IN-61
348 174

Variable Structure PID Control to Prevent Integrator Windup

C. E. Hall[*] A. S. Hodel[†] J. Y. Hung[‡]

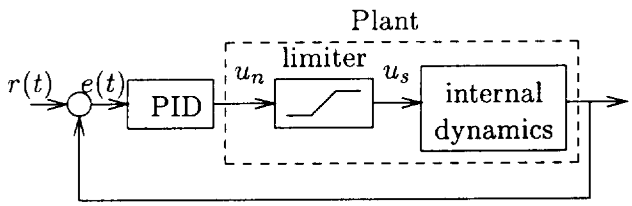

Figure 1: PID control with saturation limits

1 Introduction

PID controllers are frequently used to control systems requiring zero steady-state error while maintaining requirements for settling time and robustness (gain/phase margins). PID controllers suffer significant loss of performance due to short-term integrator wind-up when used in systems with actuator saturation (see Figure 1). We examine several existing and proposed methods for the prevention of integrator wind-up in both continuous and discrete time implementations. We may write a continuous time PID control law as

$$K(s) \quad = \quad \frac{U_n(s)}{E(s)} = K_P + sK_D + K_I/s \qquad (1.1)$$

where $u_n(t)$ is the nominal control command and $e(t) = y_{ref}(t) - y(t)$ is the error between a reference signal $y_{ref}(t)$ and output $y(t)$ of the system being controlled. The respective state-space implementation is

$$\dot{\eta} = e \qquad u_n = K_P e + K_D \dot{e} + K_I \eta$$

Control saturation occurs when u_n lies outside of actuator limits, $u_n \notin [u_{min}, u_{max}]$.

2 Background

Although numerous methods have been proposed for the prevention of windup in controller integrators and slow dynamics, very few textbooks discuss the problem, particularly at the undergraduate level (e.g., [9]). The earliest treatment of anti-windup techniques that we are aware of was done by Fertik and Ross [8]. This technique fits into the larger class of anti-windup bumpless transfer (AWBT) control, example methods of which are covered in [1], [2], [3], [4], [6], [11], [12], [13], [14], [15], [18], [22], [23]. A general theoretical framework for the parameterization, synthesis and analysis of AWBT control is provided in [18], in which it is assumed all nonlinearities are *external* to the controller; i.e., the controller is required to be linear. Stability analysis for these methods is typically performed through describing functions, see, e.g., [2].

Other anti-windup methods include conditional integration and/or integrator limiting, (e.g. [7], [10], [16]) which freezes or "clamps" the integrator value when certain conditions are not met, e.g., saturation, output not in "proportional band," etc., the use of time varying gains to avoid saturation [17], or the setting of the integrator to a prescribed value during saturation, also called preloading [21]. These methods do not fall into the class of AWBT control, since the switching action on the integrator renders the method nonlinear.

3 VSPID control

While many of the above methods are applicable to multivariable systems, we shall confine our attention in this paper to the treatment of PID control with individual saturation limits for each PID channel. We shall contrast three methods of conditional integration (CI), one method of "preloading," a simple AWBT method, and a new variable structure PID (VSPID) controller. Our discussion makes use of the following definition.

Definition 3.1 The *saturation function*

$$\mathrm{sat}(a, a_{min}, a_{max}) \triangleq \max(a_{min}, \min(a, a_{max})).$$

It will be seen that the VSPID and AWBT methods yield similar behavior when the AWBT uses a high-gain feedback of the control saturation error $u_n - u_s$, where we define

$$u_s \triangleq \mathrm{sat}(u_n, u_{min}, u_{max}) \qquad (3.2)$$

[*]Structures and Dynamics Lab, Marshall Space Flight Center, Alabama

[†]a.s.hodel@eng.auburn.edu, Dept. Elect. Eng., Auburn University.

[‡]Dept. Electrical Engineering, Auburn University

Methods that we examine here are:

CI-I Integrator limiting; see [5], p. 278. Impose hard limits (saturation) on the integrator value η:

$$\dot{\eta} = \begin{cases} 0 & \eta \notin [\eta_{min}, \eta_{max}] \text{ and } e \times (\eta - \bar{\eta}) > 0, \\ & \bar{\eta} \triangleq (\eta_{min} + \eta_{max})/2 \\ e & \text{otherwise} \end{cases}$$

The choice of design parameters η_{min}, η_{max} is not always clear; for this study, we choose

$$(\eta_{min}, \eta_{max}) = (u_{min}, u_{max})/K_I.$$

CI-II Freeze integrator input $\dot{\eta}$ at 0 when u_n is in saturation:

$$\dot{\eta} = \begin{cases} 0 & u_n \neq u_s \text{ (see Equation (3.2))} \\ e & \text{otherwise} \end{cases}$$

CI-III Freeze $\dot{\eta}$ when u_n is being driven into saturation; that is,

$$\dot{\eta} = \begin{cases} 0 & u_n \neq u_s \text{ and } e(u_n - u_s) > 0 \\ e & \text{otherwise} \end{cases} \quad (3.3)$$

($K_I > 0$ is assumed.)

