Variational Capacitance Modeling Using Orthogonal Polynomial Method *

Jian Cui[†], Gengsheng Chen[‡], Ruijing Shen[†], Sheldon X.-D. Tan[†], Wenjian Yu[§] and Jiarong Tong[‡]

[†]Department of Electrical Engineering, University of California, Riverside, CA 92521
[‡]Department of Microelectronics, Fudan University, Shanghai, 200433, China
[§]Department of Computer Science and Technology, Tsinghua University, Beijing, 100084, China

ABSTRACT

In this paper, we propose a novel statistical capacitance extraction method for interconnects considering process variations. The new method, called *statCap*, is based on the spectral stochastic method where orthogonal polynomials are used to represent the statistical processes in a deterministic way. We first show how the variational potential coefficient matrix is represented in a first-order form using Taylor expansion and orthogonal decomposition. Then an augmented potential coefficient matrix, which consists of the coefficients of the polynomials, is derived. After that, corresponding augmented system is solved to obtain the variational capacitance values in the orthogonal polynomial form. Experimental results show that our method is two orders of magnitude faster than the recently proposed statistical capacitance extraction method based on the spectral stochastic collocation approach [18] and many orders of magnitude faster than the Monte Carlo method for several practical interconnect structures.

Categories and Subject Descriptors

J.6 [**Computer-Aided Engineering**]: Computer-aided design(CAD); I.6.5 [**Simulation and Modeling**]: Model Development; G.3 [**Probability and Statistics**]: Statistical computing

General Terms

Design, Algorithms

Keywords

Capacitance extraction, process variations, random variable reduction, orthogonal decomposition

1. INTRODUCTION

It is well accepted that the process-induced variability has huge impacts on the circuit performance in the sub-100nm VLSI technologies [12, 11]. The variational consideration of process has to be assessed in the various VLSI design steps to ensure robust circuit design. Process variations consist of both systematic ones, which depend on patterns and other process parameters, and random ones, which have to be dealt with using stochastic approaches.

To consider the variation impacts on the interconnects, one has to consider the RLC extraction processes of the three-dimensional structures modeling the interconnect conductors. In this paper, we investigate the geometry variational impacts on the extracted capacitance. Statistical extraction of capacitance considering process variations has been studied recently and several approaches have been proposed [7, 18, 4, 19, 17] under different variational models. Method in [7], uses analytical formulae to consider the variations in capacitance extraction and it has only first-order accuracy. The FastSies program considers the rough surface effects of the interconnect conductors[19]. It assumes only Gaussian distributions and has high computation costs. In [4], a method combining hierarchical extraction and principle factor analysis is proposed. The capacitance extraction is done based on second-order perturbation in this method, which can generate quadratic variational capacitance for better accuracy.

Recently, a spectral stochastic collocation based capacitance extraction method was proposed [18]. This approach is based on the Hermite orthogonal polynomial representation of the variational capacitance. It applies the collocation idea, where the capacitance extraction processes (by solving the potential coefficient matrices) are performed many times by sampling so that the coefficients of orthogonal polynomials of variational capacitance can be computed using the weighted least square method. The number of samplings is O(m^2), where m is the number of variables. So if m is large, the approach will lose it efficiency compared to the Monte Carlo method.

In this paper, instead of using collocation method, we use a different spectral stochastic method, where the Galerkin scheme is used. Galerkin-based Spectral stochastic method has been applied for statisitical interconenct modeling [14, 1] and on-chip power grid analysis consider process variations in the past [9, 8].

The new method, called *statCap*, first transforms the original stochastic potential coefficient equations into a deterministic and larger one and then solves it using iterative method. It avoids the sampling process in the existing collocation-based extraction approach. As a result, the potential coefficient equations and the corresponding augmented system only need to be set up once versus many times in the collocation based sampling method. This can lead to a significant saving in CPU time. Also the augmented potential coefficient system is sparse, symmetric and low-rank, which is further exploited by an iterative solver to gain further speedup. Experimental results show that the proposed method can deliver two orders of magnitude speedup over the collocation based spectral stochastic method and many orders of magnitude over the Monte Carlo method.

