Variable Structure Rough Neural Network Control for a Class of Non-Linear Systems

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Received: November 2018 Revised: February 2019 Accepted: April 2019

ABSTRACT:
In this paper, we present a novel rough neural network control system based on the variable structure control developed for a class of SISO canonical nonlinear systems with taking the presence of bounded disturbance into account. We assume that the nonlinear functions of the system are completely unknown. The rough neural network presented here is used to approximate the unknown nonlinear functions to a desired appropriate approximation. A fuzzy soft switching structure is developed to decide the amount of efforts taken by neural network and variable structure control systems based upon the real-time error characteristics. A proper Lyapunov function is defined and used to deduce adaptive laws for tunable parameters of neural network and to achieve the closed loop stability of overall system. The rough family of neural networks have a reputation of better functionality at the presence of noise and disturbance, which comes from their interval characteristic of their parameters. In this study we utilize this property to achieve better performance. To demonstrate the effect of proposed control structure, it is applied upon three systems (one exemplary system, a dynamical, and a chaotic) and the simulated results have shown the efficiency of this hybrid variable structure control scheme.

KEYWORDS: Hybrid Control, Rough, Neural Network, Non-Linear System Stability, Variable Structure Controller.

1. INTRODUCTION

The importance of flexibility and durability in the modern architecture of nonlinear control is known to every researcher. In the past few decades, researchers exhibited interest in crafting control architectures more flexible, robust and efficient and it is no wonder that their visions are in favor of closing the gap between the machine and the human brain. Therefore, the concept of adaptive control [1, 2] with co-operation of different approaches for simulating various parts of brain's activities resulted in emerge of Fuzzy Inference Systems (FIS), Artificial Neural Networks (ANN) [3] and etc. The main problem in classical nonlinear control methods like pure Feedback Linearization, Sliding Mode or Back Stepping, arises from the fact that almost in all of these methods, a complete and comprehensive knowledge upon the nonlinear functions of system is strictly demanded, which is in contrast with actual and realistic conditions usually faced by designer. Therefore, through decades of research and with function approximation ability of FIS and ANNs coming into light, researchers started to improve and utilize these abilities in co-operation with classical nonlinear control methods.

A unique collaboration of classic sliding mode control and neural network was introduced by Sanner and Slotine [4] in which a sliding mode component and a radial basis neural network work together to keep the state of systems in a pre-specified set in the plant's state space. Later the use of variable structure schemes was raised with more researchers looking for more reliable control structures. Hogans, Homaifar and Sayyarrodsari combined variable structure control idea with fuzzy inference to obtain a control method invariant to perturbations of system and external disturbances [5]. Through different researches, it has been proven that utilizing variable control structures can help to overcome some of most difficult problems in nonlinear control [6-9]. Outside of the variable structure theory, artificial neural networks have been used in co-operation with other nonlinear control theories and are applied to subjects as vehicle stability

[10], benchmark problems as like inverted pendulum [11], robotics [12, 13], and etc. Among different subjects in soft computing, rough set theory is a new and reliable theory which has been recently introduced and is utilized alongside fuzzy and neural network theories [14-16]. Although only a few researches around application of rough set theory for control of nonlinear systems has been conducted [17-20].

In this study, a new Variable Structure Rough Neural Network Control (VSRNNC) method with utilizing a specific type of ANNs is presented. Rough family of artificial neural networks is new member of soft computing family which uses the rough set theory in neural network structure and provides the interested designer with its own flexibility in structure design. The main idea behind the rough ANN is, that based on the view of designer, different parameters inside the ANN can be taken as interval parameters. This property shapes the robust characteristic of rough ANN which comparing to other ordinary ANNs, is unique. In this hybrid structure scheme, a sliding mode controller work alongside the neural network to represent the human behavior of controlling. A fuzzy switching system is putted in the role of inference system, deciding how much effort should be taken by Rough Neural Network (RNN) and sliding mode. An adaptive disturbance compensator is introduced to the final control law to guarantee the proper performance in the presence of disturbances. Lyapunov-based stability theorem is used to achieve the global boundedness of overall system and to achieve the demanded adaptive laws. The proposed control system is used and simulated on two different systems, an inverted pendulum as an example of a mechanical system and Duffing Oscillator for a chaotic one, which performs well in both cases.

To fully recognize the novelty of proposed VSRNNC system in this study comparing to former neural – adaptive control papers mentioned earlier, the author likes to express the major discrepancies and innovations of this work by mentioning that rough neural network is combined with variable structure control for the first time, in this paper. Also, the adaptive laws are used for tuning the adjustable parameters of rough neural network. In addition to that, adaptive laws for neural network are made robust using the Projection Modification.

In the following sections of this paper, Section 2 provides the problem formulation, In Section 3 we present the formulation of rough neural network and their ability for function approximation. In Section 4, the VSRNNC system is proposed for controlling SISO nonlinear systems and the asymptotically stability of controlled systems are proven. Results for simulations are provided in Section 5, and the paper is discussed and concluded in Section 6.