Preloading Manually reset integrator value η to an off-line predetermined value η_d when u_n is in saturation. For the purposes of simulation, we implement this technique by modifying the integrator input as

$$\dot{\eta} = \begin{cases} -\alpha \times (\eta - \eta_d) & u_n \neq u_s \\ e & \text{otherwise} \end{cases}$$

where the parameter $\alpha > 0$ controls the integrator decay rate when u_n is in saturation. For this study, we select an integrator value of $\eta_d = 0$.

AWBT [9] p. 198. Include an integrator feedback term in the integrator involving the error between the nominal control u_n and its limited value u_s:

$$\dot{\eta} = e - K_a(u_n - u_s).$$

Notice that $u_n \neq u_s$ implies that the error e and the additional feedback term must "fight" one another. This property is inherent due to the linear nature of the control law.

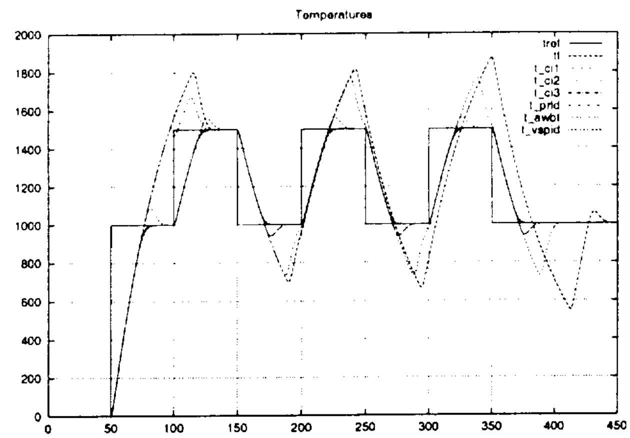

Figure 2: Closed loop temperature profiles of simulated furnace with continuous time anti-windup PID feedback laws

VSPID Rather than freeze the integrator value as shown above, dynamically drive the integrator so that u_n lies at the edge of the saturation region:

$$\dot{\eta} = \begin{cases} \frac{-\alpha(u_n - u_s)}{K_I} & u_n \neq u_s \text{ and } \frac{e(u_n - \bar{u})}{K_I} > 0, \\ & \bar{u} \triangleq (u_{min} + u_{max})/2 \\ e & \text{otherwise} \end{cases} \quad (3.4)$$

where $\alpha > 0$ is a positive constant selected such that u_n rapidly converges to the nearest extreme value of $[u_{min}, u_{max}]$.

Discrete time implementation of these control laws is straightforward.

4 Simulation examples

The anti-windup methods of the previous section were simulated in closed loop with a model of an electric furnace $P(s)$ with state-space model

$$\frac{d}{dt} \begin{bmatrix} c \\ f \end{bmatrix} = \begin{bmatrix} -0.02 & 0.02 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} c \\ f \end{bmatrix} + \begin{bmatrix} 0 \\ 250 \end{bmatrix} v(t) \quad (4.1)$$

where $v(t)$ is an input voltage constrained between 0V and 10V, $f(t)$ is the filament temperature and $c(t)$ is the chamber temperature. The uncompensated settling time of the system is 200s. PID controllers were designed to compensate the system to be critically damped with a settling time of 15 seconds.

Simulated temperatures are shown in Figure 2 (decay parameter $\alpha = 1$ for all relevant antiwindup methods).

