VAR Estimation and Forecasting When Data Are Subject to Revision

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VAR Estimation and Forecasting When Data Are Subject to Revision[*]

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Abstract:

Conventional VAR estimation and forecasting ignores the fact that economic data are often subject to revision many months or years after their initial release. This paper shows how VAR analysis can be modified to account for such revisions. The proposed approach assumes that government statistical releases are efficient with a finite lag. It takes no stand on whether earlier revisions are "noise" or "news." The technique is illustrated using data on employment and the unemployment rate, real GDP and the unemployment rate, and real GDP and the GDP/consumption ratio. In each case, the proposed procedure outperforms conventional VAR analysis and the more-restrictive methods for handling the data-revision problem that are found in the existing literature.

I. INTRODUCTION

The Problem. With the exception of securities and commodity prices and surveys of forecasts and attitudes few macroeconomic data escape revision. Admittedly, a handful of important series–household-survey employment and unemployment, the consumer price index, and the Institute for Supply Management survey, for example–are revised only as the result of changes in estimated seasonal factors, which are typically small. However, such important macroeconomic time series as payroll employment, industrial production, retail sales, the monetary aggregates, and the national income and product accounts (NIPA) are subject to large revisions spread over many months or even years. Revisions fall into several categories. Routine revisions reflect the gradual arrival of more complete source data, updates to seasonal factors, and tabulation-error corrections. Updates to the weights used to calculate NIPA price and quantity indexes once had large, predictable effects, but qualify as "routine" now that weighting is chained rather than fixed. Most difficult to deal with are methodological revisions, which occur irregularly and sometimes have far-reaching implications.

Conventional vector auto-regression (VAR) analysis uses current-vintage data throughout. Most "real-time" recursive analyses simply reproduce conventional practice after the fact, by assuming that a forecaster would have used latest-available data to obtain coefficient estimates at each point in time. As the sample period is extended, old data are thrown out, and replaced with new.

The problem with conventional practice is that it mixes apples and oranges. Data toward the tail end of the sample (oranges) have undergone little or no revision. Data early in the sample (apples) have been heavily revised. For most series and typical sample sizes, heavily revised data dominate VAR estimation. Consequently, the VAR approximates the dynamic relationship between apples and apples. However, the data that are substituted into the VAR equations to generate a forecast are end-of-sample oranges (lightly revised or first-release). Essentially, conventional practice constructs a

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cider press and then feeds oranges into it, expecting–somehow–to get cider.

Solutions. To improve on conventional VAR estimation and forecasting, one must make some assumption about the nature of data revisions. An appealing starting point is the classical measurement-error model, according to which government statistical releases equal the truth plus white-noise error. The Kalman filter can, in principle, be applied to extract estimates of the truth from the noisy government releases and to construct optimal forecasts. In empirical practice, however, the white-noise-error assumption often appears counterfactual. Moreover, convergence of coefficient estimates can be difficult to achieve if the number of unobserved state variables is large. To get around the first of these problems, Howrey (1978, 1984) allows measurement errors to be serially correlated. To get around the second, he assumes that the truth is observable with a finite lag, which allows direct ordinary-least-squares estimation of the state and observation equations.

Sargent (1989) questions the classical measurement-error model of government statistical releases more fundamentally, proposing an alternative model in which the statistical agency itself applies the Kalman filter to noisy source data to produce an efficient estimate of the truth. (Efficient estimates make full use of available information, so that subsequent revisions are completely unpredictable–i.e., are pure "news.") It is this efficient estimate–possibly contaminated by small white-noise "typos"–that is released to the public. To limit complexity, Sargent rules out the arrival of new source data directly informative about earlier states. Still, implementation is challenging, and Sargent's proposed estimation and forecasting methodology has not found widespread application.

This paper generalizes Howrey's methodology to allow for the possibility that the government statistical agency filters source data in the manner described by Sargent. Alternatively, the paper generalizes Sargent by allowing both for late-arriving source data and the possibility that early source data are not filtered by the statistical agency. More precisely, we assume that government statistical releases are fully efficient after a finite

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number, e, of rounds of revision, and we treat these efficient releases as "truth" for estimation and forecasting purposes. However, we avoid restrictive assumptions about earlier releases. They may be efficient estimates plus typos, as proposed by Sargent. They may be truth plus white noise, as in the classical measurement-error model. Or, they may be truth plus serially correlated measurement error, as proposed by Howrey.

Estimation and forecasting are accomplished in three steps. First, we use OLS to estimate a VAR in e^{th}-revision data. The VAR coefficients determine a state equation that relates apples to apples. Second, we use OLS to estimate a collection of equations that govern earlier revisions. These coefficients determine an observation equation that gives oranges as a function of apples. Third, we apply the Kalman filter to the state and observation equations to obtain estimates of current truth (convert current oranges into apples) and produce forecasts of future truth.

The efficiency-in-finite-time assumption that we employ is plausible with respect to routine revisions and, perhaps, minor methodological changes, and finds support in our empirical examples. However, major methodological revisions probably ought to be treated as structural breaks, handled by other means (e.g., dummy variables).

Other Related Work. Other recent papers concerned with estimation and forecasting using data that may be inaccurate or subject to revision include Croushore and Evans (2002); Morley, Nelson and Zivot (2003); and Koenig, Dolmas and Piger (2003).

Croushore and Evans develop an instrumental-variables methodology for estimating the VAR that relates truth to lagged truth (apples to lagged apples). They are unconcerned with the forecasting problem (which requires converting oranges into apples). The Croushore-Evans methodology assumes that after a sufficiently large number of revisions, government statistical releases are consistent with the classical measurement-error model–just the opposite of the assumption made here, that government statistical releases are eventually fully efficient.

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Morley, Nelson and Zivot question whether the white-noise measurement-error hypothesis is appropriate when trying to separate observed output into trend and cyclical components. Their analysis takes place in a uni-variate setting where the state variable (trend output) is non-stationary and observed output is never revised.

The Koenig-Dolmas-Piger analysis deals with the apples and oranges problem discussed here, but is limited to single-equation estimation and forecasting in an economy where early government estimates of the forecasted variable are fully efficient.