The rest of this paper is organized as follows: Section 2 presents statistical capacitance extraction problem to be solved. Section 3

reviews the orthogonal polynomial chaos based stochastic simulation methods and Section 4 presents our new statistical capacitance extraction method. Section 5 presents the experimental results and Section 6 concludes this paper.

2. PROBLEM FORMULATION

For m conductors system, the capacitance extraction problem based on the Boundary Element Method (BEM) formulation is to solve the following integral equation [10]:

$$\int_S \frac{1}{|r_i - r_j|} \rho(r_j) da_j = v(r_i) \tag{1}$$

where $\rho(r_j)$ is the charge distribution on the surface at conductor j, $v(r_i)$ is the potential at conductor i and $\frac{1}{|r_i-r_j|}$ is the free space Green function [1]. da_j is the surface area on conductor j. r_i and r_j are point vectors. To solve for capacitances from one conductor to the rest of others, we set the conductor's potential to be one and all other $m-1$ conductors' potential to be zero. The resulting charges computed are capacitances. BEM method divides the surfaces into N small panels and assume uniform charge distribution on each panel, which transforms (1) into a linear algebraic equation

$$Pq = v \tag{2}$$

where $P \in R^{N \times N}$ is the potential coefficient matrix, q is the charge on panels, v is the pre-set potential on each panel. By solving above linear equation, we can obtain all the panel charges (thus capacitance values). In potential coefficient matrix P, each element is defined as

$$P_{ij} = \frac{1}{s_j} \int_{S_j} G(x_i, x_j) da_j \tag{3}$$

where $G(x_i, x_j) = \frac{1}{|x_i - x_j|}$ is the Green function of point source x_j. S_j is the surface of panel j and s_j is the area of panel j.

Process variations introduce conductor geometry variations are reflected on the fact that the size of panel and distances between panels become random variables. Here we assume the panel is still a two dimensional surface. These variations will make each element in capacitance matrix follow some kinds of random distributions. The problem we need to solve now is to derive this random distribution and then to effectively compute the mean and variance of involved capacitance given geometry randomness parameters.

In this paper, we follows the variational model introduced in [4], where each point in panel i is disturbed by a vector Δn_i that has the same direction as the normal direction of panel i.

$$x_i' = x_i + \Delta n_i \tag{4}$$

where the length of the Δn_i follows Gaussian distribution $|\Delta n_i| \sim N(0, \sigma^2)$. If the value is negative, it means the direction of the perturbation is reversed. The correlation between random perturbation on each panel is governed by the empirical formulation such as the exponential model [20]

$$\gamma(r) = e^{-r^2/\eta^2} \tag{5}$$

where r is the distance between two panel centers and η is the correlation length.

The most straightforward method is to use Monte Carlo(MC) simulation to obtain distribution, mean and variance of all those capacitances. But MC method will be extremely time consuming as each sample run requires the formulation of the changed potential coefficient matrix P.

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[1] Note that the scale factor $1/(4\pi\varepsilon_0)$ can be ignored here to simplify the notation and is used in the implementation to give results in units of farads.

3. REVIEW OF SPECTRAL STOCHASTIC METHOD

In this section, we briefly review the spectral stochastic or orthogonal polynomial chaos (PC) based stochastic analysis methods.

3.1 Concept of Hermite polynomial chaos

In the following, a random variable $\xi(\theta)$ is expressed as a function of θ, which is the random event. Hermite PC utilizes a series of orthogonal polynomials (with respect to the Gaussian distribution) to facilitate stochastic analysis [3, 16]. These polynomials are used as orthogonal basis to decompose a random process.