2. PROBLEM FORMULATION

Study in this paper is focused on designing an adaptive control scheme for a class of dynamic Single Input-Single Output (SISO) nonlinear systems which could be represented with the canonical formulation:

$$\begin{aligned} &x^{(n)}(t)+f\left(x(t),\dot{x}(t),\ldots,x(n-1)(t)\right)= \\ &b(x(t),\dot{x}(t),\ldots,x(n-1))u(t)+d_{ex}(t) \\ &y(t)=x(t) \end{aligned} \tag{1}$$

Where, $f(X)$ is the unknown function, $u(t)$ is the control input, $b(X)$ is the control gain and $d_{ex}(t)$ is bounded external disturbance presumed as $|d_{ex}(t)|<D_0$. The desirable scenario here is to force the states of the system, $X=[x,\dot{x},\ldots,x^{(n-1)}]^T$ to follow a desired trajectory, $X_d=[x_d,\dot{x}_d,\ldots,x_d^{(n-1)}]^T$. We define the $\tilde{x}=X-X_d$, the tracking error vector and the desired result to achieve here is to design a suitable control law, $u(t)$ to ensure that $\tilde{x}\to 0$ as $t\to\infty$.

In this study, a rough neural network is utilized to provide the proper approximation of the unknown nonlinear function in the presence of disturbances and noise. While a lot of former researches concluded by proving the function approximation ability of ANNs, only a few has come to try to achieve the proper system performance under the mentioned conditions. In the next section, rough neural network is introduced and its unique characteristics is going to be discussed.

3. ROUGH NEURAL NETWORK

The term “rough neural networks” is used for a group of artificial neural networks which are designed based upon using rough set theory. The primary characteristic of RNNs is robustness against uncertainties in the data in hand. This robustness is a result of structural design which by the choice of designer, different parameters defining the network could be taken as interval parameters except of being crisps ones. This principle could be applied to different types of neural networks such as multilayer perceptron or radial basis networks.

Fig. 1 shows the structure of the proposed RNN in this study. It has a single hidden layer with Gaussian activation functions and the output weights, connecting the hidden layer to the output layer, has taken as interval parameters. The overall formulation [21] for the network indicated in Fig. 1 is as follows:

$$o_i = \exp(-\frac{1}{2}\sum_{k=1}^{n}\left[\frac{x_k - c_{ik}}{\sigma_{ik}}\right]^2) \qquad 1 \le i \le l \qquad (2)$$

$$u_{NN,j} = \sum_{i=1}^{l}\frac{w_{ij,u}+w_{ij,1}}{2}o_i = \sum_{i=1}^{l}\frac{w_{ij,u}+w_{ij,1}}{2} \exp(-\frac{1}{2}\sum_{k=1}^{n}\left[\frac{x_k - c_{ik}}{\sigma_{ik}}\right]^2),\ 1 \le j \le m \qquad (3)$$

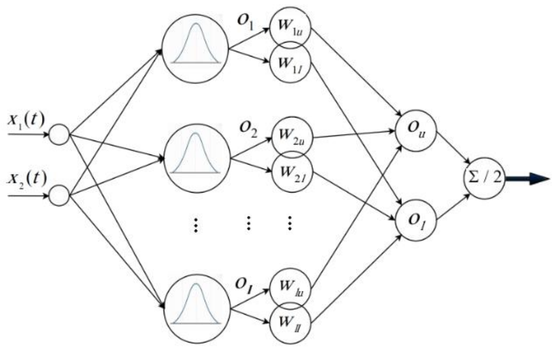

Fig. 1. Rough neural network Structure.

In which, c is the center of Gaussian neurons, σ is the standard deviation of Gaussian neurons, $x_1(t)$ and $x_2(t)$ are the inputs for the network, l and m are numbers of neurons in hidden layer and the output layer respectively, o_i is the output of i -th neuron of the hidden layer, $w_{ij,u}$ and $w_{ij,1}$ are the upper band and lower band weights of output layer, and u_{NN} is the rough neural network. The network output is average of upper band output, o_u and the lower band output o_l. The output weights of the network, both from upper band and the lower band is taken as the adjustable parameters of the network. With choosing Gaussian activation functions, overall property of function approximation of proposed network is as efficient as a simple RBF network while by taking the output weights in interval forms, we can reduce the effect of uncertainties in the process of approximation.

4. ADAPTIVE VARIABLE STRUCTURE ROUGH NEURAL NETWORK CONTROL

In this section we describe the proposed VSRNNC structure for controlling SISO nonlinear systems. First we present the structure of VSRNNC, second, using Lyapunov stability theorem, we prove VSRNNC's use in control of nonlinear systems described in (1) for both $b(X)=1$ and $b(X)\ne 1$.

4.1. Structure of VSRNN Control System

Fig. 2 shows the block diagram of the proposed VSRNN control system. In this structure, the control signal is the result of three different part working together: linear feedback, sliding mode and rough neural network. It worth to mention that using a rough neural network makes this system unique and different from structures like in [4] with utilizing simple radial basis network. The modulator block uses a fuzzy soft switching component to determine the amount of effort taken by variable structure or rough neural network in the overall control of the system.

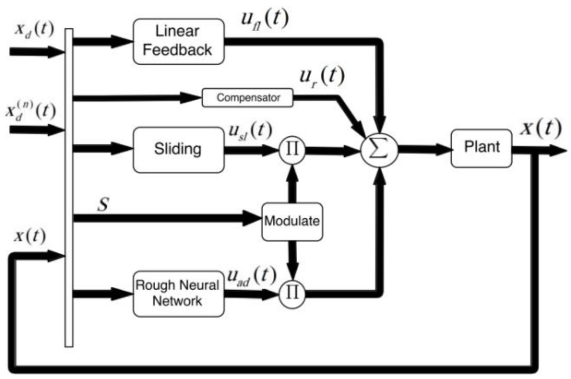

Fig. 2. VSRNN control scheme.