Outline. We start by presenting simple first-pass tests of the efficiency of two important government statistical releases. The methodology we propose is unlikely to perform well unless efficiency is a reasonable approximation after relatively few rounds of revisions. Otherwise, the dimensionality of the estimation problem becomes unmanageable. In Section III, we develop our formal model of the revision process. Estimation and forecasting are discussed in Section IV. In Section V, we consider three simple empirical examples: a model of the joint dynamics of GDP growth and the GDP/consumption ratio, a model of payroll employment and the unemployment rate, and a model of GDP growth and the unemployment rate. In each case, we find that our proposed methodology performs better, in real time, than a VAR analysis that ignores the data-revision problem. Our methodology also outperforms the Kalman filter when the filter is applied assuming white-noise or Howrey-style measurement error. Section VI concludes.

II. DATA REVISIONS: NOISE OR NEWS?

The classical measurement-error model assumes that government releases equal the truth plus noise that is orthogonal to the truth. However, the behavior of many series seems more consistent with the efficiency hypothesis, according to which revisions are unpredictable "news." Consider, for example, two of our most important coincident

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economic indicators: payroll-employment growth and real-GDP growth. As a first-pass test of the efficiency of official estimates of these variables, we regress heavily revised jobs-growth and GDP-growth data on a constant and relatively early government estimates of the same variables.[1] According to the classical model, if an early statistical release is unusually high, it's probably partly because it contains positive measurement error–error that will subsequently be revised away. Consequently, the slope coefficient in our regression should be significantly less than 1. If, on the other hand, early government releases are efficient, then revisions will be unpredictable, and the constant and slope coefficient should not be significantly different from 0 and 1, respectively.

Results are displayed in Table 1. Only for first-release GDP growth is the efficiency hypothesis rejected at the 5-percent level. Even in this case, elimination of a single statistical outlier (for the year 1975) is sufficient to reverse the test result.

These findings are, of course, not definitive. In more stringent tests presented below, we find first and second-round revisions to jobs and output growth are at least partly predictable once the information set that is brought to bear is expanded to include a measure of macroeconomic slack. The point of Table 1 is simply that the realism of the classical measurement-error model is open to serious question.

III. THE REVISIONS MODEL

Basics. We assume that government statistical estimates become fully efficient after a finite number of revisions, and model the revision process only up to that point. Trying to forecast subsequent revisions is useless–or even worse than useless, given degrees-of-freedom constraints. More controversially, perhaps, we assume that new estimates of long-ago data contain negligible marginal information about the economy's current state.

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[1] Our empirical examples also involve the unemployment rate. However, revisions to the unemployment rate–due entirely to re-estimation of seasonal factors–are tiny.

Some such assumption is implicit in any finite-dimension VAR.

Formally, let $\chi(t)$ denote the true period-t values of an $n \times 1$ vector of variables of interest, and let $x(t, s)$ denote the official estimate of $\chi(t)$ released in period $s \geq t$. We assume there exists a fixed, finite e such that $x(t, t+e) = E_{t+e}[\chi(t)]$. That is, an efficient estimate of $\chi(t)$ is announced (no more than) e periods after the initial statistical release. This assumption is trivially satisfied if the eth revision is final. Since $E_s[\chi(t)] = E_s E_{t+e}[\chi(t)] = E_s[x(t, t+e)]$ for any $s \leq t+e$, forecasting $x(t, t+e)$ amounts to forecasting $\chi(t)$. Indeed, because $\chi(t)$ equals $x(t, t+e)$ plus unforecastable noise, the finite-sample coefficient estimates obtained by relating $x(t, t+e)$ to a given information set are more precise than those obtained by relating $\chi(t)$ to the same set (Koenig, Dolmas and Piger 2003). It will be convenient to denote by $x(t)$ the efficient estimate of $\chi(t)$ that becomes available in period $t+e$.

We assume that a VAR relates e^{th} revision data to lagged e^{th} revision data (apples to apples). In particular,

$$x(t) = \mathbf{F_0}\, x(t\text{-}1) + v_0(t), \tag{1}$$

where all the roots of $\mathbf{F_0}$ are of modulus less than 1 and where $v_0(t)$ is vector white-noise error, so that $E[v_0(t)] = 0$ and $E[v_0(t)v_0'(s)] = 0$ for all $s \neq t$. It may be necessary to "stack" x to reduce its dynamics to a first-order system.

The next step in our analysis is to model the government statistical releases that precede the eth revision. We seek a formulation that is general enough to encompass the Howrey and Sargent models of how these early government statistical releases are generated.

Howrey's Revisions Model. In Howrey's revisions model, oranges are apples plus an error:

$$\hat{y}(t) = \hat{z}(t) + \eta(t), \tag{2}$$

where $\hat{y}'(t) \equiv [x'(t\text{-}e\text{+}1, t)\ x'(t\text{-}e\text{+}2, t)...\ x'(t, t)]$ is a stacked vector of first-release and lightly revised data (oranges) and $\hat{z}'(t) \equiv [x'(t\text{-}e\text{+}1)\ x'(t\text{-}e\text{+}2)...\ x'(t)]$ is a stacked vector of heavily revised data (apples). The error vector, $\eta(t)$, follows a first-order auto-regressive process with a white-noise innovation vector:[2]

$$\eta(t) = \Lambda\eta(t\text{-}1) + \upsilon(t), \tag{3}$$

where the roots of Λ are all of modulus less than 1. It follows that

$$\hat{y}(t) = \hat{z}(t) + \Lambda[\hat{y}(t\text{-}1) - \hat{z}(t\text{-}1)] + \upsilon(t), \tag{4}$$

and

$$\hat{y}(t) - \hat{z}(t) = \sum_{i=0}^{\infty} \Lambda^i \upsilon(t\text{-}i). \tag{5}$$

Note that the government's estimation errors are completely independent of the innovations in Equation 1. The classical measurement-error model is the special case where $\Lambda = \mathbf{0}$, so that the government's estimation errors are pure white-noise.

