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**ABSTRACT**

Standard representative-agent models fail to account for the weak correlation between stock returns and measurable fundamentals, such as consumption and output growth. This failing, which underlies virtually all modern asset-pricing puzzles, arises because these models load all uncertainty onto the supply side of the economy. We propose a simple theory of asset pricing in which demand shocks play a central role. These shocks give rise to valuation risk that allows the model to account for key asset pricing moments, such as the equity premium, the bond term premium, and the weak correlation between stock returns and fundamentals.

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# 1. Introduction

In standard representative-agent asset-pricing models, the expected return to an asset reflects the covariance between the asset's payoff and the agent's stochastic discount factor. An important challenge to these models is that the correlation and covariance between stock returns and measurable fundamentals, especially consumption growth, is weak at both short and long horizons. Cochrane and Hansen (1992), Campbell and Cochrane (1999), and Cochrane (2001) call this phenomenon the *correlation puzzle*. More recently, Lettau and Ludvigson (2011) document this puzzle using different methods. According to their estimates, the shock that accounts for the vast majority of asset-price fluctuations is uncorrelated with consumption at virtually all horizons.

The basic disconnect between measurable macroeconomic fundamentals and stock returns underlies virtually all modern asset-pricing puzzles, including the equity-premium puzzle, Hansen-Singleton (1982)-style rejection of asset-pricing models, violation of Hansen-Jagannathan (1991) bounds, and Shiller (1981)-style observations about excess stock-price volatility.

A central finding of modern empirical finance is that variation in asset returns is overwhelmingly due to variation in discount factors (see Campbell and Ammer (1993) and Cochrane (2011)). A key question is: how should we model this variation? In classic asset-pricing models, all uncertainty is loaded onto the supply side of the economy. In Lucas (1978) tree models, agents are exposed to random endowment shocks, while in production economies they are exposed to random productivity shocks. Both classes of models abstract from shocks to the demand for assets. Not surprisingly, it is very difficult for these models to simultaneously resolve the equity premium puzzle and the correlation puzzle.

We propose a simple theory of asset pricing in which demand shocks, arising from stochastic changes in agents' rate of time preference, play a central role in the determination of asset prices. These shocks amount to a parsimonious way of modeling the variation in discount rates stressed by Campbell and Ammer (1993) and Cochrane (2011). Our model implies that the law of motion for these shocks plays a first-order role in determining the equilibrium behavior of variables like the price-dividend ratio, equity returns and bond yields. So, our analysis is disciplined by the fact that the law of motion for time-preference shocks must be consistent with the time-series properties of these variables.

In our model, the representative agent has recursive preferences of the type considered by

Kreps and Porteus (1978), Weil (1989), and Epstein and Zin (1991). When the risk-aversion coefficient is equal to the inverse of the elasticity of intertemporal substitution, recursive preferences reduce to constant-relative risk aversion (CRRA) preferences. We show that, in this case, time-preference shocks have negligible effects on key asset-pricing moments such as the equity premium.

We consider two versions of our model. The benchmark model is designed to highlight the role played by time-preference shocks per se. Here consumption and dividends are modeled as random walks with conditionally homoscedastic shocks. While this model is very useful for expositional purposes, it suffers from some clear empirical shortcomings, e.g. the equity premium is constant. For this reason, we consider an extended version of the model in which the shocks to the consumption and dividend process are conditionally heteroskedastic. We find that adding these features improves the performance of the model.<sup>1</sup>

We estimate our model using a Generalized Method of Moments (GMM) strategy, implemented with annual data for the period 1929 to 2011. We assume that agents make decisions on a monthly basis. We then deduce the model's implications for annual data, i.e. we explicitly deal with the temporal aggregation problem.<sup>2</sup>

It turns out that, for a large set of parameter values, our model implies that the GMM estimators suffer from substantial small-sample bias. This bias is particularly large for moments characterizing the predictability of excess returns and the decomposition of the variance of the price-dividend ratio proposed by Cochrane (1992). In light of this fact, we modify the GMM procedure to focus on the plim of the model-implied small-sample moments rather than the plim of the moments themselves. This modification makes an important difference in assessing the model's empirical performance.

We show that time-preference shocks help explain the equity premium as long as the risk-aversion coefficient and the elasticity of intertemporal substitution are either both greater than one or both smaller than one. This condition is satisfied in the estimated benchmark and extended models.

Allowing for sampling uncertainty, our model accounts for the equity premium and the volatility of stock and bond returns, even though the estimated degree of agents' risk aversion is very moderate (roughly 1.5). Critically, the extended model also accounts for the mean,

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<sup>1</sup>These results parallel the findings of Bansal and Yaron (2004) who show that allowing for conditional heteroskedasticity improves the performance of long-run risk models.

<sup>2</sup>Bansal, Kiku, and Yaron (2013) pursue a similar strategy in estimating a long-run risk model. They estimate the frequency with which agents make decisions and find that it is roughly equal to one month.

variance and persistence of the price-dividend ratio and the risk-free rate. In addition, it accounts for the correlation between stock returns and fundamentals such as consumption, output, and dividend growth at short, medium and long horizons. The model also accounts for the observed predictability of excess returns by lagged price-dividend ratios.

We define *valuation risk* as the part of the excess return to an asset that is due to the volatility of the time-preference shock. According to our estimates, valuation risk is a much more important determinant of asset returns than conventional risk. We show that valuation risk is an increasing function of an asset's maturity. So, a natural test of our model is whether it can account for bond term premia and the return on stocks relative to long-term bonds. We pursue this test using stock returns as well as real returns on bonds of different maturity and argue that the model's implications are consistent with the data.

An important distinction between our model and leading alternatives lies in the predictions for the slope of the yield curve. With this distinction in mind, we discuss evidence on the slope of the yield curve obtained using data for the U.S. and the U.K. We then assess the quantitative implications of different models for the slope of the yield curve. Finally, we show that the estimated SDF for our model has the properties that Backus and Zin (1994) argue are necessary to simultaneously account for the slope of the yield curve and the persistence of bond yields.

There is a literature that models shocks to the demand for assets as arising from time-preference or taste shocks. For example, Garber and King (1983) and Campbell (1986) consider these types of shocks in early work on asset pricing. Stockman and Tesar (1995), Pavlova and Rigobon (2007), and Gabaix and Maggiori (2013) study the role of taste shocks in explaining asset prices in open economy models. In the macroeconomic literature, Eggertsson and Woodford (2003) and Eggertsson (2004), model changes in savings behavior as arising from time-preference shocks that make the zero lower bound on nominal interest rates binding.<sup>3</sup> Hall (2014) stresses the importance of variation in discount rates in explaining the cyclical behavior of unemployment. Bai, Rios-Rull and Storesletten (2014) stress the importance of demand shocks in generating business cycle fluctuations.

Time-preference shocks can also be thought of a simple way of capturing the notion that fluctuations in market sentiment contribute to the volatility of asset prices, as emphasized by authors such as in Barberis, Shleifer, and Vishny (1998) and Dumas, Kurshev and Up-

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<sup>3</sup>See also Huo and Rios-Rull (2013), Correia, Farhi, Nicolini, and Teles (2013), and Fernandez-Villaverde, Guerron-Quintana, Kuester, Rubio-Ramírez (2013).

pal (2009). Finally, in independent work, contemporaneous with our own, Maurer (2012) explores the impact of time-preference shocks in a calibrated continuous-time representative agent model with Duffie-Epstein (1992) preferences.<sup>4</sup>

Our paper is organized as follows. In Section 2 we document the correlation puzzle using U.S. data for the period 1929 to 2011 and the period 1871 to 2006. In Section 3, we present our benchmark and extended models. We discuss our estimation strategy in Section 4. In Section 5, we present our empirical results. In Section 6, we study the empirical implications of the model for bond term premia, as well as the return on stocks relative to long-term bonds. Section 7 concludes.

## 2. The correlation puzzle

In this section we examine the correlation between stock returns and fundamentals as measured by the growth rate of consumption, output, dividends, and earnings.

### 2.1. Data sources

We consider two sample periods: 1929 to 2011 and 1871 to 2006. For the first sample, we obtain nominal stock and bond returns from Kenneth French’s website. We use the measure of real consumption expenditures and real Gross Domestic Product constructed by Barro and Ursúa (2011), which we update to 2011 using National Income and Product Accounts data. We compute per-capita variables using total population (POP).<sup>5</sup> We obtain data on real S&P500 earnings and dividends from Robert Shiller’s website. We use data from Ibbotson and Associates on the nominal return to one-month Treasury bills, the nominal yield on intermediate-term government bonds (with approximate maturity of five years), and the

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<sup>4</sup>Normandin and St-Amour (1998) study the impact of preference shocks in a model similar to ours. Unfortunately, their analysis does not take into account the fact that covariances between asset returns, consumption growth, and preferences shocks depend on the parameters governing preferences and technology. As a result, their empirical estimates imply that preference shocks reduce the equity premium. In addition, they argue that they can explain the equity premium with separable preferences and preference shocks. This claim contradicts the results in Campbell (1986) and the theorem in our Appendix B.

<sup>5</sup>This series is not subject to a very important source of measurement error that affects another commonly-used population measure, civilian noninstitutional population (CNP16OV). Every ten years, the CNP16OV series is adjusted using information from the decennial census. This adjustment produces large discontinuities in the CNP16OV series. The average annual growth rates implied by the two series are reasonably similar: 1.2 for POP and 1.4 for CNP16OV for the period 1952-2012. But the growth rate of CNP16OV is three times more volatile than the growth rate of POP. Part of this high volatility in the growth rate of CNP16OV is induced by large positive and negative spikes that generally occur in January. For example, in January 2000, 2004, 2008, and 2012 the annualized percentage growth rates of CNP16OV are 14.8,  $-1.9$ ,  $-2.8$ , and 8.4, respectively. The corresponding annualized percentage growth rates for POP are 1.1, 0.8, 0.9, and 0.7.

nominal yield on long-term government bonds (with approximate maturity of twenty years). We convert nominal returns and yields to real returns and yields using the rate of inflation as measured by the consumer price index. We also use inflation-adjusted U.K. government bonds (index-linked gilts) with maturities 2.5, 5, 10, 15, and 25 years.

For the second sample, we use data on real stock and bond returns from Nakamura, Steinsson, Barro, and Ursúa (2013). We use the same data sources for consumption, expenditures, dividends and earnings as in the first sample.

In our estimation, we use two measures of real bond returns. Our primary measure is the ex-ante real return on nominal, one-year, five-year and twenty-year Treasury bonds taken from Luo (2014). Luo (2014) constructs alternative models of expected inflation for one, five and twenty-year horizons. He argues that the random walk model does a better job at forecasting one-year inflation than more sophisticated models like time-varying VAR methods of the sort considered by Primiceri (2005). It also does better than Bayesian vector autoregressions embodying Minnesota priors. However, he argues that the latter does best at forecasting inflation at the five- and twenty-year horizons.

We also report results for a second measure of real bond returns: the realized real return on nominal one-year, five-year and twenty-year Treasury bonds. The one-year rate is the measure of the risk-free rate used in Mehra and Prescott (1985) and the associated literature.

We compute dynamic correlations between stock returns and consumption growth for the G7 and OECD countries. The data on annual real stock returns for the G7 and the OECD countries are from Global Financial Statistics. The data on real per capita personal consumer expenditures and real per capita Gross Domestic Product (GDP) for these countries comes from Barro and Ursúa (2008) and was updated by these authors until 2009. The countries (beginning of sample period) included in our data set are: Australia (1901), Austria (1947), Belgium (1947), Canada (1900), Chile (1900), Denmark (1900), Finland (1923), France (1942), Germany (1851), Greece (1953), Italy (1900), Japan (1894), Korea (1963), Mexico (1902), Netherlands (1947), New Zealand (1947), Norway (1915), Spain (1941), Sweden (1900), Switzerland (1900), United Kingdom (1830), and United States (1869).

## **2.2. Empirical results**

Table 1, panel A presents results for the sample period 1929 to 2011. We report correlations at the one-, five- and ten-year horizons. The five- and ten-year horizon correlations are computed using five- and ten-year overlapping observations, respectively. We report Newey-

West (1987) heteroskedasticity-consistent standard errors computed with ten lags.

There are three key features of Table 1, panel A. First, consistent with Cochrane and Hansen (1992) and Campbell and Cochrane (1999), the growth rates of consumption and output are uncorrelated with stock returns at all the horizons that we consider. Second, the correlation between stock returns and dividend growth is similar to that of consumption and output growth at the one-year horizon. However, the correlation between stock returns and dividend growth is substantially higher at the five and ten-year horizons than the analogue correlations involving consumption and output growth. Third, the pattern of correlations between stock returns and dividend growth is similar to the analogue correlations involving earnings growth.

Table 1, panel B reports results for the longer sample period (1871-2006). The one-year correlation between stock returns and the growth rates of consumption and output are very similar to those obtained for the shorter sample. There is evidence in this sample of a stronger correlation between stock returns and the growth rates of consumption and output at a five-year horizon. But, at the ten-year horizon the correlations are, once again, statistically insignificant. The results for dividends and earnings are very similar across the two subsamples.

Table 2 assesses the robustness of our results for the correlation between stock returns and consumption using three different measures of consumption for the period 1929 to 2011, obtained from the National Product and Income Accounts. With one exception, the correlations in this table are statistically insignificant. The exception is the five-year correlation between stock returns and the growth rate of nondurables and services which is marginally significant.

Table 3 reports the correlation between stock returns and the growth rate of consumption and output for the G7, and the OECD. We report correlations at the one-, five- and ten-year horizons. We compute correlations for the G7 and the OECD countries pooling data across all countries. This procedure implies that countries with longer time series receive more weight in the calculations. Again, there is also a relatively weak correlation between consumption and output growth and stock returns at all the horizons we consider.

We have focused on correlations because we find them easy to interpret. One might be concerned that a different picture emerges from the pattern of covariances between stock returns and fundamentals. It does not. For example, using quarterly U.S. data for the period



1959 to 2000, Parker (2001) argues that one would require a risk aversion coefficient of 379 to account for the equity premium given his estimate of the covariance between consumption growth and stock returns.<sup>6</sup>

We conclude this subsection by considering the dynamic correlations between consumption growth and stock returns. Authors like Parker (2001), Campbell (2003), and Jaganathan and Wang (2007) find that in the U.S. post-war quarterly data, stock returns are correlated with future consumption growth.<sup>7</sup> Figure 1 reports the correlation between returns at time  $t$  and consumption growth at time  $t + \tau$ , where  $\tau$  ranges from  $-10$  years to  $+10$  years. The dotted lines are the limits of the 95 percent confidence interval. The figure also reports the analogue correlations involving output growth. Three key features are worth noting. First, for the U.S. and Canada there does appear to be a significant correlation between returns at time  $t$  and both consumption and output growth at time  $t + 1$ . Second, this result is not robust across the other G7 countries. Third, there is no other value of  $\tau$  for which the correlation is significant for any of the countries. So, we infer that, even taking dynamic correlations into account, there is a weak correlation between stock returns and fundamentals, as measured by consumption and output growth. Nevertheless, because we estimate our structural model using U.S. data, we assess its ability to be consistent with the correlations between stock returns at  $t$  and consumption growth at  $t + 1$ .

Viewed overall, the results in this section serve as our motivation for introducing shocks to the demand for assets. Classic representative-agent models load all uncertainty onto the supply-side of the economy. As a result, they have difficulty in simultaneously accounting for the equity premium and the correlation puzzle.<sup>8</sup> This difficulty is shared by the habit-formation model proposed by Campbell and Cochrane (1999) and the long-run risk models proposed by Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012). Rare-disaster models of the type proposed by Rietz (1988) and Barro (2006) also share this difficulty because all shocks, disaster or not, are to the supply side of the model. A model with a time-varying disaster probability, of the type considered by Wachter (2012) and Gourio (2012),

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<sup>6</sup>Parker (2001) finds that there is a larger covariance between current stock returns and the cumulative growth rate of consumption over the next 12 quarters. He shows that using this covariance measure, one would require a risk aversion coefficient of 38 to rationalize the equity premium (see also Grossman, Melino and Shiller (1987)).

<sup>7</sup>In related work, Backus, Routledge, and Zin (2010) show that equity returns is a leading indicator of the business-cycle components of consumption and output growth.

<sup>8</sup>Lynch (1996) and Gârleanu, Kogan, and Panageas (2012) provide interesting analyses of overlapping-generations models which generate an equity premium, even though the correlation between consumption growth and equity returns is low.

might be able to rationalize the low correlation between consumption and stock returns as a small-sample phenomenon. The reason is that changes in the probability of disasters induce movements in stock returns without corresponding movements in actual consumption growth. This force lowers the correlation between stock returns and consumption in a sample where rare disasters are under represented. This explanation might account for the post-war correlations. But we are more skeptical that it accounts for the results in Table 1, panel B, which are based on the longer sample period, 1871 to 2006, which includes disasters such as the Great Depression and two World Wars.

Below, we focus on demand shocks as the source of the low correlation between stock returns and fundamentals, rather than the alternatives just mentioned. We model these demand shocks in the simplest possible way by introducing shocks to the time preference of the representative agent. Consistent with the references in the introduction, these shocks can be thought of as capturing changes in agents' attitudes towards savings or, more generally, investor sentiment.

### 3. The model

In this section, we study the properties of a representative-agent endowment economy modified to allow for time-preference shocks. The representative agent has the constant-elasticity version of Kreps-Porteus (1978) preferences used by Epstein and Zin (1991) and Weil (1989). The life-time utility of the representative agent is a function of current utility and the certainty equivalent of future utility,  $U_{t+1}^*$ :

$$U_t = \max_{C_t} \left[ \lambda_t C_t^{1-1/\psi} + \delta (U_{t+1}^*)^{1-1/\psi} \right]^{1/(1-1/\psi)}, \quad (3.1)$$

where  $C_t$  denotes consumption at time  $t$  and  $\delta$  is a positive scalar. The certainty equivalent of future utility is the sure value of  $t + 1$  lifetime utility,  $U_{t+1}^*$  such that:

$$(U_{t+1}^*)^{1-\gamma} = E_t (U_{t+1}^{1-\gamma}).$$

The parameters  $\psi$  and  $\gamma$  represent the elasticity of intertemporal substitution and the coefficient of relative risk aversion, respectively. The ratio  $\lambda_{t+1}/\lambda_t$  determines how agents trade off current versus future utility. We assume that this ratio is known at time  $t$ .<sup>9</sup> We refer to

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<sup>9</sup>We obtain similar results with a version of the model in which the utility function takes the form:  $U_t = \left[ C_t^{1-1/\psi} + \lambda_t \delta (U_{t+1}^*)^{1-1/\psi} \right]^{1/(1-1/\psi)}$ . The assumption that the agents knows  $\lambda_{t+1}$  at time  $t$  is made to simplify the algebra and is not necessary for any of the key results.

$\lambda_{t+1}/\lambda_t$  as the time-preference shock. Propositions 6.9 and 6.18 in Skiadas (2009) provide a set of axioms that applies to recursive utility functions with preference shocks.<sup>10</sup>

### 3.1. The benchmark model

To highlight the role of time-preference shocks, we begin with a very simple stochastic process for consumption:

$$\log(C_{t+1}/C_t) = \mu + \sigma_c \varepsilon_{t+1}^c. \quad (3.2)$$

Here,  $\mu$  and  $\sigma_c$  are non-negative scalars and  $\varepsilon_{t+1}^c$  follows an i.i.d. standard-normal distribution.

