Variability of User Performance in Cellular DS-CDMA—Long versus Short Spreading Sequences

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***Abstract*—The uplink performance in a cellular direct-sequence code-division multiple-access system using long and short spreading sequences is compared in terms of the distribution of the bit-error probability. Three different receiver types are considered: conventional; MMSE; and interference cancellation, both with and without forward-error correction. The short code system has a slightly higher performance variability among the user population than the corresponding long code system, which requires attention when designing a short code system. Code hopping as a technique to mitigate this is investigated.**

***Index Terms*—Cochannel interference, code-division multiple access, land mobile radio cellular systems, spread spectrum communications.**

I. Introduction

RECENTLY, multiuser detection in direct-sequence code-division multiple-access (DS-CDMA) systems has received considerable attention and several different types of detectors has been proposed and analyzed, mainly in single-cell systems. Unlike the conventional correlator-type receiver, used in almost all operational DS-CDMA systems, several of the proposed interference suppressing schemes, e.g., the minimum mean-square-error (MMSE) detector [1]–[4] as well as some recently proposed subspace-based code acquisition schemes [5], [6] are based on the assumption of the received signal being cyclostationary and therefore require short spreading sequences. A short spreading sequence has a periodicity equal to the bit time, while a long sequence is essential pseudorandom. Short and long code CDMA are sometimes denoted D-CDMA (deterministic CDMA) and R-CDMA (random CDMA), respectively. In a long code system, the correlation between the users changes from bit to bit, and the multiple-access interference (MAI) therefore appears to be random in time, causing the performance for different users to be (more or less) identical and determined by the average interference level. Short codes, on the other hand, have cross correlations that remains unchanged over time, and an unfortunate user might be trapped in an inferior performance scenario due to nontime-varying cross correlations. The capacity is therefore ruled by the *distribution* of the error probability rather than by its mean solely. It should be noted that some interference suppression and cancellation techniques are applicable to long code systems as well, but the complexity is often lower in the case of short codes due to the cyclostationary interference. In virtually all operational and commercially proposed DS-CDMA systems, e.g., IS-95 and wide-band CDMA (WCDMA), long codes are used in the uplink in conjunction with conventional detectors, while a lot of academic research assumes short codes. Hence, there is a need to compare and quantify the difference, thus bridging the gap between the two approaches, which is the purpose of this paper. However, it should be noted that in the current WCDMA standard, there is a possibility of replacing the long spreading sequence used in the uplink by a short one (256 chips long) [7]. This opens up the possibility for future detectors exploiting the cyclostationarity of the received signal.

The performance distributions of three different receiver schemes—a conventional detector, an MMSE detector, and a detector cancelling all intracell interference—are computed for both long and short codes (the MMSE detector requires short codes). Uncoded as well as coded performance are considered, as forward-error correction is used extensively in wireless communication systems. The concept of *code hopping*, a scheme in which each user in a short code system switches between a predetermined set of code sequences, is investigated as a possible technique to mitigate the performance variability. Previous work in the area is scarce. Based on a discussion about finite dimensions in a short code system, it was stated in [8] that a long code system has superior capacity compared to its short code counterpart; a conclusion partially drawn upon misconceptions as pointed out in [9]. In [10], the performance variability of short code MMSE systems using random spreading sequences was investigated and a significant spread in performance was found, although the capacity of the MMSE system still was better than the corresponding long code conventional system. The signal-to-interference ratio (SIR) in a synchronous, as opposed to asynchronous in this paper, DS-CDMA system using code hopping and a conventional detector was investigated in [11], and a code hopping scheme, mainly from an hardware perspective in an indoor consumer application, was discussed in [12]. Some of the results presented herein have also been reported in [13] and [14]. Approaches other than random code hopping are of course also possible in a short code DS-CDMA system, e.g., dynamic code reassignment algorithms similar to dynamic channel allocation, although this is beyond the scope of the paper. It should also be noted that the systems described herein

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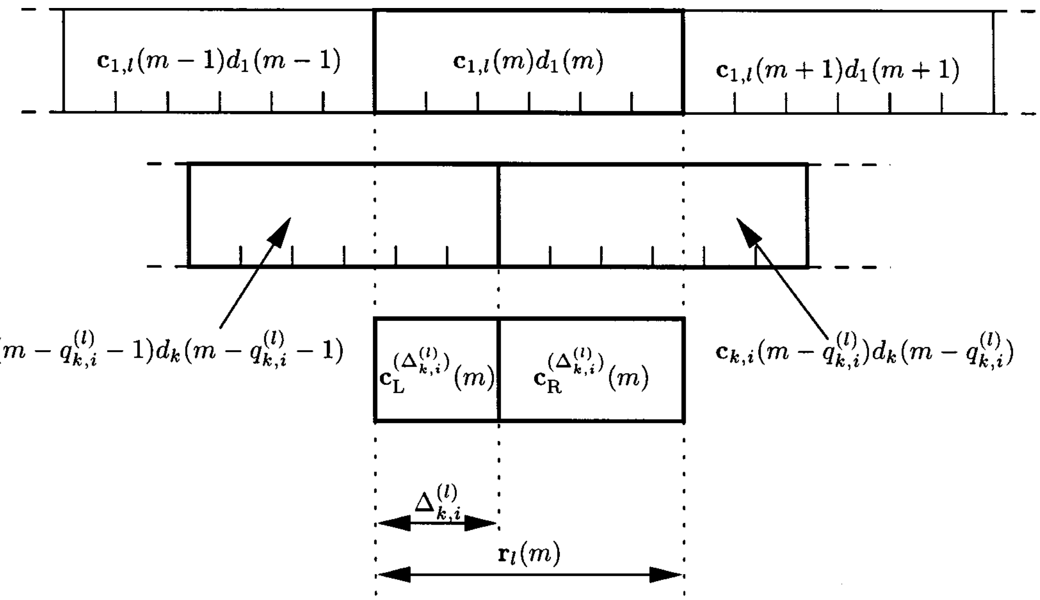

Fig. 1. Graphical illustration of the received vector. The desired contribution and one interfering user/path is shown.

are studied under idealized conditions, e.g., perfect channel state information and no pulse shaping. The impact of imperfect channel estimates on different receiver algorithms and their behavior in fast fading [3], [15] and the impact of more realistic pulse shaping [16] are among those items that need further study as the behavior of the more recent receiver algorithms are not as well known as for the plain old RAKE receiver.

