Variable-Rate Two-Phase Collaborative Communication Protocols for Wireless Networks

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Abstract

The performance of two-phase collaborative communication protocols is studied for wireless networks. All the communication nodes in the cluster are assumed to share the same channel and transmit or receive collaboratively in a quasi-static Rayleigh flat-fading environment. In addition to small-scale fading, the effect of large-scale path loss is also considered. Based on a decode-and-forward approach, we consider various variable-rate two-phase protocols that can achieve full diversity order and analyze the effect of *node geometry* on their performance in terms of the outage probability of mutual information. For the single relay node case, it is shown that if the collaborator node is close to the source node, a protocol based on space-time coding (STC) can achieve good diversity gain. Otherwise, a protocol based on receiver diversity performs better. These protocols are also compared with one based on fixed-rate repetition coding and their performance trade-offs with node geometry are studied. The second part deals with multiple relays. It is known that with N relays an asymptotic diversity order of $N+1$ is achievable with STC-based protocols in the two-phase framework. However, in the framework of collaborative STC, those relay nodes which fail to decode remain silent (this event is referred to as a *node erasure*). We show that this node erasure has the potential to considerably reduce the diversity order and point out the importance of designing the STC to be robust against such node erasure.

Index Terms

Collaborative (cooperative) communication, relay channel, space-time coding, spatial diversity, wireless networks.

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I. INTRODUCTION

In many wireless networks, the power consumption of communication nodes is a critical issue. In addition, typical wireless channels suffer from signal fading which, for a given average transmit power, significantly reduces communication capacity and range. If the channel is slow and flat fading, channel coding does not help [1, 2] and spatial diversity may be the only effective option that can either reduce the average transmit power or increase communication range. Results on space-time coding (STC) [3, 4] have shown that the use of antenna arrays at the transmitter and receiver can significantly reduce transmit energy. However, for many applications with low-cost devices such as wireless sensor networks, deployment of multiple antennas at each node is too costly to implement due to severe constraints on both the size and power consumption of analog devices.

The recently proposed collaborative (or cooperative) diversity approaches [5–14] demonstrate the potential to achieve diversity or enhance the capacity of wireless systems without deploying multiple antennas at the transmitter. Using nearby collaborators as virtual antennas, significant diversity gains can be achieved. These schemes basically require that the relay nodes share the information data of the source node, and this data sharing process is generally achieved at the cost of additional orthogonal channels (in frequency or in time). In a companion paper [15], we have shown that for a given fixed rate and under suitable node geometry conditions, there are collaborative coding schemes that can nearly achieve the same diversity as if all the relay node antennas were connected to the source node, without any additional orthogonal channels or bandwidth. The construction of such codes, however, appears to be challenging.

Among many approaches in the literature, Laneman [5, 6] analyzes several low-complexity relaying protocols that can achieve full diversity, under realistic assumptions such as half-duplex constraint and no channel state information (CSI) at the transmitting nodes. It has been shown that in the low-spectral-efficiency regime, the SNR loss relative to ideal transmit diversity system with the same information rate is 1.5 dB[5]. Multiple-relay cases are also considered in [6] and bandwidth-efficient STC-based collaborative protocols are proposed.

Collaborative diversity protocols are largely classified into *amplify-and-forward* and *decode-and-forward* schemes [5]. In the following, we will restrict our attention to decode-and-forward schemes since these may provide some salient advantages. First, there is no error propagation if the relay transmits information only when it decodes correctly. Otherwise, the relay remains silent and thus an unnecessary energy transmission can be saved[1]. Second, the information rate per symbol does not need to be the same for each phase. In other words, the relative duration of each phase can be changed according to node geometry.

[1]Even though perfect detection of the codeword is not feasible in practice, one can design a cyclic redundancy-check (CRC) or error detectable low-density parity-check (LDPC) code such that for a given system outage probability, the effect of error propagation is negligible. Many existing communication networks have this structure.

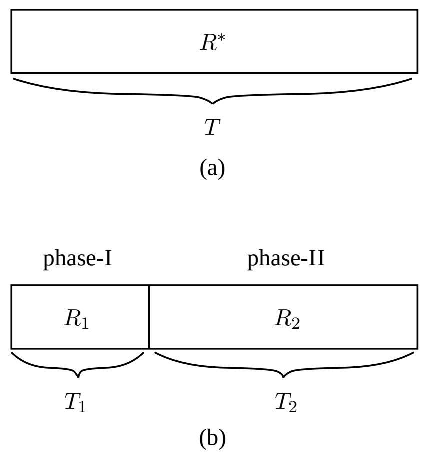


Fig. 1. Two-phase communication. (a) baseline system. (b) two-phase protocol.

It is the latter property that we shall focus on in this work. Suppose that we wish to transmit data with information rate R^* bits per second and T is the frame period, also in seconds. Then the total information transmitted during this period is R^*T bits (per frame). The baseline frame design that achieves this is shown in Fig. 1 (a). Alternatively, we may split the time interval into two phases of duration T_1 and T_2 where $T = T_1 + T_2$ and each phase is operated with information rate R_1 and R_2, respectively, as depicted in Fig. 1 (b). We assume that for both phases, the same information (but with different coding rate) is transmitted. If R_1 and R_2 are chosen such that $R_1T_1 = R_2T_2 = R^*T$, then in principle there is no loss of total transmission rate compared to the baseline system. Let the fraction of the relative time period for each phase be denoted by $\delta_1 \triangleq T_1/T = T_1/(T_1 + T_2)$, and $\delta_2 \triangleq T_2/T = 1 - \delta_1$. Then, the information rate during each phase is $R_1 = R^*/\delta_1$ and $R_2 = R^*/\delta_2$. Therefore, during each phase, information should be transmitted employing larger constellation sizes than the baseline system[2].

For ideal AWGN and interleaved fading channels under an average signal-to-noise power ratio (SNR) constraint over the entire communication process, two-phase protocols do not necessarily achieve a gain and may even result in performance loss compared to the baseline system. However, for quasi-static or block Rayleigh fading channels, it is not the constellation size but diversity that is the dominant factor for the outage behavior. Thus, if additional diversity can be achieved by two-phase methods, the resulting outage probability of the mutual information may more than offset any loss due to constellation size and yield a reduction in required SNR. (This is somewhat analogous to coded modulation which increases the signal constellation size in order to achieve coding gain. In our case, however, we shall achieve diversity gain.)

In practical collaborative wireless communication networks, node geometry is an important factor. Intuitively, if the collaborative relay node is close to the source node, it may be efficient for the relay

[2]The fraction δ_1 and δ_2, or equivalently, the coding rate R_1 and R_2 are determined based on the node location, not on each realization of fading channel coefficient as done in [15, 16].

to act as a transmit antenna. In this case, STC based protocols such as [6] may be efficient. On the other hand, if the relay is close to the destination, it should operate as a receive antenna. To capture this geometrical effect, we model the wireless network channel as an aggregate of large-scale path-loss and small-scale fading [17]. The large-scale path-loss is the decay of signal power due to the transmitter receiver separation, and is a function of the distance between the two terminals. On the other hand, the small-scale fading is a consequence of multipath which may vary randomly according to any physical change of surroundings. The overall system model is detailed in Section II.

In this paper, motivated by the rate-flexible nature of decode-and-forward protocols and the importance of node geometry, we extend the work of Laneman [5, 6] to a variable-rate framework with particular emphasis on *path-loss gain effect* of relay nodes, achieved due to the relay's proximity to source/destination nodes. Several low-complexity protocols are considered, including a simple multi-hopping protocol, the bandwidth-efficient STC-based protocol of [6], as well as its receiver diversity counterpart (Section III). Their performances with a single relay node are theoretically analyzed in terms of *achievable diversity gain for a given information rate* based on outage probability of mutual information. For this purpose, convenient simple analytical tools are developed in Section IV.

The main objective of the paper is, for a given relative location of the relay node, to determine a suitable protocol and minimize the total required power of the transmitting nodes. To that end, optimal power control factors and relative phase durations for the relay node are derived for each protocol considered. Associated with these protocols, closed-form expressions for diversity gain are derived in Section V, where it is shown that by suitably choosing the protocol and controlling the transmission rate, as a function of node geometry, the achievable diversity gain can be significantly improved. Also, it will be shown that under severe path-loss, even a simple multi-hop protocol benefits relative to direct transmission. For example, a significant gain is attained if the relay is located midway between the two communicating nodes.

In the analysis of STC-based collaborative protocols, we presume two types of STC which we denote as perfect and imperfect STC. A perfect STC refers to an STC with partial decodability, i.e., the (full) information can be retrieved from a subset of the transmitting nodes, whereas an imperfect STC refers to a system in which the receiving nodes fail to decode if any one of the transmitting nodes that constitute the STC fails to transmit. This partial erasure of an STC antenna branch may happen if the relay nodes fail to decode correctly. (This event will be referred to as a *node erasure*.) In Section VI, we show that the diversity order of an imperfect STC with N collaborative relays is at most 2, whereas that of a perfect STC can achieve diversity order of $N + 1$ as in [6].

Throughout this paper, our main focus is on the achievable diversity gain for a given information rate. The diversity-multiplexing trade-off [18] of the relay channels is also of practical importance, but this is beyond the scope of this paper. Some results in this direction are explored in [5, 6, 16]. As related work, the effect of node geometry is also considered in [12, 19, 20], but in a fixed-rate framework. Also, we do

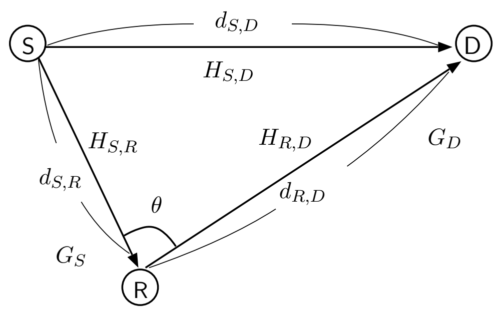


Fig. 2. Two-phase communication.

not address specific design issue of coding as many existing channel/STC techniques in the literature are applicable to our framework without major modification. Note that some practical design of collaborative codes (with implicit variable-rate coding) is proposed in [21] and its outage behavior is evaluated in [13]. The use of incremental redundancy such as [22] may be of further potential in this framework. Finally, we note that variable-rate coding for *multiple-access channels* has been recently studied in [23].

II. System and Channel Model

Fig. 2 illustrates the basic model which is considered throughout the paper. It is assumed that the three nodes source(S), relay(R), and destination(D) are located in the two dimensional plane as in Fig. 2 where θ is the angle of the line S – R – D and $d_{A,B}$ denotes the Euclidean distance between nodes A and B. We suppose that S wishes to transmit the message to D and that R has agreed to collaborate with S *a priori*.

For simplicity, we assume that all the channel links are composed of large-scale path loss and statistically independent small-scale quasi-static frequency non-selective Rayleigh fading. Consequently, the complex channel coefficients $H_{S,D}$, $H_{S,R}$, and $H_{R,D}$ in Fig. 2 are uncorrelated and circularly symmetric complex Gaussian random variables with zero mean and unit variance. They are assumed to be known perfectly to the receiver sides and unknown at the transmitter sides. Perfect timing and frequency synchronization are also assumed, even though accurate acquisition of synchronization among distributed nodes may be challenging in practice.

The path loss between two nodes, say A and B, is modeled by

$$\mathrm{PL}(A,B) = K/d_{A,B}^{\alpha}, \tag{1}$$

where K is a constant that depends on the environment and α is the path-loss exponent. For free-space path loss, we have $\alpha = 2$ and $K = G_t G_r \lambda^2/(4\pi)^2$, where G_t and G_r are antenna gains at transmitter and receiver, respectively, and λ is the wavelength [17]. Although the path-loss exponent and the constant factor K may vary for each channel link, throughout the paper it is assumed that α and K are identical for all channel links.

