being used here, this cost should not be a factor in the analysis. However, it should be remembered that a quote for an OEX option is good for only 10 options, and a quote on a stock is valid for only 100 shares.

Phillips and Smith (1980) list the explicit costs for an options market maker to range from about \$6.00 to \$20.00 per trade for stock and option portfolios: \$0.50 to \$1.00 per option contract and \$5.00 to \$12.50 per round lot of stock. In addition, there would be various fees and taxes that might add up to \$6.50 per trade. For an arbitrager employed by a brokerage firm with exchange memberships, these fees could range between \$2.50 and \$12.00. The relatively large difference in fees is due to the fact that the market maker must contract with another party to effect his stock transaction.

More recently, Fleming, Ostdiek, and Whaley [FOW] (1995) give the commission for trading OEX options to be \$2.40 per contract. This is used in this paper. FOW also suggest a commission of \$0.02 per share of stock. Phillips and Smith find the stock commission to vary between \$0.01 and \$0.04 per share. I take the FOW figure, since it is more recent.

Unfortunately, bid-ask prices are not available for the futures data (described below). Therefore, the implicit cost of the trade represented by the spread must be estimated. FOW give an average bid-ask spread for the nearby SPX future as \$27.90.

Lee and Nayar [LN] (1993) give $12.50 as one half of the bid-ask spread or one-way transaction cost. The explicit commissions on futures is found by LN to be $20 round trip for institutional investors and $10 for member firms. FOW find the figure to be slightly higher. Therefore, I use a cost of one half the bid-ask spread ($13.95) and the commission ($6.00) from FOW to be conservative for a total of $19.95 per futures trade. Since it takes 5 option contracts to equal one futures contract,[13] the total cost of this trade is $43.95. For instance, with a trade involving put-call-futures parity, five calls, five puts, and one future are traded. The next section will describe the data.

2.3 Data

The OEX option data for 1986 comes from the Berkeley option tapes. It consists of second by second bid-ask quotes. The first half hour of trading was deleted because of the thinness of trading at this time. Any set of quotes where either the bid or the ask was zero was also removed. This left 1,085,506 records, where each record consisted of the date, time of the quote, maturity, strike price, bid price, ask price, and index value. The risk-free rate was proxied by the rate on the treasury bill that matured closest to the maturity of the option and was obtained from the *Wall Street Journal.*

The intraday data for the S&P 500 futures came from the Tick Data Corporation. As mentioned above, these are actual transaction prices. Trades occur between the hours of 8:30 A.M. and 3:15 P.M. Central Time on the Chicago Mercantile Exchange. The futures expire on the third Friday of March, June, September, and December. The time period in question is 1986.

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[13]The dollar value of the OEX is one hundred times the index, and the dollar value of the SPX is 500 times the future.

An issue is that the only month that has good liquidity is the current expiration month. Therefore, it is assumed that arbitragers value liquidity, and only arbitrage trades involving the future nearest to expiration will be examined. For example, from January 2nd through the third Friday of March, only arbitrage trades affecting March expiration futures and options will be investigated. The following week, only futures and options having a June expiration will meet the criteria.

Bid-ask quotes on the relevant stocks for the proxy portfolios were obtained from the ISSM tapes. The composition of the OEX changed several times during 1986, and those changes are incorporated here. On June 11th, RCA was dropped and Bell Atlantic was added. Several changes occurred on September 17th. Sperry merged with Burroughs to form Unisys. Also, Ameritech was added. finally, on November 26th, Safeway Stores was supplanted by H.J. Heinz.

To find the weights for the mimicking portfolios, I take the last quotes in a one hour period for each stock in the OEX. The data is gathered over one week periods. Hourly returns are constructed along with hourly returns from the OEX. This data is fed into the genetic algorithm where the sum of squared errors is minimized between the two series subject to a penalty for each weight greater than zero. These weights are then used in the following week to develop mimicking portfolios.

The mimicking portfolio is designed to replicate the performance of the underlying index. Perfect replication would be optimal, but is probably not possible from a practical standpoint. However, this brings up the point that the purpose of the replicating portfolio is to substitute for the larger basket of index stocks, thus lowering transactions costs and immediacy risk. Also, the purpose of the mimicking portfolio is not to replace the index value in identifying arbitrage opportunities. Therefore, the actual index value is used to identify trading opportunities, and then the arbitrage portfolios are formed with the mimicking portfolios.

The dividend yield on the SPX used in regressing the OEX on the discounted future came from *Standard and Poor's Analysts' Handbook, 1994 Annual Edition*. With a dividend per share of 8.28 and an average SPX price during 1986 of 228.745, the annual yield was .0362. Dividends on the OEX are from Harvey and Whaley (1992a). Results and the specifics of the tests are given next.

2.4 Tests

Arbitrage opportunities are searched for with respect to violations of put-call parity boundaries, Black-Scholes option prices, Black-Scholes option prices plus the value of early exercise, and binomial option prices. If mispricings are found, a proxy index is formed to help capture the arbitrage profit.

2.4.1 American Boundaries

Any violation of option boundaries permits risk-free arbitrage profits. The boundaries for an American option are:

$$C_{bt} \leq P_{at} + I_t - Ke^{-r(T-t)} - \sum_{\tau=t}^{T} D_\tau e^{-r(\tau-t)}$$

and

$$C_{at} \geq P_{bt} + I_t - K - \sum_{\tau=t}^{T} D_\tau e^{-r(\tau-t)} \qquad (16)$$

where C_{at} is the call ask, C_{bt} is the call bid, P_{at} is the put ask, and P_{bt} is the put bid. A violation of the first equation indicates either an over-priced call or an under-priced put. A violation of the second equation implies the opposite. Since the investor knows all option prices (he may be a floor trader), he can pick those trades which are profitable and reject those which are not. I look only at those trades that a trader would identify as being profitable even though many other options may be identified as being mispriced.

For the box spread, I assume the investor looks for options that violate the put-call parity relationship outlined above. The cash index value is used in the equation. Once a violation is found, the arbitrager looks for options to complete the box. If he finds them, a trade is recorded using all the appropriate figures. If he does not find a complete box, he waits for another violation.

As can be seen from Table 10, the number of profitable arbitrage trades generated is relatively small. Of the total number of options examined, only about 1.25% are found to be mispriced and produce profitable arbitrage trades. What should be noted is that, first, although it was hypothesized that overpriced puts would cause the most frequent occurrence of mispricings, the culprit here is overpriced calls. Second, the mispricings are due almost exclusively to overpriced options relative to the boundaries.

Using the SPX futures to proxy for the index value, many more mispricings are found. However, this amounts to only about 11.35% of the options. The arbitrage trades are spread more evenly among the type of mispricings.

Finally, the stock portfolios find about 5% of the options to be mispriced. However, here the actual index value is used in the inequalities to find the mispricing. The mimicking portfolio is then substituted to find the profit. Since the value of the portfolio may deviate from the value of index, the investor cannot be assured that the trade will be profitable on initiating it. So, here I assume that the arbitrager tries to capture all mispricings. The average profit turns out to be negative. As in each category, most of the mispricings are due to overpriced calls.

When adjusting the futures trades for implicit and explicit costs, the most noticeable phenomenon is that the number of profitable opportunities drops 94% to 7400 for the futures proxy. Notice also that these are net profits. So, even though five calls and five puts are being traded with the future, it is only necessary to multiply the box trade profit by five to get a comparable figure. The mispricing is skewed towards underpriced puts.

It should be noted that the average time for a short sale to be completed for the stocks is four minutes and twenty-two seconds for the no-cost trades and four minutes and fifty-eight seconds for the costly trades. This length of time may render the trades to be too risky for arbitagers.