In Section II, the cellular system considered is presented, and in Section III, the receiver structures are discussed. Their respective performance are derived in Section IV and numerically compared with simulations in Section V. Finally, conclusions are drawn in Section VI.

II. System Model

The system considered is an asynchronous K-user DS-CDMA system using binary phase-shift keying modulation, operating in a cellular environment and encountering frequency–selective fading with L resolvable paths. The received baseband signal $r(t)$ is given by

$$r(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} \beta'_{k,l}(t) c_k(t-\tau_{k,l}) \cdot \sum_{m=-\infty}^{\infty} d_k(m) \mathrm{rect}\left(\frac{t-\tau_{k,l}-mT}{T}\right) + n(t) \quad (1)$$

where $d_k(m) \in \pm 1$ is the mth (coded) data bit, T is the bit duration, and $n(t)$ is complex white Gaussian noise with power spectral density N_0. In the following, a subscript k, l denotes that the quantity is due to the kth user's lth path. Consequently, $\tau_{k,l}$ and $\beta'_{k,l}(t)$ are the propagation delay and fading process, respectively, for the kth user's lth path. The spreading sequences $c_k(t)$ consist of chips of duration $T_c = T/N$, where N is the processing gain. In the remainder of this work, the chip pulse shape is assumed to be rectangular, although there is no fundamental reason why other pulse shapes cannot be considered.

The user of interest is, without loss of generality, assumed to be user number one, and furthermore, the receiver is assumed have perfect knowledge of the delays of this user's L resolvable paths. In the receiver, the received signal is separately chip matched filtered for each ray, sampled at the chip rate, and stacked in vectors $\mathbf{r}_l(m) \in \mathbb{C}^{N\times 1}$, where each such vector corresponds to one data bit from the user of interest, transmitted over the lth path. The received vector due to the lth path, illustrated in Fig. 1, can be written as [5]

$$\mathbf{r}_l(m) = \sum_{k=1}^{K} \sum_{i=1}^{L} \beta_{k,i}(m) \cdot \left[\mathbf{c}_R^{(\Delta_{k,i}^{(l)})} d_k(m-q_{k,i}^{(l)}) + \mathbf{c}_L^{(\Delta_{k,i}^{(l)})} d_k(m-q_{k,i}^{(l)}-1)\right] + \mathbf{n}_l(m) \quad (2)$$

where $\Delta_{k,i}^{(l)} = \tau_{k,i} - \tau_{1,l} = q_{k,i}^{(l)} T + p_{k,i}^{(l)} Tc + \delta_{k,i}^{(l)}$, $\delta_{k,i}^{(l)} \in [0, T_c)$ and $p_{k,i}^{(l)} \in [0, N-1]$ and $q_{k,i}^{(l)}$ are integers. The right and left acyclic shifts of the kth user's normalized spreading sequence $\mathbf{c}_k(m) = [c_k^{(1)}(m) \cdots c_k^{(N)}(m)]^T \in \mathbb{R}^{N\times 1}$ are defined as (3) and (4), shown at the bottom of the next page, where $m' = m - q_{k,i}^{(l)}$ and $N' = N - p$. Note that the desired term in $\mathbf{r}_l(m)$, i.e., the term with $k=1$, $i=l$, has $\Delta_{1,i}^{(l)} = 0$, and consequently, $q_{1,i}^{(l)} = 0$, $\mathbf{c}_R^{(\Delta_{k,i}^{(l)})} = \mathbf{c}_1(m)$, and $\mathbf{c}_L^{(\Delta_{k,i}^{(l)})} = 0$. The term $\mathbf{n}_l(m)$ is complex white Gaussian noise with variance σ^2, and, in general, $\mathbf{n}_l(m)$ and $\mathbf{n}_i(m)$, $l \neq i$, are correlated. Each path suffers from independent (fast) fading through the complex process $\beta_{k,l}(m)$ with an average received power $P_{k,l} = E\{|\beta_{k,l}(m)|^2\}$.

For short spreading sequences, $\mathbf{c}_k(m) = \mathbf{c}_k$, i.e., the same spreading sequence is used for each consecutive data bit, while for long spreading sequences, a new (random) vector is used for each data bit. A code hopping scheme where $\mathbf{c}_k(m)$ is cyclically switched between a set of N_{hops} code sequences will also be considered.[1] The hopping is done at the bit rate in this work, although slower hopping is feasible, for example, at the frame rate along the lines of slow frequency hopping in GSM.

The uplink in a cellular system is considered with the number of users within a certain cell being Poisson distributed. The kth user is assigned to base station b_k having the largest total received power, $P_k = \sum_{l=1}^{L} P_{k,l}$, from the kth user among all base stations. The bases are all assumed to be centered in their respective cell, and antenna sectorization is not used (although certainly possible). The user of interest is connected to base station $b_1 = 1$, and the relative received power at the first user's base station from the interfering users, both intercell and intracell interferers, is given by

$$\frac{P_k}{P_1} = \left[\frac{r_{k,b_k}}{r_{k,1}}\right]^{\alpha} 10^{(\xi_{k,1} - \xi_{k,b_k} + p_{e,k})/10} \tag{5}$$

where r_{k,b_k} is the distance between user k and its assigned base b_k, and α is the path loss exponent. The shadowing ξ_{k,b_k} is a zero-mean Gaussian random variable with standard deviation σ_s. Power control, counteracting shadowing, and path loss, but not fast fading, is used within each cell and assumed to have a random error $p_{e,k}$ (in decibels) with standard deviation σ_{PC}, where $\sigma_{\mathrm{PC}} = 0$ corresponds to perfect power control.

III. Receiver Structures

A coherent linear receiver is assumed where the lth received vector $\mathbf{r}_l(m)$ is filtered by the receiver vector $\mathbf{w}_l(m) \in \mathbb{R}^{N\times 1}$, forming the output

$$\begin{aligned} y_l(m) &= \mathbf{w}_l^*(m)\mathbf{r}_l(m) \\ &= \beta_{1,l}(m)\mathbf{w}_l^*(m)\mathbf{c}_1(m)d_1(m) + \mathbf{w}_l^*(m)\tilde{\mathbf{n}}_l(m) \end{aligned} \tag{6}$$

where

$$\begin{aligned} \tilde{\mathbf{n}}_l(m) = &\sum_{\substack{i=1\\ i\neq l}}^{L} \beta_{1,i}(m) \\ &\cdot \left[\mathbf{c}_R^{(\Delta_{1,i}^{(l)})} d_1(m - q_{1,i}^{(l)}) + \mathbf{c}_L^{(\Delta_{1,i}^{(l)})} d_1(m - q_{1,i}^{(l)} - 1)\right] \\ &+ \sum_{k=2}^{K}\sum_{i=1}^{L} \beta_{k,i}(m) \\ &\cdot \left[\mathbf{c}_R^{(\Delta_{k,i}^{(l)})} d_k(m - q_{k,i}^{(l)}) + \mathbf{c}_L^{(\Delta_{k,i}^{(l)})} d_k(m - q_{k,i}^{(l)} - 1)\right] \\ &+ \mathbf{n}_l(m). \end{aligned} \tag{7}$$

[1] A long code system can be seen as a code hopping system hopping between a very large number of different code sequences.