For isotropic antennas, the received energy at the relay can be related to the received energy at the destination according to

$$E_{S,R} = \frac{\mathrm{PL}(S,R)}{\mathrm{PL}(S,D)} E_{S,D} = \left(\frac{d_{S,D}}{d_{S,R}}\right)^{\alpha} E_{S,D} \triangleq G_S E_{S,D}, \tag{2}$$

where $E_{A,B}$ denotes the average received energy between the $\mathsf{A} \to \mathsf{B}$ channel link, and G_S is the geometrical gain achieved by the proximity advantage of the relay node over the destination node. Likewise, the gain at the destination node in communicating with the relay over the source is given by

$$G_D \triangleq \frac{E_{R,D}}{E_{S,D}} = \left(\frac{d_{S,D}}{d_{R,D}}\right)^{\alpha}. \tag{3}$$

This gain implies that if the average power of the relay is controlled in such a way that the relay transmits its signal with the same average power as the source, then the destination node receives the relay's signal with a gain of G_D compared to that of the $\mathsf{S} \to \mathsf{D}$ channel link. By the triangle equality, we have

$$\left(\frac{d_{S,R}}{d_{S,D}}\right)^2 + \left(\frac{d_{R,D}}{d_{S,D}}\right)^2 - 2\left(\frac{d_{S,R}}{d_{S,D}}\right)\left(\frac{d_{R,D}}{d_{S,D}}\right)\cos\theta = 1. \tag{4}$$

Let ζ denote the ratio of $d_{R,D}$ to $d_{S,R}$, i.e.,

$$\zeta \triangleq \frac{d_{R,D}}{d_{S,R}}. \tag{5}$$

Then the gain G_S can be expressed as a function of α, ζ, and θ:

$$G_S = \left(1 + \zeta^2 - 2\zeta\cos\theta\right)^{\frac{\alpha}{2}}. \tag{6}$$

Without loss of generality, we assume $0 \leq \theta \leq \pi$. It is easy to observe that for a given $\zeta > 0$, G_S is a monotonically increasing function with respect to α and θ. Note that if $\pi/2 \leq \theta \leq \pi$, then the relay node lies within the circle with diameter $\mathsf{S} \to \mathsf{D}$, and $\theta = \pi$ corresponds to the case where the relay lies on the line between $\mathsf{S} \to \mathsf{D}$.

III. Two-Phase Protocol

There are several variations that can be considered for two-phase protocols. We consider the four specific protocols summarized in Table I. Performance analysis of these protocols in a fixed-rate framework can be found in [5, 6, 24]. For all protocols, it will be assumed that all component codes are designed to have error detection capability, i.e., if the relay fails to decode the information correctly, it knows this and remains silent in the next phase. This may lead to power savings at the transmit side and the resulting effect is incorporated into the calculation of the SNR. In the case that a relay node is unable to collaborate during the second phase, we denote this event as a *node erasure* which corresponds to an antenna erasure in a traditional STC scenario.

TABLE I

TWO-PHASE PROTOCOLS (●: TRANSMITTING, ○: RECEIVING, -: SWITCHED OFF)

TD

	I	II
S	●	●
R	○	●
D	○	○

STD

	I	II
S	●	●
R	○	●
D	-	○

RD

	I	II
S	●	-
R	○	●
D	○	○

MH

	I	II
S	●	-
R	○	●
D	-	○

A. Descriptions

1) Transmit Diversity (TD) Protocol: In the first protocol, during phase-I, S broadcasts its information at rate R_1 and the relay node R attempts to decode this information. Node D also receives and then attempts to decode the information during this phase. If D is able to decode the message correctly, the subsequent phase will be ignored.

During phase-II, both S and R (re)encode the information using an STC with rate R_2 similar to [6]. If the decoding at node D after phase-I failed, node D reattempts to decode after phase-II. This approach is referred to as the Transmit Diversity (TD) protocol.

2) Simplified Transmit Diversity (STD) Protocol: This is a simplified alternative to the TD protocol. In this case, the destination node D is switched off during phase-I and thus ignores the signal from S. The phase-I communication link serves only the relay R. The second phase is identical to that of the TD protocol. The STD protocol may result in a simple receiver structure but in some cases, a performance loss is expected compared to the TD protocol.

3) Receiver Diversity (RD) Protocol: The third scenario we consider is similar to receiver selection diversity. In this case, during phase-I, S broadcasts information and the relay and destination decode in the same way as the TD protocol. During phase-II, the relay re-encodes the data and transmits the data at rate R_2 without STC. (The source remains silent in phase-II.) This approach is referred to as the Receiver Diversity (RD) protocol.

Further strategies such as decoding based on a combination of phase-I and phase-II data can be considered, analogous to maximum ratio combining for receiver diversity [5, 6, 13, 24]. Only in such an approach can optimal performance be achieved. In our variable-rate framework of decode-and-forward, such techniques may require special coding structures and thus impose additional complexity at the receiver side. For comparison purpose, however, the performance of this approach is also studied in Section V-H.

4) Multi-Hopping (MH) Protocol: The effectiveness of multi-hopping (MH) protocols has been widely studied[3]. For comparison a simple multi-hopping protocol will also be considered as a special case of the RD protocol where the destination node only switches on during phase-II. This approach does not

[3]For example, optimal MH distances from system energy consumption efficiency perspective are discussed in [25].

offer any diversity gain, and thus generally results in performance loss rather than gain. However, as will be shown later, if the signal decay due to path loss is severe ($\alpha > 3$), the MH protocol *does* offer an SNR gain compared to direct transmission, when the relay is between the two communicating nodes.

B. Optimization Issues

An interesting question one may ask regarding two-phase protocols is how we should choose the fraction δ_1. This depends on the geometrical location of the relay and the specific protocol. Intuitively, if the relay is located close to the source node, the path-loss of the channel link $\mathsf{S} \to \mathsf{R}$ is relatively small compared to that of the $\mathsf{S} \to \mathsf{D}$ link. Therefore, the relay receives the data with high average SNR and thus decodes the message successfully with high probability. In this case, even small δ_1 (and thus high R_1) may be sufficient for successful decoding at the relay. In phase-II, one may use the TD or STD protocols to efficiently achieve full diversity without bandwidth expansion.

On the other hand, if the relay is located close to the destination node, the situation may be reversed. In this case, the relay and destination receive the signal with equal average power. Since the relative path loss of the link $\mathsf{R} \to \mathsf{D}$ is small, the relay can transmit the received data with little power (if it is decoded correctly), and this may add additional diversity to the destination. The overall system is thus similar to an ideal receiver diversity system and the RD protocol may be efficient, provided that the relay appropriately controls its transmission power.[4] Therefore, for practical design of relay systems, it is important to consider the geometrical properties of the relay location, together with the choice of appropriate protocols, relative phase durations, and power control.

IV. Analytical Tools for Outage Probabilities

In this section, we develop our performance criterion and analytical tools for the design and evaluation of the above protocols. Our design criterion is based on the mutual information for a given realization of the fading coefficients $\mathcal{H} = \{H_{S,D}, H_{S,R}, H_{R,D}\}$. Specifically, we assume that communication is successful if the mutual information (with Gaussian code book) of the channel conditioned on $\mathcal{H}$ is greater than the information rate [27]. Otherwise, an outage event follows. The probability of an outage event defined in this way, which is commonly referred to as *outage probability*, not only has an analytically convenient form but also serves as a reasonable performance indicator for practical systems; with moderate frame length and a well-designed STC [4], the frame-error rate may fall within a few dB of the Multiple-Input Single-Output (MISO) channel outage probability [28, 29]. Also, the outage probability can be seen as a Complementary Cumulative Distribution Function (CCDF) of the non-ergodic capacity, which is a random variable of $\mathcal{H}$ [28].

[4] An alternative approach in this case may be a compress-and-forward scheme, e.g., [26], which may result in potentially better performance. Comparisons with this scheme are beyond the scope of this paper.

A. Asymptotic Diversity Order

Our main goal is to achieve a large diversity gain with minimum transmitted energy. We first define the asymptotic diversity order in the high SNR regime [18].

Definition 1 (Asymptotic Diversity Order): Let $P_{\text{out}}(\text{SNR})$ denote an outage probability as a function of the channel SNR. The asymptotic diversity order is defined as [18]

$$\mathsf{d}^*\left(P_{\text{out}}(\text{SNR})\right) \triangleq -\lim_{\text{SNR}\to\infty} \frac{\ln\left(P_{\text{out}}(\text{SNR})\right)}{\ln\left(\text{SNR}\right)}.$$

As an alternate form, in this paper we consider outage probabilities as a function of the inverse SNR, $X \triangleq \frac{1}{\text{SNR}}$, and let $f_{\text{out}}(X) = P_{\text{out}}(\text{SNR})|_{\text{SNR}=1/X}$. It will also be mathematically convenient to analytically extend $f_{\text{out}}(X)$ in a neighborhood of $X = 0$. To that end, we introduce the notion of *analytical extendibility at* $X = 0$ *for a CCDF.*

Definition 2 (Analytically Extendible CCDF): A CCDF $f(X)$ is called *analytically extendible at* $X = 0$ if all the following conditions are satisfied.

1) On $X \geq 0$, $0 \leq f(X) \leq 1$ and on this interval, $f(X)$ is a non-decreasing function with $f(0) = 0$.
2) $f(X)$ is analytic at $X = 0$. Thus, in some open interval $(-\epsilon, \epsilon)$, $\epsilon > 0$, $f(X)$ can be expressed as a power series centered at $X = 0$,

$$f(X) = \sum_{n=0}^{\infty} a_n X^n, \qquad a_n = \frac{1}{n!} f^{(n)}(0), \tag{7}$$

where $f^{(n)}(X)$ denotes the nth derivative of $f(X)$. Since $f(0) = 0$, it follows that $a_0 = 0$. When $f(X) \neq 0$, we refer to the minimum value of n where $a_n \neq 0$ as the *order* of $f(X)$. □

Since $f(X)$ is non-decreasing on $X \geq 0$, it is easy to see that $a_m > 0$ for a given minimum order m. In the rest of this paper, we do not explicitly mention 'at $X = 0$' and simply refer to such functions as analytically extendible CCDFs. With the above definition, we have the following lemma.

Lemma 1 (Diversity Order): If a CCDF $f(X)$ is analytically extendible with order m, then the asymptotic diversity order is m. □

Proof: For $m > 0$, we have $a_m > 0$ and

$$f(X) = X^m \left(a_m + \sum_{n=1}^{\infty} a_{n+m} X^n \right).$$

Thus, we obtain

$$\mathsf{d}^*\left(f(X)\right) = m + \lim_{X\to 0} \frac{\ln\left(a_m + \sum_{n=1}^{\infty} a_{m+n} X^n\right)}{\ln X}. \tag{8}$$

Since the second term of (8) can be easily shown to be zero, we obtain $\mathsf{d}^*\left(f(X)\right) = m$. ■

Also, the following corollary may be immediately obtained from the logarithmic property of the diversity definition.

Corollary 1: Let $g_i(X)$, $i = 1, 2, \ldots$, and $h_j(X)$, $j = 1, 2, \ldots$, be analytically extendible CCDFs. If $f(X) = \prod_i g_i(X) \prod_j \left(1 - h_j(X)\right)$, then $\mathsf{d}^*\left(f(X)\right) = \sum_i \mathsf{d}^*\left(g_i(X)\right)$. Furthermore, if $f(X) = \sum_i g_i(X)$, then $\mathsf{d}^*\left(f(X)\right) = \min_i \left\{\mathsf{d}^*\left(g_i(X)\right)\right\}$. □

B. *Diversity Offset Gain*

We note that for an analytically extendible CCDF $f_{\text{out}}(X)$ of order m,

$$\lim_{X \to 0} \frac{f_{\text{out}}(X)}{X^n} = \begin{cases} a_m & \text{for} \quad n = m \\ 0 & \text{for} \quad n < m. \end{cases} \tag{9}$$

Therefore, as SNR $\to \infty$ ($X \to 0$), the asymptotic outage probability is given by

$$f_{\text{out}}(X) \sim a_m X^m \quad \text{or} \quad P_{\text{out}}(\text{SNR}) \sim a_m \text{SNR}^{-m} = (c_m \text{SNR})^{-m}, \tag{10}$$

where $c_m \triangleq a_m^{-\frac{1}{m}}$. Thus,

$$\ln P_{\text{out}}(\text{SNR}) \sim \ln(a_m) - m \ln \text{SNR}. \tag{11}$$

As addressed in [30], the asymptotic diversity order m determines the slope in a plot of the log-outage probability versus SNR in decibels, whereas a_m (or c_m) determines the intercept. Therefore, our design criterion is to choose the offset term a_m as small as possible, thereby maximizing the gain c_m. Note that in [30], the term c_m is referred to as a coding gain, but since this gain is a result of spatial diversity rather than code structure, we refer to this relative gain as *diversity offset gain* (or simply, offset gain) in the following.