We define a tracking error metric to use further in sliding mode control and switching subsystem as:

$$s(t) = (\frac{d}{dt}+\lambda)^{n-1}\tilde{\boldsymbol{x}}(t) \ \ with \ \ \lambda > 0 \qquad (4)$$

With this definition, $s(t)=0$ represents a time-varying hyperplane in R^n, which by choosing λ properly , the tracking error vector decays to zero exponentially and the perfect tracking can be asymptotically achieved by maintaining this condition [4]. Hence, our goal would be designing a control that forces $s(t)=0$.

For utilizing this error metric for sliding mode component, a dead zone with the width of Φ and hence defining a continuous function s_Δ is introduced as:

$$s_\Delta(t) = s(t) - \Phi sat(s(t)/\Phi) \qquad (5)$$

This helps overcoming common sliding mode designing problems, which has a better performance from sliding mode component and the overall control structure.

By taking the time derivative of this error metric (4) we would have:

$$\dot{s}(t) = a_r(t) - f_A(x(t)) + u(t) + d_A(t) + D(t) \quad (6)$$

Where, $a_r(t) = \lambda_v^T \tilde{\boldsymbol{x}}(t) - x_d^{(n)}(t)$ in which $\lambda_v^T = [0, \lambda^{n-1}, (n-1)\lambda^{n-2}, ..., (n-1)\lambda]$ and $x_d^{(n)}(t)$ is the n th derivative of the desired trajectory. $d_A(t)$ is the approximation error defined as $d_A(t) = f_A(X) - f(X)$ and $D(t)$ is the external disturbance. Based on the above considerations, the suggested control law would be:

$$u(t) = -k_D s(t) - u_{fl}(t) + (1 - m(t))u_{ad} + m(t)u_{sl}(t) + u_r(t) \quad (7)$$

In which the following terms are defined as:

$$u_{fl} = \lambda_v^T \tilde{\boldsymbol{x}}(t) - x_d^{(n)}(t) \quad (8)$$

$$u_{ad} = \hat{f}(X) \quad (9)$$

$$u_{sl} = -k_{sl} sat(s(t)/\Phi) \quad (10)$$

$$u_r = -\rho sign(s_\Delta) \quad (11)$$

Where, the final term in the control signal (7), u_r, is the term for compensating effect of external disturbances, $m(t)$ is the output of fuzzy switching system, and $\hat{f}(X)$ is the approximation of unknown nonlinear function $f(X)$, provided by rough neural network's output which can be rewritten as :

$$u_{nn} = \sum_{i=1}^{l} \frac{w_{ij,u} + w_{ij,l}}{2} o_i \quad (12)$$

And the $sat(.)$ is defined as:

$$sat(y) = \begin{cases} -1 & if \quad y < -1 \\ y & if \quad -1 \le y \le 1 \\ 1 & if \quad y > 1 \end{cases} \quad (13)$$

4.2. Asymptotically Stability of Nonlinear Systems with Considering $b(X) = 1$

With substituting (7) in (6), $\dot{s}(t)$ can be written as:

$$\dot{s}(t) = -k_D s(t) + (1 - m(t))(\tilde{f}_A(t) + d_A(t)) + m(t)(u_{sl}(t) - f(x(t))) + u_r(t) + D(t) \quad (14)$$

Consider the Lyapunov function candidate as:

$$V(t) = \frac{1}{2}(s_\Delta^2(t) + \frac{1}{k_{a1}}\sum \tilde{w}_U^2 + \frac{1}{k_{a2}}\sum \tilde{w}_L^2 + \frac{1}{\gamma}(\rho - \rho^*)^2) \quad (15)$$

It is necessary to indicate the last term in the proposed Lyapunov function is the candidate for designing u_r in order to compensate the external disturbance. By taking the derivative of (15) we have:

$$\dot{V}(t) = s_\Delta \dot{s}(t) + \frac{1}{k_{a1}}\sum \tilde{w}_U \dot{\hat{w}}_U + \frac{1}{k_{a2}}\sum \tilde{w}_L \dot{\hat{w}}_L + \frac{1}{\gamma}\dot{\rho}(\rho - \rho^*) \quad (16)$$

Knowing $s_\Delta(t) sat(s(t)/\Phi) = |s_\Delta(t)|$ and applying (14) to (16) leads to:

$$\dot{V}(t) = -(k_D s_\Delta^2 + |s_\Delta| k_D \Phi) + s_\Delta(1-m)(\tilde{f}_A + d) + m\ s_\Delta(u_{sl} - f(x)) + \frac{1}{k_{a1}}\sum \tilde{w}_U \dot{\hat{w}}_U + \frac{1}{k_{a2}}\sum \tilde{w}_L \dot{\hat{w}}_L + s_\Delta u_r + s_\Delta D(t) + \frac{1}{\gamma}\dot{\rho}(\rho - \rho^*) \quad (17)$$

By defining the adaptive laws for neural network weights and for disturbance compensator design parameter, we have:

$$\dot{\hat{w}}_U(t) = -k_{a1} \times s_\Delta \times (1 - m(t)) \times \frac{1}{2} \times o_i \quad (18)$$

$$\dot{\hat{w}}_L(t) = -k_{a2} \times s_\Delta \times (1 - m(t)) \times \frac{1}{2} \times o_i \quad (19)$$

$$\dot{\rho} = \gamma |s_\Delta| \quad (20)$$

Assuming that the upper bound for the magnitude of nonlinear function f is known as M_0 and the uncertainty $|d_A(t)| \le \varepsilon_f$ when ε_f is the approximation error between nonlinear function f and it's neural network approximation, f_A, so that $|\hat{f} - f| \le \varepsilon_f$, and by applying (10), (18) and (19) in (17) and some mathematical simplification results in:

$$\dot{V}(t) \le -k_D s_\Delta{}^2 + (1-m)|s_\Delta|(|d| - k_D\Phi) + m|s_\Delta|\ (|f| - k_{sl} - k_D\Phi) + s_\Delta u_r + s_\Delta D(t) + \frac{1}{\gamma}\dot{\rho}(\rho - \rho^*)$$
(21)