The terms correspond to self interference and intersymbol interference (ISI), multiple-access interference (MAI), and noise, respectively. In order to ssimplify the metric used in the receiver, the ISI (the first term above) is neglected in the analysis, which is a valid approximation as long as the maximum delay spread is considerably less than T, i.e., $\max_{i,j} |\tau_{k,i} - \tau_{k,j}| \ll T$. (Self interference and ISI are, however, present in the simulations discussed in Section V.) The assumption that $y_i(m)$ and $y_j(n)$ are independent for $i \neq j$ and $m \neq n$ is also made. Clearly, this is not true in general, but is a common assumption simplifying the analysis. Assuming the MAI and noise, $\mathbf{w}_l^*(m)\tilde{\mathbf{n}}_l(m)$ can be modeled as a zero-mean Gaussian random variable, the maximum-likelihood receiver chooses the estimates of the desired user's transmitted symbols, $\hat{d}(m)$, such that the metric

$$\mathcal{M} = \sum_m \sum_l \frac{1}{\tilde{\sigma}_l^2(m)} \mathrm{Re}\{\beta_{1,l}(m)\mathbf{w}_l^*(m)\mathbf{c}_1(m)\hat{d}(m)y_l^*(m)\} \tag{8}$$

is maximized. This requires knowledge of the fading process and the noise variance for each path, which in a real system must be estimated but is assumed to be known here.

Two different linear receivers are considered: a conventional and an MMSE-type receiver. In addition to those linear receivers, an interference cancellation preprocessor followed by a conventional detector is considered, which is assumed to perfectly cancel all intracell interference without affecting neither the desired signal, nor the intercell interference. The same effect would be obtained if all users in the desired cell (unrealistically) were assigned mutual orthogonal spreading sequences.

The conventional receiver uses

$$\mathbf{w}_l(m) = \mathbf{c}_1(m), \qquad l = 1, \cdots, L \tag{9}$$

which, in essence, results in a conventional single-user RAKE receiver, not taking the MAI into account and requiring strict power control in order to perform adequately. The conventional receiver is as simple to implement for long as for short codes. The other linear structure considered, an MMSE-type receiver, can be implemented in a number of different ways, providing different advantages and disadvantages. One possibility is to use a single MMSE filter, synchronized to the strongest path, and let the MMSE filter do the RAKE combining. This requires the filter to track the relative phases of the desired user's rays, which can be troublesome. Instead, a separate MMSE filter can be used for each path, suppressing the interference independently for each ray by finding the receiver vectors $\mathbf{w}_l(m)$ minimizing

$$\mathbf{c}_R^{(\Delta_{k,i}^{(l)})} = \frac{1-\delta}{T_c}\left[0 \ \cdots \ 0 \ \ c_k^{(1)}(m') \ \cdots \ c_k^{(N')}(m')\right]^T + \frac{\delta}{T_c}\left[0 \ \cdots \ 0 \ \ c_k^{(1)}(m') \ \cdots \ c_k^{(N'-1)}(m')\right]^T \tag{3}$$

$$\mathbf{c}_L^{(\Delta_{k,i}^{(l)})} = \frac{1-\delta}{T_c}\left[c_k^{(N'+1)}(m'-1) \ \cdots \ c_k^{(N)}(m'-1) \ \ 0 \ \cdots \ 0\right]^T + \frac{\delta}{Tc}\left[c_k^{(N')}(m'-1) \ \cdots \ c_k^{(N)}(m'-1) \ \ 0 \ \cdots \ 0\right]^T \tag{4}$$

$E\{|\mathbf{w}_l^*(m)\mathbf{r}_l(m) - \beta_{1,l}(m)d_1(m)|^2\}$. The solution for each ray is given by

$$\mathbf{w}_l(m) = \mathbf{R}_l^{-1}(m)\mathbf{p}_l(m) \quad (10)$$

$$\mathbf{R}_l(m) = E\{\mathbf{r}_l(m)\mathbf{r}_l^*(m)\} \quad (11)$$

$$\mathbf{p}_l(m) = E\{\mathbf{r}_l(m)\beta_{1,l}(m)d_1(m)\} = P_{1,l}\mathbf{c}_1(m). \quad (12)$$

Note that, in theory, an MMSE-type receiver could be used both for long and short sequences, but in practice $\mathbf{R}_l(m)$ is estimated (directly or indirectly) by adaptive algorithms based on the assumption of a cyclostationary received signal and, hence, requires short codes. In the code hopping case, the equations above are solved for $\mathbf{w}_l(m)$ for each of the hops (the number of hops typically being relatively small, in the range of 2–4). This can be realized by having N_{hops} adaptive receivers operating independently, one for each hop.

IV. Performance Analysis

Exact computation of the error probability quickly becomes intractable due to the large number of parameters involved and the dependence on the transmitted bits from all users in the system. Therefore, the noise and MAI, $\mathbf{w}_l^*(m)\tilde{\mathbf{n}}_l(m)$, in the soft decision is modeled as zero-mean complex circular Gaussian noise with variance $\tilde{\sigma}_l^2(m)$. This is a common approximation due to its simplicity and is fairly accurate for MMSE detectors [17], although its application to the conventional correlation-type detector is less accurate. In the following, the average bit-error probability (BEP) for a data block, during which the average received power $P_{k,l}$ and the delays $\tau_{k,l}$ are assumed to be constant, while the fast fading $\beta_{k,l}(m)$ is time varying, will be derived. The derivation, valid for both coded and uncoded receivers, starts by deriving the pairwise-error probability conditioned on the desired users fading process, $\beta_{1,l}(m)$, and various system parameters, such as user placement, lumped into $\tilde{\mathbf{x}}$.