C. *Ideal MISO Case*

We begin by considering an ideal MISO system with m transmit antennas and let SNR denote the total received SNR. From the literature of MIMO communications systems [28, 29] the following theorem holds.

Theorem 1 (MISO Channel Diversity Order and Offset): Consider a MISO channel with m transmit antennas and suppose that we transmit the data with an information rate R^*. Under the assumption that the transmitter does not know the channel coefficients and all the m channel coefficients are circularly symmetric Gaussian random variables with zero mean and unit variance, the achievable diversity order is m, and the offset term, denoted by $\tilde{a}_m$, is given by

$$\tilde{a}_m = \frac{1}{m!}(mA_0)^m, \tag{12}$$

where $A_0 = 2^{R^*} - 1$. □

Proof: Let the m channel coefficients be denoted by $\mathbf{h} \triangleq \begin{bmatrix} H_0 & H_1 & \ldots & H_{m-1} \end{bmatrix}^T$, where the H_i are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero mean and unit variance by assumption. For a given $m \times m$ diagonal power allocation matrix with non-negative entries, denoted by $\mathbf{P}$, that satisfies trace$(\mathbf{P}) \leq 1$, the mutual information conditioned on $\mathbf{h}$ is defined as [29]

$$C(\text{SNR}, \mathbf{P}, \mathbf{h}) \triangleq \log_2 \left(1 + \text{SNR}\, \mathbf{h}^H \mathbf{P} \mathbf{h}\right). \tag{13}$$

Choosing $\mathbf{P} = \frac{1}{m}\mathbf{I}$, where $\mathbf{I}$ is an $m \times m$ identity matrix, we obtain

$$C(\text{SNR}, \mathbf{P}, \mathbf{h}) = \log_2 \left(1 + \frac{\text{SNR}}{m} Z\right), \tag{14}$$

where Z is a random variable which follows a central chi-square distribution with $2m$ degrees of freedom, each of variance $1/2$. The outage probability is given by

$$P_{\text{out}} = \Pr\left[C < R^*\right] = \Pr\left[\frac{\text{SNR}}{m} Z < A_0\right] = \Pr\left[Z < mA_0 X\right], \tag{15}$$

where $A_0 \triangleq 2^{R^*} - 1$ and $X \triangleq \frac{1}{\text{SNR}}$. We thus have the outage probability as a function of X:

$$f_{\text{out}}(X) = P(m, mA_0X) \triangleq \frac{\gamma\left(m, mA_0X\right)}{\Gamma(m)}, \tag{16}$$

where $\Gamma(m)$ is the (complete) Gamma function and $\gamma(m, x)$ is the lower incomplete Gamma function

$$\gamma(m, x) \triangleq \int_0^x e^{-t} t^{m-1} dt. \tag{17}$$

Clearly, the integrand in (17) is analytic in t and hence, so is $\gamma(m, x)$ in x. Therefore, $f_{\text{out}}(X)$ is an analytically extendible CCDF and since $P(m, x) = e^{-x} \sum_{k=m}^{\infty} x^k / k!$, one can show that

$$\left.\frac{\partial^n P(m, \kappa\, x)}{\partial x^n}\right|_{x=0} = \begin{cases} 0 & n < m \\ \kappa^n & n = m. \end{cases} \tag{18}$$

Hence, the asymptotic diversity order is m from Lemma 1, and we thus obtain (12). ■

The coefficient $\tilde{a}_m$ in (12) may serve as a reference offset for an ideal MISO system. In the following, we define a diversity offset gain with respect to an equivalent ideal MISO (or SISO) performance as follows.

Definition 3: For a system with diversity order of m and offset a_m, the diversity offset gain with respect to an equivalent ideal MISO (or SISO) performance is defined as

$$\Lambda(m) \triangleq c_m / \tilde{c}_m = \left(\tilde{a}_m / a_m\right)^{\frac{1}{m}} = mA_0 \left(m!\, a_m\right)^{-\frac{1}{m}}. \tag{19}$$

Thus, the diversity offset gain $\Lambda(m)$ serves as a measure of the relative performance of a scheme with respect to an ideal MISO system as given by the asymptotic SNR gap for a small outage probability. If $\Lambda(m) < 1$, there is a relative loss in asymptotic SNR required to achieve the same outage probability as a MISO system.

V. Performance Analysis with Single Relay

In the following, we analyze the outage probability and achievable diversity offset gain of the protocols outlined in Section III, assuming an independent Rayleigh fading plus path loss channel model. We assume that the Gaussian noise power is identical for *all the channel links* considered. Extensions to the cases with variable noise power may be tedious but straightforward.

A. Transmit SNR

For further analysis, an appropriate measure of SNR should be defined. In this paper, we shall evaluate the system in terms of the total *transmitted* power (for a given noise power). Consequently, the SNR is defined as a ratio of total *transmitted* signal power, which is the sum of the source and relay transmit power, to the Gaussian noise variance, which is assumed to be constant. We will refer to this ratio as *transmit SNR* and denote it by SNR_t, throughout the paper. Since, in the absence of relay nodes, the received SNR is given by $\mathrm{SNR} = \mathrm{PL}(S, D)\mathrm{SNR}_t$, by taking the path loss between the source and destination nodes $\mathrm{PL}(S, D)$ to be unity, the transmit and received SNRs become identical. As opposed to the more conventional notion of received SNR, transmit SNR is a more appropriate measure of wireless network performance in terms of total power consumption.

1) RD and MH Protocols: Let $\mathrm{SNR}_1^{\mathsf{S}}$ denote the *received* SNR dedicated for the communication link of phase-I, and let $\mathrm{SNR}_2^{\mathsf{R}}$ denote that of phase-II, conditioned that the relay is transmitting.

The transmit SNR can be expressed as

$$\mathrm{SNR}_t = \delta_1 \frac{\mathrm{SNR}_1^{\mathsf{S}}}{\mathrm{PL}(S, D)} + \delta_2 \beta \frac{\mathrm{SNR}_2^{\mathsf{R}}}{\mathrm{PL}(R, D)} = \frac{1}{\mathrm{PL}(S, D)} \left\{ \delta_1 \mathrm{SNR}_1^{\mathsf{S}} + \delta_2 \frac{\beta}{G_D} \mathrm{SNR}_2^{\mathsf{R}} \right\}, \tag{20}$$

where β is an average energy consumption factor that accounts for the probability that the relay is transmitting and thus $\beta \leq 1$.

Now, we suppose that the relay transmits its signal with average power Δ_R times that of the source. Then we may write $\mathrm{SNR}_2^{\mathsf{R}}/\mathrm{PL}(R, D) = \Delta_R \, \mathrm{SNR}_1^{\mathsf{S}}/\mathrm{PL}(S, D)$, or, $\mathrm{SNR}_2^{\mathsf{R}} = G_D \Delta_R \, \mathrm{SNR}_1^{\mathsf{S}}$. Thus, setting $\mathrm{PL}(S, D) = 1$, (20) reduces to

$$\mathrm{SNR}_t = (\delta_1 + \delta_2 \beta \Delta_R) \, \mathrm{SNR}_1^{\mathsf{S}} \triangleq l_{\mathrm{RD}} \mathrm{SNR}_1^{\mathsf{S}}, \qquad l_{\mathrm{RD}} = \delta_1 + \delta_2 \beta \Delta_R. \tag{21}$$

2) TD and STD Protocols: In this case, we assume that the source node employs the same average power through phases I and II for simplicity. The extension of our results to variable power cases is straightforward. Let $\mathrm{SNR}_2^{\mathsf{S}}$ denote the received SNR of phase-II due to the channel link $\mathsf{S} \rightarrow \mathsf{D}$. Then we have $\mathrm{SNR}_2^{\mathsf{S}} = \mathrm{SNR}_1^{\mathsf{S}}$ by assumption. The corresponding transmit SNR by the nodes through phases I and II with $\mathrm{PL}(S, D) = 1$ is given by

$$\mathrm{SNR}_t = (1 + \delta_2 \beta \Delta_R) \, \mathrm{SNR}_1^{\mathsf{S}} \triangleq l_{\mathrm{TD}} \mathrm{SNR}_1^{\mathsf{S}}, \qquad l_{\mathrm{TD}} = 1 + \delta_2 \beta \Delta_R. \tag{22}$$

B. Error Events

In order to derive the outage probability and associated diversity offset gain of various protocols, we first define the following events: E_1 = Event [Decoding at destination after phase-I fails], E_R = Event [Decoding at relay after phase-I fails], and E_2 = Event [Decoding at destination after phase-II fails]. If the destination receives during two phases, we have the outage probability $P_{\text{out}} = \Pr[\mathsf{E}_1 \cap \mathsf{E}_2]$. Otherwise, $P_{\text{out}} = \Pr[\mathsf{E}_2]$. Also, in the following, $\bar{\mathsf{A}}$ denotes the complement of the event A.

C. Multi-Hopping (MH) Protocol

We begin with the analysis of the MH protocol. In this protocol, since the destination listens only during phase-II, the outage probability is given by

$$P_{\text{out}} = \Pr[\mathsf{E}_2] = 1 - \Pr[\bar{\mathsf{E}}_R]\Pr[\bar{\mathsf{E}}_2|\bar{\mathsf{E}}_R]. \tag{23}$$

Let $C(\text{SNR}, H)$ denote the mutual information of the channel conditioned on channel coefficient H and received SNR, defined as $C(\text{SNR}, H) \triangleq \log_2\left(1 + \text{SNR}\,|H|^2\right)$. With this notation, we have

$$\Pr[\mathsf{E}_R] = \Pr\left[C(G_S\text{SNR}_1^{\mathsf{S}}, H_{S,R}) \le R_1\right] = \Pr\left[|H_{S,R}|^2 \le \frac{2^{R_1}-1}{G_S\text{SNR}_1^{\mathsf{S}}}\right] = 1 - e^{-\frac{A_1}{G_S\text{SNR}_1^{\mathsf{S}}}} \tag{24}$$

where $A_1 \triangleq 2^{R_1} - 1$, and $\Pr[\bar{\mathsf{E}}_R] = e^{-\frac{A_1}{G_S\text{SNR}_1^{\mathsf{S}}}}$. The conditional probability $\Pr[\bar{\mathsf{E}}_2|\bar{\mathsf{E}}_R]$ is calculated as

$$\Pr[\bar{\mathsf{E}}_2|\bar{\mathsf{E}}_R] = \Pr\left[C(\text{SNR}_2^{\mathsf{R}}, H_{R,D}) > R_2\right] = e^{-\frac{A_2}{G_D\Delta_R\text{SNR}_1^{\mathsf{S}}}}, \tag{25}$$

where $A_2 \triangleq 2^{R_2} - 1$. Consequently, we have

$$P_{\text{out}} = \Pr[\mathsf{E}_2] = 1 - e^{-\left(\frac{A_1}{G_S} + \frac{A_2}{\Delta_R G_D}\right)\frac{1}{\text{SNR}_1^{\mathsf{S}}}}. \tag{26}$$

Let $X \triangleq \frac{1}{\text{SNR}_t}$, and from (21) we obtain $\frac{1}{\text{SNR}_1^{\mathsf{S}}} = l_{\text{RD}}X$. Since the relay transmits only if the relay successfully decodes, β in (20) is given by

$$\beta = \Pr[\bar{\mathsf{E}}_R] = 1 - \Pr[\mathsf{E}_R] = e^{-\frac{A_1}{G_S\text{SNR}_1^{\mathsf{S}}}} = e^{-\frac{A_1}{G_S}l_{\text{RD}}X} \triangleq \beta(X) \tag{27}$$

which is also a function of X. From (21), we may then relate l_{RD} and X by

$$l_{\text{RD}}(X) = \delta_1 + \delta_2\Delta_R e^{-\frac{A_1}{G_S}l_{\text{RD}}(X)X}. \tag{28}$$

The outage probability can then be expressed as an analytically extendible CCDF of X:

$$f_{\text{out}}(X) = 1 - e^{-\left(\frac{A_1}{G_S} + \frac{A_2}{\Delta_R G_D}\right)l_{\text{RD}}(X)X}. \tag{29}$$

With careful manipulation of the analytic function $f_{\text{out}}(X)$, we obtain $a_0 = 0$ and

$$a_1 = \frac{A_1}{G_S}\left(1 + \frac{r}{\Delta_R}\right)(\delta_1 + \delta_2\Delta_R), \qquad r \triangleq \frac{A_2}{A_1}\frac{G_S}{G_D} = \frac{A_2}{A_1}\zeta^\alpha = \frac{2^{R^*/(1-\delta_1)}-1}{2^{R^*/\delta_1}-1}\zeta^\alpha. \tag{30}$$

Hence, the asymptotic diversity order is $m = 1$.