As in Campbell and Cochrane (1999), we allow dividends,  $D_t$ , to differ from consumption. In particular, we assume that:

$$\log(D_{t+1}/D_t) = \mu + \pi_{dc} \varepsilon_{t+1}^c + \sigma_d \varepsilon_{t+1}^d. \quad (3.3)$$

Here,  $\varepsilon_{t+1}^d$  is an i.i.d. standard-normal random variable that is uncorrelated with  $\varepsilon_{t+1}^c$ . To simplify, we assume that the average growth rate of dividends and consumption is the same ( $\mu$ ). The parameter  $\sigma_d \geq 0$  controls the volatility of dividends. The parameter  $\pi_{dc}$  controls the correlation between consumption and dividend shocks.<sup>11</sup>

The variable  $\Lambda_{t+1} = \log(\lambda_{t+1}/\lambda_t)$  determines how agents trade off current versus future utility. This ratio is known at time  $t$ . We refer to  $\Lambda_{t+1}$  as the time-preference shock. This shock evolves according to:

$$\Lambda_{t+1} = \rho_\Lambda \Lambda_t + \sigma_\Lambda \varepsilon_t^\Lambda, \quad (3.4)$$

where,  $\varepsilon_t^\Lambda$  is an i.i.d. standard-normal random variable. In the spirit of the original Lucas (1978) model, we assume, for now, that  $\varepsilon_t^\Lambda$  is uncorrelated with  $\varepsilon_t^c$  and  $\varepsilon_t^d$ . We relax this assumption in Subsection 3.4.

**The CRRA case** In Appendix A we solve this model analytically for the case in which  $\gamma = 1/\psi$ . Here preferences reduce to the CRRA form:

$$V_t = E_t \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} C_{t+i}^{1-\gamma}, \quad (3.5)$$

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<sup>10</sup>Skiadas (2009) derives a parametric SDF that satisfies the axioms in proposition 6.9 and 6.18 (see his equation 6.35). This SDF can be modified to obtain a generalized Epstein and Zin (1989) parametric utility function with stochastic risk aversion, intertemporal substitution, and time-preference shocks. We thank Soohun Kim and Ravi Jagannathan for pointing this result out to us.

<sup>11</sup>The stochastic process described by equations (3.2) and (3.3) implies that  $\log(D_{t+1}/C_{t+1})$  follows a random walk with no drift. This implication is consistent with our data.

with  $V_t = U_t^{1-\gamma}$ .

We denote the risk-free rate and the rate of return on a claim on consumption by  $R_{f,t+1}$ , and  $R_{c,t+1}$ , respectively. The unconditional risk-free rate depends on the persistence and volatility of time-preference shocks:

$$E(R_{f,t+1}) = \exp\left(\frac{\sigma_\Lambda^2/2}{1-\rho_\Lambda^2}\right) \delta^{-1} \exp(\gamma\mu - \gamma^2\sigma_c^2/2).$$

The unconditional equity premium implied by this model is proportional to the risk-free rate:

$$E(R_{c,t+1} - R_{f,t+1}) = E(R_{f,t+1}) [\exp(\gamma\sigma_c^2) - 1]. \quad (3.6)$$

Both the average risk-free rate and the volatility of consumption are small in the data. Moreover, the constant of proportionality in equation (3.6),  $\exp(\gamma\sigma_c^2) - 1$ , is independent of  $\sigma_\Lambda^2$ . So, time-preference shocks do not help to resolve the equity premium puzzle when preferences are of the CRRA form.

### 3.2. Solving the benchmark model

We define the return to the stock market as the return to a claim on the dividend process. The realized gross stock-market return is given by:

$$R_{d,t+1} = \frac{P_{d,t+1} + D_{t+1}}{P_{d,t}}, \quad (3.7)$$

where  $P_{d,t}$  denotes the ex-dividend stock price.

It is useful to define the realized gross return to a claim on the endowment process:

$$R_{c,t+1} = \frac{P_{c,t+1} + C_{t+1}}{P_{c,t}}. \quad (3.8)$$

Here,  $P_{c,t}$  denotes the price of an asset that pays a dividend equal to aggregate consumption. We use the following notation to define the logarithm of returns on the dividend and consumption claims, the logarithm of the price-dividend ratio, and the logarithm of the price-consumption ratio:

$$\begin{aligned} r_{d,t+1} &= \log(R_{d,t+1}), \\ r_{c,t+1} &= \log(R_{c,t+1}), \\ z_{dt} &= \log(P_{d,t}/D_t), \\ z_{ct} &= \log(P_{c,t}/C_t). \end{aligned}$$

In Appendix B we show that the logarithm of the stochastic discount factor (SDF) implied by the utility function defined in equation (3.1) is given by:

$$m_{t+1} = \theta \log(\delta) + \theta \Lambda_{t+1} - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \quad (3.9)$$

where  $\theta$  is defined as:

$$\theta = \frac{1 - \gamma}{1 - 1/\psi}. \quad (3.10)$$

When  $\gamma = 1/\psi$ , the case of CRRA preferences, the value of  $\theta$  is equal to one and the stochastic discount factor is independent of  $r_{c,t+1}$ .

We solve the model using the approximation proposed by Campbell and Shiller (1988), which involves linearizing the expressions for  $r_{c,t+1}$  and  $r_{d,t+1}$  and exploiting the properties of the log-normal distribution.<sup>12</sup>

Using a log-linear Taylor expansion we obtain:

$$r_{d,t+1} = \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}, \quad (3.11)$$

$$r_{c,t+1} = \kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct} + \Delta c_{t+1}, \quad (3.12)$$

where  $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$  and  $\Delta d_{t+1} \equiv \log(D_{t+1}/D_t)$ . The constants  $\kappa_{c0}$ ,  $\kappa_{c1}$ ,  $\kappa_{d0}$ , and  $\kappa_{d1}$  are given by:

$$\kappa_{d0} = \log[1 + \exp(z_d)] - \kappa_{d1} z_d,$$

$$\kappa_{c0} = \log[1 + \exp(z_c)] - \kappa_{c1} z_c,$$

$$\kappa_{d1} = \frac{\exp(z_d)}{1 + \exp(z_d)}, \quad \kappa_{c1} = \frac{\exp(z_c)}{1 + \exp(z_c)},$$

where  $z_d$  and  $z_c$  are the unconditional mean values of  $z_{dt}$  and  $z_{ct}$ .

The Euler equations associated with a claim to the stock market and a consumption claim can be written as:

$$E_t [\exp(m_{t+1} + r_{d,t+1})] = 1, \quad (3.13)$$

$$E_t [\exp(m_{t+1} + r_{c,t+1})] = 1. \quad (3.14)$$

We solve the model using the method of undetermined coefficients. First, we replace  $m_{t+1}$ ,  $r_{c,t+1}$  and  $r_{d,t+1}$  in equations (3.13) and (3.14), using expressions (3.11), (3.12) and

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<sup>12</sup>See Hansen, Heaton, and Li (2008) for an alternative solution procedure.

(3.9). Second, we guess and verify that the equilibrium solutions for  $z_{dt}$  and  $z_{ct}$  take the form:

$$z_{dt} = A_{d0} + A_{d1}\Lambda_{t+1}, \quad (3.15)$$

$$z_{ct} = A_{c0} + A_{c1}\Lambda_{t+1}. \quad (3.16)$$

This solution has the property that price-dividend ratios are constant, absent movements in  $\Lambda_{t+1}$ . This property results from our assumption that the logarithm of consumption and dividends follow random-walk processes. We compute  $A_{d0}$ ,  $A_{d1}$ ,  $A_{c0}$ , and  $A_{c1}$  using the method of undetermined coefficients.

The equilibrium solution has the property that  $A_{d1}, A_{c1} > 0$ . We show in Appendix B that the conditional expected return to equity is given by:

$$\begin{aligned} E_t(r_{d,t+1}) = & -\log(\delta) - \Lambda_{t+1} + \mu/\psi \\ & + \left[ \frac{(1-\theta)}{\theta} (1-\gamma)^2 - \gamma^2 \right] \sigma_c^2/2 + \pi_{dc} (2\gamma\sigma_c - \pi_{dc})/2 - \sigma_d^2/2 \\ & + \left\{ (1-\theta) (\kappa_{c1}A_{c1}) [2(\kappa_{d1}A_{d1}) - (\kappa_{c1}A_{c1})] - (\kappa_{d1}A_{d1})^2 \right\} \sigma_\Lambda^2/2. \end{aligned} \quad (3.17)$$

Recall that  $\kappa_{c1}$  and  $\kappa_{d1}$  are non-linear functions of the parameters of the model.

Using the Euler equation for the risk-free rate,  $r_{f,t+1}$ ,

$$E_t[\exp(m_{t+1} + r_{f,t+1})] = 1,$$

we obtain:

$$\begin{aligned} r_{f,t+1} = & -\log(\delta) - \Lambda_{t+1} + \mu/\psi - (1-\theta) (\kappa_{c1}A_{c1})^2 \sigma_\Lambda^2/2 \\ & + \left[ \frac{(1-\theta)}{\theta} (1-\gamma)^2 - \gamma^2 \right] \sigma_c^2/2. \end{aligned} \quad (3.18)$$

Equations (3.17) and (3.18) imply that the risk-free rate and the conditional expectation of the return to equity are decreasing functions of  $\Lambda_{t+1}$ . When  $\Lambda_{t+1}$  rises, agents value the future more relative to the present, so they want to save more. Since risk-free bonds are in zero net supply and the number of stock shares is constant, aggregate savings cannot increase. So, in equilibrium, returns on bonds and equity must fall to induce agents to save less.

The approximate response of asset prices to shocks, emphasized by Borovička, Hansen, Hendricks, and Scheinkman (2011) and Borovička and Hansen (2011), can be directly inferred

from equations (3.17) and (3.18). The response of the return to stocks and the risk-free rate to a time-preference shock corresponds to that of an AR(1) with serial correlation  $\rho_\Lambda$ .

Using equations (3.17) and (3.18) we can write the conditional equity premium as:

$$E_t(r_{d,t+1}) - r_{f,t+1} = \pi_{dc}(2\gamma\sigma_c - \pi_{dc})/2 - \sigma_d^2/2 + \kappa_{d1}A_{d1}[2(1-\theta)A_{c1}\kappa_{c1} - \kappa_{d1}A_{d1}]\sigma_\Lambda^2/2. \quad (3.19)$$

We define the compensation for *valuation risk* as the part of the one-period expected excess return to an asset that is due to the volatility of the time preference shock,  $\sigma_\Lambda^2$ . We refer to the part of the one-period expected excess return that is due to the volatility of consumption and dividends as the compensation for *conventional risk*.

The component of the equity premium that is due to valuation risk,  $v_d$ , is given by the last term in equation (3.19). Since the constants  $A_{c1}$ ,  $A_{d1}$ ,  $\kappa_{c1}$ , and  $\kappa_{d1}$  are all positive,  $\theta < 1$  is a necessary condition for valuation risk to help explain the equity premium (recall that  $\theta$  is defined in equation (3.10)).<sup>13</sup>

The intuition for why valuation risk helps account for the equity premium is as follows. Consider an investor who buys the stock at time  $t$ . At some later time, say  $t + \tau$ ,  $\tau > 0$ , the investor may get a preference shock, say a decrease in  $\Lambda_{t+\tau+1}$ , and want to increase consumption. Since all consumers are identical, they all want to sell the stock at the same time, so the price of equity falls. Bond prices also fall because consumers try to reduce their holdings of the risk-free asset to raise their consumption in the current period. Since stocks are infinitely-lived compared to the one-period risk-free bond, they are more exposed to this source of risk. So, valuation risk,  $v_d$ , gives rise to an equity premium. In the CRRA case ( $\theta = 1$ ) the effect of valuation risk on the equity premium is generally small (see equation 3.6)).

It is interesting to highlight the differences between time-preference shocks and conventional sources of uncertainty, which pertain to the supply-side of the economy. Suppose that there is no risk associated with the physical payoff of assets such as stocks. In this case, standard asset pricing models would imply that the equity premium is zero. In our model, there is a positive equity premium that results from the differential exposure of bonds and

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<sup>13</sup>As discussed in Epstein et al (2014), the condition  $\theta < 1$  also plays a crucial role in generating a high equity premium in long-run risk models. Because long-run risks are resolved in the distant future, they are more heavily penalized than current risks. For this reason, long-run risk models can generate a large equity premium even when shocks to current consumption are small.

stocks to valuation risk. Agents are uncertain about how much they will value future dividend payments. Since  $\Lambda_{t+1}$  is known at time  $t$ , this valuation risk is irrelevant for one-period bonds. But, it is not irrelevant for stocks, because they have infinite maturity. In general, the longer the maturity of an asset, the higher is its exposure to time-preference shocks and the larger is the valuation risk.

We conclude by considering the case in which there are supply-side shocks to the economy but agents are risk neutral ( $\gamma = 0$ ). In this case, the component of the equity premium that is due to valuation risk is always positive as long as  $\psi < 1$ . The intuition is as follows: stocks are long-lived assets whose payoffs can induce unwanted variation in the period utility of the representative agent,  $\lambda_t C_t^{1-1/\psi}$ . Even when agents are risk neutral, they must be compensated for the risk of this unwanted variation.

### 3.3. Relation to the long-run risk model

In this subsection, we briefly comment on the relation between our model and the long-run-risk model pioneered by Bansal and Yaron (2004). Both models emphasize low-frequency shocks that induce large, persistent changes in the agent's stochastic discount factor.<sup>14</sup> To see this point, it is convenient to re-write the representative agent's utility function, (3.1), as:

$$U_t = \left[ \tilde{C}_t^{1-1/\psi} + \delta (U_{t+1}^*)^{1-1/\psi} \right]^{1/(1-1/\psi)}, \quad (3.20)$$

where  $\tilde{C}_t = \lambda_t^{1/(1-1/\psi)} C_t$ . Taking logarithms of this expression we obtain:

$$\log(\tilde{C}_t) = 1/(1-1/\psi) \log(\lambda_t) + \log(C_t).$$

Bansal and Yaron (2004) introduce a highly persistent component in the process for  $\log(C_t)$ , which is a source of long-run risk. In contrast, we introduce a highly persistent component into  $\log(\tilde{C}_t)$  via our specification of the time-preference shock. From equation (3.9), it is clear that both specifications can induce large, persistent movements in  $m_{t+1}$ . Despite this similarity, the two models are *not* observationally equivalent. First, they have different implications for the correlation between observed consumption growth,  $\log(C_{t+1}/C_t)$ , and asset returns. Second, the two models have very different implications for the average return to long-term bonds, and the term structure of interest rates. We return to these points when we discuss our empirical results in Section 6. There we present empirical evidence for

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<sup>14</sup>See Dew-Becker (2014) for a version of a long-run risk model in a production economy.



the implications of different models for the slope of the yield curve. In addition, we assess whether the SDFs of the different models have the properties that Backus and Zin (1994) argue are necessary for explaining the slope of the yield curve and the persistence of bond yields.

### 3.4. The extended model

The benchmark model just described is useful to highlight the role of time-preference shocks in affecting asset returns. But its simplicity leads to two important empirical shortcomings.<sup>15</sup> First, since consumption is a martingale, the only state variable that is relevant for asset returns is  $\Lambda_{t+1}$ . This property means that all asset returns are highly correlated with each other and with the price-dividend ratio. Second, and related, the model displays constant risk premia and so it cannot generate predictability in excess returns.

In this subsection, we address the shortcomings of the benchmark model by allowing for a richer model of consumption and dividend growth:

$$\log(C_{t+1}/C_t) = \mu_c + \rho_c \log(C_t/C_{t-1}) + \alpha_c (\sigma_{t+1}^2 - \sigma^2) + \pi_{c\Lambda} \varepsilon_{t+1}^\Lambda + \sigma_t \varepsilon_{t+1}^c, \quad (3.21)$$

$$\log(D_{t+1}/D_t) = \mu_d + \rho_d \log(D_{t+1}/D_t) + \alpha_d (\sigma_{t+1}^2 - \sigma^2) + \sigma_d \sigma_t \varepsilon_{t+1}^d + \pi_{d\Lambda} \varepsilon_{t+1}^\Lambda + \pi_{dc} \sigma_t \varepsilon_{t+1}^c, \quad (3.22)$$

and

$$\sigma_{t+1}^2 = \sigma^2 + v (\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1}, \quad (3.23)$$

where  $\varepsilon_{t+1}^c$ ,  $\varepsilon_{t+1}^d$ ,  $\varepsilon_{t+1}^\Lambda$ , and  $w_{t+1}$  are mutually uncorrelated standard-normal variables. We restrict the unconditional average growth rate of consumption and dividends to be the same:

$$\frac{\mu_c}{1 - \rho_c} = \frac{\mu_d}{1 - \rho_d}.$$

Relative to the benchmark model, equations (3.21)-(3.23) incorporate three new features. First, motivated by the data, we allow for serial correlation in the growth rates of consumption and dividends. Second, as in Kandel and Stambaugh (1990) and Bansal and Yaron (2004), we allow for conditional heteroskedasticity in consumption. This feature generates time-varying risk premia: when volatility is high the stock is risky, its price is low and its expected return is high. High volatility leads to higher precautionary savings motive so that the risk-free rate falls, reinforcing the rise in the risk premium. Third, we allow for a correlation between time-preference shocks and the growth rate of consumption and dividends.

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<sup>15</sup>The shortcomings of our benchmark model are shared by other simple models like model I in Bansal and Yaron (2004), which abstract from conditional heteroskedasticity in consumption and dividends.

In a production economy, time-preference shocks would generally induce changes in aggregate consumption. For example, in a simple real-business-cycle model, a persistent increase in  $\Lambda_{t+1}$  would lead agents to reduce current consumption and raise investment in order to consume more in the future. Taken literally, an endowment economy specification does not allow for such a correlation. Importantly, only the innovation to time-preference shocks enters the law of motion for  $\log(C_{t+1}/C_t)$  and  $\log(D_{t+1}/D_t)$ . So, equations (3.21)-(3.23) do not introduce any element of long-run risk into consumption or dividend growth.

Since the price-dividend ratio and the risk-free rate are driven by a single state variable in the benchmark model, they have the same degree of persistence. But, according to our empirical estimates, the price-dividend ratio is more persistent than the risk-free rate. A straightforward way to address this shortcoming of the benchmark model is to assume that the time-preference shock is the sum of a persistent shock and an i.i.d. shock:

$$\begin{aligned}\Lambda_{t+1} &= x_t + \sigma_\eta \eta_{t+1}, \\ x_{t+1} &= \rho_\Lambda x_t + \sigma_\Lambda \varepsilon_{t+1}^\Lambda.\end{aligned}\tag{3.24}$$

Here  $\varepsilon_{t+1}^\Lambda$  and  $\eta_{t+1}$  are uncorrelated, i.i.d. standard normal shocks. We think of  $x_t$  as capturing low-frequency changes in the growth rate of the discount rate. In contrast,  $\eta_{t+1}$  can be thought of high-frequency changes in investor sentiment that affect the demand for assets (see, for example, Dumas et al. (2009)). If  $\sigma_\eta = 0$  and  $x_1 = \Lambda_1$ , we obtain the specification of the time-preference shock used in the benchmark model. Other things equal, the larger is  $\sigma_\eta$ , the lower is the persistence of the time-preference shock.