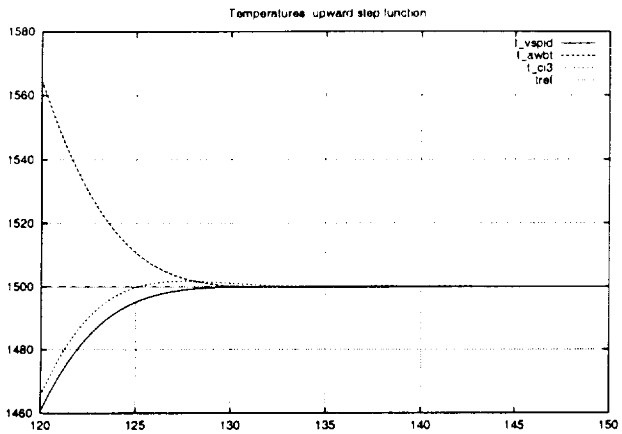

Figure 3: Closed loop temperature profiles of simulated furnace with continuous time anti-windup PID feedback laws

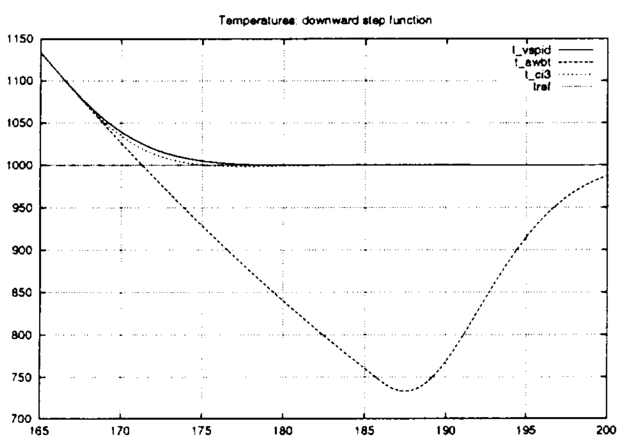

Figure 4: Closed loop temperature profiles of simulated furnace with continuous time anti-windup PID feedback laws

The curve labelled `tf` corresponds to a standard PID controller with no antiwindup law implemented. Notice that, for this example system, *any* antiwindup law is superior to an uncompensated PID. However, several plots are clearly superior to others. These signals are shown in Figures 3 and 4. Notice that the VSPID method has reduced overshoot and settling time relative to the other methods (although CI-III sometimes appears to be competitive).

Notice also that the VSPID and AWBT methods have significantly differing performance due to the nonlinearity of the VSPID method. The poor performance of the AWBT method is due to the competition between the error $e(t)$ and the feedback of the saturation error signal in the AWBT signal. This can be effectively eliminated by increasing α to 100, at which point AWBT and VSPID are nearly indistinguishable. This behavior of the AWBT method is intuitively expected from the definition of the AWBT method; a high gain feedback of the saturation error renders the error signal $e(t)$ inconsequential at the integrator summing junction. The VSPID method does not suffer from this drawback since the integrator is switched, not summed, as a function of control saturation; further, since the VSPID integrator settling time in saturation (approx 4 sec for $\alpha = 1$) is significantly faster than the designed system closed loop settling time (15 sec), the performance of the VSPID controller is not significantly changed by increasing the decay factor α.

5 Stability analysis

We analyze the stability of the VSPID method in terms of a larger class of variable structure antiwindup feedback laws.

Theorem 5.1 *Consider a linear, time-invariant system S described by*

$$\begin{aligned} \dot{x} &= Ax + Bu + B_\eta u_\eta &(5.2)\\ y(t) &= Cx(t) &(5.3)\end{aligned}$$

with $A \triangleq \mathrm{diag}(A, 0)$, $B_\eta \triangleq \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^T$, ${B_\eta}^T B = 0$ *and* $x \triangleq \begin{bmatrix} \bar{x} \\ \eta \end{bmatrix}$ *where η is a real scalar (integrator). Let the state space be denoted as $X = I\!R^n$ and impose input saturation limits u_{min}, u_{max} on the input u. Let $r(t)$ be a scalar reference signal. For each $K \in X^*$, the dual space of X [19], define the nominal (linear) state feedback* $u_n(K, x(t)) \triangleq Kx(t)$ *and the corresponding limited state feedback*

$$u_s(K, x(t)) = \mathrm{sat}(u_n(K, x(t)), u_{min}, u_{max}).$$

For each pair (K, α) in $X^* \times I\!R$ define the variable structure antiwindup feedback law (VSAFL)

$$\begin{bmatrix} u \\ u_\eta \end{bmatrix}(K, \alpha) = \begin{cases} \begin{bmatrix} u_n \\ r(t) - Cx(t) \end{bmatrix} & u_n = u_s \\ \begin{bmatrix} u_s \\ -\alpha(u_n - u_s) \end{bmatrix} & u_n \neq u_s \end{cases} \qquad (5.4)$$

(K, α, and x are omitted when clear by context.) Define

$$X_+ \triangleq \{x \in X : Kx > u_{max}\}$$

$$X_- \triangleq \{x \in X : Kx < u_{min}\}$$
$$X_\ell \triangleq \{x \in X : x \notin X_+ \cup X_-\}$$