The SIR of the decision variable for the desired user's lth path given a certain scenario is defined as

$$\gamma_l(m) = P_{1,l}\frac{[\mathbf{w}_l^*(m)\mathbf{c}_1(m)]^2}{\tilde{\sigma}_l^2(m)}, \quad (13)$$

where $\tilde{\sigma}_l^2(m) = E\{\mathbf{w}_l^*(m)\tilde{\mathbf{R}}_l(m)\mathbf{w}_l(m)\}$. When computing $\tilde{\mathbf{R}}_l(m) = E\{\tilde{\mathbf{n}}_l(m)\tilde{\mathbf{n}}_l^*(m)\}$, the choice whether a variable should be treated as a random variable or as a unknown constant to condition upon (and thus lumped into $\tilde{\mathbf{x}}$) must be made. Herein, the bits $d_k(m)$, interferers fading $\beta_{k,l}(m)$, $k = 2, \cdots, K$, and in the case of long codes, $\mathbf{c}_k(m)$ and $\tau_{k,l}$, $k = 2, \cdots, K$, are treated as random variables, while the received power $P_{k,l}$ and, for the short code case, code sequences, code hops, and delays, are treated as unknown parameters and included in $\tilde{\mathbf{x}}$. For the receiver types considered, $\gamma_l(m)$ can be expressed as

$$\gamma_l(m) = \begin{cases} P_{1,l}[\mathbf{c}_1^*(m)\tilde{\mathbf{R}}(m)\mathbf{c}_1(m)]^{-1}, & \text{conventional} \\ P_{1,l}[\mathbf{c}_1^*(m)\tilde{\mathbf{R}}_e(m)\mathbf{c}_1(m)]^{-1}, & \text{IC} \\ P_{1,l}\mathbf{c}_1^*(m)\tilde{\mathbf{R}}^{-1}(m)\mathbf{c}_1(m), & \text{MMSE} \end{cases} \quad (14)$$

where $\tilde{\mathbf{R}}_e(m)$ denotes the part of $\tilde{\mathbf{R}}(m)$ due to interference external to the cell of interest, and the last relation follows from the matrix inversion lemma.

Using the metric in (8), it is straightforward to derive the conditional pairwise-error probability $P_{2|\tilde{\mathbf{x}}}(d) = \Pr\{\Delta(d) \leq 0\}$. The quantity $\Delta(d) = \mathcal{M}_i - \mathcal{M}_j$ is the difference in metric between the correct path i and the erroneous path j of length d, and can be written as

$$\Delta(d) = \sum_{m=1}^{d}\sum_{l=1}^{L}\frac{4}{\tilde{\sigma}_l^2(m)}[|\beta_{1,l}(m)|^2(\mathbf{w}_l^*(m)\mathbf{c}_1(m))^2 + \mathbf{w}_l^*(m)\mathbf{c}_1(m)\mathrm{Re}\{\beta_{1,l}(m)\mathbf{w}_l^*(m)\tilde{\mathbf{n}}_l(m)\}]. \quad (15)$$

The BEP can then be derived from the pairwise-error probability as

$$P_{e|\tilde{\mathbf{x}}} = \begin{cases} = P_{2|\tilde{\mathbf{x}}}(1), & \text{uncoded} \\ \approx \sum_{d=d_{\text{free}}}^{\infty} b_d P_{2|\tilde{\mathbf{x}}}(d), & \text{coded} \end{cases} \quad (16)$$

where, for the coded case, the expression is a reasonably good approximation (actually, a tight upper bound) for error probabilities around 10^{-3} and below, and b_d is obtained from the code's weight spectrum [18]. Usually, only the first terms in the summation are required to get a good approximation (the first 18 were used in the numerical section). Finally, to take care of the effects from the (optional) code hopping, (16) is averaged over error events starting in all different positions in the code hopping cycle. This is necessary since the quantity of interest is $E\{P_e(\gamma)\}$ and not $P_e(E\{\gamma\})$, although the latter expression is a good approximation if d_{free} is considerably larger than the number of code hops.

Two different cases can be distinguished. Either a nonfading case, where only path loss and slow log-normal fading is taken into account, which essentially means that $\beta_{k,l}(m) = \beta_{k,l}$ has a fixed amplitude and a random, nontime-varying phase, or a fast Rayleigh fading case, where $\beta_{k,l}(m)$ is time varying. Infinite interleaving is assumed in presence of fast fading, i.e., $\beta_{k,l}(m)$ is assumed to be uncorrelated to $\beta_{k,l}(n)$, $m \neq n$. For the nonfading case, the pairwise-error probability can be written as

$$P_{2|\tilde{\mathbf{x}}}(d) = Q\left(\sqrt{\sum_{m=1}^{d_{\text{free}}}\sum_{l=1}^{L} 2\gamma_l(m)}\right). \quad (17)$$

For Rayleigh fading, a characteristic function approach is used. Since $\Delta(d)$ is a quadratic form of the complex Gaussian fading process $\beta_{k,l}(m)$, the analysis in [19] can be used. The Laplace transform of the probability density function of $\Delta(d)$ is given by

$$\Phi_\Delta(s) = \prod_{m=1}^{d}\prod_{l=1}^{L}\frac{1}{1 + 4\gamma_l(m)s(1-s)}. \quad (18)$$

The probability density function can be obtained by inverse Laplace transformation of (18), which can be subsequently used for calculating the error probability [19]. However, this can be cumbersome, especially if some, but not all, $\gamma_l(m)$ are

TABLE I
OUTAGE PROBABILITIES (IN PERCENT) DEFINED AS HAVING A BIT-ERROR RATE ABOVE THE THRESHOLD FOR A SINGLE RAY AND TWO RAYS. FOR THE CONVENTIONAL DETECTOR, DIFFERENT VALUES OF N_{hops} ARE INVESTIGATED. SIMULATIONS (ANALYSIS IN PARENTHESES). THE DIFFERENCE BETWEEN SIMULATION AND ANALYSIS CAN SOMETIMES BE QUITE LARGE SINCE THE APPROXIMATIONS DONE ARE LESS ACCURATE IN THE TAIL OF THE DISTRIBUTION. PERFECT POWER CONTROL (MOST CASES) AND IMPERFECT POWER CONTROL WITH $\sigma_{\mathrm{PC}} = 1$ dB (PC err.)