Sargent-Style Revisions. Alternatively, in the spirit of Sargent (1989), suppose that the government applies the Kalman filter to noisy source data as it arrives, and announces the filtered estimates plus "typos." In particular, suppose that new source data received in

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[2] An appendix demonstrates that a revisions model of the Howrey form is obtained if source data received each period are "truth" plus white-noise measurement error, and government statisticians pool current and past source data to obtain their public estimates.

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period t equal the "truth" plus vector white-noise measurement error:

$$w(t) = z(t) + \omega(t) \tag{6}$$

where

$$z'(t) \equiv [x'(t\text{-}e)\ x'(t\text{-}e\text{+}1)\ ...\ x'(t)] = [x'(t\text{-}e)\ \hat{z}'(t)],$$

$$w'(t) \equiv [x'(t\text{-}e,\ t)\ w'(t\text{-}e\text{+}1,\ t)\ ...\ w'(t,\ t)],$$

$$\omega'(t) \equiv [0\ \omega_{e\text{-}1}'(t)\ \omega_{e\text{-}2}'(t)\ ...\ \omega_1'(t)\ \omega_0'(t)],$$

and $w(s,\ t)$ is the new source data related to $x(s)$ that is released in period $t \geq s$. Source data are not directly available to the outside analyst. From the government's perspective, however, Equation 6 is the observation equation for a state-space model. The state equation is:

$$z(t) = \mathbf{F}z(t\text{-}1) + \nu(t), \tag{7}$$

where

$$\mathbf{F} \equiv \begin{vmatrix} 0 & \mathbf{I} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{I} & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & 0 & \cdots & \mathbf{I} \\ 0 & 0 & 0 & 0 & \cdots & \mathbf{F_0} \end{vmatrix}$$

and

$$\nu'(t) \equiv [0\ 0\ 0\ ...\ 0\ \nu_0'(t)].$$

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If government statisticians apply the Kalman filter to Equations 6 and 7, their published estimates will obey a standard updating formula:[3]

$$y(t) = \mathbf{F}y(t\text{-}1) + \mathbf{G}^*[w(t) - \mathbf{F}y(t\text{-}1)], \tag{8}$$

where

$$y'(t) \equiv [x'(t\text{-}e, t)\ x'(t\text{-}e\text{+}1, t)\ ...\ x'(t, t)] = [x'(t\text{-}e)\ \hat{y}'(t)]$$

is a stacked vector of government statistical releases,

$$\mathbf{G}^* = \mathbf{P}(\mathbf{P} + \mathbf{R})^{-1},$$
$$\mathbf{R} \equiv E[\omega(t)\omega'(t)],$$
$$\mathbf{Q} \equiv E[v(t)v'(t)],$$

and $\mathbf{P}$ is the solution to the matrix equation

$$\mathbf{P} = \mathbf{F}[\mathbf{P} - \mathbf{P}(\mathbf{P} + \mathbf{R})^{-1}\mathbf{P}]\mathbf{F}' + \mathbf{Q}.$$

Equation 8 says that the government's state-vector estimate, $y(t)$, is a weighted average of the latest source data, $w(t)$, and an extrapolation, $\mathbf{F}y(t\text{-}1)$, from last period's state estimate.

Since the first vector elements of $y(t)$ and $w(t)$ are identical to one another, the first matrix row of $\mathbf{G}^*$ must consist of an identity matrix followed by a string of zero matrices. What else can we infer about $\mathbf{G}^*$? Working backward, Equation 8 implies

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[3] See, for example, Hamilton (1994, Chap. 13).

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$$y(t) = [(\mathbf{I} - \mathbf{G}^*)\mathbf{F}]^e\, y(t\text{-}e) + \sum_{i=0}^{e-1}\; [(\mathbf{I} - \mathbf{G}^*)\mathbf{F}]^i\, \mathbf{G}^* w(t\text{-}i). \qquad (9)$$

Direct calculation establishes that the first matrix column and row of $(\mathbf{I} - \mathbf{G}^*)\mathbf{F}$ consist entirely of zero matrices. By recursion, the first matrix column and row of $[(\mathbf{I} - \mathbf{G}^*)\mathbf{F}]^e$ also consist entirely of zero matrices. Hence, $x(t\text{-}2e, t\text{-}e)$–the first element of $y(t\text{-}e)$–is irrelevant for $y(t)$ in Equation 9. The second element of $y(t\text{-}e)$ is $x(t\text{-}2e+1, t\text{-}e)$, which is an imperfect estimate of $x(t\text{-}2e+1)$. Since $x(t\text{-}2e+1)$ itself is in the government's observation set in period t [it is included in $w(t\text{-}e+1)$], $x(t\text{-}2e+1, t\text{-}e)$ must receive zero weight in Equation 9. The same argument applies to all of the other components of $y(t\text{-}e)$, up through and including the last component, $x(t\text{-}e, t\text{-}e)$. In other words, the $\mathbf{G}^*$ weighting matrix must satisfy $[(\mathbf{I} - \mathbf{G}^*)\mathbf{F}]^e = \mathbf{0}$.

We can generalize Equation 8 by allowing small errors–what Sargent calls "typos"–to creep into the government's calculations:

$$y(t) = \mathbf{F}y(t\text{-}1) + \mathbf{G}^*[w(t) - \mathbf{F}y(t\text{-}1)] + \xi(t), \qquad (10)$$

where

$$\xi'(t) \equiv [0\ \xi_{e\text{-}1}'(t)\ \xi_{e\text{-}2}'(t)\ \ldots\ \xi_0'(t)]$$

is vector white noise. Implicitly, at each stage the government doesn't realize that its previous estimate was contaminated. Consequently, typos persist (although the weight on past typos diminishes as new source information arrives).