In order to improve the achievable diversity offset gain, we wish to minimize a_1 in (30) by judiciously choosing Δ_R and δ_1. The optimization can be performed in a two step manner: first, find Δ_R for a given δ_1, and then numerically optimize δ_1.

By fixing $0 < \delta_1 < 1$ (and thus $0 < \delta_2 < 1$), the parameter Δ_R that minimizes a_1 can be found by standard calculus as $\Delta_{\text{opt}} = \sqrt{r\delta_1/\delta_2}$, which is a function of ζ, δ_1, R^*, and α from r in (30) . The overall offset gain can be given from (19) by

$$\Lambda_{\text{MH}}(1) = \left(\sqrt{\frac{\delta_1}{G_S}\frac{A_1}{A_0}} + \sqrt{\frac{\delta_2}{G_D}\frac{A_2}{A_0}}\right)^{-2}. \tag{31}$$

Using ζ notation of (5), we can write

$$\Lambda_{\mathrm{MH}}(1) = \frac{A_0 \left(1+\zeta^2 - 2\zeta\cos\theta\right)^{\frac{\alpha}{2}}}{\left(\sqrt{\delta_1 A_1} + \zeta^{\frac{\alpha}{2}}\sqrt{\delta_2 A_2}\right)^2} \tag{32}$$

where δ_1 should be numerically chosen, for given R^*, ζ, and α, according to

$$\delta_1 = \arg\min_{0<\delta_1<1} \sqrt{\delta_1\left(2^{R^*/\delta_1}-1\right)} + \zeta^{\frac{\alpha}{2}}\sqrt{(1-\delta_1)\left(2^{R^*/(1-\delta_1)}-1\right)}. \tag{33}$$

1) Geometric Effect of the Relay: If the relay is located close to the source, i.e., $\zeta \sim 0$, the optimal value of δ_1 approaches 1, which should be equivalent to direct transmission from source to destination, and we have

$$\lim_{\zeta\to 0} \Lambda_{\mathrm{MH}}(1) = \frac{A_0}{A_1} = 1. \tag{34}$$

Likewise, if the relay is located close to the destination, i.e., $\zeta \sim \infty$, the optimal value of δ_1 approaches 0, and we have

$$\lim_{\zeta\to \infty} \Lambda_{\mathrm{MH}}(1) = \frac{A_0}{A_2} = 1, \tag{35}$$

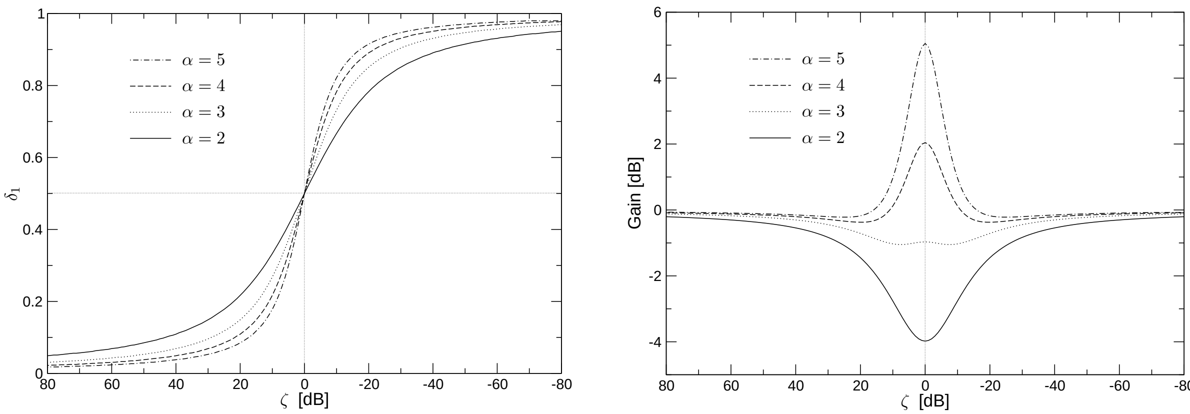
which is identical to (34). Thus, for these asymptotic cases, the path-loss exponent α does not appear in the gain expression and essentially no gain is achieved by this protocol. However, this is not the case if the relay is located between the two nodes. For example, if the relay is located midway on the line between the source and destination, i.e., $\zeta = 1$, then since a function $g(\delta_1) \triangleq \sqrt{\delta_1 A_1} = \sqrt{\delta_1\left(2^{R^*/\delta_1}-1\right)}$ is a convex $\cup$ function with respect to δ_1, the above gain is maximized at $\delta_1 = \delta_2 = 1/2$ and thus we have

$$\Lambda_{\mathrm{MH}}(1)\big|_{\zeta=1} = \frac{\left(2^{R^*}-1\right)(2-2\cos\theta)^{\frac{\alpha}{2}}}{2\left(2^{2R^*}-1\right)} = \frac{(2-2\cos\theta)^{\frac{\alpha}{2}}}{2\left(2^{R^*}+1\right)}. \tag{36}$$

Now, in this case, we observe that the offset gain is a function of path-loss exponent α, and this gain, measured in dB, increases linearly with α provided $\theta > \frac{\pi}{3}$. We shall refer to this type of gain achieved by increasing the path-loss exponent as a *MH gain* in the following.

2) Numerical Results: Fig. 3(a) and (b) show the optimal δ_1 and corresponding achievable offset gain $\Lambda_{\mathrm{MH}}(1)$, respectively, with respect to the relay position $\zeta = d_{R,D}/d_{S,R}$ and different values of the path-loss exponent α. The information rate is $R^* = 2$, and the relay is located at $\theta = \pi$, i.e., on the line between the source and destination nodes. From Fig. 3 (b), it is observed that if the path-loss exponent is less than 4, the MH protocol always results in loss compared to a traditional SISO system and thus no benefit is obtained. However, if α is at least 4, the MH protocol can offer some gain. (In fact, from (36), $\Lambda_{\mathrm{MH}}(1) > 1$ if $\alpha > 1 + \log_2 5$.) Therefore, in our scenario, a positive MH gain is possible if the path-loss exponent is sufficiently large and the relay is appropriately located[5].We also observe that the maximum

[5]It is interesting to point out that the observation of sub-optimality of multi-hopping agrees with that of [31] in terms of system energy-efficiency perspective, though the underlying performance criterion is considerably different.



(a) (b)

Fig. 3. Optimal values for the MH protocol as a function of a relay node position ζ and different value of α. Parameters: $R^* = 2$, $\theta = \pi$. (a) Relative duration δ_1. (b) Diversity offset gain $\Lambda_{\text{MH}}(1)$.

gain is achieved if the relay is located in the midway between the source and destination nodes with $\delta_1 = 1/2$ and thus routing data with the same information rate on each leg. This agrees with intuition and common observation in the literature on the MH protocol (see e.g., [25]).

D. Receiver Diversity (RD) Protocol

In the RD protocol, the destination listens during both phases. The outage probability of the RD protocol can then be expressed as

$$P_{\text{out}} = \Pr[\mathsf{E}_1 \cap \mathsf{E}_2] = \Pr[\mathsf{E}_1]\Pr[\mathsf{E}_2|\mathsf{E}_1] = \Pr[\mathsf{E}_1]\Pr[\mathsf{E}_2]. \tag{37}$$

For phase-I, we have

$$\Pr[\mathsf{E}_1] = \Pr\left[C(\text{SNR}_1^{\text{S}}, H_{S,D}) \le R_1\right] = \Pr\left[|H_{S,D}|^2 \le \frac{2^{R_1} - 1}{\text{SNR}_1^{\text{S}}}\right] = 1 - e^{-\frac{A_1}{\text{SNR}_1^{\text{S}}}}. \tag{38}$$

Also, from the result of the MH protocol, we have $\Pr[\mathsf{E}_2]$ in (26). Thus, the outage probability is given by

$$f_{\text{out}}(X) = \left(1 - e^{-A_1 l_{\text{RD}}(X) X}\right)\left(1 - e^{-\left(\frac{A_1}{G_S} + \frac{A_2}{\Delta_R G_D}\right) l_{\text{RD}}(X) X}\right). \tag{39}$$

Similar to the MH case, one can show that $a_0 = 0$, $a_1 = 0$, and

$$a_2 = \frac{A_1^2}{G_S}\left(1 + \frac{r}{\Delta_R}\right)\left(\delta_1 + \delta_2 \Delta_R\right)^2, \tag{40}$$

where r is given in (30). The asymptotic diversity order is thus $m = 2$. The value of Δ_R that minimizes a_2 can be found as

$$\Delta_{\text{opt}} = \frac{r}{4}\left(\sqrt{1 + \frac{8}{r}\frac{\delta_1}{\delta_2}} - 1\right) \tag{41}$$

and the overall offset gain is given by

$$\Lambda_{\text{RD}}(2) = \sqrt{\frac{2\,G_S}{1+\frac{r}{\Delta_{\text{opt}}}}} \cdot \frac{A_0}{A_1} \cdot \frac{1}{\delta_1 + \delta_2 \Delta_{\text{opt}}}. \tag{42}$$

We now consider the following three specific cases of node geometry.

1) Relay Close to Destination: As the relay is moved near the destination, i.e., $\zeta \to 0$, we have $G_S \to 1$, $r, \Delta_{\text{opt}} \to 0$. However, since $\lim_{r\to 0} r/\Delta_{\text{opt}} = 0$, we have

$$\lim_{\zeta\to 0} \Lambda_{\text{RD}}(2) = \sqrt{2} \cdot \frac{2^{R^*}-1}{2^{R^*/\delta_1}-1}\frac{1}{\delta_1} \leq \sqrt{2} \tag{43}$$

where the upper bound is achieved by setting $\delta_1 \to 1$. Therefore, the asymptotic offset gain achieved by this protocol, with respect to the transmit diversity bound, is 1.5 dB. Alternatively, since the receiver diversity outperforms the transmit diversity (without CSI at the transmitter) by 3 dB for the same transmit power constraint [3], the RD protocol is asymptotically 1.5 dB inferior to the two-branch receiver diversity. It is interesting to note that this is analogous to the result in [5], where the loss of collaborative diversity with respect to the ideal two-branch transmit diversity is shown to be 1.5 dB. Since the geometrical gain of the relay G_S is unity, no MH gain is achieved in this case.

2) Relay Close to Source: If ζ is large, we have $\Delta_{\text{opt}} \sim \delta_1/\delta_2$ and

$$\Lambda_{\text{RD}}(2) \sim \frac{A_0}{\sqrt{2\delta_1\,\delta_2\,A_1\,A_2}}\sqrt{G_D}. \tag{44}$$

Since $G_D \to 1$ as the relay is moved near the destination, the maximum offset gain is achieved when $\delta_1 = \delta_2 = 1/2$, and the corresponding gain is given by

$$\lim_{r\to\infty} \Lambda_{\text{RD}}(2) = \frac{\sqrt{2}}{2^{R^*}+1}. \tag{45}$$

Therefore, if the rate R^* is small, the loss relative to the MISO bound is small even if the relay is much closer to source. Again, no MH gain can be achieved in this case.