		Threshold							
		non-fading				fading			
		$2 \cdot 10^{-2}$		$3 \cdot 10^{-2}$		$9 \cdot 10^{-2}$		$1 \cdot 10^{-1}$	
one ray	long	6.7	(3.5)	1.0	(0.3)	4.4	(2.6)	1.0	(0.4)
	short 1	10.8	(9.3)	3.8	(2.8)	10.0	(7.8)	4.1	(2.8)
	short 2	10.2	(8.4)	3.2	(2.3)	8.6	(6.2)	3.1	(1.9)
	MMSE	0.1	(0.1)	0.0	(0.1)	0.1	(0.1)	0.1	(0.0)
	long, PC err.	16.8	(5.0)	6.7	(0.7)	14.9	(3.8)	6.6	(0.7)
	short 1, PC err.	17.5	(11.4)	8.8	(3.7)	16.9	(9.8)	9.5	(3.9)
	MMSE, PC err.		(0.2)		(0.1)		(0.1)		(0.1)
two rays	long	6.0	(3.5)	0.9	(0.3)	0.3	(0.0)	0.2	(0.0)
	short 1	8.9	(6.5)	2.3	(1.4)	0.3	(0.1)	0.1	(0.0)
	short 2	8.0	(5.6)	1.8	(0.9)	0.2	(0.1)	0.1	(0.0)
	MMSE	0.0	(0.0)	0.0	(0.0)	0.0	(0.0)	0.0	(0.0)
	long, PC err.	16.9	(5.3)	6.2	(0.6)	1.8	(0.1)	0.8	(0.0)
	short 1, PC err.	16.9	(8.5)	7.7	(2.0)	2.3	(0.3)	1.0	(0.1)
	MMSE, PC err.	0.3	(0.1)	0.0	(0.0)	0.0	(0.0)	0.0	(0.0)

unique, and lead to numerical problems. A way of circumventing this through a numerical approximation of $P_{2|\tilde{\mathbf{x}}}(d) = \Pr\{\Delta(d) \leq 0\} = (1/j2\pi)\int_{\varepsilon-j\infty}^{\varepsilon+j\infty} \Phi_{\Delta}(s)/s\,ds$, given by[2] $(1/\nu)\sum_{i=1}^{\nu/2} \mathrm{Re}\{\Phi_{\Delta}(\varepsilon + j\varepsilon\theta_i\} + \theta_i \mathrm{Im}\{\Phi_{\Delta}(\varepsilon + j\varepsilon\theta_i\}$, where $\theta_i = \tan((2i-1)\pi/(2i))$ is advocated in [20]. This nice technique gives good accuracy and is fairly insensitive to the choice of ν and ε.

V. NUMERICAL RESULTS

The simulated system was a cellular system using either Gold codes or random code sequences of length $N = 31$ chips/bit. A larger spreading factor would be more realistic, but in order to keep the simulation time reasonable, $N = 31$ was chosen. Noise was excluded from the simulations as the influence of the choice of spreading sequences on the performance was the main interest. Two tiers of interfering cells (6 and 12 cells in each tier, respectively) surrounded the center cell in which the user of interest was located. Numerical evidence has shown that two tiers are enough, and adding a third tier hardly affects the results. In each cell, K_{b_k} users, where K_{b_k} is a Poisson distributed random variable, were randomly placed according to a uniform distribution, and connected to the base with the smallest attenuation, as explained in Section II. The average number of users within each cell was $E\{K_{b_k}\} = 10$. Each user encountered path loss and shadowing with $\alpha = 4$ and $\sigma_s = 8$ dB, respectively, and the delays of the interfering users relative the desired user were uniformly distributed in $(-T, T)$. Perfect power control ($\sigma_{\mathrm{PC}} = 0$), i.e., all users connected to a specific base station are received with the same power, is assumed (except for some of the entries in Table I). Two different channel profiles were investigated, a flat fading channel and a two-ray channel, the latter with a second ray attenuation of 6 dB relative the first ray and the random delay of the second ray relative the main ray uniformly distributed in $[T_c, 3T_c]$. No code planning was used in the short code systems, i.e., users in different cells might use identical spreading codes, while within each cell the users are assigned unique codes. The code hopping, when used, was random and independent from cell to cell.[3] For the coded case, a simple rate 1/2 code with $d_{\mathrm{free}} = 5$ was used [21, p. 330]. The error analytical probabilities were computed according to Section IV, but for error probabilities above 10^{-3} in the coded system, table lookup based on simulations of a single-user convolutional decoder was used instead of (16) in order to improve the accuracy at low SIR values.

Plotted in Fig. 2 is a histogram of the average SIR γ, with and without code hopping, and in Fig. 3 is the two-dimensional SIR distribution of a code hopping case ($N_{\mathrm{hops}} = 2$). A total of 10^5 different scenarios, where each scenario corresponds to random realizations of the shadowing and placements of all users, were used in the averaging, and for the code hopping case, the SIR value plotted was averaged over the different hops. From the plots, it is seen that the variance in SIR is larger for the short code system compared to the corresponding long code case, suggesting a higher spread in performance. Code hopping does reduce this spread somewhat. Noteworthy is also the intracell interference cancellation receiver, in which the short code system on average performs better than the long code cancellation scheme, although with a higher variance. The MMSE is the best performing receiver with a considerable higher average SIR and almost all of the tail of the MMSE curve being better than the average conventional system. However, keep in mind that the idealized MMSE receiver is able to suppress both intercell and intracell interference.

In Figs. 4–7, the cumulative density function for the expected bit-error rate for the user of interest is shown for a nonfading (only path loss and shadowing) and a fast Rayleigh fading channel. The simulations were run over $5 \cdot 10^4$ different scenarios, and for each scenario, data blocks representing 100

[2]Note the typo in [20, eq. (5)].

[3]This implies that two users in the same cell never can be assigned identical code sequences for the same hop, although one user's code sequence could be reused by another user in the next hop, which is fine as long as the propagation delays are less than one bit time. If longer delays are present, the hopping scheme could, for instance, be designed such that a certain code is not reused until a certain number of bit periods have elapsed.