Table 10. Arbitrage Trades Generated by American Boundary Violations
Cost = $0.0

Type of Index Proxy	Number of Trades	Average $ Profit	Minimum $ Profit	Maximum $ Profit	Calls - Percent Overpriced	Calls - Percent Underpriced	Puts - Percent Overpriced	Puts - Percent Underpriced
Options	13,558	13.074	0.001	45.081	0.767	0.002	0.230	0.000
S&P 500 Future	123,218	13.192	0.002	223.45	0.336	0.124	0.289	0.251
Stocks	50,128	-111.802	-214.529	77.552	0.995	0.000	0.005	0.000

Table 11. Arbitrage Trades Generated by American Boundary Violations
Cost = $2.40 per Option, $19.95 per Future, $0.02 per Share

Type of Index Proxy	Number of Trades	Average $ Profit	Minumum $ Profit	Maximum $ Profit	Calls - Percent Overpriced	Calls - Percent Underpriced	Puts - Percent Overpriced	Puts - Percent Underpriced
Options	8,619	7.894	0.0002	35.480	0.649	0.003	0.348	0.000
S&P 500 Future	7400	22.823	0.0002	179.500	0.082	0.000	0.098	0.819
Stocks	50,128	-218.468	-325.070	-33.500	0.995	0.000	0.005	0.000

2.4.2 Black-Scholes Model Prices

The Black-Scholes option pricing model is specified for European style options on non-dividend paying individual stocks. Therefore, it would not be surprising to find OEX market prices deviating from model prices. In this test, market prices are compared to model prices. If a call (put) bid price is greater than the Black-Scholes price, then an overpriced call (put) is identified. If a call (put) ask price is less than the Black-Scholes price, then an underpriced call (put) is identified. If one of these deviations is found, a portfolio is formed with a synthetic index, and potential profits are investigated. Dividends are accounted for by subtracting the present value of all dividends to be paid between the current date and the option's maturity from the stock price. This adjusted stock price is used in the Black-Scholes formula.

Table 12 indicates that the majority of mispricings are concentrated among the overpriced puts and underpriced calls. This is more in line with the original hypothesis. Once again, on average, the stock portfolios produce losses. In fact, the average time to form the short-sale portfolios increases here to over seven minutes for both types of trades.

2.4.3 Black-Scholes Plus the Value of Early Exercise

The Black-Scholes model plus a correction for early exercise is examined. In the first section of this dissertation, the estimated average value of early exercise for calls in-the-money is found to be 4.1 percent and for puts it is 10.87 percent. Regressions were also run on the estimated value of early exercise as the dependent variable and moneyness, time to expiration, and the risk-free interest rate as the dependent variables. The coefficients that were found are used here with the degree of moneyness, time to expiration, and risk-free rate associated with each option to find an estimated value of early exercise for each option. It should be noted that the regressions were run on options in-the-money, but the coefficients are being applied to all options. Mispricings are searched for relative to a benchmark made up of the Black-Scholes model value plus this estimated value of early exercise.

Comparing Tables 12 and 14, the number of arbitrage opportunities identified for the box spread and the stock portfolios is reduced dramatically. In fact, the potential profits increase when the early exercise feature is included. However, the number of trades identified using the futures increases. The mispricings lean heavily towards the underpriced puts.

The time to form the short-sale portfolios increases significantly. Over fifteen minutes are required to complete the transaction.

2.4.4 Binomial Option Pricing Model

Finally, a binomial option pricing model that explicitly takes into account the probability of early exercise is used to search for mispriced options. Arbitrage portfolios are, once again, formed. The construction of the binomial option pricing model for index options demands a short explanation.

In using the binomial option pricing model for index options, the dividend enters into the problem in two ways. First, the index price must be adjusted to account for the discounted stream of dividends between the current date and the option's maturity. In other words, the index tree is generated using this adjusted index value.

Table 12. Arbitrage Trades Generated by the Black-Scholes Option Pricing Model
Cost = $0.0

Type of Index Proxy	Number of Trades	Average $ Profit	Minimum $ Profit	Maximum $ Profit	Calls - Percent Overpriced	Calls - Percent Underpriced	Puts - Percent Overpriced	Puts - Percent Underpriced
Options	73,167	9.911	0.001	58.356	0.099	0.357	0.338	0.207
S&P 500 Future	164,472	7.803	0.01	66.201	0.202	0.437	0.143	0.218
Stocks	356,694	-18.408	-202.505	190.774	0.526	0.000	0.396	0.077

Table 13. Arbitrage Trades Generated by the Black-Scholes Option Pricing Model
Cost = $2.40 per Option, $19.95 per Future, $0.02 per Share

Type of Index Proxy	Number of Trades	Average $ Profit	Minimum $ Profit	Maximum $ Profit	Calls - Percent Overpriced	Calls - Percent Underpriced	Puts - Percent Overpriced	Puts - Percent Underpriced
Options	16,283	6.492	0.002	33.990	0.103	0.640	0.027	0.230
S&P 500 Future	3027	6.853	0.002	33.990	0.000	0.000	0.183	0.817
Stocks	356,694	-92.828	-313.426	78.889	0.526	0.000	0.396	0.077

The dividends enter into the model again when forming the option tree. Since the tree for the options starts at the end of the option's life, those option values represent the intrinsic value of the option. One period prior to the maturity of the option, the option's value is the present value of the future values of the option. However, since this is an American option, a check must be made to see if the option's value at that node is less than the value if exercised. This is done by comparing the option (call) value with the adjusted index price plus the discounted value of dividends between that time period and the option's maturity minus the strike price. If the exercise value is greater than the option value, the exercise value is substituted onto that node. This is the method recommended in Harvey and Whaley (1992a and 1992b) and used here. The number of time periods for the binomial tree is set at twice the number of days until the option's maturity, as in Harvey and Whaley (1992a and 1992b).

For this case, the overpriced puts represent the greatest occurrence of mispricings in each category as was hypothesized. Also noteworthy is that this method generates the largest profits for the stock portfolios, but the number of profitable opportunities for the futures drops drastically when costs are included. An arbitrager needs over seven minutes on average to short sell all of the stocks required to form the portfolios.

2.5 Chapter 2 Summary

These results indicate that the S&P 100 Index option market is relatively efficient. For instance, Evnine and Rudd (1985) found approximately 16% of their sample violating American boundary conditions. Only about 1.25% are found here. However, like their study, most of the mispricings tend to be overpriced calls.

Table 14. Arbitrage Trades Generated by the Black-Scholes Option Pricing Model Plus the Estimate Value of Early Exercise, Cost = $0.0

Type of Index Proxy	Number of Trades	Average $ Profit	Minimum $ Profit	Maximum $ Profit	Calls - Percent Overpriced	Calls - Percent Underpriced	Puts - Percent Overpriced	Puts - Percent Underpriced
Options	29,390	15.341	1.299	41.085	0.059	0.0005	0.444	0.497
S&P 500 Future	208,989	12.953	0.051	87.451	0.159	0.344	0.154	0.343
Stocks	39,026	-2.619	-133.319	127.099	0.000	0.000	0.502	0.498

Table 15. Arbitrage Trades Generated by the Black-Scholes Option Pricing Model Plus the Estimate Value of Early Exercise, Cost = $2.40 per Option, $19.95, $0.02 per Share

Type of Index Proxy	Number of Trades	Average $ Profit	Minimum $ Profit	Maximum $ Profit	Calls - Percent Overpriced	Calls - Percent Underpriced	Puts - Percent Overpriced	Puts - Percent Underpriced
Options	21,787	7.904	0.004	31.485	0.011	0.001	0.490	0.499
S&P 500 Future	13,911	9.826	0.002	43.501	0.000	0.000	0.056	0.944
Stocks	39,026	-114.735	-244.644	15.324	0.000	0.000	0.502	0.498

Table 16. Arbitrage Trades Generated by the Binomial Option Pricing Model
Cost = $0.0

Type of Index Proxy	Number of Trades	Average $ Profit	Minimum $ Profit	Maximum $ Profit	Calls - Percent Overpriced	Calls - Percent Underpriced	Puts - Percent Overpriced	Puts - Percent Underpriced
Options	36,631	14.118	1.299	41.085	0.067	0.000	0.933	0.000
S&P 500 Future	84,800	10.606	0.139	223.45	0.483	0.000	0.517	0.000
Stocks	62,478	107.029	-198.366	186.845	0.011	0.000	0.989	0.000

Table 17. Arbitrage Trades Generated by the Binomial Option Pricing Model
Cost = $2.40 per Option, $19.95, $0.02 per Share

Type of Index Proxy	Number of Trades	Average $ Profit	Minimum $ Profit	Maximum $ Profit	Calls - Percent Overpriced	Calls - Percent Underpriced	Puts - Percent Overpriced	Puts - Percent Underpriced
Options	25,442	6.707	0.003	31.485	0.010	0.000	0.990	0.000
S&P 500 Future	1,864	92.051	0.002	179.501	0.327	0.000	0.673	0.000
Stocks	62,478	-5.230	-308.403	75.028	0.011	0.000	0.989	0.000

FOW have examined the lead-lag relationships among index markets and found that the cost of doing business in a particular market can affect the price discovery process. Costs are also relevant here, judging by the number of profitable trades and the net profit in each category. With no costs, the profit for trades generated by all models is $6,515,290.00 for the futures proxy and $1,870,444.00 for the options proxy. However, when costs are included, the options proxy has the greater profit over the futures proxy: $516,591.60 versus $497,906.80. The price discovery process will be further examined in the next section of the dissertation.