3) Relay Located Midway Between the Two Nodes: In this case, let $\zeta \sim 1$, and we have

$$\Lambda_{\text{RD}}(2)|_{\zeta=1} = \sqrt{\frac{2}{1+\eta}} \cdot \frac{A_0}{\delta_1\,A_1 + \frac{\delta_2}{\eta}\,A_2}\,(2-2\cos\theta)^{\frac{\alpha}{4}}, \qquad \eta = \frac{4}{\sqrt{1+8\frac{\delta_1 A_1}{\delta_2 A_2}}-1}. \tag{46}$$

Therefore, in this case, the offset gain (in dB) increases linearly with α as a consequence of the MH gain, regardless of the choice of δ_1 and δ_2.

E. Transmit Diversity (TD) Protocol

Using the notation in the previous subsections, the outage probability can be expressed as

$$P_{\text{out}} = \Pr[\mathsf{E}_1 \cap \mathsf{E}_2] = \Pr[\mathsf{E}_R]\Pr[\mathsf{E}_1 \cap \mathsf{E}_2|\mathsf{E}_R] + \Pr[\bar{\mathsf{E}}_R]\Pr[\mathsf{E}_1 \cap \mathsf{E}_2|\bar{\mathsf{E}}_R]. \tag{47}$$

The mutual information of phase-II is expressed using (13) as

$$C(\text{SNR}_2^{\text{S+R}}, \mathbf{P}, \mathbf{h}) = \log_2\left(1 + \text{SNR}_2^{\text{S+R}}\mathbf{h}^H\mathbf{P}\mathbf{h}\right), \tag{48}$$

where $\mathbf{h} = [H_{S,D}, H_{R,D}]^T$, $\text{SNR}_2^{\text{S+R}}$ is the total received power (from the source and the relay) during phase-II provided the relay is transmitting, and $\mathbf{P}$ is a diagonal matrix with $\text{trace}(\mathbf{P}) \leq 1$. The total received power with a relay power control factor Δ_R is given by

$$\text{SNR}_2^{\text{S+R}} = \text{SNR}_2^{\text{S}} + \text{SNR}_2^{\text{R}} = (1 + \Delta_R G_D)\text{SNR}_1^{\text{S}}. \tag{49}$$

Since the transmitter does not know the channel coefficient, the matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \text{diag}\left[\frac{\text{SNR}_2^{\text{S}}}{\text{SNR}_2^{\text{S+R}}}, \frac{\text{SNR}_2^{\text{R}}}{\text{SNR}_2^{\text{S+R}}}\right] = \text{diag}\left[\frac{1}{1+\Delta_R G_D}, \frac{\Delta_R G_D}{1+\Delta_R G_D}\right]. \tag{50}$$

Consequently, (48) is rewritten as

$$C(\text{SNR}_2^{\text{S+R}}, \mathbf{P}, \mathbf{h}) = \log_2\left(1 + \text{SNR}_1^{\text{S}}\left(|H_{S,D}|^2 + \Delta_R G_D |H_{R,D}|^2\right)\right). \tag{51}$$

The term $\Pr[\mathsf{E}_1 \cap \mathsf{E}_2 | \bar{\mathsf{E}}_R]$ in (47) is given by

$$\begin{aligned}\Pr[\mathsf{E}_1 \cap \mathsf{E}_2 | \bar{\mathsf{E}}_R] &= \Pr[C(\text{SNR}_1^{\text{S}}, H_{S,D}) \leq R_1 \,\cap\, C(\text{SNR}_2^{\text{S+R}}, \mathbf{P}, \mathbf{h}) \leq R_2] \\ &= \Pr\left[|H_{S,D}|^2 \leq \frac{A_1}{\text{SNR}_1^{\text{S}}} \,\cap\, |H_{S,D}|^2 + \Delta_R G_D |H_{R,D}|^2 \leq \frac{A_2}{\text{SNR}_1^{S}}\right].\end{aligned} \tag{52}$$

Let $x \triangleq |H_{S,D}|^2$ and $y \triangleq \Delta_R G_D |H_{R,D}|^2$. Then,

$$\begin{aligned}\Pr[\mathsf{E}_1 \cap \mathsf{E}_2 | \bar{\mathsf{E}}_R] &= \int_0^{\min\left(\frac{A_1}{\text{SNR}_1^{\text{S}}}, \frac{A_2}{\text{SNR}_1^{\text{S}}}\right)} \int_0^{\frac{A_2}{\text{SNR}_1^{\text{S}}} - x} e^{-x} \frac{1}{\Delta_R G_D} e^{-\frac{y}{\Delta_R G_D}}\, dy\, dx \\ &= 1 - e^{-\frac{A_{\min}}{\text{SNR}_1^{\text{S}}}} - \frac{\Delta_R G_D}{\Delta_R G_D - 1} e^{-\frac{A_2}{\Delta_R G_D \text{SNR}_1^{\text{S}}}}\left[1 - e^{-\frac{\Delta_R G_D - 1}{\Delta_R G_D}\frac{A_{\min}}{\text{SNR}_1^{\text{S}}}}\right],\end{aligned} \tag{53}$$

where $A_{\min} \triangleq \min(A_1, A_2)$. The term $\Pr[\mathsf{E}_1 \cap \mathsf{E}_2 | \mathsf{E}_R]$ in (47) is given by

$$\Pr[\mathsf{E}_1 \cap \mathsf{E}_2 | \mathsf{E}_R] = \Pr[C(\text{SNR}_1^{\text{S}}, H_{S,D}) \leq R_1 \,\cap\, C(\text{SNR}_2^{\text{S}}, H_{S,D}) \leq R_2] = 1 - e^{-\frac{A_{\min}}{\text{SNR}_1^{\text{S}}}}. \tag{54}$$

The final expression for $f_{\text{out}}(X)$ can be easily found by substituting (53), (54), and (24) with $\frac{1}{\text{SNR}_1^{\text{S}}} = l_{\text{TD}}(X)X$ into (47). We then obtain $a_0 = 0, a_1 = 0$, and

$$a_2 = \frac{A_1 A_{\min}}{G_S}\left(1 + \frac{r}{\Delta_R}\left(1 - \frac{1}{2}\frac{A_{\min}}{A_2}\right)\right)(1 + \delta_2 \Delta_R)^2. \tag{55}$$

An asymptotic diversity order of $m = 2$ is guaranteed, and the optimum value of Δ_R can be found as

$$\Delta_{\text{opt}} = \frac{r}{4}\left(1 - \frac{1}{2}\frac{A_{\min}}{A_2}\right)\left(\sqrt{1 + \frac{8}{r}\frac{1}{\delta_2\left(1 - \frac{1}{2}\frac{A_{\min}}{A_2}\right)}} - 1\right). \tag{56}$$

The overall offset gain can be obtained from (19) as

$$\Lambda_{\text{TD}}(2) = \sqrt{\frac{2\,G_S}{1 + \frac{r}{\Delta_{\text{opt}}}\left(1 - \frac{1}{2}\frac{A_{\min}}{A_2}\right)}} \cdot \frac{A_0}{\sqrt{A_1 A_{\min}}} \cdot \frac{1}{1 + \delta_2 \Delta_{\text{opt}}}. \tag{57}$$

We now consider the following three specific cases of node geometry.

1) Relay Close to Destination: As the relay approaches destination, we have

$$\lim_{\zeta\to 0}\Lambda_{\mathrm{TD}}(2)=\sqrt{2}\frac{A_0}{\sqrt{A_1A_{\min}}}. \tag{58}$$

By choosing $\delta_1>\delta_2$, we have $A_{\min}=A_1$ and thus

$$\lim_{\zeta\to 0}\Lambda_{\mathrm{TD}}(2)=\sqrt{2}\frac{A_0}{A_1}\le\sqrt{2} \tag{59}$$

where the upper bound is achieved by setting $\delta_1\to 1$. Thus, the asymptotic offset gain in this case is identical to that of the RD protocol and no MH gain is achieved.

2) Relay Close to the Source: If ζ is large, we have

$$\lim_{r\to\infty}\Lambda_{\mathrm{TD}}(2)=\frac{A_0}{\sqrt{(2A_2-A_{\min})A_{\min}\delta_2}}. \tag{60}$$

By choosing $\delta_2>\delta_1$, we have $A_{\min}=A_2$ and thus

$$\lim_{r\to\infty}\Lambda_{\mathrm{TD}}(2)=\frac{A_0}{\sqrt{\delta_2}A_2}=\frac{2^{R^*}-1}{\sqrt{\delta_2}\left(2^{R^*/\delta_2}-1\right)}\le 1 \tag{61}$$

where the upper bound is achieved by setting $\delta_2\to 1$. Therefore, the TD protocol can approach the performance of the ideal transmit diversity bound as the relay approaches the source. However, no MH gain can be achieved in this case.

3) Relay Located in the Middle of the Two Nodes: In this case, let $\zeta\sim 1$, and if we set $\delta_2>\delta_1$, then we have $A_{\min}=A_2$ and

$$\Lambda_{\mathrm{TD}}(2)|_{\zeta=1,\delta_2>\delta_1}=\sqrt{\frac{2}{1+\eta_1}}\sqrt{\frac{A_1}{A_2}}\frac{A_0}{A_1+\frac{\delta_2}{2\eta_1}A_2}\left(2-2\cos\theta\right)^{\frac{\alpha}{4}}, \qquad \eta_1=\frac{4}{\sqrt{1+16\frac{A_1}{\delta_2A_2}}-1}.$$

Alternatively, if we assume $\delta_1>\delta_2$, then we have

$$\Lambda_{\mathrm{TD}}(2)|_{\zeta=1,\delta_1>\delta_2}=\sqrt{\frac{2}{1+\eta_2}}\frac{A_0}{A_1+\frac{\delta_2}{2\eta_2}(2A_2-A_1)}\left(2-2\cos\theta\right)^{\frac{\alpha}{4}}, \quad \eta_2=\frac{4}{\sqrt{1+16\frac{A_1}{\delta_2(2A_2-A_1)}}-1}.$$

Therefore, in this case, the offset gain, measured in dB, is proportional to α and the slope does not depend on the choice of δ_1 and δ_2.

Note that the above two equations become identical if $\delta_1=\delta_2=1/2$. However, numerical calculation in the following shows that at $\zeta=1$, the parameter δ_2 that maximizes the above gain is not equal to 1/2. Therefore, there is a discontinuity in the optimal value of δ_1 around $\zeta=1$ (see Section V-G).

F. Simplified Transmit Diversity (STD) Protocol

In this case, we have

$$\begin{aligned}P_{\text{out}}&=\Pr[\mathsf{E}_2]=\Pr[\mathsf{E}_R]\Pr[\mathsf{E}_2|\mathsf{E}_R]+\Pr[\bar{\mathsf{E}}_R]\Pr[\mathsf{E}_2|\bar{\mathsf{E}}_R]\\&=\Pr[\mathsf{E}_R]\Pr[C(\mathsf{SNR}_2^{\mathsf{S}},H_{S,D})\le R_2]+\Pr[\bar{\mathsf{E}}_R]\Pr[C(\mathsf{SNR}_2^{\mathsf{S+R}},\mathbf{P},\mathbf{h})\le R_2]\end{aligned} \tag{62}$$

which can be calculated as

$$f_{\text{out}}(X) = \left(1 - e^{-\frac{A_1}{G_S} l_{\text{TD}}(X) X}\right)\left(1 - e^{-A_2 l_{\text{TD}}(X) X}\right) + e^{-\frac{A_1}{G_S} l_{\text{TD}}(X) X}\left(1 - \frac{\Delta_R G_D e^{-\frac{A_2}{\Delta_R G_D} l_{\text{TD}}(X) X} - e^{-A_2 l_{\text{TD}}(X) X}}{\Delta_R G_D - 1}\right).$$

We thus obtain $a_0 = 0$, $a_1 = 0$ and

$$a_2 = \frac{A_1 A_2}{G_S}\left(1 + \frac{1}{2}\frac{r}{\Delta_R}\right)(1 + \delta_2 \Delta_R)^2 . \tag{63}$$

Note that (63) is identical to (55) when $A_{\min} = A_2$. Therefore, when $A_2 \le A_1$, i.e., $\delta_1 \le \frac{1}{2}$, the performance of the TD and STD protocols will be identical. The asymptotic diversity order is $m = 2$, and the optimum value of Δ_R and the corresponding offset gain are given by

$$\Delta_{\text{opt}} = \frac{r}{8}\left(\sqrt{1 + \frac{16}{\delta_2 r}} - 1\right), \qquad \Lambda_{\text{STD}}(2) = \sqrt{\frac{2\, G_S}{1 + \frac{r}{2\Delta_{\text{opt}}}}} \cdot \frac{A_0}{\sqrt{A_1 A_{\min}}} \cdot \frac{1}{1 + \delta_2 \Delta_{\text{opt}}}, \tag{64}$$

which are again identical to (56) and (57), respectively, when $\delta_1 \le 1/2$. Therefore, if the relay is closer to the source than the destination, there is no loss by ignoring the phase-I signal (unless these two signals are combined upon making a decision, as will be discussed in Section V-H). The offset gain achieved by the STD and TD protocols are identical. On the other hand, if the relay is close to destination, the performance of STD becomes inferior to that of the TD protocol. As an asymptote, we have

$$\lim_{\zeta \to 0} \Lambda_{\text{STD}}(2) = \begin{cases} \sqrt{2}\frac{A_0}{\sqrt{A_1 A_2}} & \text{for } \delta_1 \ge \frac{1}{2} \\ \sqrt{2}\frac{A_0}{A_1} & \text{for } \delta_1 \le \frac{1}{2} \end{cases}$$

The above gain is maximized at $\delta_1 = 1/2$ and we have

$$\lim_{\zeta \to 0} \Lambda_{\text{STD}}(2) \le \sqrt{2}\frac{2^{R^*} - 1}{2^{2R^*} - 1} = \frac{\sqrt{2}}{2^{R^*} + 1}.$$

It is interesting to note that this asymptotic offset gain is identical to that of RD protocol (i.e., (45)) where the relay is close to the source.