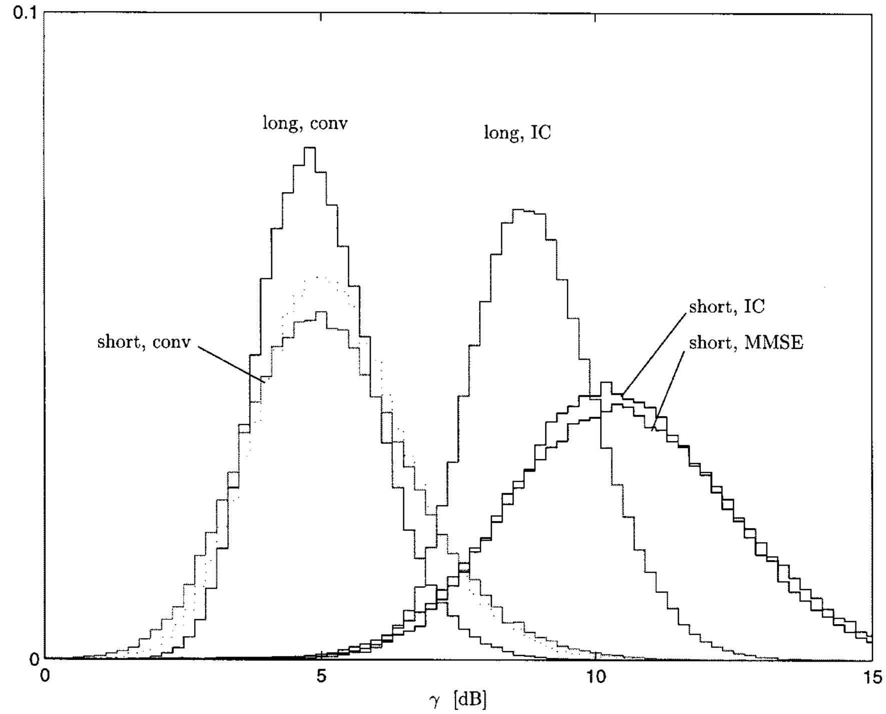

Fig. 2. Distribution of γ for conventional, IC, and MMSE receivers in a nonmultipath environment with perfect power control. No code hopping, i.e., $N_{\rm hops} = 1$ (solid) and $N_{\rm hops} = 2$ (dotted). For MMSE, code hopping does reduce the spread in a similar way to the conventional receiver (the curve is not shown in order not to clutter the plot unnecessarily).

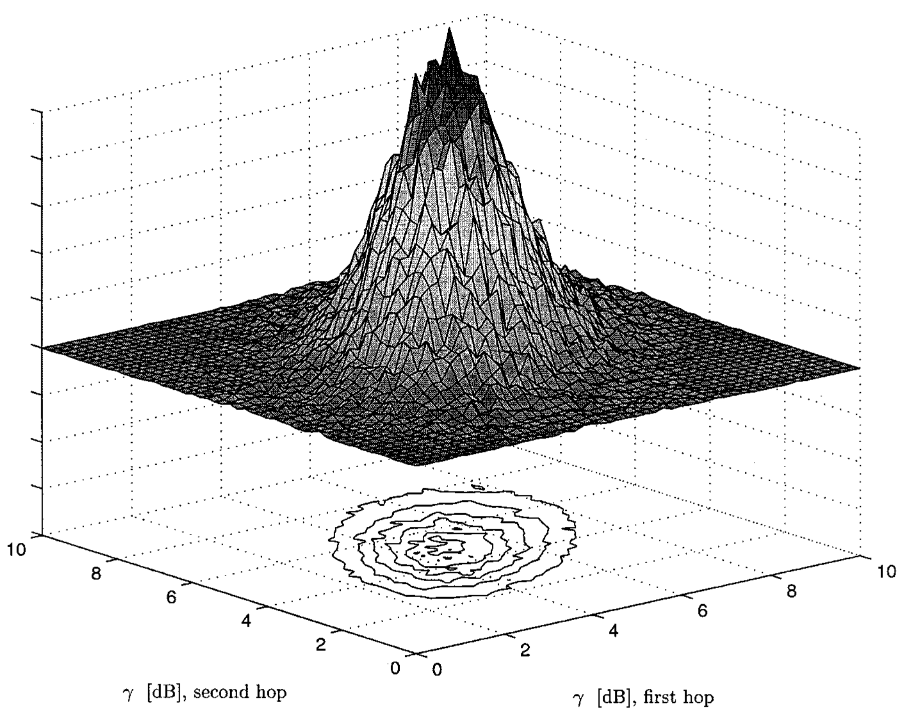

Fig. 3. Two-dimensional distribution of γ for a conventional receiver in an $N_{\rm hops} = 2$ code hopping case. No multipath and perfect power control.

uncoded bits were transmitted until at least 10 errors were recorded (a minimum of 10^4 bits was always transmitted). As seen in the plots, the difference between the long and short code systems using conventional detectors is reasonably small with a somewhat higher variance in performance for the short code case, a difference that is reduced when fast fading is present and adds more randomization to the interference, something that benefits the short code case more than the long code one. A similar effect, although less noticeable, is obtained from multipath compared to single-path propagation. Furthermore, the analytical results tend to be slightly optimistic compared to the simulations. The average error probability for the long code system is slightly higher than the short code system with conventional detection. It should be noted that for

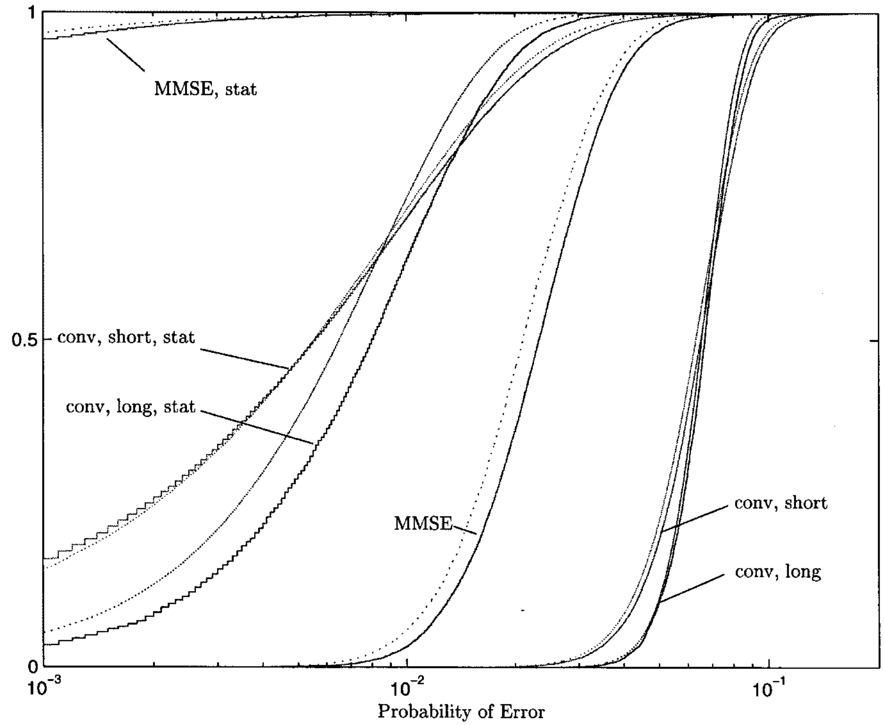

Fig. 4. Cumulative distribution of the uncoded BEP for a nonmultipath channel. Conventional and intracell interference cancellation detectors with long and short codes ($N_{\rm hops} = 1$), and the MMSE detector with $N_{\rm hops} = 1$. The three rightmost curve pairs are for fast Rayleigh fading, while the leftmost curves correspond to a static channel. Simulated (solid) and analytical (dotted).