In general, the number of mispricings found is low relative to the total number of options examined. Additionally, even though it was hypothesized that a large number of mispricings would be due to overpriced puts, the actual mispricings did not concentrate here. It is also shown that there may be too much risk in trying to capture an arbitrage profit by using a small mimicking stock portfolio to proxy for the entire basket of index stocks. Future research is indicated in this area; i.e., how large does a mispricing have to be before an arbitrager is assured of a positive average profit? Also, future research should look for concentrations of arbitrage opportunities in particular periods, such as expiration week. Finally, relationships between arbitrage opportunities and moneyness and time to maturity should be examined.

As indicated above, one impediment to forming arbitrage portfolios is the necesity to wait for an uptick. The amount of time to form the short-sale portfolios where needed is found to range from just over two minutes to over fifteen minutes on average. This is probably too risky for investors.

CHAPTER 3
PRICE DISCOVERY AND INDEX OPTIONS

3.1 Literature Review

One of the central themes of this paper is that there is no single traded security underlying index options. However, this may not be an insurmountable problem. For instance, for valuation purposes there are several proxies, as outlined previously. Also, the non-traded index itself can be used by investors to value the options. The difficulty for investors is in determining which one provides the most timely information for valuation purposes, because option traders operate in a fast moving market. The volume on a recent day for the OEX contract was 345,096 contracts. The basket of securities underlying each contract was worth approximately $52,000. This translates into about 14 contracts traded per second written on $728,000 worth of stocks or $10,920,000 every fifteen seconds. Although most option positions may be hedged, the size of the volume implies a need for current information.

Using the actual index to price the options involves several problems. First, it is updated only every fifteen seconds. So, any information revealed and incorporated in stock values at less than 15 second intervals will not show up in the index immediately. Second, even when the index changes, not all of the stocks comprising the index may have adjusted to the new information. Not only does this cause positive autocorrelation in the index returns, but it provides incomplete information to market participants.

Various studies including Stoll and Whaley (1990), Fleming, Ostdiek, and Whaley (1995), and Swinnerton, Curcio, and Bennet (1988) have shown that futures incorporate information before the cash index. In addition, Fleming, Ostdiek, and Whaley [FOW] (1995) study the lead/lag relationship between the SPX futures' and the OEX options' returns. They find that the futures slightly lead the options. They attribute this to a cost advantage in trading the futures. However, their methodology does not explicitly show a linkage between the two markets. In fact, Hasbrouck (1993) points out that this type of lead/lag analysis leaves the possibility that prices can diverge without bounds. In addition, pricing the option off of the future injects risk into the pricing function from the fact that the exact relationship between the cash OEX and the future cannot be known for more than an instant. In fact, only when the option and the future mature at the same time and when the exact pricing relationship is known can the future be used to complete a riskless hedge with the option. Even if the future and the option are written on the same index, the maturity dates must be the same to form the riskless hedge. So, valuation of the OEX option from the SPX future is risky.

The index implied by put-call parity has the disadvantage in that it is priced from options, and the goal of this chapter is to find how they are valued. So, this may be a circular argument. The advantage is that, if those options are priced by knowledgeable investors, then they contain timely and relevant information. Furtheremore, if we assume that some investors use the SPX futures to provide pricing information and others use the cash OEX or some other proxy, then investors using options to price options will be doing so using the information that other investors have deemed relevant.[14] Whether or not the information is timely is problematic.Investors who trade on information contained in futures prices might have more timely information.

Therefore, the objective here is to find where price discovery actually takes place, i.e., where information is first entering the system. Another way of saying this is, "Where is the information coming from that is moving the market?" This idea has been studied previously for other markets. For instance, some securities are traded in multiple markets but are priced differently from second to second. The share of stock in company A that trades on the New York Stock Exchange (NYSE) is the same as the share in company A trading on the Pacific Stock Exchange, but due to cost differentials or market structures or other imperfections, the prices may be different. The market with the fewest imperfections, usually in terms of cost, is the one where pricing information will usually arise first.

Garbade and Silber (1979) study this idea in the context of dominant and satellite markets. If the price of a security trading in one market adjusts to a price change of the identical security trading in another market, the first market is called a satellite market and the second is called the dominant market. The lagged adjustment is caused by some friction between the two markets, otherwise the prices would adjust simultaneously. The greater the frictions, the more the prices can drift apart. If they drift too far apart, the forces of arbitrage will force them back to some equilibrium. On the other hand, if costs are effectively zero, we would not expect the prices to differ.

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[14]An investor using this strategy would have to assume that other investors are not also using it. Otherwise, at an extreme, no new information would enter the system.

Garbade and Silber (1979) examine securities on the NYSE, the Midwest Stock Exchange, and the Pacific Stock Exchange to determine where price discovery takes place. Their method of analysis consists of studying the price variances of several stocks trading on the aforementioned exchanges both before consolidated tape reports became effective and after. The consolidated tape reports reveal trades from all markets, and, therefore may provide investors with more complete information. They find that the regional exchanges are satellites of the NYSE but not the opposite. However, they are not perfect satellites, i.e., after the consolidated tape started operating, the NYSE prices incorporated some information from the regional markets.

Hasbrouck (1995) examines the idea that there is an unknown efficient price that links a set of related securities and that by examining the variance of the change in this price, we can discover where pricing information is coming from. He shows that the mean of the price changes of this unknown efficient price follows a random walk, but the variance is influenced by the variance of the traded securities. Therefore, the objective is to discover which security's movements influences the variance of the efficient price the most. That is where new information first enters the system.

Hasbrouck (1993) uses a method described below to find the upper and lower bounds for information shares of Boeing, IBM, and Exxon over the NYSE and regional stock exchanges. He finds the bounds to be narrow and that the NYSE is largely responsible for price discovery of these stocks. Also, the S&P 500 futures and index are examined, and he finds that the maximum price discovery occurs with the futures.

Choi and Subrahmanyam (1994) use a vector autoregression-vector moving average approach to measure the degree of change in asymmetric information in stocks underlying the Major Market Index. They find that asymmetric information increases in the stocks underlying the MMI after the future is introduced. Theirs is also a bivariate system.

This paper seeks to discover where timely pricing information is coming from in the OEX option market. It will do so using the methodology of Hasbrouck (1995). Unlike the FOW study, this paper will try to determine whether or not there is an explicit link between the OEX and the SPX markets through cointegration analysis. Cointegration implies that there is a common bond linking two or more price series. This common bond is referred to here as a common efficient price.

The common efficient price refers to an equilibrium relationship between the series. Over long periods of time, this relationship will hold. This implies that shocks to the system may cause prices to deviate temporarily from equilibrium or to prevent them from ever reaching equilibrium. However, over time, the relationship assures us that deviations between the observed relationship and the theoretical one remains within certain bounds. Arbitrage possibilities and rational investors guarantee this. In the current situation, deviations from the relationship between the index variables will occur. Pricing information from one or more will inform investors as to how far apart from the equilibrium relationship the prices are. If the prices deviate far enough, arbitrage will force them back together. If one price provides more information about this relationship than the others, then this is where price discovery occurs. This price causes the system to react.

3.2 Method of Analysis

The rational for applying the above ideas to this study is as follows. The prices on the various index variables can be written in general form as $p_t = m_t + s_t$, where p_t is the price of the index, m_t is the efficient implicit price, and s_t is an error term that incorporates market imperfections. We assume that m_t is a random walk where $m_t = m_{t-1} + w_t$ and w_t is a series of homoskedastictic uncorrelated innovations.[15] w_t is the variable that we want to explain, i.e., price discovery is found in w_t. The objective is to find this implicit efficient price in actual prices and decompose its variance in such a manner that the components of the decomposition can be attributed to the individual prices. The proportion of the variance due to a particular security is a relative measure of the information it is providing.