In which, for $m<1$ we have $|d_A| \le \varepsilon_f$ and for $0 \le m(t) \le 1$, the dead zone Φ would be chosen in a way that $k_D\Phi > \varepsilon_f$, so that the second term on the right will be non-positive. Choosing the sliding controller gain as $k_{sl}(t) = M_0 - \varepsilon_f$ ensures that the third term on the right is also non-positive which results in:

$$\dot{V}(t) \le -k_D s_\Delta{}^2 + s_\Delta u_r + s_\Delta D(t) + \frac{1}{\gamma}\dot{\rho}(\rho - \rho^*) \quad (22)$$

Applying (11) in (22) gives:

$$\begin{aligned}\dot{V}(t) &\le -k_D s_\Delta{}^2 - \rho|s_\Delta| + |s_\Delta| D(t) + \frac{1}{\gamma}\dot{\rho}(\rho - \rho^*) \\ &\le -k_D s_\Delta{}^2 - \rho^*|s_\Delta| + (\rho^* - \rho)|s_\Delta| + s_\Delta D(t) + \frac{1}{\gamma}\dot{\rho}(\rho - \rho^*) \\ &\le -k_D s_\Delta{}^2 - \rho^*|s_\Delta| + s_\Delta D(t)\end{aligned} \quad (23)$$

With ρ^* being a design parameter, considering it as $D \le \rho^*$ gives:

$$\dot{V}(t) \le -k_D s_\Delta{}^2 \quad (24)$$

Therefore, for the system presented in (1) all signals are bounded. Since $s_\Delta(t)$ is bounded, it can be proven that, if $\tilde{x}(0)$ is bounded, then $\tilde{x}(t)$ will be also bounded for all t . With $X_d(t)$ being bounded by design, it results in $X(t)$ being bounded as well.

In order to prove the asymptotic convergence of the tracking error, it is necessary to prove that $s_\Delta \to 0$ as $t \to \infty$. For this, we apply Barbalat's Lemma to the continuous, non-negative function as follow:

$$V_1(t) = V(t) - \int_0^t (\dot{V}(t) + k_D s_\Delta^2(t))dt \Rightarrow \dot{V}_1 = -k_D s_\Delta^2(t)$$
(25)

It can be seen that every term on the right side (14) is bounded, hence $\dot{s}_\Delta(t)$ is bounded, which means that $\dot{V}_1(t)$ is a uniformly continuous function of the time. Since $V_1(t)$ is bounded below by zero, and $\dot{V}_1(t) \le 0$ for all t , applying the lemma results in $\dot{V}_1(t) \to 0$ as $t \to \infty$. This means that $|s(t)| \le \Phi$ is asymptotically satisfied. Also the asymptotic tracking errors are asymptotically bounded by:

$$\left|e^{(i)}(t)\right| \le 2^i \lambda^{i-n+1}\Phi, \quad i = 1,\dots,n-1 \quad (26)$$

4.3. Asymptotically Stability of Nonlinear Systems with considering *b(X) = 1*

In the following section we extend the results from last section to non-linear systems with non-unity control gains. It is assumed that the control gain $b(X)$ is finite and non-zero and is bounded as $\left|b^{-1}(X)\right| < M_1$. We define the function $h(X) = f(X)/b(X)$ to be also bounded as $|h(X)| < M_0$. It is further assumed that there is a known positive function $M_2(X)$ such that $\left|(d/dt)b^{-1}(X)\right| < M_2(X)\|X\|$.

We define the h_A and b_A^{-1} as neural network approximations to the functions h and b^{-1} so that :

$$|h(X) - h_A(X)| < \varepsilon_h \quad (27)$$

$$\left|b^{-1}(X) - b_A^{-1}(X)\right| < \varepsilon_b \quad (28)$$

Where, ε_h and ε_b are as small as desired. We utilize a single network with two outputs as shown in Fig. 3.

By taking the above definitions in mind, the time derivative of the error metric (6) would give us:

$$b^{-1}(X)\dot{s}(t) = b^{-1}(X)a_r(t) - h_A(x(t)) + u(t) + d_A(t) + D(t)$$
(29)

Where, by using the neural network approximations of functions, the disturbance $d_A(t)$ is given by:

$$d_A(t)=\left[b^{-1}(X)-b_A^{-1}(X)\right]a_r(t)$$
$$-\left[h(X)-h_A(X)\right]$$
(30)

And satisfies $\left|d_A(t)\right|\le \varepsilon_h+\varepsilon_b\left|a_r(t)\right|$.

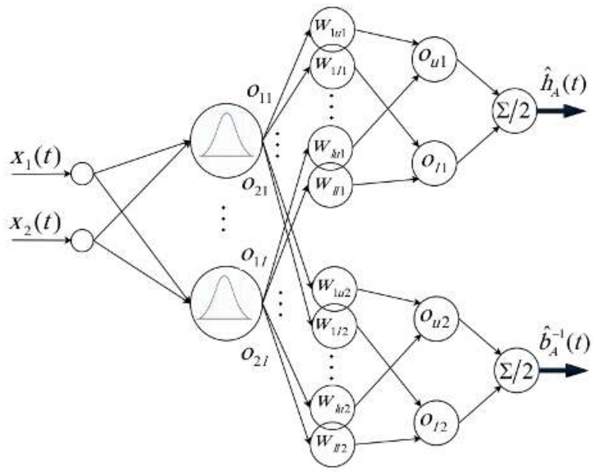

Fig. 3. Rough neural network Structure for non-unity control gain case.