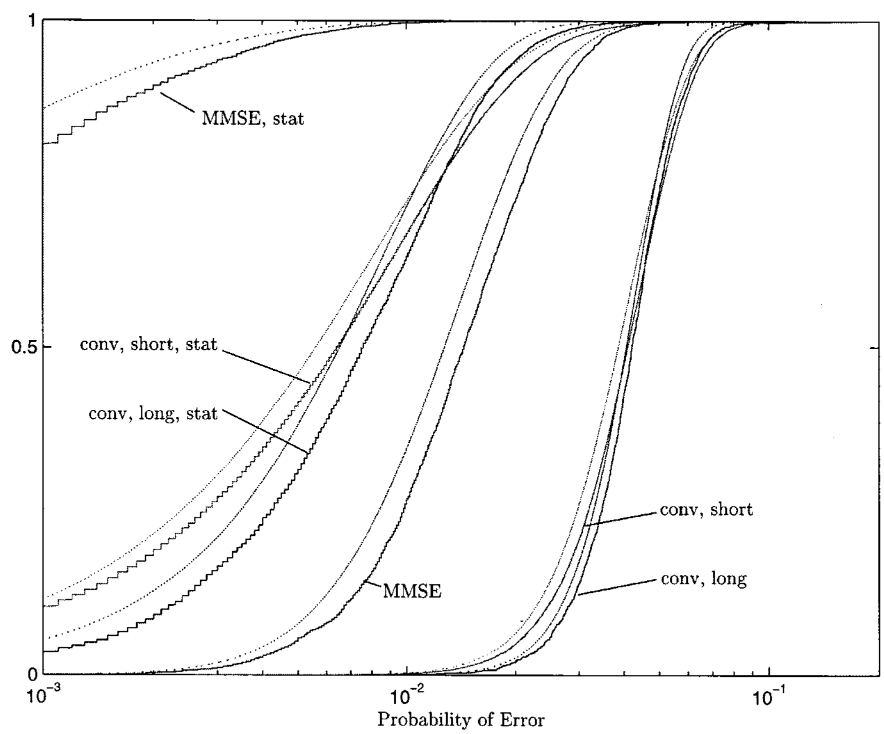

Fig. 5. Cumulative distribution of the uncoded BEP for a multipath channel (second ray is 6 dB weaker than first). Conventional and intracell interference cancellation detectors with long and short codes ($N_{\rm hops} = 1$), and the MMSE detector with $N_{\rm hops} = 1$. The three rightmost curve pairs are for fast Rayleigh fading, while the leftmost curves correspond to a static channel. Simulated (solid) and analytical (dotted).

a specific realization of shadowing and placements of users, the performance for all the users connected to the cell of interest is (on average) identical in a long code system, while it can vary for the short code case. For nonselective channels, the short code IC and MMSE detectors appears to have similar performance, while the MMSE receiver suffers relatively more when multipath propagation is present. The more rays present, the harder the task of suppressing the interference for the

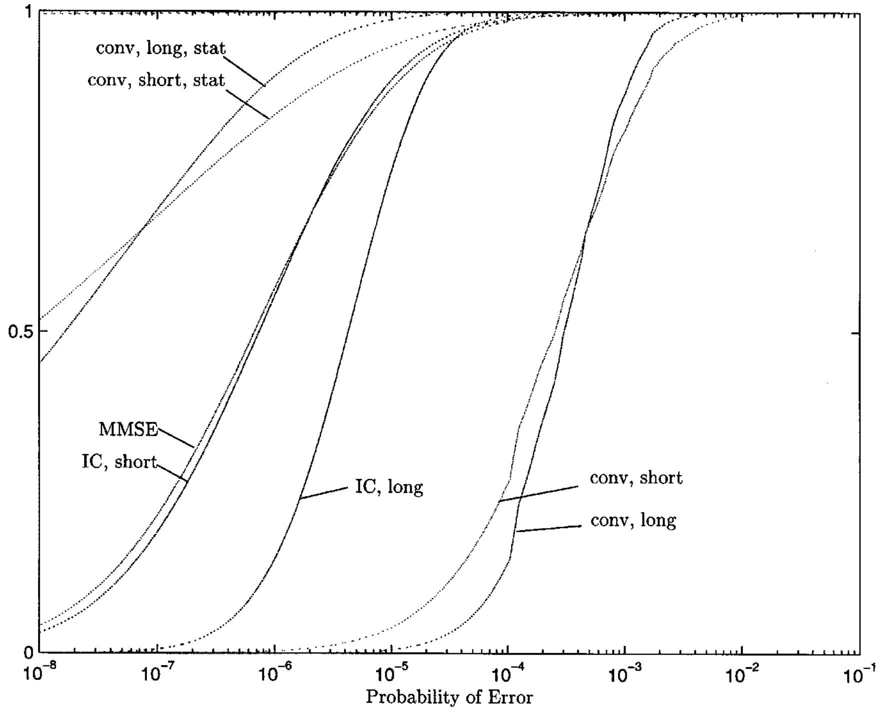

Fig. 6. Cumulative distribution of the coded (rate $1/2$) BEP for a nonmultipath channel. Conventional and interference cancellation detectors with long and short codes ($N_{\rm hops} = 1$), and the MMSE detector with $N_{\rm hops} = 1$. The five rightmost curves are for fast Rayleigh fading, while the leftmost curves correspond to a static channel. Analytical results.