This analysis relies on vector error correction models (VECMs) and vector moving averages (VMAs). A vector error correction model is related to the concept of a vector autoregression and consists of a system of equations. The dependent variable of one equation is regressed on its own lagged values and those of the other dependent variables. This makes all of the variables are endogenous. The difference between a VECM and a vector autoregression (VAR) is that one or more error correction terms are included in each equation of the VECM.

[15]In a more general model, this would become a martingale.

Granger (1981) points out the relationship between cointegration and error correction models. Furthermore, Engle and Granger (1987) prove the Granger Representation Theorem which states that a cointegrated system can be represented as an error correction model, a vector autoregression, or a moving average. A general vector autoregression can be written as

$$y_t = k + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \cdots + \beta_p y_{t-p} + \epsilon_t, \qquad (17)$$

where the y_t's are price variables, the β's are (n x n) matrices of coefficients, and the ε_t are (n x 1) vectors of white noise where cov(ε) is Ω. Including the error correction terms transforms it into a VECM. The error correction terms constrain the variables involved to move together over time. The VECM describes the interactions among the (n x 1) price vector. If we are examining price changes, then $y_t = p_t - p_{t-1}$. Each equation in the model can be estimated separately by ordinary least squares. Although the lagged independent variables may be correlated with previous lagged errors, it is assumed that they are not correlated with contemporaneous errors. So, ordinary least squares produces consistent estimators.

One criticism is that in many applications VAR systems have no theoretical foundation. Sims (1980) answers the criticism by pointing out that even if a theoretical model is derived, tested, and rejected, this does not necessarily mean that there is no relationship among the variables in question. It just means that the theoretical model is wrong. A VAR might point the researcher to various relationships that are not theorized or are counter-theoretical. Vector autoregressions allow the estimating equations to take forms permitted by the VAR format that the data imply, rather than theoretically imposed forms. This is consistent with Granger's (1969) assertion that the summary statistics of the data should aid the researcher in model selection. Since the present analysis seeks to discover the underlying relationship rather than to test a theoretical model, this technique seems to be particularly appropriate.

As mentioned above, the prices may be cointegrated. The prices are cointegrated if they are nonstationary and do not drift too far apart. That is what may be found here and a VECM will be used. If not, the VAR will suffice. Cointegration is evidence of an equilibrium relationship, and the literature indicates that index futures and options belong to an equilibrium relationship. The intuition is that both the future and the option are written on some index, perhaps not the same one, that represents the market. Therefore, arbitrage will force to follow a common trend. If this is the case, a cointegrating vector (error correction term) will have to be added to the VAR regressions, and we get the VECM. The error correction term can be thought of as the amount the price series are away from equilibrium and brings long-run dynamics to the system.

The defining paper on cointegrated time series in economics and finance is Granger (1981). Granger uses the gold markets in New York and London as an example of how cointegration works. If the two markets were completely isolated from each other, factors in one economy might cause the price of gold in that market to drift randomly. The price in the other economy might drift randomly also, and there would be no relationship between the two price series. However, consider the real world.

Prices in one market are known by traders in the other almost as quickly as they are updated. Costs are low, so any price disparity between the two series is quickly arbitraged away. Therefore the two series drift together over time and are said to be cointegrated. The same may hold here with the index derivatives. Whether the SPX proxies for the OEX or vice versa, they are both market indexes. And, to the extent they correctly represent the "market," they will move together; hence, they should be cointegrated.

Of course, this presupposes that both are at least I(1) series. An I(1) variable is one that must be differenced once to achieve stationarity. Stationarity is a concept that indicates the parameters of the distribution are invariant to time. This is important, because various test statistics lose their interpretability if constructed with nonstationary data. Therefore, all price series are tested for unit roots and multiple cointegration.

In addition to single equation models, Engle and Granger (1983) point out that if the data series are cointegrated, there could be problems with a vector autoregression representation. First, if the system is modelled in levels, important constraints will have been excluded. Second, the model will be misspecified if only differenced data is used. Therefore, both levels and differenced data must be used to obtain consistent and efficient estimators. That is done here, and the cointegrating vectors are developed with reference to the first lag of the prices.

Finally, the VECM can be inverted into a vector moving average. This representation best describes how an innovation affects the system and can be written as:

$$y_t = \mu + \psi_0 \epsilon_t + \psi_1 \epsilon_{t-1} + \psi_2 \epsilon_{t-2} + \cdots, \qquad (17)$$

The Judge, Hill, Griffiths, Lütkepohl, and Lee (1985) show that the ψs can be calculated as

$$\psi_0 = I$$

$$\psi_1 = B_1 - I$$

$$\psi_j = B_j - B_{j-1} \quad \text{for } j = 2, \ldots$$

where the *B*s are the coefficients from a VMA that has been derived from a VAR in price levels using recursive substitution.

Sims (1980) believes the VMA is more easily interpreted than the autoregression, since the coefficients of the VAR are difficult to characterize due to the oscillations of the coefficients on successive lags and feedbacks between the equations. Therefore, he suggests examining the system's moving average representation. In this form, we can get a description of how the system reacts to a random shock, which in this paper is the equivalent to new information entering the market. In a mathematical sense, this is equivalent to shocking the system with a positive residual of one unit or standard deviation to each equation in the system. This is most easily seen by examining the ψs.

These are referred to as impulse response functions. They can be interpreted as the response by one variable to a unit increase in another variable's innovation holding all else constant or

$$\frac{\partial y_{i,\,t+s}}{\partial \varepsilon_{jt}}. \tag{18}$$

However, since the errors in the VECM may be contemporaneously correlated, the errors are usually orthogonalized.

The orthogonalization takes the innovations and transposes them into uncorrelated elements. Therefore, the impulses are in terms of the orthogonalized innovations. If Ω is the covariance matrix of the VECM and is symmetric and positive definite, then there exist two triangular matrices such that $PP' = \Omega$ and P is the lower triangle. This is the Cholesky decomposition of Ω. The standard deviation of the orthogonalized innovation lies along the diagonal. The new orthogonalized innovation is $v_t = P^{-1}\varepsilon_t$. So instead of speaking in terms of a unit increase in one variable's innovation, it now becomes an increase in one standard deviation. In other words,

$$\frac{\partial y_{i,\,t+s}}{\partial v_{jt}} = \psi_s p_j. \tag{19}$$

The right-hand side of this equation will become important in the calculation of the information shares.

The process takes the following route. First, we compute the total variance of the implicit efficient price. This is defined by Hasbrouck (1995) as

$$\sigma_w^2 = var(w) = \psi\,\Omega\,\psi' \tag{20}$$

where ψ represents a row vector of the matrix Ψ. Since it is assumed that the variables are related by cointegration, each row vector, ψ, sums to the same value. The reason is that cointegration constrains the price series to have the same reactions to shocks over time. The objective is to find the portion of this variance that can be assigned to each price.

Second, as mentioned before, Ω might include positive values on the off-diagonals (i.e., the errors may be correlated across price series). This will yield a non-unique solution. Therefore, the matrix undergoes a Cholesky decomposition to orthogonalize the errors. This has the effect of establishing bounds on the information shares but has the drawback that the ordering of the series matters. In other words, the first price variable in the system is going to have its share of price discovery maximized, and the last is going to have its share minimized. Therefore, the analysis will require each price series to rotate in the order of appearance.

Finally, the share of the variance attributable to a particular price series is shown in equation (23)

$$S_j = \frac{(\psi P)_j^2}{\sigma_w^2} \qquad (21)$$

and can be referred to as the information share of the *j*th price variable. $(\psi P)^2_j$ is the *j*th element of the corresponding row vector. It should be noted for better understanding that σ^2_w can be written as in equation (22) or as $(\psi P)I(\psi P)'$ with an equivalent value since $\Omega = PP'$. But, equation (21) shows that the impulse response function after orthogonalizing the innovations is (ψP). So, once (ψP) is found, the calculation is relatively easy.

So, the sequence is this: first, test whether the price series are integrated. Second, if they are, test for cointegration. Third, form the basic VECM and test for the number of lags. Fourth, given the number of lags, estimate the VECM and find the variance-covariance matrix of the system. Fifth, do a Cholesky decomposition of this matrix. Sixth, invert the VECM to form a VMA. Since the series are related by arbitrage and, in this case, formally through cointegration, the sums of the rows of the coefficient matrix will be identical. Very simply, this means that since the prices move together over time, their responses over time to new information or shocks have to be equivalent. So, the rows of the coefficient matrix of the VMA are summed to form ψ. Finally, equation 23 is used to find the information shares.