G. Numerical Comparison of the Three Protocols

In the following, we numerically evaluate the performance of the three protocols of diversity order 2, in terms of achievable offset gains. The information rate is set to be $R^* = 2$.

1) Achievable Offset Gains for the RD, TD, and STD Protocols: Fig. 4(a) shows the optimal δ_1 with respect to the relay position ζ for the three protocols of diversity order 2, with the path-loss exponent $\alpha = 2$ and the relay location at $\theta = \pi$. Changing α does not significantly affect the curves. As observed, the optimal fraction δ_1 for the RD protocol ranges from 0.5 near the source node and increases as the relay approaches the destination. Contrastingly, the optimal value of δ_1 for the STD protocol ranges from 0.5 near the destination node and decreases as the relay approaches the source node.

Fig. 4 (b) shows the corresponding optimal offset gain $\Lambda(2)$ with $\alpha = 2$ and 4. It can be seen that the offset gain of the TD protocol is identical to that of the STD protocol when $\zeta \ge 0$ dB, and approximates

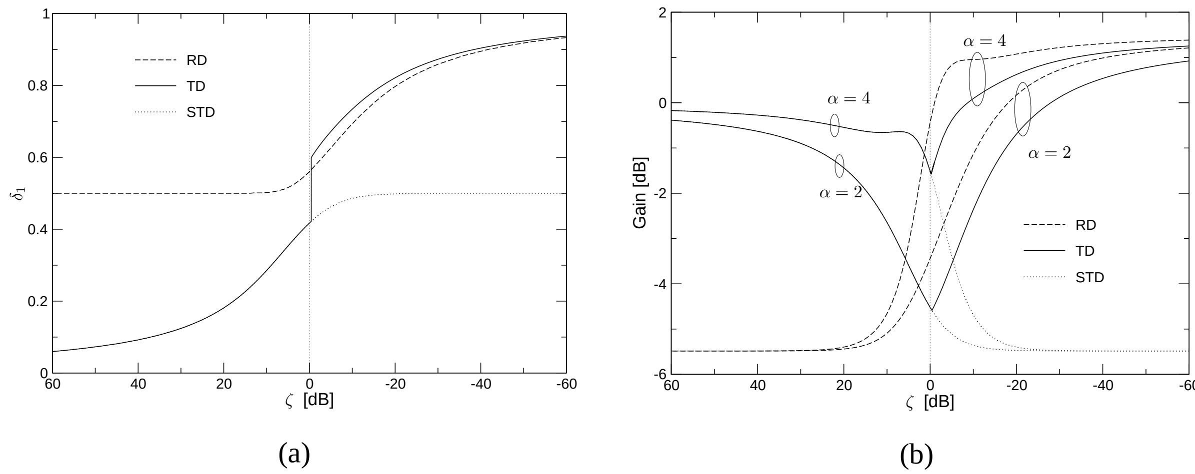


Fig. 4. Optimal values for the three protocols as a function of a relay node position ζ. Parameters: $R^* = 2$, $\alpha = 2$, $\theta = \pi$. (a) Relative duration δ_1. (b) Diversity offset gain $\Lambda(2)$. Note that the gains of the TD and STD protocols are identical and thus their curves overlap when $\delta_1 < 1/2$.

that of the RD protocol when $\zeta < 0$ dB. It is also observed that as the relay approaches the destination, the offset gain of the RD and TD protocols can approach the predicted 1.5 dB bound. On the other hand, if the relay approaches the source, the offset gain of the TD and STD protocols can approach the predicted 0 dB bound.

Consequently, in terms of minimizing complexity without sacrificing performance, the suggested strategy is that if the relay is close to the source ($\zeta > 0$ dB), it should employ the STD protocol and otherwise use the RD protocol. Although the TD protocol may result in stable performance results for both cases (which may be suitable if the exact node geometry is unknown), if the protocols are switched appropriately, it outperforms neither.

2) Effect of Node Angle: So far, we have evaluated the performance with $\theta = \pi$, which may be optimal in the sense of the achievable diversity offset gain for a given ζ. Changing θ for a given ζ may be expected to result in a performance loss. The SNR loss with $\mathrm{d}^* = m$, relative to the case with $\theta = \pi$, can be expressed from (6) as

$$10 \log_{10} \frac{\Lambda(m)}{\Lambda(m)|_{\theta=\pi}} = 10 \log_{10} \left(\frac{G_S}{G_S|_{\theta=\pi}} \right)^{\frac{1}{m}} = \frac{\alpha}{2m} \times 10 \log_{10} \frac{1+\zeta^2 - 2\zeta\cos\theta}{1+\zeta} \triangleq \frac{\alpha}{2m} L(\theta).$$

The relative loss $L(\theta)$ [dB] defined above is plotted in Fig. 5 for several instances of the geometric ratio ζ. As observed, as ζ deviates from 1, the loss becomes small. Therefore, in many cases of interest, the performance is not sensitive to the value of θ compared to that of ζ.

H. Comparison with Two-Phase Combining Approach

In the previous analysis, we have not exploited the fact that the information transmitted during phase-I and phase-II are the same, and thus combining the signals of the two phases may potentially improve performance.

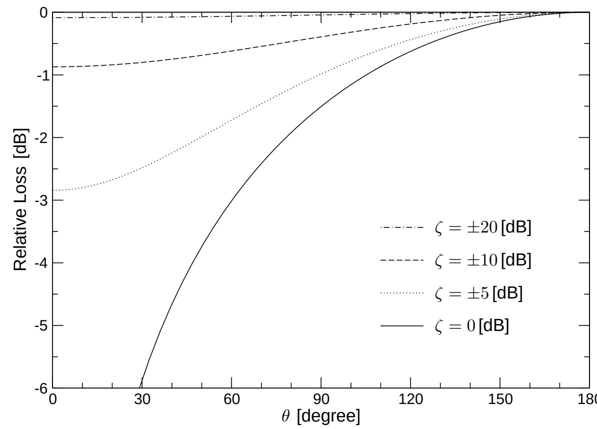


Fig. 5. Relative loss $L(\theta)$ in achievable offset gain with respect to the case with relay node position $\theta = \pi$.

Therefore, in the following, we first derive an upper bound for the achievable diversity offset gain for such a case based on the sum of mutual information method similar to [6, 15] for the RD protocol. (Similar analysis for the TD protocol may be possible, but this is not considered here for simplicity.) It should be noted that in contrast to the previous protocols which can make use of existing channel coding or STC and thus are attractive from a practical viewpoint, designing codes that allow combining received codewords with different codes in an optimal manner (as this method suggests) may be challenging.

Another approach that can combine the two signals and that is much easier to implement is the use of repetition coding [6]. In this case, the same encoding with the same information rate should be transmitted from the relay in phase-II. In our scenario, this is possible when $\delta_1 = \delta_2 = 1/2$. We also analyze the repetition coding in terms of diversity offset gain and then derive a bound for the variable-rate case using parallel channel coding argument.

Note that if the decision is made after combining the two phases, the decision after phase-II will be better than that at phase-I. Hence the event that phase-I fails is a subset of the event that phase-II fails. Hence, the outage probability is expressed as

$$P_{\text{out}} = \Pr[\mathsf{E}_2] = \Pr[\mathsf{E}_2|\bar{\mathsf{E}}_R]\Pr[\bar{\mathsf{E}}_R] + \Pr[\mathsf{E}_1]\Pr[\mathsf{E}_R]$$

where $\Pr[\bar{\mathsf{E}}_R], \Pr[\mathsf{E}_R]$, and $\Pr[\mathsf{E}_1]$ are identical to those of the MH protocol.

1) Parallel Channel Coding: Assuming that independent channel codes are employed for the phase-I and phase-II, the probability of the event that the destination fails to decode conditioned that the relay successfully decodes is given by

$$\Pr[\mathsf{E}_2|\bar{\mathsf{E}}_R] = \Pr\left[\delta_1 \log_2\left(1 + \text{SNR}_1^{\text{S}} |H_{S,D}|^2\right) + \delta_2 \log_2\left(1 + \text{SNR}_2^{\text{R}} |H_{R,D}|^2\right) < R^*\right]. \tag{65}$$

By Jensen's inequality, we obtain

$$\Pr[\mathsf{E}_2|\bar{\mathsf{E}}_R] \geq \Pr\left[\log_2\left(1+\delta_1\mathrm{SNR}_1^{\mathsf{S}}\left|H_{S,D}\right|^2+\delta_2\mathrm{SNR}_2^{\mathsf{R}}\left|H_{R,D}\right|^2\right)<R^*\right] \tag{66}$$

$$= 1-\frac{\delta_1 e^{-\frac{A_0}{\delta_1\mathrm{SNR}_1^{\mathsf{S}}}}-\delta_2 G_D\Delta_R e^{-\frac{A_0}{\delta_2 G_D\Delta_R\mathrm{SNR}_1^{\mathsf{S}}}}}{\delta_1-\delta_2 G_D\Delta_R} \triangleq P_{\text{low}}. \tag{67}$$

Note that this bound becomes tight as $\delta_1 \to 1$ or $\delta_1 \to 0$. The outage probability is lower bounded as

$$P_{\text{out}} \geq P_{\text{low}}\Pr[\bar{\mathsf{E}}_R]+\Pr[\mathsf{E}_2|\mathsf{E}_R]\Pr[\mathsf{E}_R] = P_{\text{low}}\Pr[\bar{\mathsf{E}}_R]+\Pr[\mathsf{E}_1]\Pr[\mathsf{E}_R].$$

We then have for this lower bound: $a_1 = 0$, and

$$a_2 = \frac{A_1^2}{G_S}\left(1+\frac{q}{\Delta_R}\right)(\delta_1+\delta_2\Delta_R)^2, \qquad q \triangleq \frac{1}{2\delta_1\delta_2}\frac{A_0^2}{A_1^2}\frac{G_S}{G_D}. \tag{68}$$

Therefore, the optimum value of Δ_R is given by (41) with r replaced by q of (68). The corresponding offset gain is given by (42) with r replaced by q. Note that since this gain is that of the lower bound of the outage probability, it serves as an upper bound in terms of the diversity offset gain.