An Encompassing Model. A state-space formulation that encompasses the Howrey and Sargent models of the early-revision process is:

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$$z(t) = \mathbf{F}z(t\text{-}1) + \nu(t) \tag{7}$$

$$y(t) = (\mathbf{I} - \mathbf{G})\mathbf{F}y(t\text{-}1) + \mathbf{G}z(t) + \varepsilon(t), \tag{11}$$

where

$$\mathbf{G} \equiv \begin{vmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{G}_{e\text{-}1,e} & \mathbf{G}_{e\text{-}1,e\text{-}1} & \mathbf{G}_{e\text{-}1,e\text{-}2} & \cdots & \mathbf{G}_{e\text{-}1,0} \\ \mathbf{G}_{e\text{-}2,e} & \mathbf{G}_{e\text{-}2,e\text{-}1} & \cdot & \cdots & \mathbf{G}_{e\text{-}2,0} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \mathbf{G}_{1,e} & \mathbf{G}_{1,e\text{-}1} & \cdot & \cdots & \mathbf{G}_{1,0} \\ \mathbf{G}_{0,e} & \mathbf{G}_{0,e\text{-}1} & \cdot & \cdots & \mathbf{G}_{0,0} \end{vmatrix},$$

and

$$\varepsilon'(t) \equiv [0 \ \varepsilon_{e\text{-}1}'(t) \ \varepsilon_{e\text{-}2}'(t) \ ... \ \varepsilon_0'(t)].$$

The state-equation and observation-equation error vectors are assumed to be uncorrelated with one another at all leads and lags, and to be serially uncorrelated. The Sargent model in the special case where $\mathbf{G} = \mathbf{G}^*$, $[(\mathbf{I} - \mathbf{G})\mathbf{F}]^e = \mathbf{0}$, and $\varepsilon(t) = \mathbf{G}\omega(t) + \xi(t)$.

To see that the Howrey model is also a special case, note that Equations 7 and 11 imply that

$$y(t) - z(t) = (\mathbf{I} - \mathbf{G})\mathbf{F}[y(t\text{-}1) - z(t\text{-}1)] + \varepsilon(t) - (\mathbf{I} - \mathbf{G})\nu(t). \tag{12}$$

The corresponding equation in the Howrey model is Equation 4. The two formulations are equivalent when $\varepsilon'(t) = [0 \mid \upsilon'(t)]$ and

$$\mathbf{G} = \mathbf{I} - \begin{vmatrix} 0 & 0 & \cdots & 0 & 0 \\ \multicolumn{3}{c}{\text{-------}} & & 0 \\ & & & | & \cdot \\ & \Lambda & & | & \cdot \\ & & & | & \cdot \\ & & & | & 0 \end{vmatrix}, \tag{13}$$

so that

$$(\mathbf{I} - \mathbf{G})\mathbf{F} = \begin{vmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & \multicolumn{4}{c}{\text{----------}} \\ \cdot & | & & & \\ \cdot & | & & \Lambda & \\ \cdot & | & & & \\ 0 & | & & & \end{vmatrix}.$$

As already noted, the classical model is obtained from Howrey's model by setting $\Lambda = \mathbf{0}$, in which case $\mathbf{G} = \mathbf{I}$.

Equation 12 implies that the government's estimation errors are governed by

$$y(t) - z(t) = \sum_{i=0}^{\infty} \ [(\mathbf{I} - \mathbf{G})\mathbf{F}]^{i}[\varepsilon(t\text{-}i) - (\mathbf{I} - \mathbf{G})\nu(t\text{-}i)]. \tag{14}$$

A key, distinguishing feature of the Howrey special case is that $(\mathbf{I} - \mathbf{G})\nu(t\text{-}i) = 0$, so that the right-hand side of Equation 14 is uncorrelated with the error in the state equation. Since it requires that $[(\mathbf{I} - \mathbf{G})\mathbf{F}]^{e} = \mathbf{0}$, an important implication of the Sargent model is that the right-hand side of Equation 14 is a MA(e - 1) process.

Summary. As long as one is willing to treat government estimates as "final" after a sufficiently large number of revisions (even if the estimates are really *not* final), then the dynamics of earlier revisions can be captured using a state-space model in which both the

state-equation and observation-equation errors are vector white noise. The state-space specification (Equations 7 and 11) is general enough to encompass the classical white-noise-measurement-error model, the Howrey model of serially correlated measurement error, and the Sargent "news plus typos" model of early revisions as special cases. Moreover, as long as the government estimates that are being treated as final are fully efficient, then the law of iterated projections says that forecasting these estimates is equivalent to forecasting the truth, even if the truth is never directly observed.[4] Hence, the key to good forecasting is obtaining accurate estimates of the $\mathbf{F}$ and $\mathbf{G}$ matrices that enter Equations 7 and 11. It is to this problem that we now turn.

IV. ESTIMATION AND FORECASTING

Proposed Approach. We begin by writing out the relevant portions of Equations 7 and 11 (rearranging slightly):[5]

$$x(t) = \mathbf{F_0}\, x(t\text{-}1) + v_0(t) \tag{15}$$

$$x(t\text{-}e\text{+}1, t) - x(t\text{-}e\text{+}1, t\text{-}1) = \sum_{j=0}^{e\text{-}1} \mathbf{G}_{e\text{-}1,e\text{-}j}[x(t\text{-}e\text{+}j) - x(t\text{-}e\text{+}j, t\text{-}1)] +$$
$$\mathbf{G}_{e\text{-}1,0}[x(t) - \mathbf{F_0}x(t\text{-}1, t\text{-}1)] + \varepsilon_{e\text{-}1}(t) \tag{16.1}$$

$$x(t\text{-}e\text{+}2, t) - x(t\text{-}e\text{+}2, t\text{-}1) = \sum_{j=0}^{e\text{-}1} \mathbf{G}_{e\text{-}2,e\text{-}j}[x(t\text{-}e\text{+}j) - x(t\text{-}e\text{+}j, t\text{-}1)] +$$
$$\mathbf{G}_{e\text{-}2,0}[x(t) - \mathbf{F_0}x(t\text{-}1, t\text{-}1)] + \varepsilon_{e\text{-}2}(t) \tag{16.2}$$
$$.$$
$$.$$

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[4] Admittedly, by treating partly revised data as final, one artificially restricts the size of the information set upon which forecasts will be conditioned. (Revisions to data more than e periods back are excluded.) There's nothing to prevent one from choosing a larger value for e, however, if the current value is too limiting.