Based on (29), we suggest a control law as:

$$u(t)=-k_D s(t)-\frac{1}{2}M_2(X)\left\|X\right\|s_\Delta(t)+m(t)u_{sl}(t)$$
$$-(1-m(t))\left[\hat{b}_A^{-1}(X)a_r(t)-\hat{h}_A(X)\right]+u_r(t) \quad (31)$$

Where, $u_{sl}(t)$ is same as (10) with k_{sl} as:

$$k_{sl}(t)=(M_0(X)-\varepsilon_h)+(M_1(X)-\varepsilon_b)\left|a_r(t)\right|$$
(32)

And $u_r(t)$ is given in (11), and we define $\hat{h}_A(X)$ and $\hat{b}_A^{-1}(X)$ as the two outputs of rough Gaussian network, respectively:

$$h_A(X)=\sum_{i=1}^{l}\frac{w_{ij,u1}+w_{ij,l1}}{2}o_i \quad (33)$$

$$\hat{b}_A^{-1}(X)=\sum_{i=1}^{l}\frac{w_{ij,u2}+w_{ij,l2}}{2}o_i$$
(34)

By using the control law (31) in (29) we have:

$$b^{-1}\dot{s}=-k_D s-\frac{1}{2}M_2\left\|X\right\|s_\Delta+(1-m)(\tilde{h}_A-\tilde{b}_A^{-1}a_r+d_A)$$
$$+m(u_{sl}+b^{-1}a_r-h)+d_{ex} \quad (35)$$

In which, $\tilde{h}_A(X)=\hat{h}_A(X)-h_A(X)$ and similarly for $\tilde{b}_A^{-1}(X)=\hat{b}_A^{-1}(X)-b_A^{-1}(X)$. We choose the adaptive laws for output weights of network as follows:

$$\dot{\hat{w}}_{U1}(t)=-k_{a1}\times s_\Delta\times(1-m(t))\times\frac{1}{2}\times o_i$$
$$\dot{\hat{w}}_{L1}(t)=-k_{a2}\times s_\Delta\times(1-m(t))\times\frac{1}{2}\times o_i$$
$$\dot{\hat{w}}_{U2}(t)=k_{a3}\times a_r(t)\times s_\Delta\times(1-m(t))\times\frac{1}{2}\times o_i$$
$$\dot{\hat{w}}_{L2}(t)=k_{a4}\times a_r(t)\times s_\Delta\times(1-m(t))\times\frac{1}{2}\times o_i$$
(36)

Next, we propose a non-negative function to prove stability and convergence:

$$V(t)=\frac{1}{2}\left[b^{-1}(X)s_\Delta(t)^2+\frac{1}{k_{a1}}\sum\tilde{w}_{U1}^2+\frac{1}{k_{a2}}\sum\tilde{w}_{L1}^2\right.$$
$$\left.+\frac{1}{k_{a3}}\sum\tilde{w}_{U2}^2+\frac{1}{k_{a4}}\sum\tilde{w}_{L2}^2+\frac{1}{\gamma}(\rho-\rho^*)^2\right]$$
(37)

Where, $\tilde{w}_{U1}^2=\hat{w}_{U1}^2-w_{U1}^2$ and $\tilde{w}_{L1}^2=\hat{w}_{L1}^2-w_{L1}^2$, and for second group of output weights $\tilde{w}_{U2}^2=\hat{w}_{U2}^2-w_{U2}^2$ and $\tilde{w}_{L2}^2=\hat{w}_{L2}^2-w_{L2}^2$.

By taking the derivative of proposed function and using (35) we would have:

$$\dot{V}=\frac{1}{2}(\dot{b}^{-1}-M_2\left\|X\right\|-2k_D)s_\Delta^2$$
$$-\left|s_\Delta\right|k_D\Phi+m(s_\Delta(b^{-1}a_r-h)-\left|s_\Delta\right|k_{sl})$$
$$+(1-m)s_\Delta(\tilde{h}_A-\tilde{b}_A^{-1}a_r+d_A)$$
$$+\frac{1}{k_{a1}}\sum\tilde{w}_{U1}\dot{\hat{w}}_{U1}+\frac{1}{k_{a2}}\sum\tilde{w}_{L1}\dot{\hat{w}}_{L1}$$
$$+\frac{1}{k_{a3}}\sum\tilde{w}_{U2}\dot{\hat{w}}_{U2}+\frac{1}{k_{a4}}\sum\tilde{w}_{L2}\dot{\hat{w}}_{L2}$$
$$+s_\Delta u_r+s_\Delta D(t)+\frac{1}{\gamma}\dot{\rho}(\rho-\rho^*)$$
(38)

In which, $\dot{b}^{-1}$ is the derivative of b^{-1}. By using the assumed bound on derivative of b^{-1} and adaptive laws (36) in (38) we will have:

$$\dot{V} \le -k_D s_\Delta^2 + (1-m)|s_\Delta|(|d_A| - k_D\Phi) + m|s_\Delta|(|b^{-1}||a_r| + |h| - k_D\Phi - k_{sl}) + s_\Delta u_r + s_\Delta D(t) + \frac{1}{\gamma}\dot{\rho}(\rho - \rho^*) \quad (39)$$