## 4. Estimation methodology

We estimate the parameters of our model using the Generalized Method of Moments (GMM). Our estimator is the parameter vector  $\hat{\Phi}$  that minimizes the distance between a vector of empirical moments,  $\Psi_D$ , and the corresponding model moments,  $\Psi(\hat{\Phi})$ . Our estimator,  $\hat{\Phi}$ , is given by:

$$\hat{\Phi} = \arg \min_{\Phi} [\Psi(\Phi) - \Psi_D]' \Omega_D^{-1} [\Psi(\Phi) - \Psi_D].$$

We found that, for a wide range of parameter values, the model implies that there is small-sample bias in terms of various moments, especially the predictability of excess returns. We therefore focus on the plim of the model-implied small-sample moments when constructing  $\Psi(\Phi)$ , rather than the plim of the moments themselves. For a given parameter vector,  $\Phi$ , we

create 500 synthetic time series, each of length equal to our sample size. For each sample, we calculate the sample moments of interest. The vector  $\Psi(\Phi)$  that enters the criterion function is the median value of the sample moments across the synthetic time series.<sup>16</sup> In addition, we assume that agents make decisions at a monthly frequency and derive the model's implications for variables computed at an annual frequency. We estimate  $\Psi_D$  using a standard two-step efficient GMM estimator with a Newey-West (1987) weighting matrix that has ten lags. The latter matrix corresponds to our estimate of the variance-covariance matrix of the empirical moments,  $\Omega_D$ .

When estimating the benchmark model, we include the following 19 moments in  $\Psi_D$ : the mean and standard deviation of consumption growth, the mean and standard deviation of dividend growth, the contemporaneous correlation between the growth rate of dividends and the growth rate of consumption, the mean and standard deviation of real stock returns, the mean, standard deviation and autocorrelation of the price-dividend ratio, the mean, standard deviation and autocorrelation of the real risk-free rate, the correlation between stock returns and consumption growth at the one, five and ten-year horizon, the correlation between stock returns and dividend growth at the one, five and ten-year horizon. We constrain the growth rate of dividends and consumption to be the same. In practice, when we estimate the benchmark model we find that the standard deviation of the point estimate of the risk-free rate across the 500 synthetic time series is very large. So here we report results corresponding to the case where we constrain the median risk-free rate to exactly match our point estimate.

For the benchmark model, the vector  $\Phi$  includes nine parameters:  $\gamma$ , the coefficient of relative risk aversion,  $\psi$ , the elasticity of intertemporal substitution,  $\delta$ , the rate of time preference,  $\sigma_c$ , the volatility consumption growth shocks,  $\pi_{dc}$ , the parameter that controls the correlation between consumption and dividend growth shocks,  $\sigma_d$ , the volatility of dividend growth shocks,  $\rho_\Lambda$ , the persistence of time-preference shocks,  $\sigma_\Lambda$ , the volatility of the innovation to time-preference shocks, and  $\mu$ , the mean growth rate of dividends and consumption.

In the extended model, the vector  $\Phi$  includes nine additional parameters:  $\rho_c$  and  $\rho_d$ , which control the serial correlation of consumption and dividend growth,  $\alpha_c$  and  $\alpha_d$ , which control the effect of volatility on the growth rate of consumption and dividends, respectively,  $\pi_{c\Lambda}$  and  $\pi_{d\Lambda}$ , which control the effect of time preference shock innovations on consumption and dividends, respectively,  $\nu$ , which governs the persistence of volatility,  $\sigma_w$ , the volatility

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<sup>16</sup>As the sample size grows, our estimator becomes equivalent to a standard GMM estimator so that the usual asymptotic results for the distribution of the estimator apply.

of innovations to volatility, and  $\sigma_\eta$ , the volatility of transitory shocks to time preference. In estimating the extended model, we add to  $\Psi_D$  the following six moments: the first-order serial correlation of consumption growth, the average yield for 5 and 20-year bonds, and the slope coefficients from the regressions of the excess-equity returns over holding periods of 1, 3 and 5 years on the lagged price-dividend ratio.

## 5. Empirical results

In the main text, we discuss the results that we obtain estimating the models using the ex-ante measures of real bond yields. In appendix D, we report the analogue results obtained estimating the models with ex-post measures of real bond yields.

Table 4 reports our parameter estimates along with standard errors. Several features are worth noting. First, the coefficient of risk aversion is quite low, 1.5 and 2.4, in the benchmark and extended models, respectively. We estimate this coefficient with reasonable precision. Second, for both models, the intertemporal elasticity of substitution is somewhat larger than one. Third, for both models, the point estimates satisfy the necessary condition for valuation risk to be positive,  $\theta < 1$ . Fourth, the parameter  $\rho_\Lambda$  that governs the serial correlation of the growth rate of  $\Lambda_t$  is estimated to be close to one in both models, 0.991 and 0.992 in the benchmark and extended model, respectively. Fifth, the parameter  $\nu$ , which governs the persistence of consumption volatility in the extended model, is also quite high (0.997). The high degree of persistence in both the time-preference and the volatility shock are the root cause of the small-sample biases in our estimators.

Table 5 compares the small-sample moments implied by the benchmark and extended models with the estimated data moments. Recall that in estimating the model parameters we impose the restriction that the unconditional average growth rate of consumption and dividends are the same. To assess the robustness of our results to this restriction, we present two versions of the estimated data moments, with and without the restriction. With one exception, the constrained and unconstrained data moment estimates are similar, taking sampling uncertainty into account. The exception is the average growth rate of consumption, where the constrained and unconstrained estimates are statistically different.

**Implications for the equity premium** Table 5 shows that both the benchmark and extended model give rise to a large equity premium, 5.81 and 3.88, respectively. This result

holds even though the estimated degree of risk aversion is quite moderate in both models. In contrast, long-run risk models require a high degree of risk aversion to match the equity premium.

Recall that in order for valuation risk to contribute to the equity premium,  $\theta$  must be less than one. This condition is clearly satisfied by both our models: the estimated value of  $\theta$  is  $-1.6$  and  $-2.6$  in the benchmark and extended model, respectively (Table 4). In both cases,  $\theta$  is estimated quite accurately. Taking sampling uncertainty into account, the benchmark model easily accounts for the equity premium, while the extended model does so marginally. We can easily reject the null hypothesis of  $\theta = 1$ , which corresponds to the case of constant relative risk aversion.

The basic intuition for why our model generates a high equity premium despite a low coefficient of relative risk aversion is as follows. From the perspective of the model, stocks and bonds are different in two ways. First, the model embodies the conventional source of an equity premium, namely a differential covariance of bonds and stocks with the SDF. Since  $\gamma$  is relatively small, this traditional covariance effect is also small. Second, the model embodies a compensation for valuation risk that is particularly pronounced for stocks because they have longer maturities than bonds. Recall that, given our timing assumptions, when an agent buys a bond at time  $t$ , the agent knows the value of  $\Lambda_{t+1}$ , so the only source of risk are movements in the marginal utility of consumption at time  $t + 1$ . In contrast, the time- $t$  stock price depends on the value of  $\Lambda_{t+j}$ , for all  $j > 1$ . So, agents are exposed to valuation risk, a risk that is particularly important because time-preference shocks are very persistent.

**Implications for the risk-free rate** A problem with some explanations of the equity premium is that they imply counterfactually high levels of volatility for the risk-free rate (see e.g. Boldrin, Christiano and Fisher (2001)). Table 5 shows that the volatility of the risk-free rate and stock market returns implied by our model are similar to the estimated volatilities in the data.

An empirical shortcoming of the benchmark model is its implication for the persistence of the risk-free rate. Recall that, according to equation (3.18), the risk-free rate has the same persistence as the growth rate of the time-preference shock. Table 5 shows that the AR(1) coefficient of the risk-free rate, as measured by the ex-ante real returns to one-year treasury bills, is only 0.52, with a standard error of 0.07. This estimate is substantially smaller than the analogue statistic implied by the benchmark model (0.90). The extended model does a

much better job at accounting for the persistence of the risk-free rate (0.51). In this model there are both transitory and persistent shocks to the risk-free rate. The former account for roughly 44 percent of the variance of  $\Lambda_t$ .

**Implications for the correlation puzzle** Table 6 reports the model’s implications for the correlation of stock returns with consumption and dividend growth.<sup>17</sup> Recall that in the benchmark model consumption and dividends follow a random walk. In addition, the estimated process for the growth rate of the time-preference shock is close to a random walk. So, the correlation between stock returns and consumption growth implied by the model is essentially the same across different horizons. A similar property holds for the correlation between stock returns and dividend growth.

In the extended model, persistent changes in the variance of the growth rate of consumption and dividends can induce persistent changes in the conditional mean of these variables. As a result, this model produces correlations between stock returns and fundamentals that vary across different horizons.

The benchmark model does well at matching the correlation between stock returns and consumption growth in the data, because this correlation is similar at all horizons. In contrast, the empirical correlation between stock returns and dividend growth increases with the time horizon. The estimation procedure chooses to match the long-horizon correlations and does less well at matching the yearly correlation. This choice is dictated by the fact that it is harder for the model to produce a low correlation between stock returns and dividend growth than it is to produce a low correlation between stock returns and consumption growth. This property reflects the fact that the dividend growth rate enters directly into the equation for stock returns (see equation (3.11)).

The extended model does better at capturing the fact that the correlations between equity returns and dividend growth rises with the horizon for two reasons. When volatility is high, the returns to equity are high. Since  $\alpha_d < 0$ , the growth rate of dividends is low. As a result, the one-year correlation between dividend growth and equity returns is negative. The variance of the shock to the dividend growth rate is mean reverting. So, the effect of a negative value of  $\alpha_d$  becomes weaker as the horizon extends. The direct association

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<sup>17</sup>The moments reported in this table are slightly different from those reported in Table 1. The reason is that the moments in the two tables were computed using different samples. In Table 6, we compute a leaded and a lagged correlation, so we lose one observation in the beginning of the sample and another in the end of the sample.

between equity returns and dividend growth (see equation (3.11)), which induces a positive correlation, eventually dominates as the horizon gets longer.

An additional force that allows the extended model to generate a lower short-term correlation between equity returns and dividend growth, is that the estimated value of  $\pi_{d\Lambda}$  is negative. The estimation algorithm chooses parameters that allow the model to do reasonably well in matching the one- and five-year correlation, at the cost of doing less well at matching the ten-year correlation. Presumably, this choice reflects the greater precision with which the one-year and five-year correlations are estimated relative to the ten-year correlation. Taking sampling uncertainty into account, the extended model matches the correlation between stock returns and consumption growth at different horizons. Interestingly, the correlation between stock returns and consumption growth increases with the horizon.

We conclude by highlighting an important difference between our model and two leading alternatives. The first is the external-habit model proposed by Campbell and Cochrane (1999).<sup>18</sup> Working with their parameter values, we find that the correlation between stock returns and consumption growth are equal to 0.53, 0.69, and 0.62 at the one-, five- and ten-year horizon, respectively.

The second alternative is the long-run risk model proposed by Bansal, Kiku, and Yaron (2012). Working with their parameter values, we find that the correlation between stock returns and consumption growth are equal to 0.34, 0.49, and 0.54 at the one-, five- and ten-year horizon, respectively. Their model also implies correlations between stock returns and dividend growth equal to 0.64, 0.90, and 0.93 at the one-, five- and ten-year horizon, respectively.

Our estimates reported in Table 1 imply that, for both models, the correlations between fundamentals and returns are counterfactually high. The source of this empirical shortcoming is that all the uncertainty in the long-run risk model stems from the endowment process.

**Implications for the price-dividend ratio** In Table 5 we see that both the benchmark and the extended models match the average price-dividend ratio very well. The benchmark model somewhat underpredicts the persistence and volatility of the price-dividend ratio. The extended model does much better at matching those moments. The moments implied by this model are within two standard errors of their sample counterparts.

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<sup>18</sup>As in Wachter (2005), we consider a version of the model in which equities are a claim to consumption. We thank Jessica Wachter for sharing her program for solving the Campbell-Cochrane model with us.

Table 7 presents evidence reproducing the well-known finding that excess returns are predictable based on lagged price-dividend ratios. We report the results of regressing excess-equity returns over holding periods of 1, 3 and 5 years on the lagged price-dividend ratio. The slope coefficients are  $-0.10$ ,  $-0.27$ , and  $-0.42$ , respectively, while the R-squares are 0.07, 0.14, and 0.26, respectively.

The analogue results for the benchmark model are shown in the top panel of Table 7. In this model, consumption is a martingale with conditionally homoscedastic innovations. So, by construction, excess returns are unpredictable in population at a monthly frequency. Since we aggregate the model to annual frequency, temporal aggregation produces a small amount of predictability (see column titled “Model (plim)”).

Stambaugh (1999) and Boudoukh et al. (2008) argue that the predictability of excess returns may be an artifact of small-sample bias and persistence in the price-dividend ratio. Our results are consistent with this hypothesis. The column labeled “Model (median)” reports the plim of the small moments implied by our model. The slope coefficients for the 1, 5, and 10 year horizons, are  $-0.05$ ,  $-0.14$  and  $-0.21$ , respectively. In each case, the median Monte Carlo point estimates are contained within a two-standard deviation band of the respective data estimates.

Table 7 also presents results for the extended model. Because of conditional heteroskedasticity in consumption, periods of high volatility in consumption growth are periods of high expected equity returns and low equity prices. So, in principle, the model is able to generate predictability in population. At our estimated parameter values this predictability is quite small. But, once we allow for the effects of small-sample bias, the extended model does quite well at accounting for the regression slope coefficients. Finally, we note that the benchmark model understates the regression R-squares. Taking sampling uncertainty into account, the extended model does better on this dimension.

Cochrane (1992) proposes a decomposition of the variation of the price-dividend ratio into three components: excess returns, dividend growth, and the risk-free rate.<sup>19</sup> While this decomposition is not additive, authors like Bansal and Yaron (2004) use it to compare the importance of these three components in the model and data.

In our sample, the point estimate for the percentage of the variation in price-dividend

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<sup>19</sup>The fraction of the volatility of the price-dividend ratio attributed to variable  $x$  is given by  $\sum_{j=1}^{15} \Omega^j cov(z_{dt}, x_{t+j})/var(z_{dt})$ , where  $\Omega = 1/(1 + E(R_{d,t+1}))$ . See Cochrane (1992) for details.



ratio due to excess return fluctuations is 102.2 percent, with a standard error of 30 percent. Dividend growth accounts for  $-14.5$  with a standard error of 13 percent. Finally, the risk-free rate accounts for  $-20.4$  percent with a standard error of 14.8 percent. These results are similar to those in Cochrane (1992) and Bansal and Yaron (2004).

Based on the small-sample moments implied by the benchmark model, the fraction of the variance of the price-dividend ratio accounted for by excess returns, dividend growth, and the risk-free rate is 34.6,  $-2.8$ , and 54.2, respectively. So, this model clearly overstates the importance of the risk-free rate and understates the importance of excess returns in accounting for the variance of the price-dividend ratio.

The extended model does substantially better than the benchmark model. Based on the small-sample moments implied by the extended model, the fraction of the variance of the price-dividend ratio accounted for by excess returns, dividend growth, and the risk-free rate is 51.2,  $-38.5$ , and 5.6, respectively. The fraction attributed to excess returns is just within two standard errors of the point estimate. The fraction attributed to dividend growth is well within two standard errors of the point estimate.

It is interesting to contrast these results to those in Bansal and Yaron (2004). Their model also attributes a large fraction of the variance of the price-dividend ratio to excess returns. At the same time, their model substantially overstates the role of dividend growth in accounting for the variance of the price-dividend ratio.

**Correlation between the price-dividend ratio and the risk-free rate** We now discuss the implications of our model for the correlation between the price-dividend ratio and the risk-free rate. Table 5 reports that our estimate of this correlation is 0.27 with a standard error of 0.11. The benchmark model predicts that this correlation should be sharply negative ( $-0.95$ ). The intuition for this result is clear. When there is a shock to the rate of time preference, agents want to save more, both in the form of bonds and stocks. So, the price-dividend ratio must rise to clear the equity market and the risk-free rate must fall to clear the bond market. The extended model does much better along this dimension of the data ( $-0.20$ ). Recall that a positive shock to  $\varepsilon_t^\Lambda$  drives the risk-free rate down. Since  $\pi_{d\Lambda}$  is negative, the same shock lowers the growth rate of dividends which causes the price dividend ratio to fall. So the extended mode embodies a force that generates a positive correlation between the risk-free rate and the price dividend ratio. This forces helps to substantially lower the sharp negative correlation between these variables in the benchmark model.

It is interesting to compare our model with Bansal, Kiku and Yaron (2012). The long-run risk shock in that model induces a counterfactually sharp positive correlation: 0.65. The intuition is straightforward. A shock that induces a persistent change in the growth rate of dividends causes stock prices to go up since equity now represents a claim to a higher flow of dividends. Dividends do not change much in the short run, so the price-dividend ratio goes up. At the same time, the shock causes agents to want to shift away from bond holdings and into equity. So, the risk-free rate must rise to clear the bond market. Other things equal, the long-run risk shock induces a positive correlation between the price-dividend ratio and the risk-free rate. In contrast to our model, the Bansal, Kiku and Yaron (2012) does not embody any forces that would lead to a negative correlation between the price-dividend ratio and the risk-free rate. The net result is that their model overstates this correlation.

We are hesitant to use this correlation as a litmus test to choose between our model and Bansal, Kiku and Yaron’s for two reasons. First, the point estimate of this correlation is sensitive to sample period selection. The correlation is positive (0.17) before 1983 but negative ( $-0.44$ ) after 1983. Second, the sign of this correlation varies substantially across the G7 countries and is again sensitive to breaking the sample in 1983 (see Table 8).

## 6. Bond term premia

As we emphasize above, the equity premium in our estimated models results primarily from the valuation risk premium. Since this valuation premium increases with the maturity of an asset, a natural way to assess the plausibility of our model is to evaluate its implications for the slope of the real yield curve.

This section is organized as follows. First, we discuss evidence on the slope of the yield curve obtained using data for the U.S. and the U.K. Second, we discuss the implications of different models for the yield curve. Third, we evaluate whether the estimated SDF in our extended model has the properties that Backus and Zin (1994) argue are necessary to simultaneously account for the slope of the yield curve and the persistence of bond yields. Finally, we discuss and evaluate the implications of our model for the long-term equity premium.

**Empirical evidence** Table 9 reports the mean and standard deviation of ex-ante real yields on short-term (one year) Treasury Bills, intermediate-term government bonds (with

approximate maturity of five years), and long-term government bonds (with approximate maturity of twenty years).<sup>20</sup> The ex-ante yields on five (twenty) year bonds are computed as the difference between the five (twenty) year nominal yield and the ex-ante five (twenty) year inflation rate (yields are expressed on an annualized basis), which are taken from Luo (2014).

A number of features are worth noting. First, consistent with Alvarez and Jermann (2005), the term structure of real yields is upward sloping. Second, the real yield on long-term bonds is positive. This result is consistent with Campbell, Shiller and Viceira (2009) who report that the real yield on long-term TIPS has always been positive and is usually above two percent.

We use data on inflation-index U.K. gilts to shed further light on the slope of the real yield curve. Table 10 reports the difference between the yield of a bond with 2.5 years maturity and yields of bonds with 5, 10, 15, and 20 years of maturity. Our benchmark sample period is 1985 to 2015. In every case, the longer duration bond has, on average, a higher yield than the bond with duration 2.5 years.<sup>21</sup> One might be concerned about the effect of the recent financial crisis on these results. As Table 10 indicates, the qualitative results are unaffected if we use a sample period from 1985 to 2006.<sup>22</sup>

**Model implications** Our model implies that long-term bonds command a positive risk premium that increases with the maturity of the bond (see Appendix C for details on the pricing of long-term bonds). The latter property reflects the fact that longer maturity assets are more exposed to valuation risk. Table 9 shows that, taking sampling uncertainty into account, the benchmark and extended models are consistent with the observed mean yields, except that the former model generates slightly larger yields for long-term bonds than in the data. The table also shows that the estimated models account for the volatility of the yields on short-, intermediate-, and long-term bonds.