[5] When $e = 1$, only Equations 15 and 16e apply.

$$x(t\text{-}1,\ t) - x(t\text{-}1,\ t\text{-}1) = \sum_{j=0}^{e\text{-}1} \mathbf{G}_{1,e\text{-}j}[x(t\text{-}e\text{+}j) - x(t\text{-}e\text{+}j,\ t\text{-}1)] +$$
$$\mathbf{G}_{1,0}[\mathrm{x}(t) - \mathbf{F_0}x(t\text{-}1,\ t\text{-}1)] + \varepsilon_1(t) \qquad\qquad (16.e\text{-}1)$$

$$x(t,\ t) = \mathbf{F_0}\,x(t\text{-}1,\ t\text{-}1) + \sum_{j=0}^{e\text{-}1} \mathbf{G}_{0,e\text{-}j}[x(t\text{-}e\text{+}j) - x(t\text{-}e\text{+}j,\ t\text{-}1)] +$$
$$\mathbf{G}_{0,0}[x(t) - \mathbf{F_0}x(t\text{-}1,\ t\text{-}1)] + \varepsilon_0(t). \qquad\qquad (16.e)$$

Because $\varepsilon(t)$ and $\nu(t)$ are serially uncorrelated, and uncorrelated with one another at all leads and lags, estimation of Equations 15 and 16 does not pose any special problems, apart from accounting for the cross-equation restrictions on $\mathbf{F_0}$. Recalling that each x vector is $n\times1$, there are a total of n^2 $\mathbf{F}$-matrix coefficients and $e\times(e+1)\times n^2$ $\mathbf{G}$-matrix coefficients to estimate. Obviously, degrees of freedom will suffer if e is large. The Howrey model is obtained in the special case where $\mathbf{G}_{0,0} = \mathbf{I}$ and $\mathbf{G}_{1,0} = \mathbf{G}_{2,0} = \ldots = \mathbf{G}_{e\text{-}1,0} = \mathbf{0}$; while $\mathbf{G} = \mathbf{I}$ in the classical measurement-error model (c.f. Equation 13). These restrictions are easily tested. Unfortunately, the same cannot be said of the Sargent-model restrictions, which are highly non-linear in the estimated coefficients.

The analyst more interested in forecasting than in hypothesis testing can avoid non-linear, simultaneous estimation of Equations 15 and 16 by using a two-step procedure. First, apply OLS to Equation 15 to obtain an estimate, $\langle\mathbf{F_0}\rangle$, of $\mathbf{F_0}$. This amounts to estimating a VAR in e^{th}-revision data. Second, substitute $\langle\mathbf{F_0}\rangle$ into each of the remaining equations and apply OLS. Despite potential correlation between disturbance terms, SUR is unnecessary, because each equation in 16 has the same set of right-hand-side variables. The presence of generated regressors means that coefficient standard errors from the second-step regressions cannot be trusted.

Suppose that estimates of $\mathbf{F}$ and $\mathbf{G}$ have been obtained from Equations 15 and 16 using data through period T. It is then a simple matter to apply the Kalman filter to the state-space model defined by Equations 7 and 11 to obtain an estimate, $\mathrm{K}_T z'(T) = [x'(T\text{-}e)$ $\mathrm{K}_T x'(T\text{-}e+1)\ \mathrm{K}_T x'(T\text{-}e+2) \ldots \mathrm{K}_T x'(T)]$, of the current state vector. Forecasts are generated

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in the obvious way:

$$_T\langle\chi(T+i)\rangle = {}_T\langle x(T+i)\rangle = {}_T\langle\mathbf{F_0}\rangle^i \, \mathbf{K}_T \, x(T) \tag{17}$$

for $i = 0, 1, 2, \ldots$, where $_T\langle\chi(T+i)\rangle$ and $_T\langle x(T+i)\rangle$ are the period-T forecasted values of $\chi(T+i)$ and $x(T+i)$, respectively, and where $_T\langle\mathbf{F_0}\rangle$ is the estimated coefficient matrix from Equation 15. If desired, state-vector forecasts can be substituted back into Equation 11 to produce forecasts of future initial-release and lightly revised data.

Conventional Approach. If available data run from $t = 0$ through $t = T$, standard econometric procedure would be to apply OLS to estimate a VAR in vintage-T data:

$$x(t, T) = \mathbf{F_0} \, x(t\text{-}1, T) + \mathbf{v}_0(t) \tag{18}$$

for $t = 1, 2, \ldots T$. Forecasts are prepared by substituting latest-available data into the right-hand-side of the estimated equation:

$$_T\langle\chi(T+i)\rangle = {}_T\langle\mathbf{F_0}\rangle^i \, x(T, T) \tag{19}$$

for $i = 1, 2, \ldots$, where $_T\langle\chi(T+i)\rangle$ is the period-T forecasted value of $\chi(T+i)$ and $_T\langle\mathbf{F_0}\rangle$ is the estimated coefficient matrix from Equation 18. When they become available, the VAR is re-estimated, and new forecasts prepared, with period-T+1-vintage data. Diebold-Rudebusch-style real-time analysis simply reproduces conventional practice after the fact (Diebold and Rudebusch 1991).

As noted in the introduction, conventional practice mixes apples and oranges. For example, the sample used to estimate Equation 18 contains both data that have seen many rounds of revisions ($t \ll T$) and data that are first release or only lightly revised ($t \approx T$), even though the dynamics of the former will not generally be the same as the dynamics of

the latter. Our approach eliminates this estimation-stage inconsistency by including only e^{th}-revision data in Equation 15.

In practice, heavily revised data so dominate most samples that estimates of $\mathbf{F_0}$ obtained from Equation 18 will often not be much different from those obtained from Equation 15. However, heavily revised data's dominance during estimation creates a problem for forecasting because the data that are substituted into the estimated VAR to generate a forecast are lightly revised or first release (c.f. Equation 19). Our approach eliminates this forecast-stage inconsistency by converting end-of-sample oranges into apples before substituting them into the VAR (c.f. Equation 17).