We define $\varepsilon_r = \varepsilon_h + \varepsilon_b(2\|\lambda\| r_A + |x_d^{(n)}|_{\max})$ with r_A being the smallest n-ball completely containing the set of plant state space where both neural component and sliding component work together, and $|x_d^{(n)}|_{\max}$ is an upper bound on the magnitude of the n th derivative of the reference trajectory. Then for $m<1$, we have $|a_r| \le \|\lambda\|(2r_A) + |x_d^{(n)}|_{\max}$ and the bounds (27) and (28) hold in this region. In addition, we have $|d_A| \le \varepsilon_r$ holding in $m<1$. By taking $\Phi \ge \varepsilon_r / k_D$, when $0 \le m \le 1$, the second term on the right is less than or equal to zero. By choosing proper sliding mode controller gains, we can ensure that the third term on the right side is also non-positive. For the remaining terms of u_r and external disturbance, it follows the same as in (22) for unity control gain case. Hence using the proposed control and adaption laws we produce $\dot{V}(t) \le -k_D s_\Delta^2$ for all $t \ge 0$, so if the initial values for the states of plant and for parameter estimates are bounded , they should remain bounded for all positive time. Finally, by having the constrains assumed on b^{-1}, same as the case for unity gain control in (24) to (26) can be used to prove that $s_\Delta \to 0$ as $t \to \infty$.

4.4. Fuzzy Switching System

In this study a fuzzy switching system is used as a modulator component, which determines the amount of participation of hybrid control and the neural network in overall control signal. Fig. 3 shows the structure of this fuzzy system. The input to this fuzzy system is the error metric, it is a single input zero-order Takagi-Sugeno fuzzy system with two Gaussian membership functions. An example of membership functions used in this system is shown in Fig. 4, which together with (7) it can be seen that with error metric being more distanced from $s(t)=0$, the more effort is taken by sliding mode component rather than neural network.

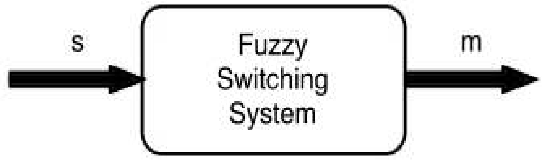

Fig. 4. Diagram of Fuzzy Switching System.

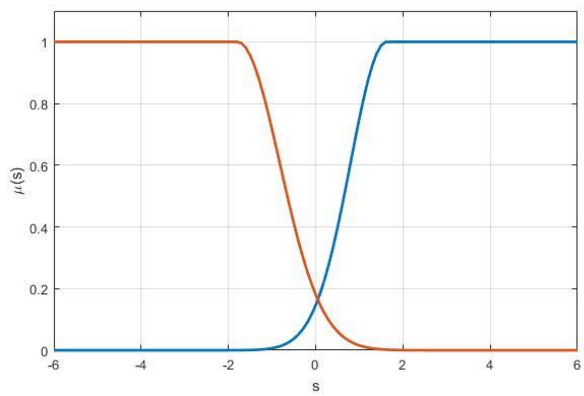

Fig. 5. Example of membership functions for Fuzzy Switching System.

4.5. Projection Modification Method

In most of adaptive neural network control studies, in order to simplify the control problem, it is usually assumed that the only uncertainty in the dynamical system is due to unknown nonlinear elements, while in practice, the function approximation of neural networks may not be able to match the modeling uncertainty exactly, due to other modeling errors caused by such as un-modeled dynamics, measurement noise or external disturbances. This may cause adaptive components of control to exhibit parameter drift, which is the result of attempting to adjust the tunable parameters to match a function which its exact match does not exist. In this study, in order to prevent the tunable parameters from drifting, projection modification method is utilized to ensure that parameter estimates are restrained inside a predefined and convex region. It is important to note that the projection modification method is one of robust adaptive learning techniques which does not affect the stability properties obtained using the standard adaptive laws.

Considering for a generic adaptive law as:

$$\dot{\hat{\theta}}(t) = -\gamma\xi(t)\varepsilon(t) \quad (40)$$

Where, γ is the learning rate, $\xi(t)$ to be the regressor vector and $\varepsilon(t)$ to be training error, with defining $\bar{\theta}$ and $\underline{\theta}$ as respectively the upper and lower limits for the changing parameters, a form of mentioned method is implemented as below:

$$\dot{\hat{\theta}}(t) = \begin{cases} -\gamma\xi\varepsilon & if \;\; \underline{\theta} < \hat{\theta} < \overline{\theta} \\ & or \; if \;\; \hat{\theta} = \underline{\theta} \; and \;\; \gamma\xi\varepsilon \le 0 \\ & or \; if \;\; \hat{\theta} = \overline{\theta} \; and \;\; \gamma\xi\varepsilon \ge 0 \\ 0 & otherwise \end{cases} \quad (41)$$

The initial conditions also need to be chosen in a way that $\underline{\theta} < \hat{\theta}(0) < \overline{\theta}$.

5. SIMULATION AND RESULTS

In this section, three examples of using the proposed control method will be presented: First we utilize our control scheme on an exemplary system with unity control gain to prove the efficiency of VSRNN as described in 4.2. As for non-unity control gain, the second system is a mechanical system, an inverted pendulum and the third one is chaotic one, the famous Duffing oscillator, in both cases in the control gain; $b(X) \neq 1$. The results from proposed method is being compared to a more simpler hybrid control structure with a simple RBF as the adaptive component of control as proposed in [4] (HSRBF). The initial conditions for the plant will be as same for both methods.