Again, if $\zeta \to 0$, then we have $G_S \to 1$ and the upper bound of the gain is expressed as

$$\lim_{\zeta\to 0}\Lambda(2)_{\text{UB}} = \frac{A_0}{A_1}\sqrt{2}\frac{1}{\delta_1} = \frac{2^{R^*}-1}{2^{R^*/\delta_1}-1}\sqrt{2}\frac{1}{\delta_1} \leq \sqrt{2} \tag{69}$$

which is the same asymptotic bound of the RD protocol. On the other hand, if $\zeta \to \infty$, we obtain $\Delta_{\text{opt}} = \delta_1/\delta_2$ and

$$\lim_{\zeta\to\infty}\Lambda(2)_{\text{UB}} = 1 \tag{70}$$

regardless of the value of δ_1. The reason that this upper bound does not depend on the value of δ_1 is as follows. As the relay is located close to the source, the relay is likely to decode correctly with high probability, and the channel links between the source and relay to the destination also becomes equally reliable. Therefore, if we choose $\Delta_R = \Delta_{\text{opt}} = \delta_1/\delta_2$, the received *energy* for each phase becomes identical, regardless of the choice of δ_1 and this equal-energy assignment should maximize the mutual information for a given total received energy. Hence, this asymptotic performance should be also equivalent to that of 2×1 MISO system.

Some Remarks: It is interesting to note that for the case with $\zeta \to 0$, the upper bound is 1.5 dB inferior to that of the receiver diversity bound as in (69), whereas the case with $\zeta \to \infty$ can achieve that of the transmitter diversity bound as in (70). This is because for the latter case, due to the broadcasting nature of the channel, the communication link between $\mathsf{S} \to \mathsf{R}$ is free in terms of energy and bringing $\delta_1 \to 0$ cancels the loss of bandwidth efficiency required for the phase-I communication. Therefore, a virtual transmit diversity system can be achieved without loss of efficiency. On the other hand, for the former case, additional energy is required for the communication link between the relay and the destination, whereas this is not required in the receiver diversity system. This accounts for the 1.5 dB loss in terms of SNR that holds regardless of the information rate.

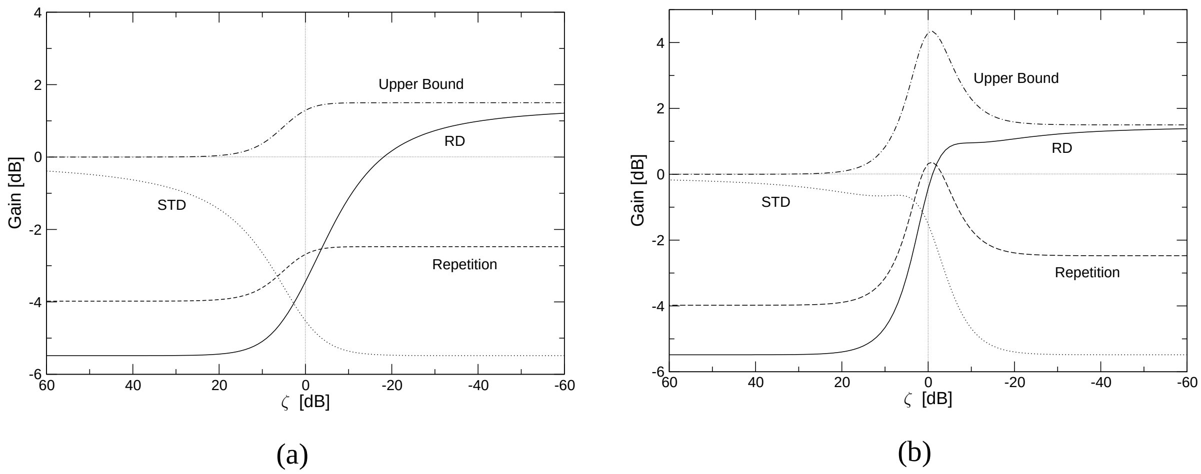


Fig. 6. Diversity offset gain of the repetition codes and parallel channel coding, as a function of a relay node position ζ. Parameters: $R^* = 2$, $\alpha = 2$, $\theta = \pi$. (a) $\alpha = 2$. (b) $\alpha = 4$.

2) Repetition Coding: In this case, we set $\delta_1 = \delta_2 = 1/2$, and the received SNR is the sum of the two phases. Thus from [6]

$$\Pr[\mathsf{E}_2|\bar{\mathsf{E}}_R] = \Pr\left[\log_2\left(1 + \mathsf{SNR}_1^{\mathsf{S}}\left|H_{S,D}\right|^2 + \mathsf{SNR}_2^{\mathsf{R}}\left|H_{R,D}\right|^2\right) < 2R^*\right] = 1 - \frac{e^{-\frac{2^{2R^*}-1}{\mathsf{SNR}_1^{\mathsf{S}}}} - G_D\Delta_R e^{-\frac{2^{2R^*}-1}{G_D\Delta_R\mathsf{SNR}_1^{\mathsf{S}}}}}{1 - G_D\Delta_R}.$$

Consequently, we have $a_1 = 0$ and

$$a_2 = \left(2^{2R^*} - 1\right)^2 \left(\frac{1}{G_S} + \frac{1}{2\Delta_R G_D}\right) \frac{1}{4}\left(1 + \Delta_R\right)^2. \qquad (71)$$

The optimal factor Δ_R and corresponding diversity offset gain is given by

$$\Delta_{\text{opt}} = \frac{1}{8}\zeta^\alpha\left(\sqrt{1 + 16\zeta^{-\alpha}} - 1\right), \quad \Lambda_{\text{REP}} = \frac{2\sqrt{2G_S}}{2^{R^*}+1}\frac{1}{(1+\Delta_{\text{opt}})\sqrt{1 + \frac{1}{2\Delta_{\text{opt}}}\zeta^\alpha}}.$$

Note that as the relay node is moved closer to the source and destination, we have, respectively,

$$\lim_{\zeta\to\infty}\Lambda_{\text{REP}} = \frac{2}{2^{R^*}+1}, \qquad \lim_{\zeta\to 0}\Lambda_{\text{REP}} = \frac{2\sqrt{2}}{2^{R^*}+1}.$$

Therefore, unlike the parallel channel coding upper bound, in repetition coding the diversity offset gain decreases rapidly as the information rate increases. This observation agrees with that in [6].

3) Numerical Results: Fig. 6(a) and (b) show the diversity offset gain of the repetition codes as well as upper bound of the parallel channel coding for path-loss exponents $\alpha = 2$ and 4, respectively. Along with these, those of the STD and RD protocols are also shown for comparison. As we can observe, if the relay is close to the source or destination, the STD and RD protocols can approach the upper bound of parallel channel coding. On the other hand, if the relay is located midway between the source and the destination, the repetition coding is better. Therefore, selecting between the STD/RD/repetition protocols, depending on the relay's location, is an inexpensive yet powerful approach in practical design.

VI. Multiple Relays and The Effect of Imperfect STCs on Diversity Performance

One can extend the above ideas to the multiple relay node case. Suppose that we have N relays. For the RD protocol, each relay must forward its data using dedicated channels, and N additional channels may be required. On the other hand, for the TD and STD protocols, as discussed in [6], a single additional channel suffices. For this reason, the TD and STD protocols are attractive especially when N is large.

In this section, we focus on the performance of the TD and STD protocols. In the previous analysis of these protocols, it was assumed that even if only a subset of the transmitting nodes during phase-II actually transmit, the destination can decode correctly provided the mutual information given this fact is above the required rate. This implicitly assumes that the destination always knows which relay nodes are transmitting and which are not, or equivalently, which antennas are undergoing an erasure on a frame by frame basis. (In our scenario, a node erasure is a probabilistic event that depends on the channel link(s) of phase-I. However, this model is also applicable to a sudden change of node status, such as a battery failure, node failure or a change in shadowing state.) Therefore, in a practical STC system, unless the STC is appropriately designed, the destination node may fail to correctly identify a node erasure and thus cause a decoding error[6]. The effect of the node erasure may become salient, especially when the number of relay nodes increases. Therefore, we consider the achievable diversity order of these protocols when the STC is perfect and imperfect. Note that conventional coherent STCs such as [3, 4] require the knowledge of CSI at the receiver and thus are not necessarily perfect. On the other hand, it is easy to see that non-coherent versions of STCs that do not require any CSI, such as [32], are perfect by nature.

A. System Model and Outage Probability

Fig. 7 summarizes the system model with two relays and the associated notation. The case of three or more relays is analogous. Following the single relay case in the previous sections, we assume that H_{S,R_i} and $H_{R_i,D}$, which denote the complex channel coefficients of each channel link, are uncorrelated and circularly symmetric complex Gaussian random variables with zero mean and unit variance. Let $G_{S,i}$ and $G_{D,i}$ denote the corresponding geometrical gains achieved by the ith node R_i, where $i = 1, 2, \ldots, N$. Also, let $\mathsf{E}_{R,i}$ denote the event that the node R_i fails to decode after phase-I. For brevity we use notation as $\mathsf{F}_i^{(1)} = \mathsf{E}_{R,i}$ and $\mathsf{F}_i^{(0)} = \bar{\mathsf{E}}_{R,i}$ (where F_i stands for the failure of ith relay).

[6]In practice, if channel estimation is performed on a frame by frame basis, a node erasure can be easily detected at the destination. If the relay ceases transmitting a signal for any reason then the destination will simply assume that the corresponding channel link suffers severe fading. Conventional STCs can thus be used without modification. However, since channel fading is slow by assumption and this generally precludes the use of short interval channel estimation, per frame channel estimation is expensive. Furthermore, the additional overhead becomes substantial as the number of relay nodes increases.

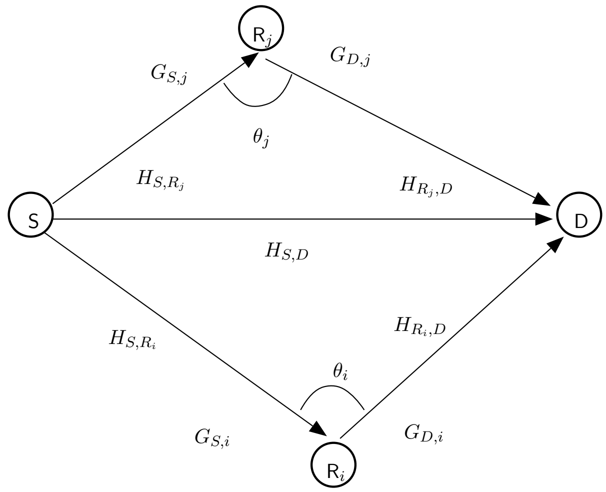


Fig. 7. Two-phase communication with multiple relays.

In the case of the TD protocol, the outage probability is expressed as

$$P_{\text{out}} = \sum_{(x_1,x_2,\ldots,x_N)\in\{0,1\}^N} \underbrace{\left(\prod_{i=1}^{N} \Pr\left[\mathsf{F}_i^{(x_i)}\right]\right) \Pr\left[\mathsf{E}_1 \cap \mathsf{E}_2 | \mathsf{F}_1^{(x_1)}, \mathsf{F}_2^{(x_2)}, \ldots, \mathsf{F}_N^{(x_N)}\right]}_{\triangleq B_{x_1 x_2 \cdots x_N}}. \tag{72}$$

The above summation is the sum of 2^N terms of the product of $N+1$ probability events. Consider the specific term in which exactly n out of N relay nodes have correctly decoded the message and suppose that their indices are $i = 1, 2, \ldots, n$. Then this term can be expressed as

$$B_{(0)_n(1)_{N-n}} = \prod_{i=1}^{n} e^{-\frac{A_1}{G_{S,i}} l_{\text{TD}}(X) X} \prod_{i=n+1}^{N} \left(1 - e^{-\frac{A_1}{G_{S,i}} l_{\text{TD}}(X) X}\right) \Pr\left[\mathsf{E}_1 \cap \mathsf{E}_2 | \mathsf{F}_1^{(0)}, \ldots, \mathsf{F}_n^{(0)}, \mathsf{F}_{n+1}^{(1)}, \ldots, \mathsf{F}_N^{(1)}\right],$$

where $l_{\text{TD}} = 1 + \delta_2 \sum_{i=1}^{N} \beta_i \Delta_{R,i}$, $\beta_i = \Pr[\mathsf{F}_i^{(0)}]$, and $\Delta_{R,i}$ is the power control factor of the ith relay node. Note that $l_{\text{TD}}(X)$ is analytic about $X = 0$ with $l_{\text{TD}}(0) = 1 + \delta_2 \sum_{i=1}^{N} \Delta_{R,i}$. The outage probability for the STD protocol can be expressed in a similar form.