V. EMPIRICAL EXAMPLES

The Models. We use three simple, two-variable dynamic linear systems to demonstrate the usefulness of our procedures. The first bivariate model includes real GDP growth and the output–consumption ratio. The motivation comes from Cochrane (1994), who shows that real output and consumption are co-integrated in the U.S. economy and that any deviation from the long-run relationship between output and consumption has strong predictive power for output growth. The second model includes payroll employment growth and the unemployment rate. Here, the motivation comes from the observation that deviations in the unemployment rate tend to be corrected, over time, through unusually rapid or unusually sluggish subsequent employment growth. Substitute real GDP growth for employment growth in this story, and you get our third model.[6]

In each case, we want to see whether our estimation and forecasting method yields more accurate real-time forecasts than methods which either ignore the apples and

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[6] See Blanchard and Quah (1989) and Evans (1989) for more thorough analyses of the joint dynamics of output and unemployment.

oranges problem or which make restrictive assumptions about the revisions process. We also want to see how badly recursive forecasting exercises that use current-vintage data distort the forecasting performance that is actually achievable in real time.

We use only one observation per year for each variable. Thus, we measure real GDP growth from Q2 to Q2, jobs growth from March to March, and the output–consumption ratio in Q2. We use the Q2 average unemployment rate in our GDP–unemployment model and the March unemployment rate in our jobs–unemployment model. Our "first revision" data consist of government estimates available one year after the initial release. Our "second revision" data consist of government estimates available two years after the initial release.[7] For purposes of measuring forecast performance, our "truth" is 2003-vintage data (the latest data available when most of this analysis was completed). By using annual observations, we keep lag lengths short. Also, official GDP, consumption, and jobs estimates are all subject to major annual revisions, which are easiest to model when data are sampled at an annual frequency.

Preliminaries. Our method assumes that government statistical releases become efficient estimates of the truth after some finite number, e, of revisions. To test the efficiency assumption and find a realistic value for e, we ran orthogonality tests, regressing $\chi(t) - x(t, t+e)$ on $y(t+e)$ for alternative values of e. (Here, $y(t)$ is defined as in Equation

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[7] All data are from the real-time data set compiled by the Federal Reserve Bank of Philadelphia. They span the period from 1966 through 2003. "Initial release" second-quarter GDP and consumption data are true initial releases, published at the end of July. The July release typically also reflects annual revisions to the NIPA. We use March data in our model of jobs growth and unemployment because March is the benchmark month for payroll employment revisions. However, we use data for March *as they appeared at the time of the benchmark revision*, which has typically had a June release date. The consumption measure we use in constructing the output–consumption ratio is the sum of household expenditures on non-durables and on services. The real-time output–consumption ratio has discontinuities in 1976, 1986 and 1992 due to changes in base years and in 1996 due to the move to chain weights. In each case, we shift the new data upward or downward, as necessary, to eliminate any jump in the series.

11, and $\chi(t)$ is, in practice, 2003-vintage data.) Results, displayed in Table 2, are consistent across the models. Efficiency is rejected (at the 5-percent level) for $e = 1$, but not for $e = 2$. Accordingly, all of our estimations assume $e = 2$.

To reduce the error term in Equation 15 to vector white noise, it was necessary to stack $x(t)$ to include both current and one lag of the variables in each model.

Forecasting the Truth. Table 3 presents results from recursive, one-step-ahead forecasting exercises over the ten years from 1992 through 2001. In every case, forecasts are compared with 2003-vintage data. The first row of results, labeled "Current Vintage," shows root-mean-square errors obtained from a recursive analysis in which all data used in estimation and forecasting are the latest currently available. Thus, 2003-vintage data running through 1991 are used to estimate a VAR that is, in turn, used to forecast 1992. The sample is extended through 1992 to produce a new VAR estimate, used to forecast 1993, and so forth. Obviously, 2003-vintage data would not actually have been available in 1991 or 1992, so the root-mean-square errors obtained in this way are likely to be unrealistically low. The results are of interest only because similar forecasting exercises are common in the empirical macroeconomics literature.

Many real-time analyses use end-of-sample vintage data for estimation. Thus, 1992 forecasts are generated from a VAR estimated using 1991-vintage data, 1993 forecasts from a VAR estimated using 1992-vintage data, and so forth (Diebold and Rudebusch 1991). This approach is useful for showing how conventional estimation, using latest-available data, performs in real time. In Table 3, estimation using end-of-sample vintage data yields the recursive forecasting results labeled "Diebold-Rudebusch." As expected, root-mean-square errors are uniformly larger than those in the "Current Vintage" row, where real-time data limitations are ignored.

End-of-sample vintage data are likely to be dominated by government estimates that have undergone many rounds of revisions. However, the most recent observations in

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any sample will be first release, and forecasts generated by substituting first-release data into an equation fitted to heavily revised data are not likely to be optimal. This is the apples-and-oranges problem that is the focus of our paper. In principle, the Kalman filter can be used to convert end-of-sample oranges into the apples needed for proper forecasting. How well the Kalman filter performs in practice, however, depends on whether one correctly models the data-revision process. The results in the final three rows of Table 3 are obtained when the Kalman filter is applied to the state-space model defined by Equations 7 and 11 under various assumptions about the **G** matrix in Equation 11. Results in the row labeled "Classical" are obtained when **G** = **I**, as assumed by the classical measurement error model. Results in the row labeled "Howrey" are obtained when **G** takes the form specified in Equation 13. Finally, results in the row labeled "Kishor-Koenig" are obtained when **G** is left entirely unrestricted, which allows for the possibility that government estimation errors are correlated with innovations in the state-equation VAR, as implied by Sargent's data-revision model. In all three cases, we use the simple two-step procedure described in Section IV to estimate first $\mathbf{F_0}$, then **G**.