We say that the plant S (5.2) is in linear operation *when $x(t) \in X_\ell$ and that S is* in saturation *if $X \in X_- \cup X_+$. Let $\mathcal{P}$ be the cone of positive definite, symmetric matrices. For each $K \in X^*$ define*

$$\mathcal{P}(K) = \{P \in \mathcal{P} \;:\; A_c{}^T P + PA_c < 0, \quad A_c = (A + BK - B_\eta C)\}$$

$\mathcal{P}(K)$ is the set of positive definite matrices corresponding to quadratic Lyapunov functions $V(x) = x^T Px$ for the system S in linear operation. For each $(K, P, \alpha, u) \in (X^* \times \mathcal{P} \times I\!R \times I\!R)$ define the set

$$\begin{aligned}\mathcal{V} \quad & (K, P, \alpha, u) = \{x \in X : \\ & x^T \left[(A - \alpha B_\eta K)^T P + P(A - \alpha B_\eta K)\right] x \\ & +2x^T P(B + \alpha B_\eta)u < 0\}\end{aligned}$$

Finally, for each $(Q, \gamma) \in \mathcal{P} \times I\!R^+$, let

$$X_s(Q, \gamma) = \{x \in X : x^T Qx < \gamma\} \subset X$$

be the local region in X in which we wish to stablize X. Then the VSAFL (5.4) with parameters K and α stabilizes the plant on $X_s(Q, \gamma)$ in the sense of Lyapunov if the following three conditions hold.

1. *$P \in \mathcal{P}(K)$*
2. *$X_- \cap X_s \subset \mathcal{V}(K, P, \alpha, u_{min})$*
3. *$X_+ \cap X_s \subset \mathcal{V}(K, P, \alpha, u_{max})$*

The controller globally stabilizes the plant if $X_s(Q, \gamma) = X_s(Q, \infty) = X$.

Proof: Observe that the sets X_+, X_-, X_ℓ, $\mathcal{P}$, $\mathcal{P}(K)$, $\mathcal{V}(K, P, \alpha, u)$, and $X_s(P, \gamma)$ are all convex. The boundaries of X_+, X_-, X_ℓ are hyperplanes normal to the vector K (see Figure 5). When S is in linear operation the closed loop dynamics are

$$\begin{aligned}\dot{x} &= Ax + BKx + B_\eta(R - Cx) \\ &= (A + BK - B_\eta C)x + B_\eta r \qquad (5.5)\end{aligned}$$

When the system S is in saturation, the closed loop dynamics become

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu_s(t) + \alpha B_\eta(u_s(t) - Kx(t)) \\ &= (A - \alpha B_\eta K)\, x(t) + (B + \alpha B_\eta)u_s(t) \quad (5.6)\end{aligned}$$

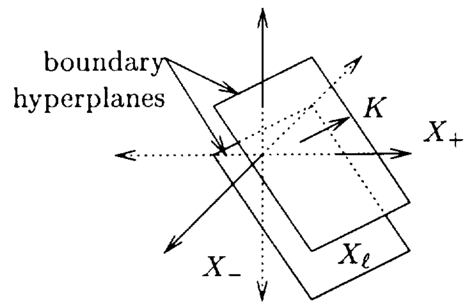

Figure 5: Sets X_+, X_-, and X_ℓ for VSAFL analysis

For each $P \in \mathcal{P}$, consider the quadratic function

$$V(x) = x^T Px \qquad P \in \mathcal{P}.$$

A nominal state feedback $u_n(K, x)$ stabilizes S if the set $\mathcal{P}(K)$ is not empty. When S is in saturation

$$\begin{aligned}\frac{d}{dt}V(x) &= x^T P[(A - \alpha B_\eta K)x + (B + \alpha B_\eta)u_s] \\ &\quad +[(A - \alpha B_\eta K)x + (B + \alpha B_\eta)u_s]^T Px \\ &= x^T \left[(A - \alpha B_\eta K)^T P + P(A - \alpha B_\eta K)\right] x \\ &\quad +2x^T P(B + \alpha B_\eta)u_s\end{aligned}$$

with $u_s = u_{min}, u_{max}$ for $x \in X_-, X_+$, respectively.