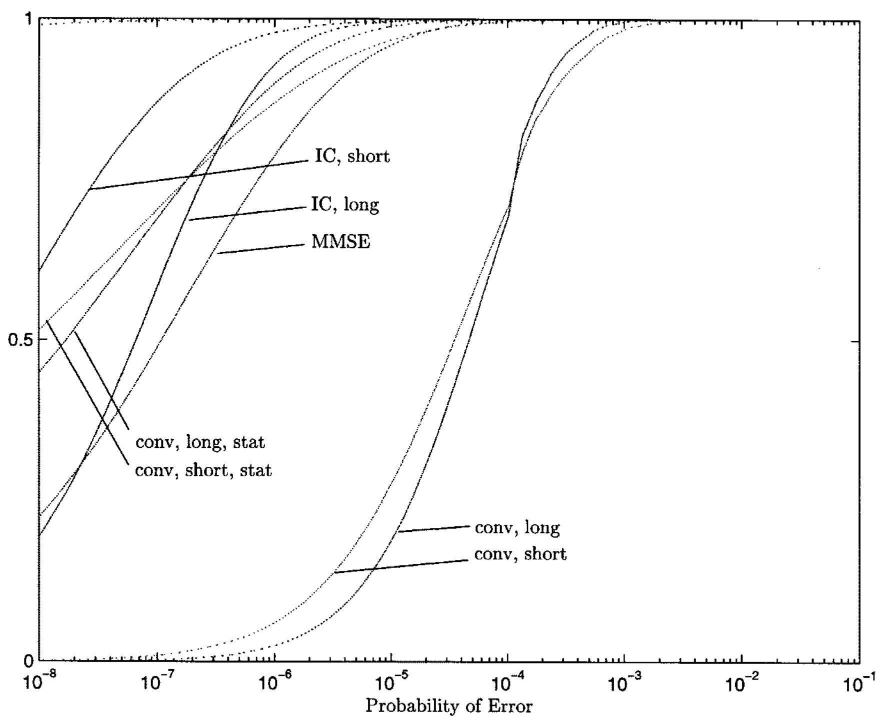

Fig. 7. Cumulative distribution of the coded (rate $1/2$) BEP for a multipath channel (second ray is 6 dB weaker than first). Conventional and interference cancellation detectors with long and short codes ($N_{\rm hops} = 1$), and the MMSE detector with $N_{\rm hops} = 1$. The five rightmost legends correspond to fast Rayleigh fading, while the two leftmost legends correspond to a static channel (for a static channel, IC and MMSE are outside the plot area). Analytical results.

MMSE receiver, which has a fixed number of filter taps. The IC detector, on the other hand, does not have this problem as it is based on cancellation rather than suppression. Code hopping results in a performance somewhere in between the long and short code cases, as illustrated in Fig. 8, although most of the effect seems to be in the low BER region, and not in the high region, which is what rules the outage probability. In Table I, the outage probability is listed for long and short codes with conventional detection. Note the relatively larger degradation in performance for long codes with conventional detection

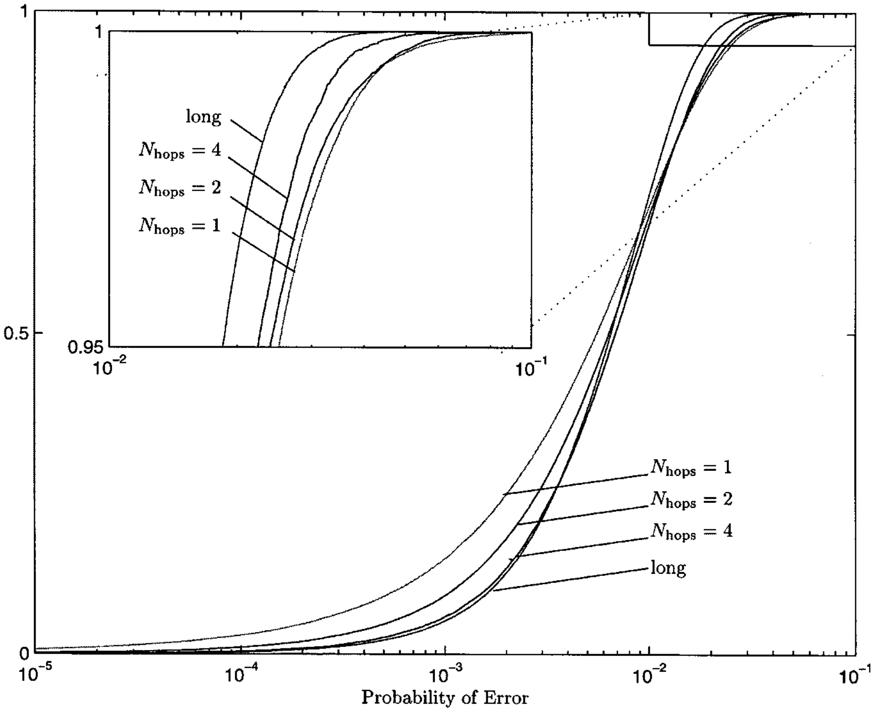

Fig. 8. Cumulative distribution function of the uncoded BEP for conventional detectors with long and short codes ($N_{\text{hops}} = 1, 2, 4$). Analytical results, nonfading single-path channel.

in the presence of power control errors compared to short codes. The corresponding plot of the cumulative distribution functions, omitted due to space limitations, is similar to Figs. 4–7, although less steep and with smaller difference between long and short codes.

If the load of the system in increased from $E\{K_{b_k}\} = 10$ to $E\{K_{b_k}\} = 20$ users, the general relative behavior remains unchanged. For the nonfading case, the slope is steeper as the number of users increase, both for long and short spreading codes, while for the fast fading case, the slope is approximately unchanged. The average BEP is of course increased when the number of users increases. The lack of change in the slope for the fast fading case can be explained by the randomization in mutual correlation introduced by the fading.

VI. Conclusions

The difference between long and short spreading codes in a cellular DS-CDMA system has been investigated, and a somewhat larger performance spread was found for short codes than for long codes when conventional detectors were used, while the short code MMSE detector clearly outperformed the two systems with conventional detectors. Perfect cancellation of intracell interference was investigated as well. Code hopping, a scheme in which each user's short spreading code is switched between a predetermined set of code sequences, was suggested and found to narrow the gap between the long and short code systems. If the slight difference between the long and short sequences can be accepted, a viable approach is to use conventional detectors in a short code system while developing low-complexity multiuser detectors. Future issues to consider are, for example, imperfections in channel estimates, as the current study considered the idealized case with perfect knowledge of most parameters.

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