Example 1. For the $b(X) = 1$ case example, as it has been introduced in [4] we use an arbitrary system with $n = 2$ and unity control gain and nonlinear system function as:

$$f(X) = -4\left(\frac{\sin(4\pi x)}{\pi x}\right)\left(\frac{\sin(\pi \dot{x})}{\pi \dot{x}}\right)^2 \quad (42)$$

Two different input signals, step signal and sinusoidal signal is used in different simulation runs. For step signal test $[0.1, 0]$ and for sinusoidal test, $[0.7, 0]$ is chosen for system states initial values.

The results for simulation are shown in Fig. 6 and Fig.7. Table 1 shows the numerical values used for different parameters used in this example simulation. Table 2 shows the mean squared error calculated from simulation results for example 1. As it can be seen in figures and the Table 2, the proposed VSRNN control scheme exhibits a better performance in high output noise while it has compensated the effect of external disturbance injected into system very well.

Table 1. Numerical values of parameters in VSRNN control design.

Parameters	Numerical values
k_d	10
λ	3
k_{sl}	10
Φ	0.05

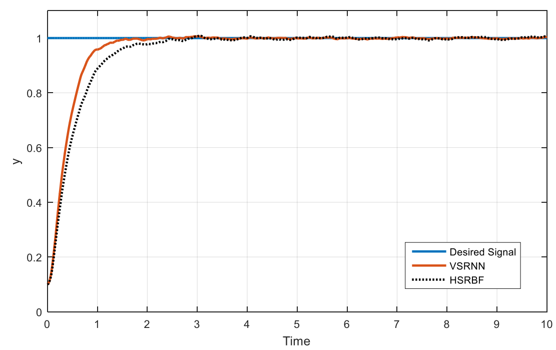

Fig. 6. Time response of the adaptive system; proposed method y_{VSRNN} ('_') and method in [4] y_{HSRBF} ('..'); Tracking of the step signal.

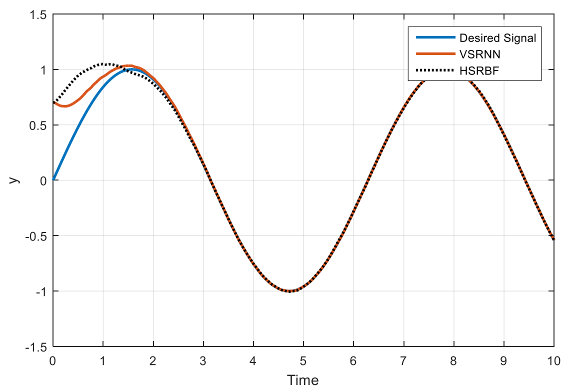

Fig. 7. Time response of the adaptive system; proposed method y_{VSRNN} ('_') and method in [4] y_{HSRBF} ('..'); Tracking of the sinusoidal signal.

Table 2. Mean Squared Error for results of VSRNN against HSRBF

Signal / Method	VSRNN	HSRBF
Step	0.0202	0.0245
Sinusoidal	0.0122	0.0231

Example 2. Fig. 4. shows the inverted pendulum system, which is a typical nonlinear system used to evaluate the efficiency of control algorithms. In this system, the input signal u forces the pendulum to keep an angular position θ , which is the output of this system. The nonlinear equations of this system will be as follow:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{g \sin x_1 - Mlx_2^2 \cos x_1 \sin x_1 / (m_c + M)}{l(4/3 - M \cos^2 x_1 / (m_c + M))} + \frac{\cos x_1 / (m_c + M)}{l(4/3 - M \cos^2 x_1 / (m_c + M))} u \quad (42)$$

Where, x_1 and x_2 will be, respectively, swing angle θ and swing rate, $g = 9.8 m/s^2$, $\dot{\theta}$ and $M = 0.1\,kg$ is the vehicle mass and $m_c = 1\,kg$ is mass for pendulum. $l = 0.5\,m$ is the half of the pendulum length and the u is the control input.

The parameters values for control of this plant is depicted in Table 1. The numerical values used for the controllers are achieved with trial and error.

Simulation results for this example is shown in Fig. 8 and Fig. 9. To verify the proper performance of proposed control, two different desired signals are used in separate simulation runs, step and sinusoidal signals. Random initial values are chosen for tunable parameters. For step signal test $[0.1, 0]$ and for sinusoidal test $[0.7, 0]$ is chosen for system states initial values.

For this example, Table 3 shows the design parameters values used in simulations and Table 4 shows the results of mean squared error calculated.

Table 3. Numerical values of parameters in VSRNN control design.

Parameters	Numerical values
k_d	10
λ	3
k_{sl}	10
Φ	0.05

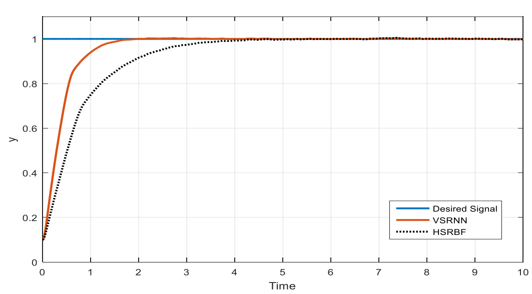

Fig. 8. Time response of the adaptive system; proposed method y_{VSRNN} ('_') and method in [4] y_{HSRBF} ('..'); Tracking of the step signal.