3.3 Data

Three time series are analyzed here. The first is the OEX cash series in five minute increments each trading day for 1986. It comes from the Berkeley Options Database.

The second is an implied OEX five minute series. This is found by retaining all at-the-money OEX options during the day. At-the-money here is defined as being no more than two percent in-the-money nor less than two percent out-of-the-money. Observations are kept at five minute intervals. Then, using the dividends and T-bill rates of return defined in Chapter 1, an implied index series is found by isolating the index variable on one side of the put-call parity equation. The call and put prices are the midpoints of the bid-ask prices.

The last series is the OEX index implied by the SPX futures. A five minute series is formed of SPX futures prices. The contract is the one closest to maturity, because it will have the greatest liquidity. It is rolled over the last day prior to the expiration week. This mitigates the effects of increased volatility as the expiration approaches. Otherwise, the variance would be a function of time, and the series would be nonstationary. The expiration of the futures matches the expiration of the option series. This was done, because, at maturity, the future equals the index value. Using a future with a different maturity from the option introduces more risk and would be much less likely to be used to form a hedge.

To form the OEX implied from the SPX futures, the cash five minute OEX series is regressed on the SPX futures everyday. Then, the day *t-1* coefficients are applied to the day *t* SPX futures. Also, since the day *t-1* coefficients are being used, the analysis will begin with the second day in the database, January 3, 1986. There are 17,368 observations for each of the three series in 1986.

3.4 Tests

Initially, the series are tested for unit roots. The null hypothesis is that there is a unit root in the series. The Phillips-Perron [PP] unit root test is a nonparametric test. The critical values of the test statistics do not have to be calculated for different data generating processes. The PP test takes the test statistic and modifies it to correct for autocorrelation. To be more specific, the PP test uses a nonparametric correction in the test statistic for the serial correlation. So, it can be used with many different data generating processes (for instance, see Banerjee, Dolado, Galbraith, and Hendry, (1993).[16] The test is run both with and without a trend in the model. With the time trend in the model, the null hypothesis is that the coefficient on the time trend and the lagged dependent variable are jointly equal to zero. This results in an *F*-statistic. The results in Table 18 show that the null hypothesis of a unit root cannot be rejected for all series in either model. Therefore, a search for cointegrating vectors is undertaken.

Table 18. Results of the Phillips-Perron Test

Phillips-Perron Test	OEX Cash	OEX from SPX	OEX from Put-Call Parity
With Constant, no Trend	-2.1654	-2.0295	-2.0958
With Constant and Trend	2.7654	2.4534	2.6045

The values of the statistics do not allow the rejection of the null hypothesis of a unit root. The critical value of the Phillips-Perron test the .01 level for an infinite sample size is: -3.43 (*t* value) with constant but no trend and 6.25 (*F* value) with constant and trend.

It is important to determine the number of cointegrating vectors in a multivariate system. If the number of vectors is underestimated, important information is being left out of the system in the nature of the omitted cointegrating vector(s). If the number is overestimated, Banerjee, Dolado, Galbraith, and Hendry (1993) state that the statistics'

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[16]A study by Tae-Hwy Lee and Yiuman Tse (1995) suggests that the Dickey-Fuller test for stationarity is relatively robust to GARCH. They find it tends to over-reject slightly the null of no cointegration. No cointegration means that the residuals of the cointegrating regression are I(1). So, over-rejecting the null of no cointegration is tantamount to over-rejecting the null that a variable is I(1) or nonstationary. The Phillips-Perron test may be more robust since it is based on nonparametrics. This is a worthy topic for future study.

distributions will not be standard. Johansen and Juselius (1990) suggest using a likelihood ratio test. Two test statistics are examined:

$$\eta_r = -(T-1)\sum_{i=r+1}^{n}\log(1-\lambda_i),\ where\ r = 0,1,\ldots,n-1 \quad (22)$$

and

$$\zeta_r = -(T-1)\log(1-\lambda_{r+1}),\quad where\ r = 0,1,\ldots,n-1 \quad (23)$$

where T is the number of observations, r is the hypothesized number of cointegrating vectors, n is the number of time series, and the λ's are the eigenvalues from the log-likelihood function of the differenced series. η is the trace statistic, and the null hypothesis is that there are r cointegrating vectors. ζ is the maximal-eigenvalue statistic, and it tests the significance of the largest λ_r. The critical values of the test statistics are from Osterwald-Lenum (1992). Osterwald-Lenum generates several tables for different data generating processes. The data generation process that is used to calculate the test statistics corresponds most closely to the one used in his Table 1.1*, which includes no intercept and no time trend. The Osterwald-Lenum calculates the tables, because each data generation process has a different asymptotic distribution. So, care must be taken in selecting the set of critical values by which to judge the test statistic. The tests agree that there are at most two cointegrating vectors. Table 19 gives the test statistics and critical values. The eigenvalues are λ_1=0.3038201, λ_2=0.0836322, and λ_3=0.0000369.

The cointegrating vectors are chosen and interpreted as follows. For the equation with the dependent variable being the price change of the cash OEX, the vectors are $(P_{PCP,t-1}-P_{OEX,t-1})$ and $(P_{SPX,t-1}-P_{OEX,t-1})$. The coefficients of the vectors can be thought of as the degree the cash OEX is out of equilibrium with the other price series. The equation with the OEX implied from put-call parity will incorporate the vectors $(P_{PCP,t-1}-P_{OEX,t-1})$ and $(P_{PCP,t-1}-P_{SPX,t-1})$, and the final equation will use $(P_{SPX,t-1}-P_{OEX,t-1})$ and $(P_{PCP,t-1}-P_{SPX,t-1})$. Like the first equation, the coefficients on the error correction vectors in these final two equations can be thought of as a measure of arbitrage that has been generated by the difference in the two prices.

Table 19. Test Results for the Number of Cointegrating Vectors

	ζ Maximal-Eigenvalue Test	Critical Value for ζ	η Trace Test	Critical Value for η
n-3 = r = 0	5565.48	25.75	7807.29	37.22
n-2 = r = 1	1516.88	19.19	1517.52	23.52
n-1 = r = 2	0.64	11.65	0.64	11.65

Critical values are for the .01 level of significance obtained from Osterwald-Lenum's (1992) Table 1.1*. This was chosen, because the data generation process in this paper most closely matches the process in his Table 1.1*.

The VECM consists of three equations. In each, a differenced variable is regressed on the two error correction terms and a finite number of lags of the differenced variable along with the lags of the other differenced variables. This calls for a test to determine the appropriate number of lags. To test for the number of lags to include in a vector autoregression, Sims (1980) suggests using a likelihood ratio test of the form

$$(T-k)\ (\log|D_R|-\log|D_U|) \tag{24}$$

where T is the number of observations, k is the number of regression coefficients being estimated divided by the number of equations, and D_R and D_U are the matrices of cross products of residuals for the restricted and unrestricted models respectively. He demonstrates that the usual form of the likelihood ratio test is biased against the null hypothesis. It is found that five lags is sufficient. The model is as follows:

$$dp_{OEX,t} = \alpha_1(P_{PCP,t-1}-P_{OEX,t-1}) + \alpha_2(P_{SPX,t-1}-P_{OEX,t-1}) + \beta_1 dp_{OEX,t-1} + \ldots + \beta_7 dp_{OEX,t-5} + \beta_8 dp_{SPX,t-1} + \ldots + \beta_{14} dp_{SPX,t-5} + \beta_{15} dp_{PCP,t-1} + \ldots + \beta_{21} dp_{PCP,t-5} + \varepsilon_{1,t}$$

$$dp_{PCP,t} = \alpha_3(P_{PCP,t-1}-P_{OEX,t-1}) + \alpha_4(P_{PCP,t-1}-P_{SPX,t-1}) + \delta_1 dp_{PCP,t-1} + \ldots + \delta_7 dp_{PCP,t-5} + \delta_8 dp_{SPX,t-1} + \ldots + \delta_{14} dp_{SPX,t-5} + \delta_{15} dp_{OEX,t-1} + \ldots + \delta_{21} dp_{OEX,t-5} + \varepsilon_{2,t}$$

$$dp_{SPX,t} = \alpha_5(P_{PCP,t-1}-P_{SPX,t-1}) + \alpha_6(P_{SPX,t-1}-P_{OEX,t-1}) + \lambda_1 dp_{SPX,t-1} + \ldots + \lambda_7 dp_{SPX,t-5} + \lambda_8 dp_{PCP,t-1} + \ldots + \lambda_{14} dp_{PCP,t-5} + \lambda_{15} dp_{OEX,t-1} + \ldots + \lambda_{21} dp_{OEX,t-5} + \varepsilon_{3,t} \tag{25}$$

where $dp_{OEX,t}$ is the difference between the current value of the cash OEX and the value one period (five minutes) earlier. $dp_{PCP,t}$ and $dp_{SPX,t}$ are similar values for the OEX index implied by put-call parity and the SPX futures respectively. A constant term is not included. The constant would represent an average return. Since the data is measured over five minute intervals, it would be negligible and is therefore left out. In a simpler model without error correction terms and with only differenced prices, each equation could be viewed as the difference between one equation containing the current prices and its lags and another equation containing those prices lagged one period. The constant would drop out naturally in this case.