With classical measurement-error restrictions imposed, the Kalman filter fails to consistently outperform conventional real-time estimation and forecasting. (Compare the root-mean-square errors reported in the second and third rows of Table 3.) Allowing measurement errors to be serially correlated (the Howrey model) yields improvement relative to the classical model, but not always enough to beat out the conventional (Diebold-Rudebusch) approach. However, when restrictions on the observation equation are entirely eliminated–as proposed here–the Kalman filter comes into its own, producing superior forecasting performance across the board. (See the results in Table 3, row 5.)

Note that proper estimation not only improves the forecasting performance of each particular economic model, but also can change the rank ordering of alternative models. For forecasting GDP growth, the results in rows 1-4 of Table 3 all suggest that Model #1, which uses the output/consumption ratio as an error-correction term, outperforms Model

#3, which uses the unemployment rate. (Admittedly, the performance differences are sometimes small.) As shown in row 5, however, this result is strongly reversed when the models are estimated using the methodology proposed here.

As noted above, the "Classical," "Howrey," and "Kishor-Koenig" estimations used to produce Table-3 forecasting results were all accomplished using the simple two-step procedure outlined in Section IV: The state equation (a VAR in e^{th}-revision data) was estimated first; then variables generated using the state equation were substituted into the observation equations, and these were estimated by OLS. However, to obtain the unbiased variance-covariance matrix needed to test the Howrey revisions model against our own, more general model, we also did a one-off, full-sample, simultaneous estimation of the state and observation equations. Test results indicate that the Howrey-model restrictions are strongly rejected.[8]

Forecasting First Releases. Table 3 assumes that the analyst is interested in forecasting the truth (or, in any event, data that have undergone many rounds of revision). Under some circumstances, however, it may make sense to forecast a relatively early government statistical release. Agents' behaviors may be influenced by early statistical releases, for example. Alternatively, a sudden deterioration in one's ability to forecast early-release data may be the first sign of a structural shift in the economy. Accordingly, Table 4 looks at forecasts of the government's initial statistical estimates.[9] Of course, the Diebold-Rudebusch methodology–used by most real-world forecasters–makes no distinction between initial-release and heavily revised data either in estimation or in forecasting. So, the "Diebold-Rudebusch" forecasts that are compared with initial-release

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[8] *P* values for the Howrey restrictions on **G** are 0.0035, 0.0001, and 0.0000 for Models 1, 2, and 3, respectively.

[9] Qualitatively similar results were obtained when forecasting first-revision or second-revision data. Results are available on request.

data in Table 4 are exactly the same as those compared with true (2003-vintage) data in Table 3. This statement also applies to the "Classical" forecasts in Tables 3 and 4, because if government statistical estimates are the truth plus white noise, then the best forecast of the truth is always also the best forecast of the initial release.

Comparing corresponding entries in Tables 3 and 4, root-mean-square errors are lower for forecasts of initial-release GDP and initial-release jobs growth than for current-vintage GDP and jobs growth. For these variables, in other words, it's easier to predict the initial release than to predict what the data will eventually look like. Within Table 4, just as within Table 3, our approach ("Kishor-Koenig") produces consistently lower recursive root-mean-square forecast errors than estimation methods which ignore the apples-and-oranges problem ("Diebold-Rudebusch"), or which make restrictive assumptions about the nature of the revision process ("Classical" and "Howrey").

VI. SUMMARY AND CONCLUSIONS

Data revisions are problematic for VAR forecasting. Typically, the data which are substituted into the VAR to generate a forecast have undergone little, if any, revision, while the sample used to estimate the VAR is dominated by heavily revised data. Existing methods for correcting this mis-match are quite restrictive in their assumptions about the data revision process and, in practice, do not perform consistently better than methods that ignore the problem entirely.

The VAR estimation and forecasting methodology proposed in this paper is more flexible than existing approaches, yet easily implemented. It recognizes that government statistical releases are subject to multiple rounds of revision, and it allows early revisions to have both noise and news elements. A key assumption is that government statistical releases, at some point, become efficient estimates of the truth. The methodology dominates alternatives in empirical forecasting exercises.

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APPENDIX: Serial Correlation of Government Estimation Errors

Serial correlation of government estimation errors arises naturally if newly arriving source data is the truth plus white-noise measurement error, and officials pool new source data with old. To see this, consider, the case where $e = 2$, so that $x(t) \equiv x(t, t+2)$ is a fully efficient estimate of $\chi(t)$. In period t, the government collects source data $w(t, t)$ pertaining to $x(t)$, and source data $w(t-1, t)$ pertaining to $x(t-1)$. We assume that

$$w(t, t) = x(t) + \omega_0(t) \tag{A.1}$$

and

$$w(t-1, t) = x(t-1) + \omega_1(t) \tag{A.1'}$$

where $\omega_0(t)$ and $\omega_1(t)$ are white-noise error vectors that may be contemporaneously correlated, but which are otherwise unrelated at all leads and lags. For its first release, the government takes the source data at face value:

$$x(t, t) = w(t, t). \tag{A.2}$$

But for its second statistical release, the government pools current source data with that previously collected:

$$\begin{aligned} x(t-1, t) &= \mathbf{A}_1 w(t-1, t-1) + (\mathbf{I} - \mathbf{A}_1)w(t-1, t) \\ &= \mathbf{A}_1 x(t-1, t-1) + (\mathbf{I} - \mathbf{A}_1)w(t-1, t), \end{aligned} \tag{A.2'}$$

where $\mathbf{A}_1$ has roots of modulus less than 1. With a little algebra, Equations A.1, A.1', A.2 and A.2' imply the following formulas for period-t first-release and first-revision data:

$$x(t, t) = x(t) + \omega_0(t) \tag{A.3}$$

$$x(t-1, t) = x(t-1) + \mathbf{A}_1[x(t-1, t-1) - x(t-1)] + (\mathbf{I} - \mathbf{A}_1)\omega_1(t). \tag{A.3'}$$

It follows that

$$x(t\text{-}1,\ t) - x(t\text{-}1) = \mathbf{A}_1\omega_0(t\text{-}1) + (\mathbf{I} - \mathbf{A}_1)\omega_1(t). \tag{A.4}$$

While first-release data differ from $x(t)$ by white-noise error, first-*revision* data differ from $x(t)$ by an MA(1) error.