Piazzesi and Schneider (2007) and Beeler and Campbell (2012) argue that the bond term

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<sup>20</sup>The moments reported in this table are slightly different from those reported in Table 5. The reason is that the moments in the two tables were computed using different samples. In Table 5, we compute a leaded and a lagged correlation, so we lose one observation in the beginning of the sample and another in the end of the sample.

<sup>21</sup>The statistical significance of these average differences depend on whether we use zero or 12 lags in computing Newey-West standard errors.

<sup>22</sup>Evans (1998) and Piazzesi and Schneider (2006) finds that the U.K. real yield curve is downward sloping for the periods January 1983-November 1995 and December 1995-March 2006. Our results indicate that their findings depend on the sample period that they work with.

premium and the yield on long-term bonds are useful for discriminating between competing asset pricing models. For example, they stress that long-run risk models, of the type pioneered by Bansal and Yaron (2004), imply negative long-term bond yields and a negative bond term premium. The intuition is as follows: in a long-run risk model agents are concerned that consumption growth may be dramatically lower in some future state of the world. Since bonds promise a certain payout in all states of the world, they offer insurance against this possibility. The longer the maturity of the bond, the more insurance it offers and the higher is its price. So, the term premium is downward sloping. Indeed, the return on long-term bonds may be negative. Beeler and Campbell (2012) show that the return on a 20-year real bond in the Bansal, Kiku and Yaron (2012) model is  $-0.88$ .

Standard rare-disaster models also imply a downward sloping term structure for real bonds and a negative real yield on long-term bonds. See, for example the benchmark model in Nakamura et al. (2013). According to these authors, these implications can be reversed by introducing the possibility of default on bonds and assuming that probability of partial default is increasing in the maturity of the bond.<sup>23</sup> So, we cannot rule out the possibility that other asset-pricing models can account for bond term premia and the rate of return on long-term bonds. Still, it seems clear that valuation risk is a natural explanation of these features of the data.

### **Accounting for the slope of the yield curve and the persistence of bond yields**

Backus and Zin (1994) investigate the properties of the log of the SDF that are consistent with two key features of nominal yields: the yield curve is upward sloping and yields are very persistent. They estimate various ARMA representations for the log of the SDF using data on the mean and autocovariance of bond yields.<sup>24</sup> Their best statistical fit is an ARMA(2,3). These properties can be summarized as follows. First, the log of the SDF must have negative serial correlation to account for the positive slope of the nominal yield curve. Second, the log of the SDF must be close to i.i.d. but still have a small predictable component.<sup>25</sup> The log of the real SDF in both our benchmark and extended model satisfy these two properties. For an ARMA(2,3), these properties are most easily seen in the impulse response function

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<sup>23</sup>Nakamura et al (2013) consider a version of their model in which the probability of partial default on a perpetuity is 84 percent, while the probability of partial default on short-term bonds is 40 percent. This model generates a positive term premium and a positive real return on long-term bonds.

<sup>24</sup>Bansal and Viswanathan (1993) follow a similar approach but use a semi-nonparametric approximation to the SDF.

<sup>25</sup>See Ljungqvist and Sargent (2000) for a detailed exposition of these two properties.

of the extended model's log SDF, which we display in Panel A of Figure 2.<sup>26</sup>

The first-order serial correlation of the extended model's log of the real SDF is  $-0.07$ . Interestingly, the first-order serial correlation of the log SDF in Bansal, Kiko and Yaron (2012) is  $0.002$ . These results are consistent with the fact that our model implies an upward-sloping yield curve, while Bansal, Kiko and Yaron (2012) implies a downward-sloping yield curve.

To deduce the implications of our model for the nominal yield curve, we proceed as follows. First, we generate 1000 synthetic time series for the log of the real SDF, each of length equal to our sample size. We think of the first synthetic time-series observation as corresponding to 1929. Second, for each synthetic time series, we obtain the nominal SDF by dividing each realized value of the real SDF by the corresponding actual gross rate of inflation. By working with ex-post inflation rates, we avoid taking a stand on the stochastic process for inflation.<sup>27</sup> For each of the 1000 synthetic time series, we estimate an ARMA(2,3) for the log of the nominal SDF and compute the impulse response function to an innovation.

Panel B of Figure 2 displays the average impulse response functions of the log of the nominal SDF along with a 95 percent confidence interval computed across the 1000 synthetic time series. The impulse response function of the log of the real and the nominal SDFs are very similar. The first-order serial correlation of the log of the nominal SDF is  $-0.073$ , which is very similar to first-order serial correlation of the log of the real SDF. A 95 percent confidence interval computed across the 1000 simulations spans the interval from  $-0.134$  to  $-0.008$ . So we can exclude, at conventional significant levels, that the first-order serial correlation of the log of the nominal SDF is positive.

In sum, the estimated SDF for our extended model has the two characteristics that Backus and Zin (1994) argue to be required to explain an upward sloping nominal and real yield curve and the persistence of bond yields.

**Long-term equity premium** According to Table 9, the benchmark and extended models imply that the difference between stock returns and 20-year bond yield is roughly 1 percent. In the data, the difference between stock returns and the 20-year bond yields is roughly 4.16 percent with a standard error of 2.39. So, taking sampling uncertainty into account, both

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<sup>26</sup>The dotted lines are the limits of the 95 percent confidence interval.

<sup>27</sup>The results for  $m_t$  are calculating using a very long simulation, so they are close to the plim of the ARMA(2,3). In contrast, the results for  $M_t$ , we rely on realized inflation, correspond to the plim of the small sample for the ARMA(2,3).

the benchmark and extended models are consistent with the data.

In our model, the positive premium that equity commands over long-term bonds reflects the difference between an asset of infinite and twenty-year maturity. Consistent with this perspective, Binsbergen, Hueskes, Koijen, and Vrugt (2011) estimate that 90 (80) percent of the value of the S&P 500 index corresponds to dividends that accrue after the first five (ten) years.

It is important to emphasize that the equity premium in our model is not solely driven by the term premium. One way to see this property is to consider the results of regressing the equity premium on two alternative measures of excess bond yields. The first measure is the difference between yields on bonds of 20-year and 1-year maturities. The second measure is the difference between yields on bonds of 5-year and 1-year maturities. Table 12 reports our results. Not surprisingly, the benchmark model does poorly since the expected equity premium and the term premia are constant. The extended model does better in the sense that the model slopes are within one standard error of the slope point estimates. Also, both models are consistent with the fact that the  $R^2$  in these regressions are quite low.

We conclude with an interesting observation made by Binsbergen, Brandt, and Koijen (2012). Using data over the period 1996 to 2009, these authors decompose the S&P500 index into portfolios of short-term and long-term dividend strips. The first portfolio entitles the holder to the realized dividends of the index for a period of up to three years. The second portfolio is a claim on the remaining dividends. Binsbergen et al (2012) find that the short-term dividend portfolio has a higher risk premium than the long-term dividend portfolio, i.e. there is a negative stock term premium. They argue that this observation is inconsistent with habit-formation, long-run risk models and standard of rare-disaster models. Our model, too, has difficulty in accounting for the Binsbergen et al (2012) negative stock term premium.<sup>28</sup> Of course, our sample is very different from theirs and their negative stock term premium result is heavily influenced by the recent financial crisis (see Binsbergen et al (2011)). Also, Boguth, Carlson, Fisher, and Simutin (2012) argue that the Binsbergen et al (2012) results may be significantly biased because of the impact of small pricing frictions.

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<sup>28</sup>Recently, Nakamura et al (2013) show that a time-varying rare disaster model in which the component of consumption growth due to a rare disaster follows an AR(1) process is consistent with the Binsbergen et al (2012) results. Belo et al. (2013) show that the Binsbergen et al. (2012) result can be reconciled in a variety of models if the dividend process is replaced with processes that generate stationary leverage ratios.

## 7. Conclusion

In this paper we argue that allowing for demand shocks substantially improves the performance of representative-agent, asset-pricing models. Specifically, it allows the model to account for the equity premium, bond term premia, and the correlation puzzle with low degrees of estimated risk aversion. According to our estimates, valuation risk is by far the most important determinant of the equity premium and the bond term premia.

The recent literature has incorporated many interesting features into standard asset-pricing models to improve their performance. Prominent examples, include habit formation, long-run risk, time-varying endowment volatility, and model ambiguity. We abstract from these features to isolate the empirical role of valuation risk. But they are, in principle, complementary to valuation risk and could be incorporated into our analysis. We leave this task for future research.

## References

- [1] Alvarez, Fernando, and Urban J. Jermann, “Using Asset Prices to Measure the Persistence of the Marginal Utility of Wealth,” *Econometrica*, 73 (6): 1977–2016, 2005.
- [2] Backus, David K., and Stanley E. Zin. Reverse engineering the yield curve. No. w4676. National Bureau of Economic Research, 1994.
- [3] Backus, David, Bryan R. Routledge, and Stanley E. Zin, The Cyclical Component of US Asset Returns, manuscript, New York University, April 2010.
- [4] Bai, Yan, José Victor Ríos-Rull and Kjetil Storesletten “Demand Shocks as Productivity Shocks,” manuscript, University of Minnesota, 2014.
- [5] Bansal, Ravi and S. Viswanathan “No Arbitrage and Arbitrage Pricing: A New Approach,” *The Journal of Finance*, 48:4, 1231-1262, 1993.
- [6] Bansal, Ravi and Amir Yaron, “Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles,” *The Journal of Finance*, 59 (4): 1481-509, 2004.
- [7] Bansal, Ravi, Dana Kiku and Amir Yaron, “An Empirical Evaluation of the Long-Run Risks Model for Asset Prices,” *Critical Finance Review*, 1: 183–221, 2012.
- [8] Bansal, Ravi, Dana Kiku, and Amir Yaron. “Risks for the Long Run: Estimation with Time Aggregation,” manuscript, Wharton School, 2013.
- [9] Barberis, Nicholas, Andrei Shleifer, and Robert Vishny. “A Model of Investor Sentiment,” *Journal of Financial Economics* 49 (3): 307-43, 1998.
- [10] Barro, Robert J. “Rare Disasters and Asset Markets in the Twentieth Century,” *Quarterly Journal of Economics*, 121: 823–66, 2006.
- [11] Barro, Robert J., and José F. Ursúa. “Rare Macroeconomic Disasters,” working paper No. 17328, National Bureau of Economic Research, 2011.
- [12] Beeler, Jason, and John Y. Campbell. “The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment,” *Critical Finance Review*, 1: 141–82, 2012.
- [13] Belo, Frederico, Pierre Collin-Dufresne, and Robert S. Goldstein. “Dividend Dynamics and the Term Structure of Dividend Strips,” manuscript, University of Minnesota, 2013.



- [14] Binsbergen, Jules H. van, Michael W. Brandt, and Ralph S.J. Koijen. “On the Timing and Pricing of Dividends,” *American Economic Review*, 102 (4): 1596-618, 2012.
- [15] Binsbergen, Jules H. van, Wouter Hueskes, Ralph Koijen, and Evert Vrugt. “Equity Yields,” working paper N. 17416, National Bureau of Economic Research, 2011.
- [16] Boldrin, Michele, Lawrence J. Christiano, and Jonas Fisher. “Habit Persistence, Asset Returns, and the Business Cycle,” *American Economic Review*, 91: 149-66, 2001.
- [17] Borovička, Jaroslav, and Lars Peter Hansen. “Examining Macroeconomic Models through the Lens of Asset Pricing,” manuscript, University of Chicago, 2011.
- [18] Borovička, Jaroslav, Lars Peter Hansen, Mark Hendricks, and José A. Scheinkman. “Risk-Price Dynamics,” *Journal of Financial Econometrics* 9 (1): 3-65, 2011.
- [19] Boguth, Oliver, Murray Carlson, Adlai Fisher, and Mikhal Simutin. “Leverage and the Limits of Arbitrage Pricing: Implications for Dividend Strips and the Term Structure of Equity Risk Premia,” manuscript, Arizona State University, 2012.
- [20] Boudoukh, Jacob, Matthew Richardson, and Robert F. Whitelaw. “The Myth of Long-Horizon Predictability,” *Review of Financial Studies* 24 (4): 1577-605, 2008.
- [21] Campbell, John Y. “Bond and Stock Returns in a Simple Exchange Model,” *The Quarterly Journal of Economics* 101 (4): 785-803, 1986.
- [22] Campbell, John Y. “Consumption-Based Asset Pricing,” in Handbook of the Economics of Finance, Eds. G.M. Constantinides, M. Harris and R. Stulz, 801-85, 2003.
- [23] Campbell, John Y., and John Ammer. “What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns,” *The Journal of Finance* 48 (1): 3-37, 1993.
- [24] Campbell, John Y., and John H. Cochrane. “By Force of Habit: A Consumption-based Explanation of Aggregate Stock Market Behavior,” *Journal of Political Economy*, 107 (2): 205-51, 1999.
- [25] Campbell, John Y., and Robert J. Shiller. “The Dividend-price Ratio and Expectations of Future Dividends and Discount Factors,” *Review of Financial Studies* 1 (3): 195-228, 1988.

- [26] Campbell, John, Robert Shiller, and Luis Viceira. “Understanding Inflation-Indexed Bond Markets,” *Brookings Papers on Economic Activity*, 79-120, 2009.
- [27] Cochrane, John H. “Explaining the Variance of Price Dividend Ratios,” *Review of Financial Studies* 5: 243-80, 1992.
- [28] Cochrane, John H. *Asset Pricing*, Princeton University Press, 2001.
- [29] Cochrane, John H. “Presidential Address: Discount Rates,” *The Journal of Finance*, 66 (4): 1047-108, 2011.
- [30] Cochrane, John H., and Lars Peter Hansen. “Asset Pricing Explorations for Macroeconomics,” *NBER Macroeconomics Annual 1992*, Volume 7, pp. 115-82. MIT Press, 1992.
- [31] Correia, Isabel, Emmanuel Farhi, Juan Pablo Nicolini, and Pedro Teles. “Unconventional Fiscal Policy at the Zero Bound,” *American Economic Review*, 103 (4): 1172-211, 2013.
- [32] Dew-Becker, Ian “A Model of Time-Varying Risk Premia with Habits and Production,” manuscript, Northwestern University, 2014.
- [33] Dumas, Bernard, Alexander Kurshev, and Raman Uppal. “Equilibrium Portfolio Strategies in the Presence of Sentiment Risk and Excess Volatility,” *The Journal of Finance*, 64 (2): 579-629, 2009.
- [34] Duffie, Darrell, and Larry G. Epstein. “Stochastic Differential Utility,” *Econometrica*, 60: 353-94, 1992.
- [35] Eggertsson, Gauti. “Monetary and Fiscal Coordination in a Liquidity Trap,” Chapter 3 of *Optimal Monetary and Fiscal Policy in the Liquidity Trap*, Ph.D. dissertation, Princeton University, 2004.
- [36] Eggertsson, Gauti B., and Michael Woodford. “Zero Bound on Interest Rates and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, 1: 139-233, 2003.
- [37] Epstein, Larry G., and Stanley E. Zin. “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework,” *Econometrica*, 57 (4): 937-69, 1989.

- [38] Epstein, Larry G., and Stanley E. Zin. “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis,” *Journal of Political Economy*, 99: 263-86, 1991.
- [39] Epstein, Larry, Emmanuel Farhi, and Tomasz Strzalecki “How Much Would You Pay to Resolve Long-Run Risk?,” *American Economic Review*, 104, (2014) 2680–2697
- [40] Fernandez-Villaverde, Jesus, Pablo Guerron-Quintana, Keith Kuester, and Juan Rubio-Ramírez. “Fiscal Volatility Shocks and Economic Activity,” manuscript, University of Pennsylvania, 2013.
- [41] Gabaix, Xavier, and Matteo Maggiori. “International Liquidity and Exchange Rate Dynamics,” manuscript, New York University, 2013.
- [42] Garber, Peter M., and Robert G. King. “Deep Structural Excavation? A Critique of Euler Equation Methods,” working paper No. 31, National Bureau of Economic Research, 1983.
- [43] Gârleanu, Nicolae, Leonid Kogan, and Stavros Panageas. “Displacement Risk and Asset Returns,” *Journal of Financial Economics*, 105 (3): 491-510, 2012.
- [44] Gourio, François. “Disaster Risk and Business Cycles,” *American Economic Review*, 102 (6): 2734-66, 2012.
- [45] Grossman, Sanford J., Angelo Melino, and Robert J. Shiller. “Estimating the Continuous Time Consumption Based Asset Pricing Model”, *Journal of Business and Economic Statistics*, 5: 315-28, 1987.
- [46] Hall, Robert E. “High Discounts and High Unemployment,” working paper No. 19871, National Bureau of Economic Research, 2014.
- [47] Hansen, Lars Peter, John C. Heaton, and Nan Li. “Consumption Strikes Back? Measuring Long-Run Risk,” *Journal of Political Economy*, 116 (2): 260-302, 2008.
- [48] Hansen, Lars Peter, and Ravi Jagannathan. “Implications of Security Market Data for Models of Dynamic Economies,” *Journal of Political Economy*, 99 (2): 225-62, 1991.

- [49] Hansen, Lars Peter, and Kenneth J. Singleton. “Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models,” *Econometrica*, 50, 1269-86, 1982.
- [50] Huo, Zhen, and José-Víctor Ríos-Rull. “Paradox of Thrift Recessions,” working paper No. 19443, National Bureau of Economic Research, 2013.
- [51] Kandel, Shmuel, and Robert F. Stambaugh. “Expectations and Volatility of Consumption and Asset Returns,” *Review of Financial Studies* 3 (2): 207-32, 1990.
- [52] Kreps, David M., and Evan L. Porteus. “Temporal Resolution of Uncertainty and Dynamic Choice Theory,” *Econometrica*, 46, 185-200, 1978.
- [53] Lettau, Martin, and Sydney C. Ludvigson. “Shocks and Crashes,” manuscript, New York University, 2011.
- [54] Lucas Jr, Robert E. “Asset Prices in an Exchange Economy,” *Econometrica*, 46, 1429-45, 1978.
- [55] Luo, Victor. “Using Bond Yields to Discriminate Between Long-run and Valuation Risk Models,” manuscript, Northwestern University, 2014.
- [56] Lynch, Anthony W. “Decision frequency and synchronization across agents: Implications for aggregate consumption and equity return,” *The Journal of Finance* 51.4: 1479-1497, 1996.
- [57] Ljungqvist, Lars and Thomas Sargent *Recursive Macroeconomic Theory*, 2nd edition, Massachusetts Institute of Technology, 2000.
- [58] Maurer, Thomas A. “Is Consumption Growth Merely a Sideshow in Asset Pricing?,” manuscript, London School of Economics, 2012.
- [59] Mehra, Rajnish, and Edward C. Prescott. “The Equity Premium: A Puzzle,” *Journal of Monetary Economics*, 15 (2): 145-61, 1985.
- [60] Nakamura, Emi, Jón Steinsson, Robert Barro, and José Ursúa. “Crises and Recoveries in an Empirical Model of Consumption Disasters,” *American Economic Journal: Macroeconomics*, 5 (3), 35-74, 2013.