Table 20 gives the results of one ordering of the VECM. This particular ordering has the OEX cash index first followed by the index implied by the SPX and then the PCP index. Although the purpose of this paper is not the interpretation of the VECM, some comments may be in order. First, the cointegrating vectors relative to the cash OEX index are both positive. Cointegration implies that the difference between the two variables in the vector is constant except for error. This indicates that as errors from the implied indexes get "too big," the vector becomes larger, and because the coefficients are positive, the change in the cash OEX is increased, thus correcting the error. Or, in finance terminology, an investor would short the implied index and buy the cash.

Second, the vectors relative to the index implied by put-call parity (PCP) show that as the PCP index becomes larger, the error correction term grows, and since the coefficient is negative, the correction is a reduction in the size of the change in PCP. In other words, short the PCP and buy the other two indexes. This is in accordance with the interpretation of the OEX vectors above.

Table 20. Vector Error Correction Model Results.

	Dependent Variable					
	$dp_{OEX,t}$		$dp_{PCP,t}$		$dp_{SPX,t}$	
Variable	Coeff.	T-stat.	Coeff.	T-stat.	Coeff.	T-stat.
$p_{SPX,t-1}-p_{OEX,t-1}$	0.020	8.034**			-0.019	-8.179**
$p_{PCP,t-1}-p_{SPX,t-1}$			-0.109	-12.180**	-0.002	-1.348
$p_{PCP,t-1}-p_{OEX,t-1}$	0.007	4.368**	-0.194	-7.508**		
$dp_{OEX,t-1}$	-0.518	-64.092**	0.046	0.951	-0.001	-0.121
$dp_{OEX,t-2}$	0.182	20.115**	0.324	6.045**	-0.008	-1.082
$dp_{OEX,t-3}$	-0.140	-15.456**	0.076	1.425	-0.004	-0.577
$dp_{OEX,t-4}$	0.047	5.411**	0.129	2.486*	0.127	1.711
$dp_{OEX,t-5}$	-0.003	-0.444	0.098	2.186*	0.007	1.087
$dp_{PCP,t-1}$	-0.004	-2.232*	-0.425	-38.906**	0.003	1.932
$dp_{PCP,t-2}$	-0.001	-0.514	-0.296	-27.361**	0.004	2.855**
$dp_{PCP,t-3}$	0.0002	0.119	-0.238	-23.163**	0.004	2.570*
$dp_{PCP,t-4}$	-0.001	-0.510	-0.158	-16.861**	0.002	1.855
$dp_{PCP,t-5}$	-0.001	-1.103	-0.090	-11.898**	0.002	1.625
$dp_{SPX,t-1}$	0.463	49.099**	0.314	5.623**	0.019	2.319*
$dp_{SPX,t-2}$	0.159	15.806**	0.199	3.323**	-0.017	-1.951
$dp_{SPX,t-3}$	0.102	10.004**	0.125	2.072*	-0.012	-1.356
$dp_{SPX,t-4}$	0.023	2.307*	0.084	1.403	-0.002	-0.187
$dp_{SPX,t-5}$	0.029	2.910**	-0.025	-0.418	-0.004	-0.471

**Significant at the .01 level, *Significant at the .05 level; $dp_{OEX,t}$ is the price change in the cash OEX index, $dp_{PCP,t}$ is the price change in the OEX index implied by put-call parity, and $dp_{SPX,t}$ is the price change in the OEX index implied by the SPX futures

Third, the SPX implied index has a negative coefficient on the cointegrating vector relative to the cash OEX. This is a good fit to what the same vector is saying in the cash OEX equation: short the SPX implied index and buy the cash index. The coefficient on the vector relative to the PCP index is also negative as expected but insignificant. Except for this last piece of information, the relationships are all consistent.

Fourth, Huang and Stoll (1995) test the predictive power of the SPX futures' lagged returns relative to stocks. They find that the futures' ability to predict stock quote and price returns is significantly greater than zero. As can be seen in Table 20, all of the lagged SPX implied index variables are significantly influencing the current cash OEX. This is consistent with Huang and Stoll's findings. On the other hand, the OEX cash index does not seem to be a significant contributor to movements in the SPX implied index. However, it should be remembered that the SPX implied index is based on the previous day's relationship between the SPX futures and the cash OEX.

As the coefficients of the VECM may be difficult to interpret, and the effect of an innovation is more easily determined from a VMA, a the VECM is converted into a VMA. The impulse response functions are carried out over twenty periods to aid in the search for the information shares. As mentioned before, because of the particular factorization being used, it matters which price series comes first and which comes last. One such set of VMA coefficients is found in Table 21, but the model is run three times. This allows all price series to appear once as the first series and once as the last. The results are in Table 22. A step by step description of the procedure can be found

Table 21. Impulse Response Functions for the Orthogonalized Innovations. Each column represents a particular variable's response to an innovation plus the response by the two cointegrating vectors associated with that variable.

5 minute increments	Implied Indices		
	OEX	SPX	PCP
1	0.19495	0.04093	0.05549
2	-0.03657	-0.01280	-0.01126
3	0.09152	0.00690	0.02540
4	-0.07499	-0.01156	-0.02117
5	0.09171	0.02222	0.02327
6	-0.08252	-0.01919	-0.02056
7	0.07194	0.01347	0.01986
8	-0.06837	-0.01266	-0.01863
9	0.06510	0.01224	0.01777
10	-0.06094	-0.01129	-0.01667
.11	0.05857	0.01163	0.01580
12	-0.05508	-0.01073	-0.01486
13	0.05158	0.00970	0.01405
14	-0.02212	0.01742	-0.01330
15	0.04613	0.00879	0.01256
16	-0.04350	-0.00829	-0.01181
17	0.04117	0.00784	0.01113
18	-0.03885	-0.00742	-0.01056
19	0.03664	0.00692	0.00990
20	-0.03465	-0.00660	-0.00943
	0.23172	0.05752	0.05698

The last row in each column is the sum of that column.

Table 22. Information Shares

	Maximum Share	Minimum Share
Cash OEX	0.8931	0.8912
OEX Implied by Put-Call Parity	0.0868	0.0539
OEX Implied by SPX Futures	0.0224	0.0049

in Appendix B. It should be noted that the responses are to shocks that have gone through the system including the cointegrating terms. The summed columns here correspond to a summed row in the ψ matrix. The time interval represents five minutes. These and different orderings are used to compute the information shares in Table 22.

The OEX cash index and the PCP index each contribute a significant amount of information. The importance that investors are putting on the OEX is understandable. It is the stated underlying variable for the options, and if the options are exercised, they will be exercised into a cash value representing the OEX, not the SPX. Also, and most important, the OEX is made up of very liquid and highly monitored stocks. So. information relevant to the pricing of these securities is rapidly impounded into their prices.