More generally, we have

$$w(t\text{-}i,\ t) = x(t\text{-}i) + \omega_i(t) \tag{A.5}$$

and

$$x(t\text{-}i,\ t) = \mathbf{A}_i x(t\text{-}i,\ t\text{-}1) + (\mathbf{I} - \mathbf{A}_i)w(t\text{-}i,\ t), \tag{A.6}$$

for $i = 1, 2, ..., e - 1$, respectively. Equations A.3 and A.3' are replaced by

$$\hat{y}(t) = \hat{z}(t) + \mathbf{A}[\hat{y}(t\text{-}1) - \hat{z}(t\text{-}1)] + \upsilon(t) \tag{A.7}$$

where $\hat{y}'(t) \equiv [x'(t\text{-}e+1,\ t)\ x'(t\text{-}e+2,\ t)...\ x'(t,\ t)]$ is a stacked vector of first-release and lightly revised data, $\hat{z}'(t) \equiv [x'(t\text{-}e+1)\ x'(t\text{-}e+2)...\ x'(t)]$ is a stacked vector of heavily revised data, $\upsilon'(t) \equiv [\omega_{e\text{-}1}'(t)(\mathbf{I} - \mathbf{A}_{e\text{-}1})'\ \omega_{e\text{-}2}'(t)(\mathbf{I} - \mathbf{A}_{e\text{-}2})'...\omega_1'(t)(\mathbf{I} - \mathbf{A}_1)'\ \omega_0'(t)]$ is vector white noise, and

$$\mathbf{A} \equiv \begin{vmatrix} \mathbf{0} & \mathbf{A}_{e\text{-}1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{e\text{-}2} & \mathbf{0} & \cdots & \mathbf{0} \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{vmatrix}.$$

Equation A.7 is clearly a special case of Equation 4 in the main text. It's readily verified that $\mathbf{A}^e = \mathbf{0}$, so that the statistical agency's estimation errors, $\hat{y}(t) - \hat{z}(t)$, follow a MA(e-1) process.

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TABLE 1. Univariate tests suggest that early government estimates of jobs and output growth are approximately efficient. Results are from a regression of the form $x(t) = \alpha_0 + \alpha_1 x^e(t)$, where $x(t)$ is heavily revised and $x^e(t)$ is relatively lightly revised. Sample: 1965-1998.

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$e =$	α_0	α_1	Test $\alpha_0 = 0$, $\alpha_1 = 1$
Jobs Growth (March/March)			
0	-0.002 (0.118)	1.067 (0.042)	$P = 0.056$
1	0.064 (0.056)	1.001 (0.019)	$P = 0.211$
3	0.020 (0.019)	0.990 (0.006)	$P = 0.272$
GDP Growth (Q2/Q2)			
0	0.562 (0.183)	0.891 (0.049)	$P = 0.016$
0 (excluding 1975)	0.241 (0.187)	0.983 (0.051)	$P = 0.252$
1	0.270 (0.133)	0.940 (0.034)	$P = 0.134$
3	0.005 (0.076)	0.990 (0.019)	$P = 0.782$

Notes:

In the jobs-growth regressions:

$x^0(t) \equiv$ percent growth in non-farm jobs, as first released

$x^1(t) \equiv$ percent growth in non-farm jobs, measured in August of year $t + 1$

$x^3(t) \equiv$ percent growth in non-farm jobs, measured in August of year $t + 3$

$x(t) \equiv$ percent growth in non-farm jobs, as measured in August of year $t + 5$.

In the GDP-growth regressions:

$x^0(t) \equiv$ percent growth in real GDP, as first released

$x^1(t) \equiv$ percent growth in real GDP, as it appeared in August of year $t + 1$

$x^3(t) \equiv$ percent growth in real GDP, as it appeared in August of year $t + 3$

$x(t) \equiv$ percent growth in real GDP, as measured in August of year $t + 5$.

TABLE 2. Multivariate orthogonality tests suggest that second-release government estimates of jobs growth, real GDP growth, and the unemployment rate are approximately efficient. Results from a regression of χ(*t*) - *x*(*t*, *t+e*) on *y*(*t+e*).

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Model	*e*	*P* Value
1. GDP growth; GDP/PCE	1	0.027
	2	0.076
2. Jobs growth; unemployment rate	1	0.019
	2	0.593
3. GDP growth; unemployment rate	1	0.000
	2	0.180

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Note: Sample runs from $t = 1966$ to $t = 2001$.

TABLE 3. Forecasting the truth.[*] Recursive forecasting performance of alternative estimation techniques, one-period-ahead root mean-square errors, 1992-2001.

Technique	Model #1		Model #2		Model #3	
	ΔGDP	GDP/PCE	ΔJobs	Unemploy.	ΔGDP	Unemploy.
Current Vintage	1.413	0.0109	0.711	0.530	1.437	0.547
Diebold-Rudebusch	1.598	0.0893	0.939	0.800	1.973	0.742
Classical	1.523	0.0645	1.238	1.316	1.858	1.649
Howrey	1.507	0.0621	1.027	0.707	1.513	0.749
Kishor-Koenig	1.490	0.0493	0.903	0.596	1.328	0.716

* In every case, root-mean-square errors were obtained by comparing forecasts with 2003-vintage data.

TABLE 4. Forecasting the government's initial statistical release. Recursive forecasting performance of alternative real-time estimation techniques, one-period-ahead root mean-square errors, 1992-2001.

Technique	Model #1		Model #2		Model #3	
	ΔGDP	GDP/PCE	ΔJobs	Unemploy.	ΔGDP	Unemploy.
Diebold-Rudebusch	1.360	0.0866	0.738	0.760	1.808	0.772
Classical	1.343	0.0603	1.129	0.965	1.816	1.270
Howrey	1.255	0.0621	0.847	0.675	1.426	0.840
Kishor-Koenig	1.085	0.0366	0.492	0.535	1.262	0.729