- [61] Newey, Whitney K., and Kenneth D. West. "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55 (3): 703-8, 1987.
- [62] Normandin, Michel, and Pascal St-Amour. "Substitution, Risk Aversion, Taste Shocks and Equity Premia," *Journal of Applied Econometrics*, 13 (3): 265-81, 1998.
- [63] Parker, Jonathan A. "The Consumption Risk of the Stock Market," *Brookings Papers on Economic Activity*, 2: 279-348, 2001.
- [64] Pavlova, Anna, and Roberto Rigobon. "Asset Prices and Exchange Rates," *Review of Financial Studies*, 20 (4): 1139-81, 2007.
- [65] Piazzesi, Monika, and Martin Schneider. "Equilibrium Yield Curves," *NBER Macroeconomics Annual 2006*, 21: 389-472, 2007.
- [66] Primiceri, Giorgio. "Time Varying Structural Vector Autoregressions and Monetary Policy," *The Review of Economic Studies*, 72: 821-852, 2005.
- [67] Rietz, Thomas A. "The Equity Risk Premium a Solution," *Journal of Monetary Economics*, 22 (1): 117-31, 1988.
- [68] Shiller, Robert. "Do Stock Prices Move Too Much to be Justified by Subsequent Dividends," *American Economic Review*, 71: 421-36, 1981.
- [69] Skiadas, Costis. "Asset Pricing Theory," Princeton University Press, 2009.
- [70] Stambaugh, Robert F. "Predictive Regressions," *Journal of Financial Economics*, 54, 375-421, 1999.
- [71] Stockman, Alan C., and Linda L. Tesar. "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements," *The American Economic Review*, 85 (1): 168-185, 1995.
- [72] Wachter, Jessica A. "Solving models with external habit," *Finance Research Letters* 2.4: 210-226, 2005.
- [73] Wachter, Jessica A. "Can Time-varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?," *Journal of Finance*, forthcoming.

- [74] Weil, Philippe. “The Equity Premium Puzzle and the Risk-free Rate Puzzle,” *Journal of Monetary Economics*, 24 (3): 401-421, 1989.

Table 1

Correlation Between Stock Returns and Per Capita  
Growth Rates of Fundamentals

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*Panel A, 1929-2011*

Horizon	Consumption	Output	Dividends	Earnings
1 year	-0.05 (0.12)	0.05 (0.10)	0.05 (0.11)	0.10 (0.10)
5 years	0.00 (0.14)	0.00 (0.12)	0.30 (0.13)	0.20 (0.13)
10 years	-0.11 (0.20)	-0.09 (0.14)	0.59 (0.14)	0.30 (0.11)

*Panel B, 1871-2006*

Horizon	Consumption	Output	Dividends	Earnings
1 year	0.09 (0.09)	0.14 (0.10)	-0.04 (0.10)	0.13 (0.10)
5 years	0.40 (0.18)	0.25 (0.14)	0.38 (0.15)	0.44 (0.18)
10 years	0.25 (0.18)	0.00 (0.11)	0.64 (0.17)	0.41 (0.13)

Table 2

Correlation Between Stock Returns and Per Capita  
Growth Rates of Fundamentals

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NIPA measures of consumption, 1929-2011			
Horizon	Durables	Non-durables	Non-durables and services
1 year	-0.04 (0.13)	0.05 (0.14)	0.27 (0.14)
5 years	0.07 (0.11)	-0.08 (0.09)	0.18 (0.08)
10 years	0.21 (0.15)	-0.20 (0.13)	0.06 (0.14)



Table 3

Correlation Between Stock Returns and Per Capita  
Growth Rates of Fundamentals

Horizon	Consumption	Output	Horizon	Consumption	Output
	G7			OECD	
1 year	0.013 (0.061)	0.178 (0.079)	1 year	0.038 (0.033)	0.125 (0.045)
5 years	0.189 (0.105)	0.345 (0.091)	5 years	0.130 (0.067)	0.272 (0.067)
10 years	0.280 (0.130)	0.397 (0.117)	10 years	0.151 (0.103)	0.262 (0.104)

Table 4

Parameter Estimates and Standard Errors  
Models estimated with ex-ante real bond yields

Parameter	Benchmark model	Extended model	Parameter	Benchmark model	Extended model
$\gamma$	1.516 (0.029616)	2.3961 (0.11244)	$\rho$	0.99132 (0.00056369)	0.99168 (0.00078787)
$\psi$	1.4567 (0.041753)	2.2107 (0.17437)	$\sigma_s$	0.00058631 (2.8373e-05)	0.00038594 (1.8036e-05)
$\delta$	0.99795 (0.0017074)	0.99796 (8.0205e-05)	Implied value of $\theta$	-1.6459 (0.19603)	-2.5492 (0.29743)
$\sigma_c$	0.0069004 (0.00014956)	0.0046033 (0.00013664)	$\nu$	0.00	0.99717 (0.00030196)
$\mu$	0.0015644 (4.4976e-05)	0.0010428 (7.8579e-05)	$\sigma_w$	0.00	1.68E-06 (4.9166e-08)
$\pi_{c_s}$	0.00	-0.0029185 (0.00023329)	$\alpha_c$	0.00	-95.3767 (4.1769)
$\sigma_d$	0.017103 (0.00039626)	0.025274 (0.19243)	$\alpha_d$	0.00	-151.8814 (22.7162)
$\pi_{dc}$	0.00049127 (0.00044218)	-1.3205 (0.2236)	$\rho_c$	0.00	0.094708 (0.045391)
$\pi_{d_s}$	0.00	-0.010907 (0.0015436)	$\rho_d$	0.00	0.33427 (0.11204)
$\sigma_v$	0.00	0.0077828 (0.00024356)			

Table 5

Model estimated with ex-ante real bond yields

Moments	Data (constrained)	Data (unconstrained)	Benchmark model	Extended model
Average growth rate of consumption	1.4941 (0.30)	2.2197 (0.22413)	1.87	1.38
Average growth rate of dividends	1.4941 (0.30395)	0.12538 (0.65732)	1.87	1.38
Standard deviation of the growth rate of consumption	2.1865 (0.34494)	2.1624 (0.31345)	2.38	2.65
Standard deviation of the growth rate of dividends	6.914 (1.0409)	6.5181 (1.1219)	5.85	7.25
Contemporaneous correlation between consumption growth and dividend growth	0.2215 (0.1006)	0.23722 (0.10649)	0.03	0.29
First-order serial correlation of consumption	0.20014 (0.1475)	0.1584 (0.1240)	-0.01	0.30
Correlation between stock returns and one-period lagged consumption growth	-0.1555 (0.1070)	-0.26065 (0.0999)	0.00	-0.03
Correlation between stock returns and one-period lead consumption growth	0.49504 (0.0670)	0.50929 (0.0490)	0.01	0.17
Average return to equities	7.8252 (1.7396)	6.3948 (1.8804)	5.94	4.00
Standard deviation of return to equities	17.2537 (1.3783)	17.6831 (1.461)	17.37	19.23
Average risk-free rate	0.12613 (0.8088)	0.12087 (0.8088)	0.13	0.13
Standard deviation of the risk-free rate	3.5615 (0.81818)	3.4989 (0.83447)	4.30	4.06
First-order serial correlation of the risk-free rate	0.51802 (0.068326)	0.52154 (0.070816)	0.90	0.51
Equity premium	7.6991 (1.8141)	6.2739 (1.9485)	5.81	3.88
Average price-dividend ratio	3.3761 (0.14887)	3.3775 (0.14887)	3.22	3.32
Standard deviation of price-dividend ratio	0.47324 (0.074827)	0.44502 (0.081116)	0.31	0.46
First-order serial correlation of price dividend ratio	0.95041 (0.034499)	0.93257 (0.042679)	0.84	0.89
Contemporaneous correlation between risk free rate and price-dividend ratio	0.2736, (0.11356)	0.30705 (0.11909)	-0.95	-0.20

Table 6

Correlation Between Stock Returns and Per Capita  
Growth Rates of Consumption and Dividends

Model estimated with ex-ante real bond yields

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Moments	<i>Consumption growth</i>			
	Data (constrained)	Data (unconstrained)	Benchmark model	Extended model
1-year correlation between equity returns and consumption growth	-0.07 (0.11)	-0.05 (0.12)	0.02	-0.19
5-year correlation between equity returns and consumption growth	-0.01 (0.16)	-0.05 (0.13)	0.02	-0.05
10-year correlation between equity returns and consumption growth	-0.08 (0.30)	-0.21 (0.21)	0.03	0.11
	<i>Dividend growth</i>			
1-year correlation between equity returns and dividend growth	0.08 (0.11)	0.05 (0.11)	0.34	-0.10
5-year correlation between equity returns and dividend growth	0.22 (0.14)	0.29 (0.14)	0.32	0.03
10-year correlation between equity returns and dividend growth	0.51 (0.22)	0.59 (0.15)	0.38	0.20

Table 7

## Predictability of Excess Returns by Price-dividend Ratio at Various Horizons

Model estimated with ex-ante real bond yields

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<i>Benchmark model</i>						
	Data	Model (median) Slope coefficient	Model (plim)	Data R-square	Model (median) (% of values larger than R-square in data)	Model (plim)
1 year	-0.10 (0.03)	-0.05	0.008	0.07	0.01 (0.03)	0.0002
3 years	-0.27 (0.06)	-0.14	0.027	0.14	0.03 (0.08)	0.001
5 years	-0.42 (0.12)	-0.21	0.034	0.26	0.04 (0.05)	0.001
<i>Extended model</i>						
1 year	-0.10 (0.03)	-0.07	-0.04	0.07	0.03 (0.13)	0.01
3 years	-0.27 (0.06)	-0.19	-0.10	0.14	0.08 (0.23)	0.04
5 years	-0.42 (0.12)	-0.30	-0.16	0.26	0.12 (0.15)	0.06

Table 8

Correlation of price-dividend ratio and ex-ante risk-free rate

	Sample	Full-sample	Pre 1983	Post 1983
Canada	1952-2014	0.0223 (0.139)	0.288 (0.136)	-0.213 (0.239)
France	1932-2014	-0.484 (0.152)	-0.576 (0.124)	0.132 (0.179)
Germany	1968-2014	-0.115 (0.105)	-0.382 (0.189)	-0.117 (0.277)
Italy	1940-2014	-0.588 (0.166)	-0.725 (0.0823)	0.344 (0.2)
Japan	1960-2014	0.342 (0.176)	-0.128 (0.16)	0.682 (0.109)
U.K.	1934-2014	0.246 (0.122)	0.147 (0.0819)	-0.253 (0.216)
US	1939-2011	0.30705 (0.11909)	0.39 (0.113)	-0.42 (0.227)

Table 9

## Term Structure of Bond Yields

Model estimated with ex-ante real bond yields  
(sample 1929-2011)

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Moments	Data (ex-post real yields)	Data (ex-ante real yields)	Benchmark model	Extended model
	<i>Average yield</i>			
Long-term bonds	1.32 (1.10)	2.90 (0.84)	4.92	3.00
Intermediate-term bond	1.39 (0.91)	1.93 (0.99)	2.10	1.64
Short-term bond	0.42 (0.80)	0.46 (0.78)	0.13	0.13
Return to equity minus long-term bond yield	4.16 (2.39)	2.54 (2.09)	1.02	1.00
	<i>Standard deviation of yield</i>			
Long-term bonds	3.02 (0.65)	2.59 (0.52)	1.84	1.40
Intermediate-term bond	3.29 (0.53)	3.14 (0.57)	3.42	2.46
Short-term bond	3.88 (0.77)	3.85 (0.77)	4.30	4.06
Return to equity minus long-term bond yield	20.2 (2.47)	20.09 (1.96)	16.96	19.10

Table 10

Difference between long-term yields and 2.5 year yields  
Inflation-indexed U.K. gilts

	Sample	Mean	Newey-West standard errors	
			Zero lags in weighting matrix	12 lags in weighting matrix
5 year minus 2.5 year	1985-2015	0.19	0.03	0.10
	1985-2006.Q3	0.09	0.03	0.11
10 year minus 2.5 year	1985-2015	0.38	0.05	0.17
	1985-2006.Q3	0.22	0.05	0.17
15 year minus 2.5 year	1985-2015	0.42	0.06	0.20
	1985-2006.Q3	0.26	0.06	0.19
20 year minus 2.5 year	1985-2015	0.42	0.08	0.26
	1985-2006.Q3	0.24	0.08	0.27



Table 11

## Regressions of Excess Stock Returns on Long-term Bond Yields in Excess of Short Rate

Model estimated with ex-ante real bond yields

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	Data 1929-2011	Benchmark model	Extended model
	<i>Long-term government bonds (20 years)</i>		
R-square	0.03	0.13	0.03
Slope	0.91 (0.46)	3.48	0.98
	<i>Intermediate term government bonds (5 years)</i>		
R-square	0.02	0.07	0.02
Slope	0.72 (0.51)	3.36	1.01

## 8. Appendix

### 8.1. Appendix A

In this appendix, we solve the model in Section 3 analytically for the case of CRRA utility. Let  $C_{a,t}$  denote the consumption of the representative agent at time  $t$ . The representative agent solves the following problem:

$$U_t = \max E_t \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} \frac{C_{a,t+i}^{1-\gamma}}{1-\gamma},$$

subject to the flow budget constraints

$$W_{a,i+1} = R_{c,i+1} (W_{a,i} - C_{a,i}),$$

for all  $i \geq t$ . The variable  $R_{c,i+1}$  denotes the gross return to a claim that pays the aggregate consumption as in equation (3.8), financial wealth is  $W_{a,i} = (P_{c,i} + C_i) S_{a,i}$ , and  $S_{a,i}$  is the number of shares on the claim to aggregate consumption held by the representative agent. The first-order condition for  $S_{a,t+i+1}$  is:

$$\delta^i \lambda_{t+i} C_{a,t+i}^{-\gamma} = E_t (\delta^{i+1} \lambda_{t+i+1} C_{a,t+i+1}^{-\gamma} R_{c,i+1}).$$

In equilibrium,  $C_{a,t} = C_t$ ,  $S_{a,t} = 1$ . The equilibrium value of the intertemporal marginal rate of substitution is:

$$M_{t+1} = \delta \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}. \quad (8.1)$$

The Euler equation for stock returns is the familiar,

$$E_t [M_{t+1} R_{c,t+1}] = 1.$$

We now solve for  $P_{c,t}$ . It is useful to write  $R_{c,t+1}$  as

$$R_{c,t+1} = \frac{(P_{c,t+1}/C_{t+1} + 1)}{P_{c,t}/C_t} \left( \frac{C_{t+1}}{C_t} \right).$$

In equilibrium:

$$E_t \left[ M_{t+1} \left( \frac{P_{c,t+1}}{C_{t+1}} + 1 \right) \left( \frac{C_{t+1}}{C_t} \right) \right] = \frac{P_{c,t}}{C_t}. \quad (8.2)$$

Replacing the value of  $M_{t+1}$  in equation (8.2):

$$E_t \left[ \delta \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{P_{c,t+1}}{C_{t+1}} + 1 \right) \left( \frac{C_{t+1}}{C_t} \right) \right] = \frac{P_{c,t}}{C_t}.$$

Using the fact that  $\lambda_{t+1}/\lambda_t$  is known as of time  $t$  we obtain:

$$\delta \frac{\lambda_{t+1}}{\lambda_t} E_t \left[ \exp(\mu + \sigma_c \varepsilon_{t+1}^c)^{1-\gamma} \left( \frac{P_{ct+1}}{C_{t+1}} + 1 \right) \right] = \frac{P_{ct}}{C_t}.$$

We guess and verify that  $P_{ct+1}/C_{t+1}$  is independent of  $\varepsilon_{t+1}^c$ . This guess is based on the fact that the model's price-consumption ratio is constant absent time-preference shocks.

Therefore,

$$\delta \frac{\lambda_{t+1}}{\lambda_t} \exp \left[ (1-\gamma)\mu + (1-\gamma)^2 \sigma_c^2 / 2 \right] E_t \left( \frac{P_{c,t+1}}{C_{t+1}} + 1 \right) = \frac{P_{c,t}}{C_t}. \quad (8.3)$$

We now guess that there are constants  $k_0, k_1, \dots$ , such that

$$\frac{P_{c,t}}{C_t} = k_0 + k_1 (\lambda_{t+1}/\lambda_t) + k_2 (\lambda_{t+1}/\lambda_t)^{1+\rho} + k_3 (\lambda_{t+1}/\lambda_t)^{1+\rho+\rho^2} + \dots \quad (8.4)$$

Using this guess,

$$\begin{aligned} & E_t \left( \frac{P_{c,t+1}}{C_{t+1}} + 1 \right) \\ &= E_t \left( k_0 + k_1 ((\lambda_{t+1}/\lambda_t)^\rho \exp(\sigma_\lambda \varepsilon_{t+1}^\lambda)) + k_2 ((\lambda_{t+1}/\lambda_t)^\rho \exp(\sigma_\lambda \varepsilon_{t+1}^\lambda))^{1+\rho} + \dots + 1 \right) \\ &= k_0 + k_1 (\lambda_{t+1}/\lambda_t)^\rho \exp(\sigma_\lambda^2/2) + k_2 (\lambda_{t+1}/\lambda_t)^{\rho(1+\rho)} \exp((1+\rho)^2 \sigma_\lambda^2/2) + \dots + 1. \end{aligned} \quad (8.5)$$

Substituting equations (8.4) and (8.5) into equation (8.3) and equating coefficients leads to the following solution for the constants  $k_i$ :

$$\begin{aligned} k_0 &= 0, \\ k_1 &= \delta \exp \left[ (1-\gamma)\mu + (1-\gamma)^2 \sigma_c^2 / 2 \right], \end{aligned}$$

and for  $n \geq 2$

$$k_n = k_1^n \exp \left\{ \left[ 1 + (1+\rho)^2 + (1+\rho+\rho^2)^2 + \dots + (1+\dots+\rho^{n-2})^2 \right] \sigma_\lambda^2 / 2 \right\}.$$

We assume that the series  $\{k_n\}$  converges, so that the equilibrium price-consumption ratio is given by equation (8.4). Hence, the realized return on the consumption claim is

$$R_{c,t+1} = \frac{C_{t+1}}{C_t} \frac{k_1 (\lambda_{t+2}/\lambda_{t+1}) + k_2 (\lambda_{t+2}/\lambda_{t+1})^{1+\rho} + \dots + 1}{k_1 (\lambda_{t+1}/\lambda_t) + k_2 (\lambda_{t+1}/\lambda_t)^{1+\rho} + \dots}. \quad (8.6)$$

The equation that prices the one-period risk-free asset is:

$$E_t [M_{t+1} R_{f,t+1}] = 1.$$

Taking logarithms of both sides of this equation and noting that  $R_{f,t+1}$  is known at time  $t$ , we obtain:

$$r_{f,t+1} = -\log E_t (M_{t+1}).$$

Using equation (8.1),

$$\begin{aligned} E_t (M_{t+1}) &= \delta \frac{\lambda_{t+1}}{\lambda_t} E_t [\exp(-\gamma (\mu + \sigma_c \varepsilon_{t+1}^c))] \\ &= \delta \frac{\lambda_{t+1}}{\lambda_t} \exp(-\gamma \mu + \gamma^2 \sigma_c^2 / 2). \end{aligned}$$

Therefore,

$$r_{f,t+1} = -\log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \gamma \mu - \gamma^2 \sigma_c^2 / 2.$$

Using equation (3.4), we obtain

$$E [(\lambda_{t+1}/\lambda_t)^{-1}] = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right).$$

We can then write the unconditional risk-free rate as:

$$E (R_{f,t+1}) = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right) \delta^{-1} \exp(\gamma \mu - \gamma^2 \sigma_c^2 / 2).$$

Thus, the equity premium is given by:

$$E [(R_{c,t+1}) - R_{f,t+1}] = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right) \delta^{-1} \exp(\gamma \mu - \gamma^2 \sigma_c^2 / 2) [\exp(\gamma \sigma_c^2) - 1],$$

which can be written as:

$$E [(R_{c,t+1}) - R_{f,t+1}] = E (R_{f,t+1}) [\exp(\gamma \sigma_c^2) - 1].$$

## 8.2. Appendix B

This appendix provides a detailed derivation of the equilibrium of the model economy where the representative agent has Epstein-Zin preferences and faces time-preference shocks. The agent solves the following problem:

$$U (W_t) = \max_{C_t} \left[ \lambda_t C_t^{1-1/\psi} + \delta (U_{t+1}^*)^{1-1/\psi} \right]^{1/(1-1/\psi)}, \quad (8.7)$$

where  $U_{t+1}^* = [E_t (U (W_{t+1})^{1-\gamma})]^{1/(1-\gamma)}$ . The optimization is subject to the following budget constraint:

$$W_{t+1} = R_{c,t+1} (W_t - C_t).$$

The agent takes as given the stochastic processes for the return on the consumption claim  $R_{c,t+1}$  and the preference shock  $\lambda_{t+1}$ . For simplicity, we omit the dependence of life-time utility on the processes for  $\lambda_{t+1}$  and  $R_{c,t+1}$ .