It is surprising that the SPX proxy index is contributing virtually nothing to price discovery, because it is also liquid and possibly less costly than the OEX options market. In fact, Hasbrouck (1993) finds the SPX futures' information share to range between 86.5% and 96.7% relative to the SPX cash index. But, it should be remembered that the SPX futures' information share is being measured against the OEX here and that the method used to form the SPX proxy for the OEX may not be the same as used by actual traders. Their actual thought processes are not known, but they must observe the price changes in the SPX and then transform it into some measure relative to the OEX. In addition, SPX futures traders must get their trading information from somewhere. If they price the futures off of the cash SPX, they will run into the same problems as the OEX option traders. However, FOW (1995) show that this is not the case. Therefore, OEX option traders who price off of the SPX futures must assume that not only are the SPX traders correctly pricing the futures, but that their own conversion from the SPX future to the OEX is correct. This injects two sources of risk into the situation. It may be less risky to just use the cash OEX.

The index implied by PCP also provides a pricing information. The analysis in Chapter 2 of this dissertation shows that index options can proxy for the index and used to identify and capture mispricings through arbitrage trades. These trades are providing information about the relative values of the options to other traders. And, as long as there are few constraints on arbitragers, the options will be priced efficiently, hence investors will be able to imply a timely index price from them. Also, transactions costs are very low. Fleming and Whaley (1995) show that transactions costs are much lower to trade a portfolio of index options than it is to trade the basket of stocks replicating the index. Chapter 2 of this dissertation also points that out. Informed investors are going to place their trades in low cost liquid markets. The OEX option market is such a market, so it should not be surprising that some price discovery takes place there.

Perhaps it is more surprising that a larger proportion of price discovery takes place in the higher cost cash OEX market.

3.5 Chapter 3 Summary

OEX options have no single traded security underlying them. This makes valuation difficult. It obscures the source of relevant pricing information. For a market to be efficient, timely information must be available to investors to act on. Various pricing proxies have been suggested as being suitable for the purpose, such as the SPX futures.

The methodology used here to discover where investors are obtaining their information relies on the concept that a group of securities is linked by arbitrage. In a statistical sense, they are cointegrated. Both terms imply that the price movements of all of the securities will move together over time. Therefore, there must be an equilibrium price or implied efficient price that is common to all of the price series. The technique seeks to identify the efficient price variance, and once that is accomplished, to attribute portions of that variance to the actual price series. The series claiming the highest percentage of the variance is the one that is providing the most timely information to the market.

The price series in question are the OEX cash index and two proxies: one formed from the SPX futures and the other from index options. Extant literature has shown impirically (i.e., Fleming, Ostdiek, and Whaley (1995)) that the SPX futures provide price discovery to the OEX options. However, here it is found that little of the variance in the implied efficient price can be attributed to the futures. This could be due to the way the index was constructed. Future research should investigate the system using the actual SPX futures prices rather than the implied OEX index.

The largest source of price discovery is found in the OEX index itself. Since the OEX market is very liquid, this should not be too surprising. It is somewhat counterintuitive, because of the three markets, this is the most costly. The next largest source of price discovery is found in the index implied by put-call parity. Since the market is relatively efficient and low-cost, it should not be surprising that option traders can imply an index from put-call parity that provides timely information.

CONCLUSION

This dissertation has the following major contributions. First, it finds a reasonable estimate of early exercise for American options. In finding this value, it is demonstrated how data collected at different times and over different cross-sections can affect the results. Then, it is shown how this value can be used to with the Black-Scholes option pricing model to value American style options more efficiently. If these securities are priced closer to their efficient values, resources can be allocated more effectively to other sectors of the economy. Also, to the extent that investors use these options for price discovery, other assets will be priced more efficiently.

Second, various proxies have been suggested in the literature for the underlying index to be used in forming arbitrage portfolios. The effect of arbitrage is to identify and capture mispricings. By doing so, option prices are forced back toward equilibrium levels. It is shown that forming optimal mimicking portfolios of stocks is risky due to the tracking error. However, proxies made up from other options or index futures are viable alternatives.

Third, a source of price discovery is found. Using three proxies for the OEX, it is found that the cash OEX is the variable investors look to for most of their information. The actual index options are also important sources. The SPX provides little or no contribution to price discovery. This is the reverse of what other researchers have found, but this discrepancy may be due to the way in which the OEX proxy was formed from the SPX futures.

The dissertation also implies several extensions. First, further work is indicated in the index arbitrage study. Index fund managers face a tradeoff between tracking error and transactions costs. Optimal mimicking portfolios could be developed with these variables in mind. Also other methods of developing optimal portfolios could be developed. For instance, instead of forming a portfolio whose return tracks the index's return, a portfolio whose securities have the same beta as the index could be tried.

Second, a study could be done on the effect of SPDR's on index arbitrage. While the volume is currently less than the SPX futures, these are becoming more important to fund managers. The study could look at index arbitrage both before and after SPDR's became available.

And, third, the methodology used in the price discovery chapter has never been used with equity options. Currently, researchers have found conflicting directions of price discovery between equity options and the underlying options. This technique could be used to help resolve the issue.

BIBLIOGRAPHY

Banerjee, Anindya, Juan Dolado, John W. Galbraith, and David F. Hendry, 1993. *Co-integration, Error Correction, and the Econometric Analysis of Non-stationary Data*(Oxford University Press, New York, NY).

Black, Fischer, 1975, Fact and Fantasy in the Use of Options, *Financial Analysts Journal*, 31, 36-41, 61-72.

______________ and Myron Scholes, 1973, The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, 81, 637-654.

Blomeyer, Edward C. and Herb Johnson, 1988, An Empirical Examination of the Pricing of American Put Options, *Journal of Financial and Quantitative Analysis*, 23, 13-22.

Bodurtha, James N. Jr. and Georges R. Courtadon, 1986, Efficiency Tests of the Foreign Currency Options Market, *Journal of Finance*, 41, 151-162.

Brenner, Menachem and Dan Galai, 1986, Implied Interest Rates, *Journal of Business*, 59, 493-507.

Chan, Kalok, Y. Peter Chung, and Herb Johnson, 1993, Why Option Prices Lag Stock Prices: A Trading-based Explanation, *Journal of Finance*, 48, 1957-1967.

Chance, Don, 1988, Boundary Condition Tests of Bid and Ask Prices of Index Call Prices, *Journal of Financial Research*, 11, 21-31.

Chen, Nai-fu and Herb Johnson, 1985, Hedging Options, *Journal of Financial Economics*, 14, 317-321.

Chiras, Donald P. and Steven Manaster, 1978, The Information Content of Option Prices and a Test of Market Efficiency, *Journal of Financial Economics*, 6, 213-234.

Choi, Hong and Avanidhar Subrahmanyam, 1994, Using Intraday Data to Test for Effects of Index Futures on the Underlying Stock Markets, *Journal of Futures Markets*, 14, 293-322.

Corrado, Charles J. and Thomas W. Miller, Jr., 1993, Optimal Volatility Estimates Based on Black-Scholes Implied Standard Deviations, Working Paper, University of Missouri, 1-22.

Cox, John C. and Mark Rubinstein, 1985. *Options Markets* (Prentice-Hall, Englewood Cliffs, NJ).

Day, Theodore E. and Craig M. Lewis, 1988, The Behavior of the Volatility Implicit in the Prices of Stock Index Options, *Journal of Financial Economics*, 22, 103-122.

Diz, Fernando and Thomas J. Finucane, 1993, The Rationality of Early Exercise Decisions: Evidence from the S&P 100 Index Options Market, *The Review of Financial Studies*, 6, 765-797.

Engle, Robert F. and C.W.J. Granger, 1987, Co-integration and Error Correction: Representation, Estimation, and Testing, *Econometrica*, 55, 251-276.

Evnine, Jeremy and Andrew Rudd, 1985, Index Options: The Early Evidence, *The Journal of Finance*, 40, 743-756.

______, ______ and ______ ____, 1983, "Stock Index Options: Preliminary Results." Paper presented at the Fall Seminar of the Fall Seminar of the Berkeley Program in Finance, Silverado, California, September 11-14.

Figlewski, Steven, 1984, Hedging Performance and Basis Risk in Stock Index Futures, *Journal of Finance*, 39, 657-669.

_________, ______, 1989, Options Arbitrage in Imperfect Markets, *Journal of Finance*, 44, 1289-1311.

Fleming, Jeff and Robert E. Whaley, 1994, The Value of Wildcard Options, *Journal of Finance*, 49, 215-236.

_______, ____ Barbara Ostdiek, and Robert E. Whaley, 1995, Trading Costs and the Relative Rates of Price Discovery in Stock, Futures, and Option Markets, working paper, Rice University, 44 pages.