Now suppose that the conditions (1)-(3) hold for a given set $X_s(Q, \gamma)$. Then, for every $x \in X_s$ we have that $V(x) \geq 0$ and $\dot{V}(x) \leq 0$ with equality holding only at the origin. □

6 Conclusions

We have presented the VSPID technique for the prevention of integrator windup in PID feedback control. Stability conditions are presented in terms of the larger class of VSAFL systems with state-feedback and integrator control in Theorem 5.1. All of the sets in the theorem are convex; further, the problem of computing a stabilizing nominal state feedback matrix K is a convex programming problem. Hence, we believe that VSAFL design problem can be posed as a convex programming problem[19], which can be solved in polynomial time [20]. However, since the class of VSPID controllers are not a a subset of VSAFL controllers, the resulting stability conditions do not lead to a convex programming problem.

The combinatorial complexity of the VSAFL design problem increases exponentially with the number of inputs subject to saturation; there are effectively $2^n + 1$

input laws acting in tandem, two for each input channel (saturation limit) and one for the linear region of operation.

References

[1] K. J. Åström and T. Hägglund. *Automatic Tuning of PID Controllers.* Instrument Society of America, Research Triangle Park, 1988.

[2] Karl Johan Åström and Lars Rundqwist. Integrator windup and how to avoid it. In *Proceedings of the 1989 American Control Conferences*, pages 1693–1698, Pittsburgh, PA, 1989.

[3] P. S. Buckley. Designing override and feedforward controls. *Control Engineering*, 18, 1971.

[4] P. J. Camp and M. Morari. Robust control of processes subject to saturation nonlinearities. *Comput. Chemical Engineering*, 1990.

[5] John Van de Vegte. *Feedback Control Systems.* Prentice-Hall, 3rd edition, 1994.

[6] J. C. Doyle, R. S. Smith, and D. F. Enns. Control of plants with input saturation nonlinearities. In *Proceedings of the 1987 American Control Conference*, Minneapolis, 1987.

[7] L. H. Dreinhoefer. Controller tuning for a slow nonlinear process. *IEEE Control Systems Magazine*, 8(2):56–60, 1988.

[8] H. A. Fertik and C. W. Ross. Direct digital control algorithm with anti-windup feature. *ISA Trans.*, 6, 1967.

[9] G. F. Franklin, J. D. Powell, and A. Emami-Naeini. *Feedback Control of Dynamic Systems.* Addison-Wesley, 3rd edition, 1994.

[10] S. E. Gallun, C. W. Matthews, C. P. Senyard, and B. Slater. Windup protection and initialization for advanced digital control. *Hydrocarbon Processing*, pages 63–68, June 1985.

[11] A. H. Glattfelder and W. Schaufelberger. Stability analysis of single loop systems with saturation and antireset-windup circuits. *IEEE Transactions on Automatic Control*, AC-28:1074–1081, 1983.

[12] A. H. Glattfelder and W. Schaufelberger. Stability of discrete override and cascade-limiter single-loop control systems. *IEEE Transactions on Automatic Control*, AC-33:532–540, 1988.

[13] R. Hanus and M. Kinnaert. Control of constrained multivariable systems using the conditioning technique. In *Proceedings of the 1989 American Control Conference*, pages 1711–1718, Pittsburgh, 1989.

[14] R. Hanus, M. Kinnaert, and J. L. Henrotte. Conditioning technique, a general anti-windup and bumpless transfer method. *Automatica*, 23:729–739, 1987.

[15] R. Hanus and Y. Peng. Conditioning technique for controllers with time delays. *IEEE Transactions on Automatic Control*, 37:689–692, 1992.

[16] G. Howes. Control of overshoot in plastics-extruder barrel zones. In *EI Technology*, number 3, pages 16–17. Eurotherm International, Brighton, England, 1986.

[17] P. Kapasouris, M. Athans, and G. Stein. Design of feedback control systems for stable plants with saturating actuators. In *Proceedings of the 27^{th} IEEE Conference on Decision and Control*, Austin, Texas, 1988.

[18] Mayuresh V. Kothare, Peter J. Campo, Manfred Morari, and Carl N. Nett. A unified framework for the study of anti-windup designs. *Automatica*, 30(12):1869–1883, 1994.

[19] D. G. Luenberger. *Optimization by Vector Space Methods.* Wiley and Sons, Inc., New York, NY, 1969.

[20] Yurii Nesterov and Arkadii Nemirovskii. *Interior-Point Polynomial Algorithms in Convex Programming.* SIAM, 1994.

[21] Shinkskey. *Process-Control Systems.* McGraw-Hill, New York, 3 edition, 1988.

[22] K. S. Walgama, S. Rönbaäck, and J. Sternby. Generalization of conditioning technique for anti-windup compensators. *IEE Proceedings Part D*, 139:109–118, 1992.

[23] K. S. Walgama and J. Sternby. Inherent observer property in a class of anti-windup compensators. *International Journal of Control*, 52:705–724, 1990.