The first-order condition with respect to consumption is,

$$\lambda_t C_t^{-1/\psi} = \delta (U_{t+1}^*)^{-1/\psi} [E_t (U (W_{t+1})^{1-\gamma})]^{1/(1-\gamma)-1} E_t (U (W_{t+1})^{-\gamma} U' (W_{t+1}) R_{c,t+1}),$$

and the envelope condition is:

$$U' (W_t) = U (W_t)^{1/\psi} \delta (U_{t+1}^*)^{-1/\psi} [E_t (U (W_{t+1})^{1-\gamma})]^{1/(1-\gamma)-1} E_t (U (W_{t+1})^{-\gamma} U' (W_{t+1}) R_{c,t+1}).$$

Combining the first-order condition and the envelope condition we obtain:

$$U' (W_t) = U (W_t)^{1/\psi} \lambda_t C_t^{-1/\psi}. \quad (8.8)$$

This equation can be used to replace the value of  $U' (W_{t+1})$  in the first order condition:

$$\lambda_t C_t^{-1/\psi} = \delta (U_{t+1}^*)^{-1/\psi} [E_t (U (W_{t+1})^{1-\gamma})]^{1/(1-\gamma)-1} E_t \left( U (W_{t+1})^{1/\psi-\gamma} \lambda_{t+1} C_{t+1}^{-1/\psi} R_{c,t+1} \right).$$

Using the expression for  $U_{t+1}^*$ , this last equation can be written after some algebra as,

$$1 = E_t (M_{t+1} R_{c,t+1}). \quad (8.9)$$

Here,  $M_{t+1}$  is the stochastic discount factor, or intertemporal marginal rate of substitution, which is given by:

$$M_{t+1} = \delta \frac{\lambda_{t+1}}{\lambda_t} \frac{U (W_{t+1})^{1/\psi-\gamma}}{(U_{t+1}^*)^{1/\psi-\gamma}} \frac{C_{t+1}^{-1/\psi}}{C_t^{-1/\psi}}.$$

We guess and verify the policy function for consumption and the form of the utility function. As in Weil (1989) and Epstein and Zin (1991), we guess that:

$$\begin{aligned} U (W_t) &= a_t W_t, \\ C_t &= b_t W_t. \end{aligned}$$

Replacing these guesses in equation (8.8) and simplifying yields:

$$a_t^{1-1/\psi} = \lambda_t b_t^{-1/\psi}. \quad (8.10)$$

Using the same guesses in the Hamilton-Jacobi-Bellman equation (8.7) and simplifying, we obtain:

$$a_t = \left[ \lambda_t b_t^{1-1/\psi} + \delta \left( \left[ E_t \left( \left( a_{t+1} \frac{W_{t+1}}{W_t} \right)^{1-\gamma} \right) \right]^{1/(1-\gamma)} \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}.$$

Finally, using the budget constraint to replace  $W_{t+1}/W_t$  we obtain:

$$a_t = \left[ \lambda_t b_t^{1-1/\psi} + \delta \left( \left[ E_t \left( (a_{t+1} (1-b_t) R_{c,t+1})^{1-\gamma} \right) \right]^{1/(1-\gamma)} \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}. \quad (8.11)$$

Equations (8.10) and (8.11) give a solution to  $a_t$  and  $b_t$ .

Combining equations (8.10) and (8.11) we obtain:

$$\lambda_t b_t^{-1/\psi} (1-b_t) = \delta \left( \left[ E_t \left( (a_{t+1} (1-b_t) R_{c,t+1})^{1-\gamma} \right) \right]^{1/(1-\gamma)} \right)^{1-1/\psi},$$

which we can replace in the expression for the stochastic discount factor together with (8.10) to obtain:

$$M_{t+1} = \left( \delta \frac{\lambda_{t+1}}{\lambda_t} \right)^{(1-\gamma)/(1-1/\psi)} \left( \frac{b_{t+1}}{b_t} (1-b_t) \right)^{-((1/\psi-\gamma)/\psi)/(1-1/\psi)} \left( \frac{C_{t+1}}{C_t} \right)^{-1/\psi} (R_{c,t+1})^{1/\psi-\gamma}.$$

Now note that  $\theta = (1-\gamma)/(1-1/\psi)$ , and that

$$\frac{C_{t+1}}{R_{c,t+1}} = \frac{b_{t+1} R_{c,t+1} (W_t - C_t) / (b_t W_t)}{R_{c,t+1}} = \frac{b_{t+1} (1-b_t)}{b_t},$$

to finally get,

$$M_{t+1} = \left( \delta \frac{\lambda_{t+1}}{\lambda_t} \right)^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} (R_{c,t+1})^{\theta-1}.$$

Taking logarithms of both sides and equating the consumption of the representative agent to aggregate consumption yields equation (3.9).

The rest of the equilibrium derivation involves solving for  $r_{c,t+1}$ ,  $r_{d,t+1}$ , and  $r_{f,t+1}$ . Until this point, we did not need to specify the process for the time-preference, consumption growth, and dividend growth shocks. We solve the rest of the model assuming the general processes of Subsection 3.4 given in equations (3.21) through (3.24). Recovering the equilibrium values for the benchmark model is easily done by setting  $\pi_{c\Lambda} = \pi_{d\Lambda} = \sigma_\eta = 0$  and  $\nu = \sigma_w = \alpha_c = \alpha_d = \rho_c = \rho_d = 0$ .

To price the consumption claim, we must solve the pricing condition:

$$E_t [\exp(m_{t+1} + r_{c,t+1})] = 1.$$

We guess that the logarithm of the price consumption ratio,  $z_{ct} \equiv \log(P_{c,t}/C_t)$ , is

$$z_{ct} = A_{c0} + A_{c1}x_t + A_{c2}\eta_{t+1} + A_{c3}\sigma_t^2 + A_{c4}\Delta C_t,$$

and approximate

$$r_{c,t+1} = \kappa_{c0} + \kappa_{c1}z_{ct+1} - z_{ct} + \Delta C_{t+1}. \quad (8.12)$$

Replacing the approximation and the guessed solution for  $z_{ct}$  on the pricing condition gives

$$E_t [\exp(\theta \log(\delta) + \theta \log(\lambda_{t+1}/\lambda_t) + (1 - \gamma) \Delta c_{t+1} + \theta \kappa_{c0} + \theta \kappa_{c1} z_{ct+1} - \theta z_{ct})] = 1.$$

Computing this expectation requires some algebra and yields the equation

$$\begin{aligned} 0 = & \theta \log(\delta) + (1 - \gamma + \theta \kappa_{c1} A_{c4}) \mu_c - (1 - \gamma + \theta \kappa_{c1} A_{c4}) \alpha_c \sigma^2 + ((1 - \gamma + \theta \kappa_{c1} A_{c4}) \alpha_c + \theta \kappa_{c1} A_{c3}) (1 - \nu) \\ & + ((1 - \gamma + \theta \kappa_{c1} A_{c4}) \pi_{c\Lambda} + \theta \kappa_{c1} A_{c1} \sigma_\Lambda)^2 / 2 + (\theta \kappa_{c1} A_{c2})^2 / 2 + ((1 - \gamma + \theta \kappa_{c1} A_{c4}) \alpha_c + \theta \kappa_{c1} A_{c3})^2 \sigma_w^2 / 2 \\ & + \theta x_t + \theta \kappa_{c1} A_{c1} \rho x_t - \theta A_{c1} x_t + \theta \sigma_\eta \eta_{t+1} - \theta A_{c2} \eta_{t+1} + (1 - \gamma + \theta \kappa_{c1} A_{c4}) \rho_c \Delta c_t - \theta A_{c4} \Delta c_t - \theta A_{c0} \\ & + ((1 - \gamma + \theta \kappa_{c1} A_{c4}) \alpha_c + \theta \kappa_{c1} A_{c3}) v \sigma_t^2 - \theta A_{c3} \sigma_t^2 + (1 - \gamma + \theta \kappa_{c1} A_{c4})^2 \sigma_t^2 / 2 + \theta \kappa_{c1} A_{c0} + \theta \kappa_{c0}. \end{aligned}$$

In equilibrium, this equation must hold in all possible states resulting in the restrictions:

$$\begin{aligned} A_{c1} &= \frac{1}{1 - \kappa_{c1} \rho}, \\ A_{c2} &= \sigma_\eta, \\ A_{c3} &= \frac{\alpha_c v + \frac{(1-\gamma)/2}{1-\kappa_{c1} \rho_c} (1 - 1/\psi)}{1 - \kappa_{c1} v} \frac{1 - 1/\psi}{1 - \kappa_{c1} \rho_c}, \\ A_{c4} &= \frac{(1 - 1/\psi) \rho_c}{1 - \kappa_{c1} \rho_c}, \end{aligned}$$

and

$$\begin{aligned} A_{c0} = & \frac{\log(\delta) + (1 - 1/\psi + \kappa_{c1} A_{c4}) (\mu_c - \alpha_c \sigma^2) + \kappa_{c0} + ((1 - 1/\psi + \kappa_{c1} A_{c4}) \alpha_c + \kappa_{c1} A_{c3}) (1 - \nu) \sigma^2}{1 - \kappa_{c1}} \\ & + \frac{\theta ((1 - 1/\psi + \kappa_{c1} A_{c4}) \pi_{c\Lambda} + \kappa_{c1} A_{c1} \sigma_\Lambda)^2 / 2 + \theta (\kappa_{c1} A_{c2})^2 / 2 + \theta ((1 - 1/\psi + \kappa_{c1} A_{c4}) \alpha_c + \kappa_{c1} A_{c3})^2 \sigma_w^2 / 2}{1 - \kappa_{c1}} \end{aligned}$$

To solve for the risk-free rate, we again use the stochastic discount factor to price the risk-free asset. In logarithms, the Euler equation is:

$$\begin{aligned} r_{f,t+1} &= -\log(E_t(\exp(m_{t+1}))) \\ &= -\log\left(E_t\left(\exp\left(\theta \log(\delta) + \theta \log(\lambda_{t+1}/\lambda_t) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}\right)\right)\right). \end{aligned}$$

Using equation (8.12), we get:

$$r_{f,t+1} = -\log(E_t(\exp(\theta \log(\delta) + \theta \log(\lambda_{t+1}/\lambda_t) - \gamma \Delta c_{t+1} + (\theta - 1) (\kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct}))))).$$

Substituting in the consumption process and the solution for the price-consumption ratio, and after much algebra, we obtain,

$$\begin{aligned}
r_{f,t+1} = & -\theta \log(\delta) + \gamma_1 \mu_c - \gamma_1 \alpha_c \sigma^2 - (\theta - 1) \kappa_{c0} - (\theta - 1) \kappa_{c1} A_{c0} - ((\theta - 1) \kappa_{c1} A_{c3} - \gamma_1 \alpha_c) (1 - \nu) \sigma^2 \\
& + (\theta - 1) A_{c0} - ((\theta - 1) \kappa_{c1} A_{c1} \sigma_\Lambda - \gamma_1 \pi_{c\Lambda})^2 / 2 - ((\theta - 1) \kappa_{c1} A_{c2})^2 / 2 - ((\theta - 1) \kappa_{c1} A_{c3} - \gamma_1 \alpha_c)^2 \\
& - x_t - \sigma_\eta \eta_{t+1} + (1/\psi) \rho_c \Delta c_t - ((\theta - 1) \kappa_{c1} A_{c3} - \gamma_1 \alpha_c) \nu \sigma_t^2 + (\theta - 1) A_{c3} \sigma_t^2 - \gamma_1^2 \sigma_t^2 / 2,
\end{aligned}$$

where

$$\gamma_1 = \gamma - (\theta - 1) \kappa_{c1} A_{c4}.$$

Setting  $\pi_{c\Lambda} = \pi_{d\Lambda} = \sigma_\eta = 0$  and  $\nu = \sigma_w = \alpha_c = \alpha_d = \rho_c = \rho_d = 0 = 0$ , we get the benchmark-model value of the risk-free rate (3.18).

Finally, we price a claim to dividends. Again, we assume the price-dividend ratio is given by

$$z_{dt} = A_{d0} + A_{d1} x_t + A_{d2} \eta_{t+1} + A_{d3} \sigma_t^2 + A_{d4} \Delta c_t + A_{d5} \Delta d_t,$$

and approximate the log-linearized return to the claim to the dividend:

$$r_{d,t+1} = \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}. \quad (8.13)$$

The pricing condition is

$$E_t [\exp(m_{t+1} + r_{d,t+1})] = 1.$$

Using the expressions for  $m_{t+1}$ ,  $r_{c,t+1}$  and  $r_{d,t+1}$ ,

$$\begin{aligned}
1 = & E_t \left[ \exp \theta \log(\delta) + \theta \log(\lambda_{t+1}/\lambda_t) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) (\kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct} + \Delta c_{t+1}) \right. \\
& \left. + \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1} \right].
\end{aligned}$$

Replacing the consumption and dividend growth processes and of the price-consumption and price-dividend ratios, and solving for the expectations, we obtain:

$$\begin{aligned}
A_{d1} &= \frac{1}{1 - \kappa_{d1} \rho}, \\
A_{d2} &= \sigma_\eta, \\
A_{d4} &= \frac{-(1/\psi) \rho_c}{1 - \kappa_{d1} \rho_c}, \\
A_{d5} &= \frac{\rho_d}{1 - \kappa_{d1} \rho_d},
\end{aligned}$$



$$A_{d3} = \frac{(1 + \kappa_{d1}A_{d5})\alpha_d v - \widehat{\gamma}\alpha_c v - \left(\alpha_c v + \frac{1-\gamma}{1-\kappa_{c1}\rho_c}/2\right)\frac{1/\psi-\gamma}{1-\kappa_{c1}\rho_c} + (1 + \kappa_{d1}A_{d5})^2\sigma_d^2/2 + ((1 + \kappa_{d1}A_{d5})\pi_{dc} - \widehat{\gamma})^2}{1 - \kappa_{d1}v}$$

where

$$\widehat{\gamma} = \gamma - (\theta - 1)\kappa_{c1}A_{c4} - \kappa_{d1}A_{d4},$$

and

$$\begin{aligned} A_{d0}(1 - \kappa_{d1}) &= \theta \log(\delta) - \widehat{\gamma}\mu_c + (1 + \kappa_{d1}A_{d5})\mu_d - ((1 + \kappa_{d1}A_{d5})\alpha_d - \widehat{\gamma}\alpha_c)\sigma^2 + (\theta - 1)\kappa_{c0} + (\theta - 1)\kappa_{c1} \\ &+ ((1 + \kappa_{d1}A_{d5})\alpha_d - \widehat{\gamma}\alpha_c + (\theta - 1)\kappa_{c1}A_{c3} + \kappa_{d1}A_{d3})(1 - v)\sigma^2 + ((\theta - 1)\kappa_{c1}A_{c2} + \kappa_{d1}A_{d2}) \\ &+ ((1 + \kappa_{d1}A_{d5})\pi_{d\Lambda} - \widehat{\gamma}\pi_{c\Lambda} + ((\theta - 1)\kappa_{c1}A_{c1} + \kappa_{d1}A_{d1})\sigma_\Lambda)^2/2 \\ &+ ((1 + \kappa_{d1}A_{d5})\alpha_d - \widehat{\gamma}\alpha_c + (\theta - 1)\kappa_{c1}A_{c3} + \kappa_{d1}A_{d3})^2\sigma_w^2/2 - (\theta - 1)A_{c0}. \end{aligned}$$

Equality of the long run mean of consumption growth and of dividend growth requires that

$$\frac{\mu_c}{1 - \rho_c} = \frac{\mu_d}{1 - \rho_d}.$$

Having solved for these constants, we can compute the expected return on the dividend claim,  $E_t(r_{d,t+1})$ . Setting the relevant parameters to zero, we obtain the benchmark-model value of  $E_t(r_{d,t+1})$  given in equation (3.17). The expression for the conditional risk premium in the benchmark model can be obtained by solving:

$$E_t(r_{d,t+1}) - r_{f,t+1} = E_t(\kappa_{d0} + \kappa_{d1}z_{dt+1} - z_{dt} + \Delta d_{t+1}) - r_{f,t+1}.$$

### 8.3. Appendix C

In this appendix we solve for the prices of zero-coupon bonds of different maturities. Again, we present the equilibrium prices for the model of Subsection 3.4. Recovering the equilibrium values for the benchmark model is easily done by setting  $\pi_{c\Lambda} = \pi_{d\Lambda} = \sigma_\eta = 0$  and  $\nu = \sigma_w = \alpha_c = \alpha_d = \rho_c = \rho_d = 0$ . Let  $P_t^{(n)}$  be the time- $t$  price of a bond that pays one unit of consumption at  $t+n$ , with  $n \geq 1$ . The Euler equation for the one-period risk-free bond price  $P_t^{(1)} = 1/R_{f,t+1}$  is

$$P_t^{(1)} = E_t(M_{t+1}).$$

The price for a risk-free bond maturing in  $n > 1$  periods can be written recursively as:

$$P_t^{(n)} = E_t\left(M_{t+1}P_{t+1}^{(n-1)}\right).$$

In Appendix B we derive the value of the risk-free rate:

$$r_{f,t+1} = -\ln \left[ P_t^{(1)} \right].$$

It is useful to write the risk-free rate as:

$$r_{f,t+1} = -\log \left( \frac{\lambda_{t+1}}{\lambda_t} \right) + (1/\psi) \rho_c \Delta c_t - p^1 - q_1 \sigma_t^2,$$

where

$$\begin{aligned} p^1 &= \theta \log(\delta) - \gamma_1 \mu_c + \gamma_1 \alpha_c \sigma^2 + (\theta - 1) \kappa_{c0} + (\theta - 1) \kappa_{c1} A_{c0} - (\theta - 1) A_{c0} \\ &\quad + ((\theta - 1) \kappa_{c1} A_{c3} - \gamma_1 \alpha_c) (1 - \nu) \sigma^2 \\ &\quad + ((\theta - 1) \kappa_{c1} A_{c2})^2 / 2 + ((\theta - 1) \kappa_{c1} A_{c1} \sigma_\Lambda - \gamma_1 \pi_{c\Lambda})^2 / 2 \\ &\quad + ((\theta - 1) \kappa_{c1} A_{c3} - \gamma_1 \alpha_c)^2 \sigma_w^2 / 2, \end{aligned}$$

and

$$q_1 = \gamma_1^2 / 2 + ((\theta - 1) \kappa_{c1} A_{c3} - \gamma_1 \alpha_c) \nu - (\theta - 1) A_{c3}.$$