Frankfurter, George M. and Wai K. Leung, 1991, Further Analysis of the Put-Call Parity Implied Risk-Free Interest Rate, *Journal of Financial Research*, 14, 217-232.

French, Dan W. and Edwin D. Maberly, 1992, Early Exercise of American Index Options, *The Journal of Financial Research*, 2, 127-137.

Fuller, W.A. and G.E. Battese, 1974, Estimation of Linear Models with Crossed-Error Structure, *Journal of Econometrics*, 2, 67-78.

Garbade, Kenneth D. and William L. Silber, 1979, Dominant and Satellite Markets: A Study of Dually-Traded Securities, *Review of Economics and Statistics*, 61, 455-460.

_______________, 1981, Some Properties of Time Series Data and Their Use in Econometric Model Specification, *Journal of Econometrics*, 16, 121-130.

Gastineau, Gary L., 1991, A Short History of Program Trading, *Financial Analysts Journal*, September-October, 4-7.

Geske, R. and Herb Johnson, 1984, The American Put Option Valued Analytically, *Journal of Finance*, 39, 1511-1524.

_________ and R. Roll, 1984, On Valuing American Call Options with the Black-Scholes European Formula, *Journal of Finance*, 39, 443-455.

Granger, C. W. J., 1969, Investigating Causal Relations by Econometric Models and Cross-Spectral Methods, *Econometrica*, 37, 424-438.

Grossman, Sanford J., 1988, An Analysis of the Implications for Stock and Futures Price Volatility of Program Trading and Dynamic Hedging Strategies, *Journal of Business*, 61, 275-298.

Hamilton, James D., 1993. *Econometric Analysis of Time Series* (MacMillan, New York, New York).

Harvey, Campbell R. and Robert E. Whaley, 1992a, Dividends and S&P 100 Index Option Valuation, *Journal of Futures Markets*, 12, 123-137.

______, ________ __ and ______ __ ______, 1992b, Market Volatility Prediction and the Efficiency of the S&P 100 Index Option Market, *Journal of Financial Economics*, 31, 43-73.

Hasbrouck, Joel, 1993, One Security, Many Markets: Determining the Contributions to Price Discovery, Working paper, New York University, 1-37.

_________, ____, 1995, One Security, Many Markets: Determining the Contributions to Price Discovery, forthcoming in *Journal of Finance*, 1-40.

Hsiao, C., 1975, Some Estimation Methods for a Random Coefficient Model, *Econometrica*, 43, 305-325.

Johansen, S. and K. Juselius, 1990, Maximum Likelihood Estimation and Inference on Cointegration -- with Applications to the Demand for Money, *Oxford Bulletin of Economics and Statistics*, 52, 169-210.

Judge, George G., W.E. Griffiths, R. Carter Hill, Helmut Lütkepohl, Tsoung-Chao Lee, 1985. *The Theory and Practice of Econometrics* (John Wiley and Sons, New York).

Kane, Alex and Alan Marcus, 1986, Valuation and Optimal Exercise of the Wild Card Option in the Treasury Bond Futures Market, *Journal of Finance*, 41, 195-207.

Lee, Jae Ha and Nandkumar Nayar, 1993, A Transactions Data Analysis of Arbitrage between Index Options and Index Futures, *Journal of Futures Markets*, 13, 889-902.

Lee, Tae-Hwy, and Yiuman Tse, 1995, Cointegration with Conditional Heteroskedasticity, forthcoming in *The Journal of Econometrics*.

Leland, Hayne E., 1980, Who Should Buy Portfolio Insurance?, *Journal of Finance*, 35, 581-594.

Merton, Robert C., 1973a, The Theory of Rational Option Pricing, *Bell Journal of Economics and Management Science*, 4, 141-183.

__________________, 1973b, The Relationship Between Put and Call Option Prices: Comment, *Journal of Finance*, 28, 183-184.

Osterwald-Lenum, Michael, 1992, A Note with Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics, *Oxford Bulletin of Economics and Statistics*, 54, 461-471.

Phillips, P. and P. Perron, 1988, Testing for a Unit Root in Time Series Regression, *Biometrika*, 75, 335-346.

Phillips, Susan M. and Clifford W. Smith, Jr., 1980, Trading Costs for Listed Options, *Journal of Financial Economics*, 8, 179-201.

Rubinstein, Mark, 1989, Market Basket Alternatives, *Financial Analysts Journal*, September-October, 20-61.

Sheikh, Aamir M., 1991, Transaction Data Tests of S&P 100 Call Option Pricing, *Journal of Financial and Quantitative Analysis*, 26, 459-475.

Sims, Christopher A., 1980, Macroeconomics and Reality, *Econometrica*, 48, 1-49.

Stoll, Hans R., 1969, The Relationship Between Put and Call Option Prices, *Journal of Finance*, 24, 801-824.

______________, 1973, Reply, *Journal of Finance*, 28, 185-187.

______________, 1988, Futures and Options on Stock Indexes: Economic Purpose, Arbitrage, and Market Structure, *Review of Futures Markets*, 7, 224-248.

______________ and Robert Whaley, 1990, The Dynamics of Stock Index and Stock Index Futures Returns, *Journal of Financial and Quantitative Analysis*, 25, 441-468.

Swinnerton, Eugene A., Richard J. Curcio, and Richard E. Bennett, 1988, Index Arbitrage Program Trading and the Prediction of Intraday Stock Price Changes, *Review of Futures Markets*, 7, 301-323.

Valerio, Nicholas III, 1993, Valuation of Cash-Settlement Call Options Containing a Wild-Card Exercise Feature, *Journal of Financial Engineering*, 2, 335-364.

Whaley, Robert E., 1982, Valuation of American Call Options on Dividend-Paying Stocks, *Journal of Financial Economics*, 10, 29-58.

Zivney, Terry L., 1991, The Value of Early Exercise in Option Prices: An Empirical Investigation, *Journal of Financial and Quantitative Analysis* 26, 129-138.

APPENDIX A

Table A1. Summary of Initial Five Minute Interval Data

Month 1986	Initial Number of Prices	Number of Non-synchronous Prices	Final Number of Prices Matching for Put-Call Parity[1]
January	103967	3741	100226
February	91664	2930	88734
March	81136	1628	79508
April	90784	2598	88186
May	82940	2700	80240
June	85121	1847	83274
July	83967	5187	78780
August	79691	2707	76984
September	78607	2525	76082
October	82588	1766	80822
November	70795	1391	69404
December	76777	2487	74290
Total	1,008,038	31,508	976,530

[1]These figures are before deleting expiration week observations.

Table A2. Statistics on the Variables

Variable	Mean	Standard Dev.	Minimum	Maximum
S&P 100 Index	224.957	10.5292	195.840	242.030
Strike Price	223.446	14.1981	175.000	255.000
Call Bid	8.6837	6.215	0.5000	34.000
Put Ask	6.9092	5.473	0.5625	35.000
T-bill Rates	0.0602	0.007	0.0375	0.0782
Disc. Divs.	1.4234	0.7422	0.2223	3.2332
Days to Mat.	66.05	0.084	8.00	123.999

APPENDIX B

To find the information shares, the innovations are orthogonalized as outlined in the text. The rows of the impulse response functions of the orthogonalized innovations are summed. One such summation can be seen in Table 22. This leads to the ψ summed matrix values, 0.23172, 0.05752, and 0.0698 corresponding to the cash OEX, SPX implied OEX, and PCP implied OEX, respectively.

Then, using the ψP matrix, the variance of the system is found:

$$\sigma^2 = (\psi P)I(\psi P)' = 0.060249$$

Finally, the information shares are derived as $(\psi P)^2_j \,/\, \sigma^2$, and in this case are 0.8912, 0.0549, and 0.0539 for the cash OEX, the OEX implied by the futures, and the OEX implied by put-call parity.

VITA

Bruce Kelvin Grace received his B.S. in Finance and M.B.A. degrees from the University of New Orleans. He has worked for Merrill Lynch as a Financial Consultant and has operated his own management consulting firm. On receiving a Graduate School Fellowship from Louisiana State University, he entered and completed the Ph.D. program in Business Administration with a concentration in Finance in 1995. His first position after having completed the requirements for the Ph.D. was with Clarkson University in Potsdam, New York as a Visiting Assistant Professor.

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