B. Diversity Order for Multiple Relays with Perfect and Imperfect Constituent STC

We first assume that there are N collaborating relays and the STC used in the TD or STD protocol is perfect. In this case, we have the following lemma [6].

Lemma 2 (Asymptotic Diversity Order with Multiple Relay Nodes): For the N relay node TD and STD protocols with perfect constituent STCs, the asymptotic diversity order is $\mathsf{d}^* = N + 1$. □

The proof is omitted as it can be inferred from [6]. Now, as a worst case scenario, we assume that an outage event occurs even if a single relay fails to decode. In other words, unless all the relay nodes correctly decode the message, the phase-II link fails. Specifically, we assume that

$$\Pr\left[\mathsf{E}_2 | \ldots, \mathsf{F}_i^{(1)}, \ldots\right] = 1 \qquad \text{for any } i. \tag{73}$$

In the following discussion, the STC having this property will be referred to as an *imperfect STC*. The outage performance based on this assumption may serve as an upper bound for the TD and STD protocols. In this case we have the following theorem:

Theorem 2: For the N relay node TD and STD protocols with imperfect constituent STCs, i.e., (73) holds, the asymptotic diversity orders are 2 and 1, respectively. □

Proof: Considering the outage probability expression in (72), it is easy to see that the worst-case terms are those with $n = N-1$. Then, we have for the TD protocol

$$B_{(0)_{N-1}(1)_1} = \prod_{i=1}^{N-1} e^{-\frac{A_1}{G_{S,i}\mathsf{SNR}_1^{\mathsf{S}}}} \left(1 - e^{-\frac{A_1}{G_{S,N}\mathsf{SNR}_1^{\mathsf{S}}}}\right) \Pr\left[\mathsf{E}_1 \cap \mathsf{E}_2 | \mathsf{F}_1^{(0)}, \ldots, \mathsf{F}_{N-1}^{(0)}, \mathsf{F}_N^{(1)}\right]$$
$$= \prod_{i=1}^{N-1} e^{-\frac{A_1}{G_{S,i}\mathsf{SNR}_1^{\mathsf{S}}}} \left(1 - e^{-\frac{A_1}{G_{S,N}\mathsf{SNR}_1^{\mathsf{S}}}}\right) \left(1 - e^{-\frac{A_1}{\mathsf{SNR}_1^{\mathsf{S}}}}\right), \tag{74}$$

where the last equality is from $\Pr\left[\mathsf{E}_1 \cap \mathsf{E}_2 | \mathsf{F}_1^{(0)}, \ldots, \mathsf{F}_{N-1}^{(0)}, \mathsf{F}_N^{(1)}\right] = \Pr\left[\mathsf{E}_2 | \mathsf{E}_1, \mathsf{F}_1^{(0)}, \ldots, \mathsf{F}_{N-1}^{(0)}, \mathsf{F}_N^{(1)}\right] = 1$. Therefore, by Corollary 1 we obtain $\mathsf{d}^*\left(P_{\text{out}}\right) = \mathsf{d}^*\left(B_{(0)_{N-1}(1)_1}\right) = 2$.

For the STD protocol, we have

$$B_{(0)_{N-1}(1)_1} = \prod_{i=1}^{N-1} e^{-\frac{A_1}{G_{S,i}\mathsf{SNR}_1^{\mathsf{S}}}} \left(1 - e^{-\frac{A_1}{G_{S,N}\mathsf{SNR}_1^{\mathsf{S}}}}\right), \tag{75}$$

and thus $\mathsf{d}^*\left(P_{\text{out}}\right) = \mathsf{d}^*\left(B_{(0)_{N-1}(1)_1}\right) = 1$. ■

Therefore, in the high SNR regime, the asymptotic diversity orders of the TD and STD protocols with imperfect STC are $\mathsf{d}^* = 2$ and $\mathsf{d}^* = 1$, respectively. This suggests that in the high SNR regime, it is important that collaborative STCs for distributed nodes be designed such that the information can be decoded with only a partial subset of the code (i.e., robustness against transmit antenna erasures in the traditional multiple-antenna STC scenario). Practical design issues in this direction are addressed in [33].

C. Outage Probabilities and Diversity Offset Gains for Single Relay Node Case

In the case of a single relay with imperfect STC, it is straightforward to obtain the outage probabilities and their associated diversity offset gain expression based on the approach outlined in Section V.

1) TD Protocol: From Theorem 2, it follows that the diversity order is 2. The corresponding outage probability is given by

$$P_{\text{out}} = \Pr[\mathsf{E}_R]\Pr[\mathsf{E}_1] + \Pr[\bar{\mathsf{E}}_R]\Pr[\mathsf{E}_1 \cap \mathsf{E}_2 | \bar{\mathsf{E}}_R]. \tag{76}$$

The closed-form expression can be found by using (38), (24), and (53). It follows that $a_0, a_1 = 0$ and

$$a_2 = \frac{A_1^2}{G_S}\left(1 + \frac{r}{\Delta_R}\left(1 - \frac{1}{2}\frac{A_{\min}}{A_2}\right)\frac{A_{\min}}{A_1}\right)\left(1 + \delta_2\Delta_R\right)^2. \tag{77}$$

It is observed that a_2 is similar to (55). In fact, if $\delta_1 > 1/2$ and thus $A_{\min} = A_1$, the offset gain of the TD protocol with imperfect STC, denoted by $\Lambda_{\text{IM-TD}}(2)$, is equivalent to $\Lambda_{\text{TD}}(2)$. On the other hand, if $A_{\min} = A_2$, the gap becomes (assuming the same Δ_{opt} is applied)

$$\Lambda_{\text{TD}}(2)/\Lambda_{\text{IM-TD}}(2) = \sqrt{A_1\left(1+\frac{A_2}{A_1}\frac{r}{2\Delta_{\text{opt}}}\right)/A_2\left(1+\frac{r}{2\Delta_{\text{opt}}}\right)}, \tag{78}$$

which is significantly large if $A_1 \gg A_2$ (and thus $\delta_1 \ll \frac{1}{2}$).

2) STD Protocol: In this case, we have

$$P_{\text{out}} = 1 - \Pr[\bar{\mathsf{E}}_R \cap \bar{\mathsf{E}}_2] = 1 - \Pr\left[C(G_S \text{SNR}_1^{\mathsf{S}}, H_{S,R}) > R_1\right]\Pr\left[C(\text{SNR}_2^{\mathsf{S+R}}, \mathbf{P}, \mathbf{h}) > R_2\right], \tag{79}$$

$$f_{\text{out}}(X) = 1 - e^{-\frac{A_1}{G_S}l_{\text{TD}}(X)X}\left[\frac{\Delta_R G_D\, e^{-\frac{A_2}{\Delta_R G_D}l_{\text{TD}}(X)X} - e^{-A_2 l_{\text{TD}}(X)X}}{\Delta_R G_D - 1}\right]. \tag{80}$$

Consequently, we obtain $a_0 = 0$,

$$a_1 = \frac{A_1}{G_S}(1+\delta_2\Delta_R) \qquad \text{and} \qquad a_2 = \frac{A_1A_2}{2G_S}\left(\frac{r}{\Delta_R} - \frac{A_1}{A_2}\frac{1}{G_S}\right)(1+\delta_2\Delta_R)^2. \tag{81}$$

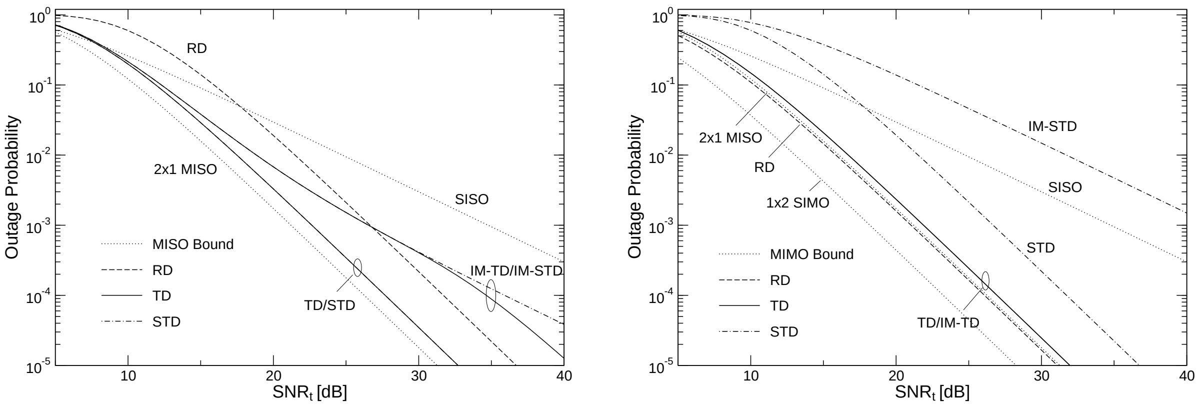
Therefore, the asymptotic diversity order is 1, which agrees with Theorem 2. However, it should be noted that the offset gain with respect to the SISO system is given by

$$\Lambda_{\text{IM-STD}}(1) = \frac{A_0 G_S}{A_1(1+\delta_2\Delta_R)} \tag{82}$$

and this indicates that if G_S is large, one may still achieve significant gain over the baseline system. In particular, for the SNR region where $\text{SNR}_t \ll \Lambda_{\text{IM-STD}}(1)$ ($X \gg 1/\Lambda_{\text{IM-STD}}(1)$), the outage probability has a *local* slope of order 2 since the term a_1X in $f_{\text{out}}(X)$ is dominated by a_2X^2. The following section elucidates this effect numerically.

3) Numerical Results: We numerically compare the performance of the two protocols with imperfect and perfect STC in terms of outage probability. Fig. 8 (a) and (b) show the outage probabilities of these protocols with relay node locations ζ = 20 and -20dB, respectively. The performance of the RD protocols is also shown as a reference. In these results, it is assumed that the relay performs the optimal power control algorithm.

From Fig. 8 (a), it is observed that the two protocols with imperfect STC are almost identical in the low SNR region with a *local* slope of (diversity) order 2, but for high SNR, the bound for the STD protocol shows a slope of order 1, whereas that of the TD protocol maintains a slope of diversity order 2. The gap between the two bounds becomes noticeable in Fig. 8 (b), where the outage probability of the TD protocol with an imperfect STC is identical to that of the ideal TD, whereas the STD protocol with an imperfect STC is much worse than the ideal SISO bound. Therefore, if the STC is designed imperfectly, then the use of the TD protocol can offer stable performance and is thus preferable.



(a) (b)

Fig. 8. Outage probability of the TD and STD protocols with perfect or imperfect STC. The result of the RD protocol and associated MISO/SIMO bounds are also shown. Parameters: $R^* = 2$, $\alpha = 2$, $\theta = \pi$. (a) $\zeta = 20$ dB. (b) $\zeta = -20$ dB.

VII. Conclusion

We have analyzed the performance of various variable-rate two-phase collaborative diversity protocols for wireless networks. These protocols can be implemented in a straightforward manner using standard variable-rate channel coding and STC. Theoretical analysis of the outage probability has shown that these protocols, if properly designed based on the node geometry, can achieve full diversity order and considerable offset gains. Our conclusion is that if the relay is close to the source and destination, the STD and RD protocols, respectively, achieve good performance. If the relay is midway between the source and the destination, fixed-rate repetition coding with signal combining at the destination [6] is a good candidate considering its simplicity of implementation.

It is also shown that for a system with N relays, a diversity order of $N + 1$ is achievable for the TD based protocol using STC as in [6]. However, if the STC fails to be decoded whenever node erasure occurs, their diversity order is considerably reduced and for the STD protocol with an imperfect STC, no diversity offset gain can be achieved. Therefore, the design of STCs that are robust against node erasures is an important area of future research.

Finally, even though perfect synchronizations are assumed throughout the paper, accurate timing and frequency acquisitions among distributed nodes are difficult to achieve in practice. Further research in this direction is of critical importance for implementation of these protocols.

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