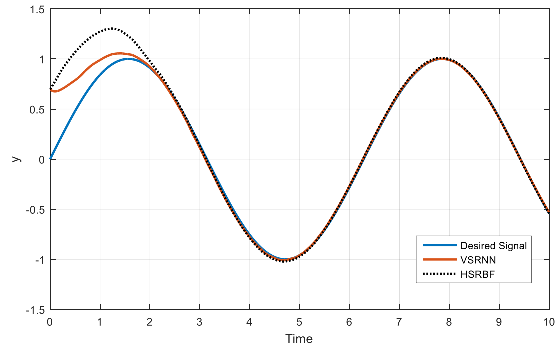

Fig. 9. Time response of the adaptive system; proposed method y_{VSRNN} ('_') and method in [4] y_{HSRBF} ('..'); Tracking of the sinusoidal signal.

Table 4. Mean Squared Error for results of VSRNN against HSRBF.

Signal / Method	VSRNN	HSRBF
Step	0.0193	0.0345
Sinusoidal	0.0149	0.0421

Again, with results showing in Fig. 8 and Fig. 9 and from Table 4, it can be seen that the proposed VSRNN for a dynamical benchmark system as an inverted pendulum is showing a far better performance especially under effect of output noise and external disturbance introduced to system.

Example 3. The second system used for simulations here is the Duffing oscillator which is a nonlinear second-order differential equation used for modeling certain damped and driven oscillators. The describing equations for this system in state space is as follows:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \gamma x_1 + \alpha x_1^3 + \delta x_2 + \varepsilon \cos(wt) + (3 + \cos(x_1))u(t) \quad (43)$$

In which, $u(t)$ is the control input to the system, $\varepsilon \cos(wt)$ is the bounded external disturbance. Other parameters in the equation (43) is defined as follows:

$$\gamma = 1,\ \alpha = \text{-}1,\ \delta = \text{-}0.15,\ \varepsilon = 0.15,\ w = 1 \quad (44)$$

The proposed control method is implemented for the Duffing oscillator. Again, two different desired signals, sinusoidal and pulse signal is used in different simulation runs and output noise is introduced to the system dynamics. The initial values for plants states are chosen same as the previous example. The results of controlling the system with proposed hybrid structure method is compared with a simple RBF neural network control and are shown in Fig. 10 and Fig. 11.

Like previous examples, Table 5 and Table 6 are showing information around design parameters and results respectively. It can be seen that for a chaotic system in hand, the VSRNN is proving to be a better and faster controller.

Table 5. Numerical values of parameters in VSRNN control design.

Parameters	Numerical values
k_d	10
λ	3
k_{sl}	10
Φ	0.05

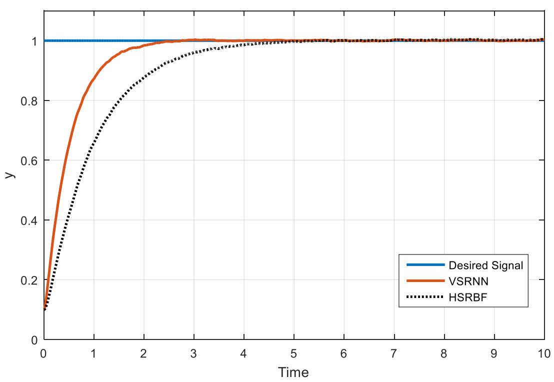

Fig. 10. Time response of the adaptive system; proposed method y_{VSRNN} ('_') and method in [4] y_{HSRBF} ('..'); Tracking of the step signal.

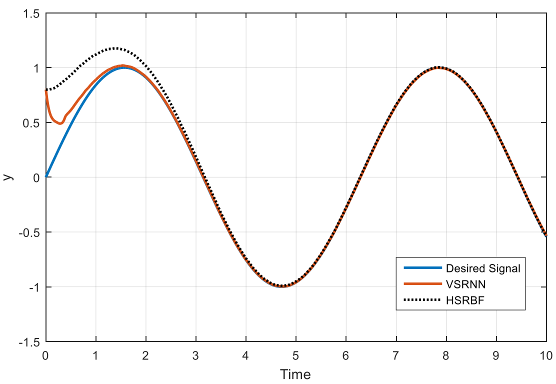

Fig. 11. Time response of the adaptive system; proposed method y_{VSRNN} ('_') and method in [4] y_{HSRBF} ('..'); Tracking of the sinusoidal signal.

Table 6. Mean Squared Error for results of VSRNN against HSRBF.

Signal / Method	VSRNN	HSRBF
Step	0.0226	0.0443
Sinusoidal	0.0069	0.0293

6. CONCLUSION

In this paper, we discussed the output tracking control for a class of high-order SISO nonlinear systems with canonical structure and we proposed a novel hybrid structure control method, utilizing nonlinear control skills, fuzzy and neural network properties to efficiently control two different exemplary benchmark systems. The nonlinear functions in the systems are considered to be completely unknown. A radial basis function rough neural network with excellent functionality in presence of output noise and external disturbances is used to achieve a proper tracking result. Additional terms are added to control signal to compensate the effect of external disturbances. Projection method is used for keeping the control parameters bounded. Overall closed loop stability of the system is guaranteed using Lyapunov theorem.

The proposed control is applied to two different benchmark system, a mechanical inverted pendulum and Duffing oscillator as a chaotic system. Results of simulations demonstrate that the proposed control method is capable of controlling the nonlinear systems with completely unknown functions in the presence of output noise and external disturbances. Against previous control methods, the comparing results show better precision and more robustness against real-life scenario.

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