Let  $p_t^{(n)} = \ln \left[ P_t^{(n)} \right]$ . Therefore,

$$\begin{aligned} p_t^{(1)} &= -r_{f,t+1} \\ &= p^1 + x_t + \sigma_\eta \eta_{t+1} - (1/\psi) \rho_c \Delta c_t + q_1 \sigma_t^2. \end{aligned}$$

We now compute the price of a risk-free bond that pays one unit of consumption in two periods:

$$p_t^{(2)} = \ln E_t \left[ \exp \left( m_{t+1} + p_{t+1}^{(1)} \right) \right].$$

Using the expression for  $m_{t+1}$  and the solution for  $r_{c,t+1}$  and  $z_{ct}$  we obtain, after much algebra,

$$p_t^{(2)} = p^2 + (1 + \rho) x_t + \sigma_\eta \eta_{t+1} + q_2 \sigma_t^2 - (\gamma_2 \rho_c + (\theta - 1) A_{c4}) \Delta c_t,$$

with

$$\gamma_2 = \gamma_1 + (1/\psi) \rho_c,$$

$$\begin{aligned} p^2 &= \theta \log(\delta) - \gamma_2 \mu_c + \gamma_2 \alpha_c \sigma^2 + (\theta - 1) \kappa_{c0} + (\theta - 1) \kappa_{c1} A_{c0} - (\theta - 1) A_{c0} \\ &\quad + ((\theta - 1) \kappa_{c1} A_{c3} - \gamma_2 \alpha_c + q_1) (1 - \nu) \sigma^2 \\ &\quad + ((\theta - 1) \kappa_{c1} A_{c2} + \sigma_\eta)^2 / 2 + (((\theta - 1) \kappa_{c1} A_{c1} + 1) \sigma_\Lambda - \gamma_2 \pi_{c\Lambda})^2 / 2 \\ &\quad + (-\gamma_2 \alpha_c + (\theta - 1) \kappa_{c1} A_{c3} + q_1)^2 \sigma_w^2 / 2 + p^1, \end{aligned}$$

$$q_2 = \gamma_2^2/2 + ((\theta - 1) \kappa_{c1} A_{c3} - \gamma_2 \alpha_c) v - (\theta - 1) A_{c3} + v q_1.$$

Following the same procedure, we obtain the general formula for  $n \geq 2$ :

$$p_t^{(n)} = p^n + (1 + \rho + \dots + \rho^{n-1}) x_t + \sigma_\eta \eta_{t+1} + q_n \sigma_t^2 - (\gamma_n \rho_c + (\theta - 1) A_{c4}) \Delta c_t,$$

where:

$$q_n = \gamma_n^2/2 + ((\theta - 1) \kappa_{c1} A_{c3} - \gamma_n \alpha_c) v - (\theta - 1) A_{c3} + \nu q_{n-1},$$

$$\begin{aligned} p^n &= \theta \log(\delta) - \gamma_n \mu_c + \gamma_n \alpha_c \sigma^2 + (\theta - 1) \kappa_{c0} + (\theta - 1) \kappa_{c1} A_{c0} - (\theta - 1) A_{c0} \\ &+ ((\theta - 1) \kappa_{c1} A_{c3} - \gamma_n \alpha_c + q_{n-1}) (1 - \nu) \sigma^2 \\ &+ ((\theta - 1) \kappa_{c1} + 1)^2 \sigma_\eta^2/2 + (((\theta - 1) \kappa_{c1} A_{c1} + (1 + \rho + \dots + \rho^{n-2})) \sigma_\Lambda - \gamma_n \pi_{c\Lambda})^2/2 \\ &+ ((\theta - 1) \kappa_{c1} A_{c3} - \gamma_n \alpha_c + q_{n-1})^2 \sigma_w^2/2 + p^{n-1}, \end{aligned}$$

with  $p^1$  and  $q_1$  given above, and the sequence of  $\gamma$  given by

$$\begin{aligned} \gamma_n &= \gamma_1 (1 + \rho_c + \dots + \rho_c^{n-2}) + (1/\psi) \rho_c^{n-1} + (1 + \rho_c + \dots + \rho_c^{n-3}) (\theta - 1) A_{c4} \\ &\dots \\ \gamma_3 &= \gamma_1 (1 + \rho_c) + (1/\psi) \rho_c^2 + (\theta - 1) A_{c4} \\ \gamma_2 &= \gamma_1 + (1/\psi) \rho_c \\ \gamma_1 &= \gamma - (\theta - 1) \kappa_{c1} A_{c4}. \end{aligned}$$

Finally, we define the yield on an  $n$ -period zero-coupon bond as  $y_t^{(n)} = -\frac{1}{n} p_t^{(n)}$ .

#### 8.4. Appendix D

This Appendix contains a set of tables that provide versions of the results discussed in the main text that use ex-post, instead of ex-ante versions of the real interest rate.

Table 4 (appendix)

Parameter Estimates and Standard Errors  
Models estimated with ex-post real bond yields

Parameter	Benchmark model	Extended model	Parameter	Benchmark model	Extended model
$\gamma$	1.7005 (0.017871)	2.2228 (0.22671)	$\rho$	0.99143 (0.00047649)	0.99108 (0.00059552)
$\psi$	1.4623 (0.043679)	2.3876 (0.62451)	$\sigma_\lambda$	0.00054228 (2.0143e-05)	0.00040323 (2.6299e-05)
$\delta$	0.99784 (0.0021963)	0.9978 (0.00013075)	Implied value of $\theta$	-2.2158 (0.099733)	-2.1041 (0.50544)
$\sigma_c$	0.0068554 (0.00015674)	0.0046005 (0.00046367)	$\nu$	0.00	0.99673 (0.00057417)
$\mu$	0.0015234 (5.2956e-05)	0.0009117 (5.7211e-05)	$\sigma_w$	0.00	1.70E-06 (2.6272e-07)
$\pi_{c\lambda}$	0.00	-0.0018895 (0.00029618)	$\alpha_c$	0.00	-119.3912 (28.7094)
$\sigma_d$	0.017153 (0.00041145)	0.025564 (0.52335)	$\alpha_d$	0.00	-145.5293 (68.6766)
$\pi_{dc}$	0.0015658 (0.00041998)	-1.3161 (0.80412)	$\rho_c$	0.00	0.090348 (0.045439)
$\pi_{d\lambda}$	0.00	-0.010369 (0.0054912)	$\rho_d$	0.00	0.33438 (0.39606)
$\sigma_\eta$	0.00	0.0082606 (0.00043981)			

Table 5 (appendix)

## Models estimated with ex-post real bond yields

Moments Matched in Estimation	Data (constrained, ex-post bond yields)	Data (unconstrained, ex- post bond yields)	Benchmark model	Extended model
Average growth rate of of consumption	1.4941 (0.30395)	2.2197 (0.22413)	1.84	1.20
Average growth rate of dividends	1.4941 (0.30395)	0.12538 (0.65732)	1.84	1.20
Standard deviation of the growth rate of consumption	2.1865 (0.34494)	2.1624 (0.31345)	2.36	2.72
Standard deviation of the growth rate of dividends	6.914 (1.0409)	6.5181 (1.1219)	5.89	6.95
Contemporaneous correlation between consumption growth and dividend growth	0.2215 (0.1006)	0.23722 (0.10649)	0.09	0.21
First-order serial correlation of consumption	0.2001 (0.1475)	<b>0.1584</b> (0.1240)	-0.01	0.43
Correlation between stock returns and one-period lagged consumptio	-0.1555 (0.1070)	-0.2607 (0.0999)	0.00	-0.04
Correlation between stock returns and one-period lead consumption $\xi$	0.4950 (0.0670)	0.5093 (0.0490)	0.01	0.22
Average return to equities	7.8252 (1.74)	6.3948 (1.8804)	6.25	3.87
Standard deviation of return to equities	17.2537 (1.3783)	17.6831 (1.461)	15.93	16.67
Average risk-free rate	0.19347 (0.84341)	0.10476 (0.84457)	0.19	0.19
Standard deviation of the risk-free rate	3.4656 (0.82769)	3.5054 (0.83489)	3.99	4.23
First-order serial correlation of the risk-free rate	0.59242 (0.082348)	0.59736 (0.078284)	0.90	0.49
Equity premium	7.6318 (1.758)	6.29 (1.8812)	6.06	3.68
Average price-dividend ratio	3.3761 (0.14887)	3.3775 (0.14887)	3.12	3.30
Standard deviaton of price- dividend ratio	0.47324 (0.074827)	0.44502 (0.081116)	0.28	0.40
First-order serial correlation of price dividend ratio	0.95041 (0.034499)	0.93257 (0.042679)	0.84	0.89
Contemporaneous correlation between risk free rate and price dividend ratio	<b>0.20139</b> (0.12648)	0.21274 (0.12982)	-0.95	-0.24

Table 6 (appendix)

Correlation Between Stock Returns and Per Capita  
Growth Rates of Consumption and Dividends

Model estimated with ex-post bond returns

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Moments	<i>Consumption growth</i>			
	Data (constrained)	Data (unconstrained)	Benchmark model	Extended model
1-year correlation between equity returns and consumption growth	-0.07 (0.11)	-0.05 (0.12)	0.04	-0.14
5-year correlation between equity returns and consumption growth	-0.01 (0.16)	-0.05 (0.13)	0.05	0.01
10-year correlation between equity returns and consumption growth	-0.08 (0.30)	-0.21 (0.21)	0.05	0.18
	<i>Dividend growth</i>			
1-year correlation between equity returns and dividend growth	0.08 (0.11)	0.05 (0.11)	0.37	-0.11
5-year correlation between equity returns and dividend growth	0.22 (0.14)	0.29 (0.14)	0.35	0.07
10-year correlation between equity returns and dividend growth	0.51 (0.22)	0.59 (0.15)	0.41	0.26

Table 7 (apendix)

## Predictability of Excess Returns by Price-dividend Ratio at Various Horizons

Model estimated with ex-post real bond yields

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<i>Benchmark model</i>						
	Data	Model (median) Slope coefficient	Model (plim)	Data R-square	Model (median) (% of values larger than R-square in data)	Model (plim)
1 year	-0.10 (0.03)	-0.05	0.008	0.08	0.01 (0.02)	0.0003
3 years	-0.27 (0.07)	-0.14	0.027	0.14	0.03 (0.08)	0.001
5 years	-0.41 (0.12)	-0.21	0.034	0.26	0.04 (0.04)	0.001
<i>Extended model</i>						
1 year	-0.10 (0.03)	-0.06	-0.03	0.08	0.02 (0.05)	0.01
3 years	-0.27 (0.07)	-0.18	-0.08	0.14	0.06 (0.19)	0.02
5 years	-0.41 (0.12)	-0.26	-0.13	0.26	0.09 (0.12)	0.04

Table 8 (appendix)

Correlation of price-dividend ratio and ex-post risk-free rate

	Sample	Full-sample	Pre 1983	Post 1983
Canada	1952-2014	0.02 (0.14)	0.55 (0.06)	-0.31 (0.25)
France	1932-2014	-0.56 (0.17)	-0.67 (0.13)	0.07 (0.17)
Germany	1968-2014	-0.11 (0.08)	-0.10 (0.13)	-0.20 (0.20)
Italy	1940-2014	-0.54 (0.14)	-0.63 (0.08)	0.35 (0.21)
Japan	1960-2014	0.39 (0.16)	-0.01 (0.20)	0.66 (0.08)
U.K.	1934-2014	0.32 (0.11)	0.27 (0.06)	-0.25 (0.22)
US	1939-2011	0.21274 (0.12982)	0.172 (0.148)	-0.44 (0.21)



Table 9 (appendix)

## Term Structure of Bond Yields

Model estimated with ex-post yields

Moments	Data (ex-post real yields)	Data (ex-ante real yields)	Benchmark model	Extended model
<i>Average yield</i>				
Long-term bonds (sample 1929-1992)	1.32 (1.10)	2.90 (0.84)	5.30	2.53
Intermediate-term bond (sample 1929-2007)	1.39 (0.91)	1.93 (0.99)	2.25	1.53
Short-term bond (sample 1929-2011)	0.42 (0.80)	0.46 (0.78)	0.19	0.19
Return to equity minus long-term bond yield (sample 1929-1992)	4.16 (2.39)	2.54 (2.09)	0.95	1.34
<i>Standard deviation of yield</i>				
Long-term bonds (sample 1929-1992)	3.02 (0.65)	2.59 (0.52)	1.73	1.39
Intermediate-term bond (sample 1929-2007)	3.29 (0.53)	3.14 (0.57)	3.19	2.49
Short-term bond (sample 1929-2011)	3.88 (0.77)	3.85 (0.77)	3.99	4.23
Return to equity minus long-term bond yield (sample 1929-1992)	20.2 (2.47)	20.09 (1.96)	15.59	16.53

Table 11 (appendix)

Regressions of Excess Stock Returns on Long-term Bond Yields in Excess of Short Rate

Model estimated with ex-post real yields

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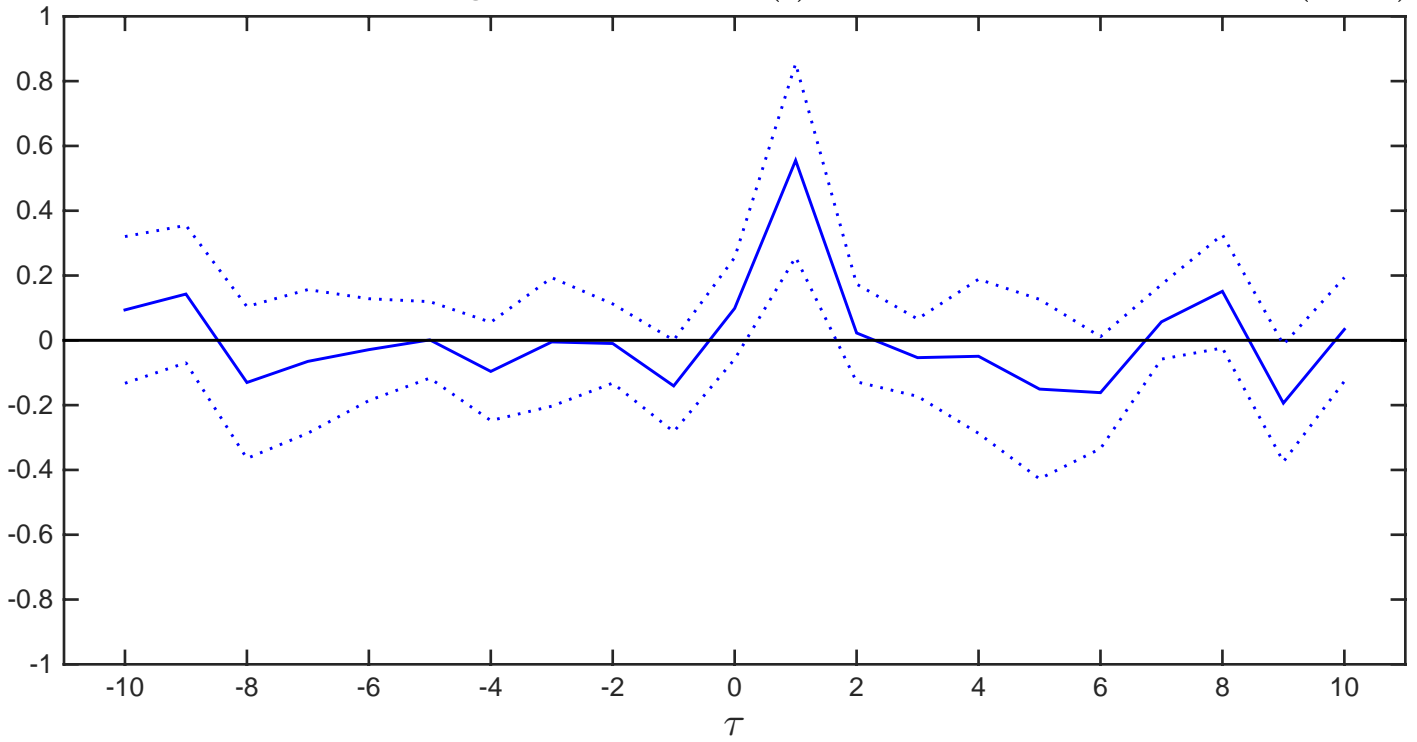
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	Data 1929-2011	Benchmark model	Extended model
<i>Long-term government bonds (20 years)</i>			
R-square	0.02	0.12	0.04
Slope	0.60 (0.96)	3.40	0.97
<i>Intermediate term government bonds (5 years)</i>			
R-square	0.002	0.07	0.03
Slope	-0.30 (1.10)	3.30	0.97

Figure 1: Correlograms of stock returns and fundamentals

Panel A

United States - Correlogram of Returns ( $t$ ) and Consumption Growth ( $t + \tau$ )



United States - Correlogram of Returns ( $t$ ) and GDP Growth ( $t + \tau$ )

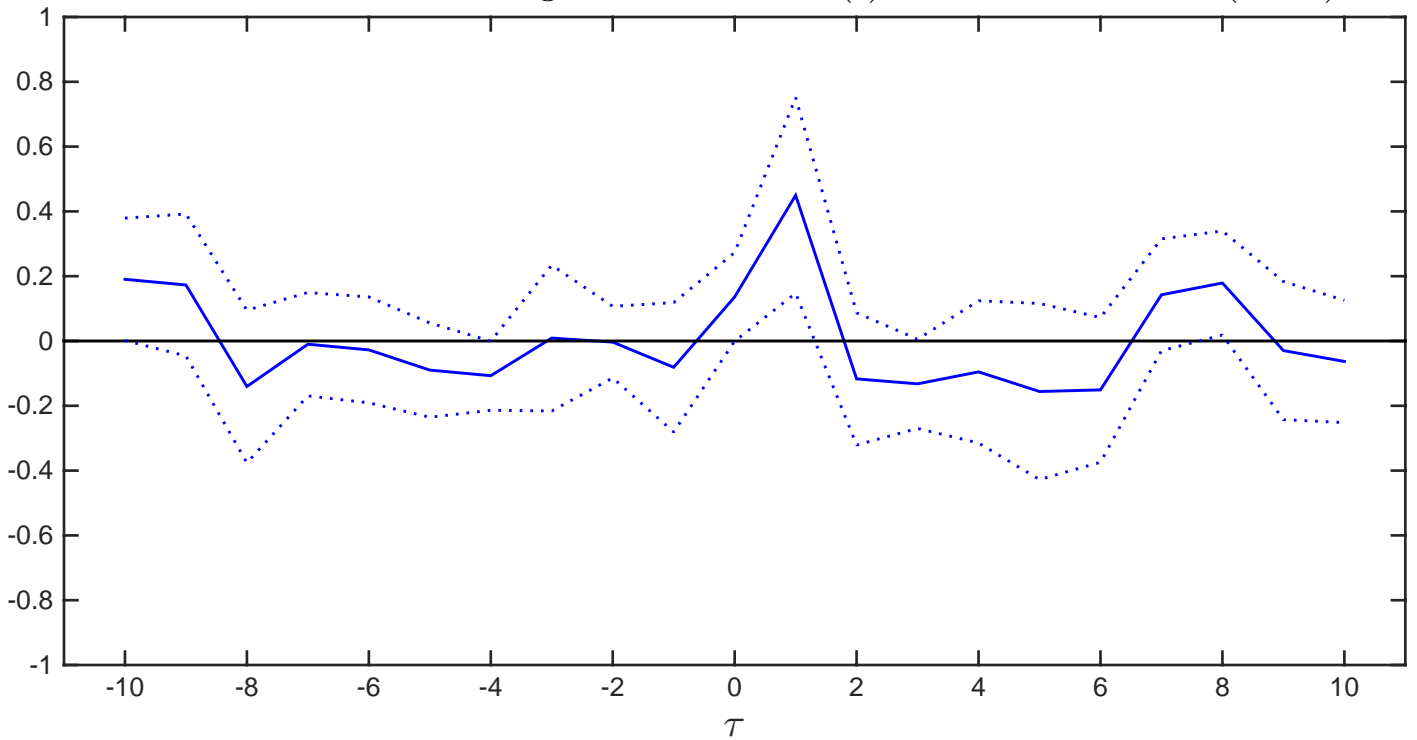


Figure 1: Correlograms of stock returns and fundamentals

Panel B

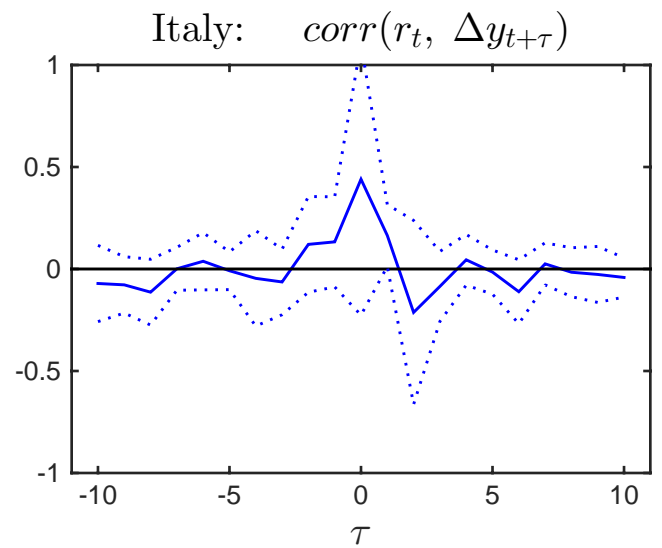
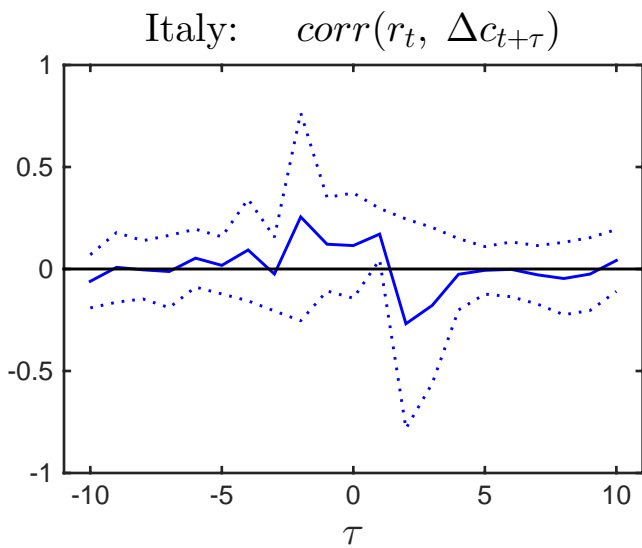
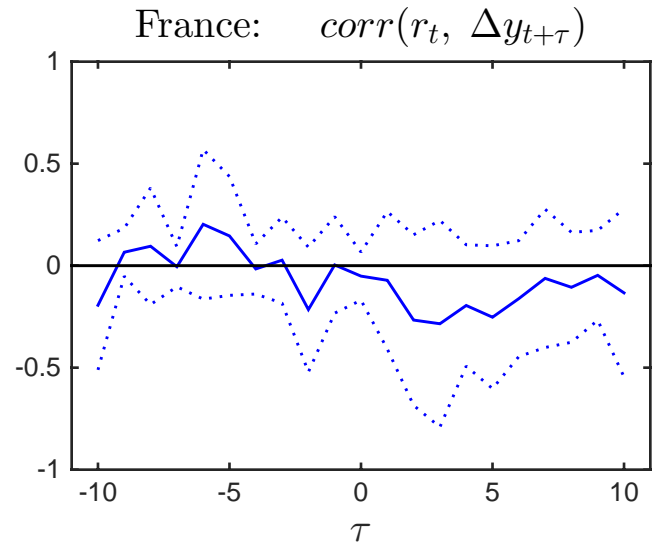
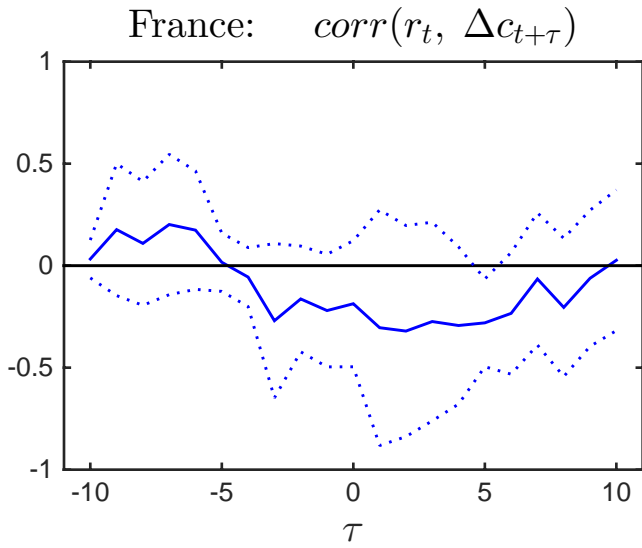
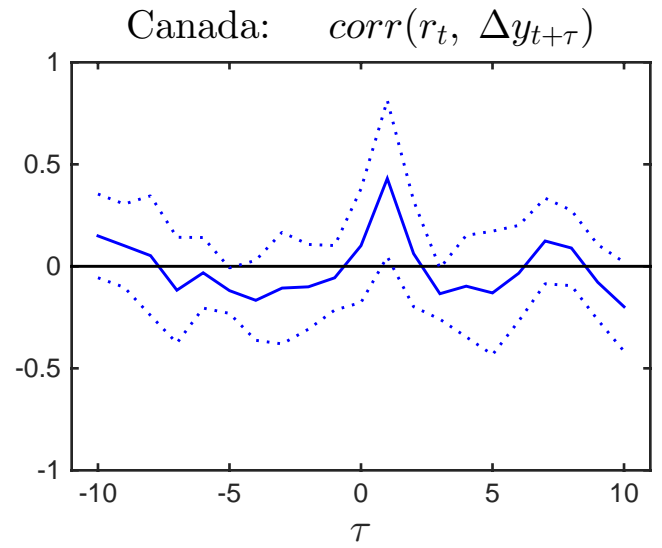
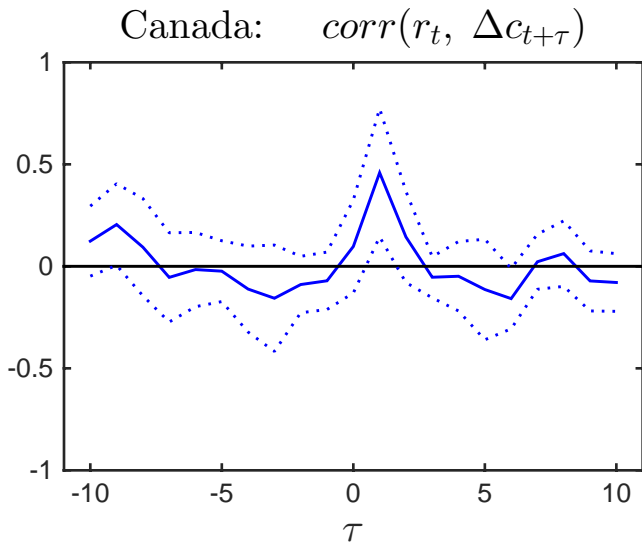


Figure 1: Correlograms of stock returns and fundamentals

Panel C

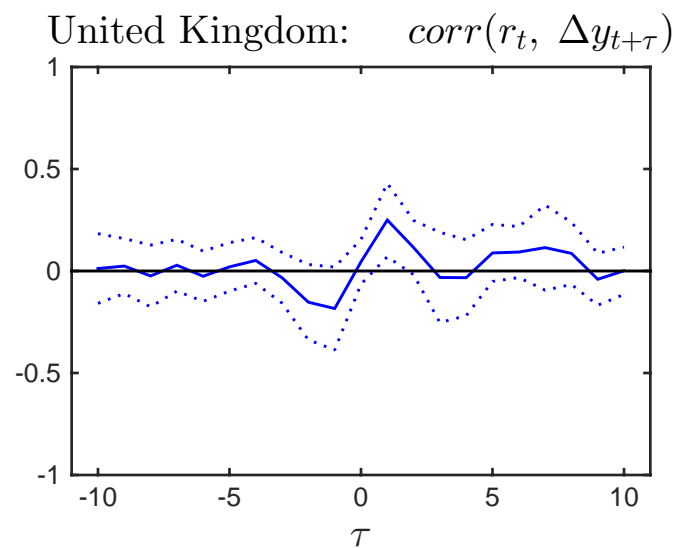
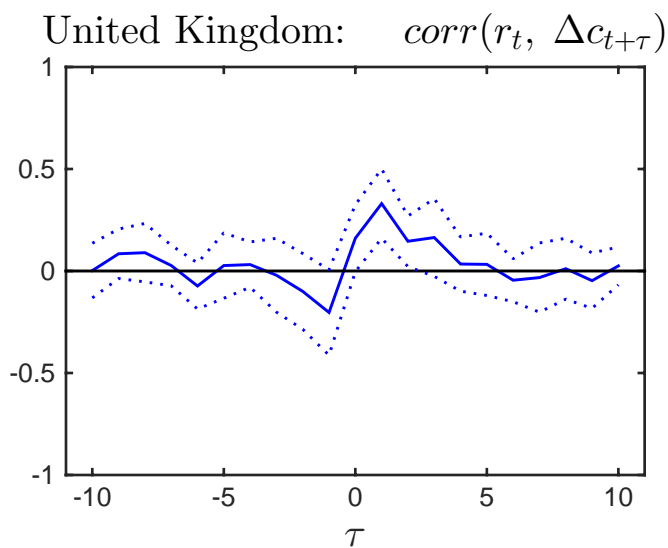
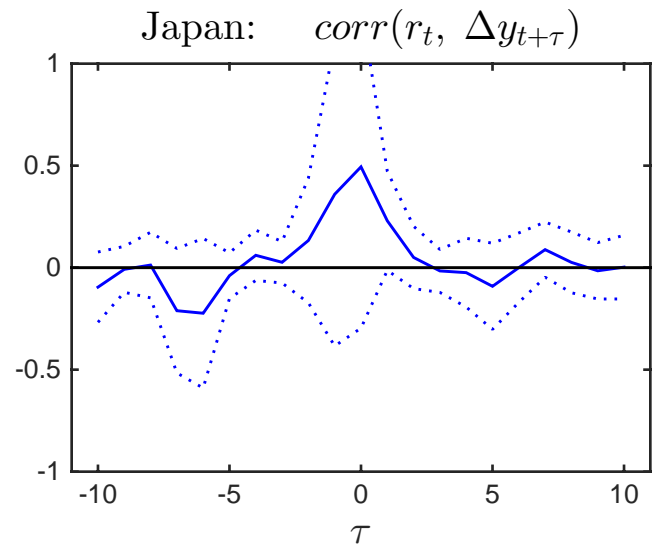
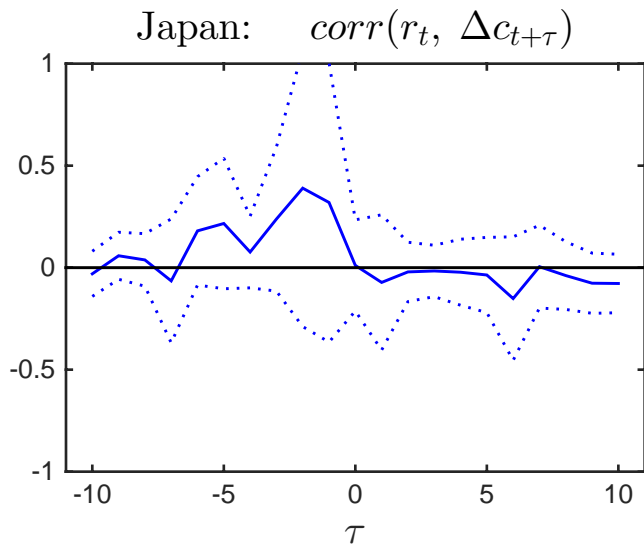
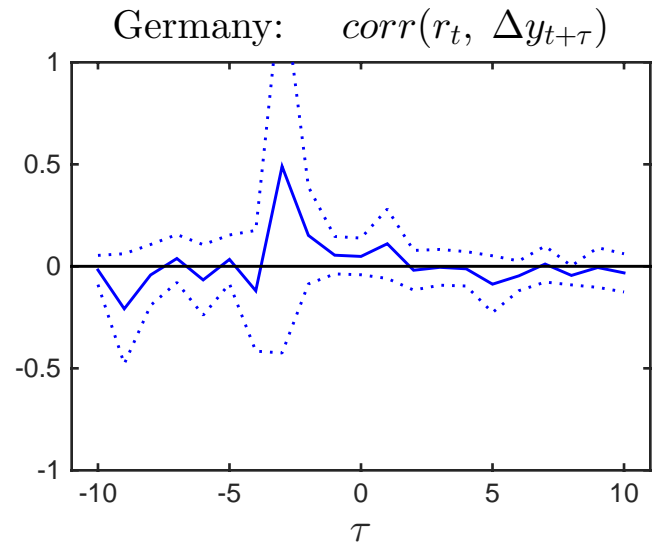
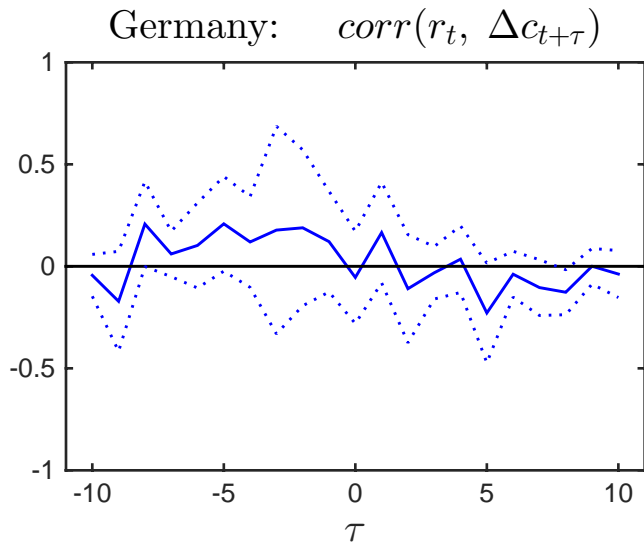


Figure 2: Impulse Response Function of ARMA(2,3) model estimated from synthetic log SDF series

Panel A: real log SDF

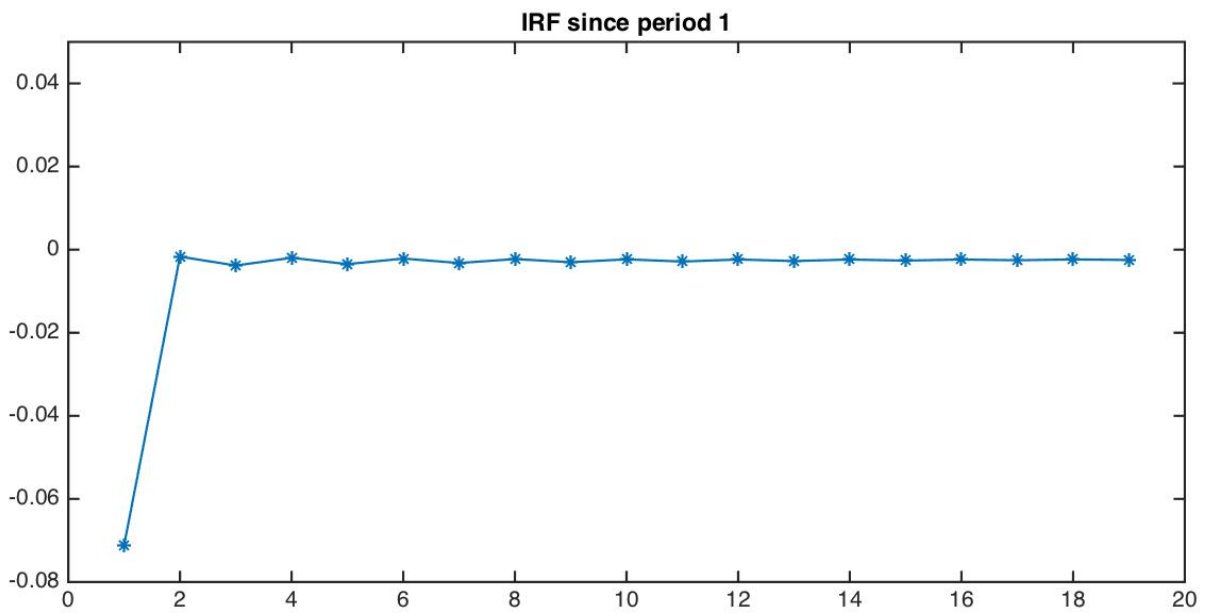
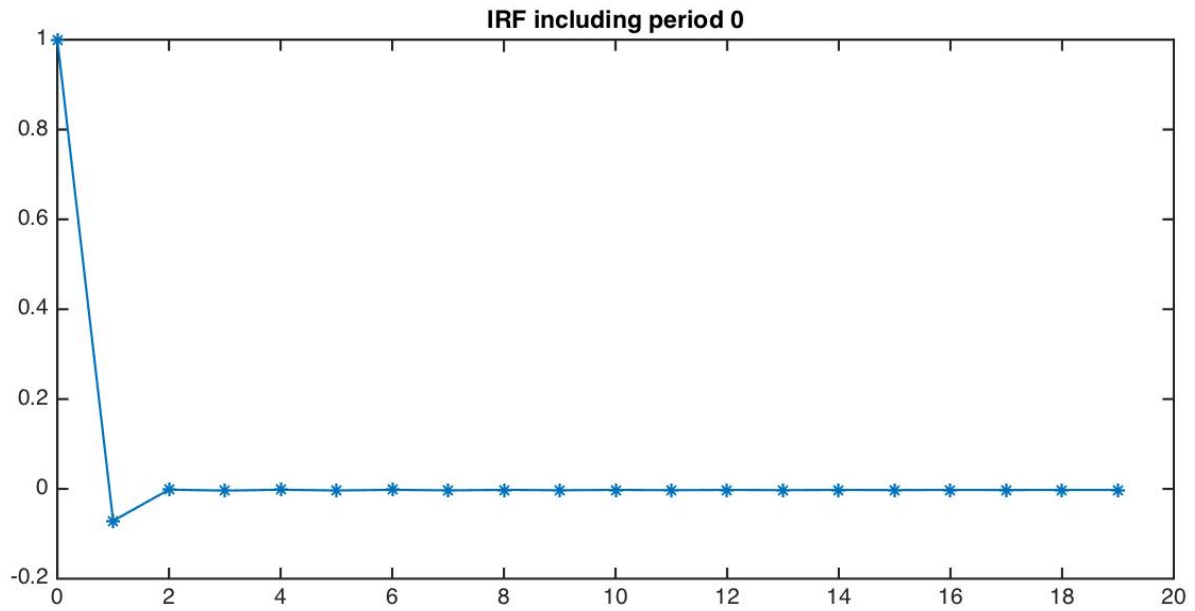


Figure 2: Impulse Response Function of ARMA(2,3) model estimated from synthetic log SDF series

Panel B: nominal log SDF