We remark that for the Gaussian and log-normal distributions, using Hermite polynomials is the best choice as they lead to exponential convergence rate [3]. For non Gaussian and non log-normal distributions, there are other orthogonal polynomials such as Legendre for uniform distribution, Charlier for Poisson distribution and Krawtchouk for Binomial distribution, etc [2, 14].

Given a random variable $v(t, \xi)$ with variation, where $\xi = [\xi_1, ..., \xi_n]$ denotes a vector of orthonormal Gaussian random variables with zero mean, the random variable can be approximated by a truncated Hermite PC expansion as follows: [3]

$$v(t, \xi) = \sum_{k=0}^{P} a_k H_k^n(\xi), \tag{6}$$

where n is the number of independent random variables, $H_k^n(\xi)$ are n-dimensional Hermite polynomials, and a_k are the deterministic coefficients. The number of terms P is given by

$$P = \sum_{k=0}^{p} \frac{(n-1+k)!}{k!(n-1)!}, \tag{7}$$

where p is the order of the Hermite PC. For simplification of explanation, only one random variable is considered, and the one-dimensional Hermite polynomials are expressed as follows:

$$H_0^1(\xi) = 1, H_1^1(\xi) = \xi, H_2^1(\xi) = \xi^2 - 1, H_3^1(\xi) = \xi^3 - 3\xi, \dots \tag{8}$$

The Hermite polynomials are orthogonal with respect to Gaussian weighted expectation (the superscript n is dropped for simple notation):

$$< H_i(\xi), H_j(\xi) >=< H_i^2(\xi) > \delta_{ij}, \tag{9}$$

where δ_{ij} is the Kronecker delta and $< *, * >$ denotes an inner product defined as:

$$< f(\xi), g(\xi) >= \frac{1}{\sqrt{(2\pi)^n}} \int f(\xi) g(\xi) e^{-\frac{1}{2}\xi^T \xi} d\xi. \tag{10}$$

Thus, the coefficients, a_k, are evaluated by the projection operation onto the Hermite PC basis:

$$a_k(t) = \frac{< v(t, \xi), H_k(\xi) >}{< H_k^2(\xi) >}, \forall k \in \{0, \dots, P\}. \tag{11}$$

3.2 Computing coefficients, the mean and variance from Hermite PCs

In case that $q(\xi)$ in (2) is unknown random variable vector (with normal distribution), then potential coefficient equation become

$$P(\xi)q(\xi) = v \tag{12}$$

Where both $P(\xi)$ and $q(\xi)$ are in Hermite PC form. Then the coefficients can be computed by using Galerkin method. The principle of orthogonality states that the best approximation of $v(\xi)$ is obtained when the error, $\Delta(\xi)$, defined as

$$\Delta(\xi) = P(\xi)q(\xi) - v \tag{13}$$

is orthogonal to the approximation. That is

$$< \Delta(\xi), H_k(\xi) >= 0, k = 0, 1, \dots, P, \tag{14}$$

where, $H_k(\xi)$ are Hermite polynomials. In this way, we have transformed the stochastic analysis process into a deterministic form, whereas we only need to compute the corresponding coefficients of the Hermite PC.

For the illustration purpose, considering two Gaussian variable $\xi = [\xi_1, \xi_2]$, we assume that the charge vector in panels can be written as a second order ($p = 2$) Hermite PC, we have

$$
\begin{aligned}
q(\xi) &= q_0 + q_1\xi_1 + q_2\xi_2 + q_3(\xi_1^2 - 1) + \\
&\quad q_4(\xi_2^2 - 1) + q_5(\xi_1\xi_2).
\end{aligned} \tag{15}
$$

which will be solved by using augmented potential coefficient matrices to be discussed in Section 4. Once the Hermite PC of $q(\xi)$ is known, the mean and variance of $q(\xi)$ can be evaluated trivially. Given an example, for one random variable, the mean and variance are calculated as:

$$
\begin{aligned}
E(q(\xi)) &= q_0 \\
Var(q(\xi)) &= q_1^2 Var(\xi) + q_2^2 Var(\xi^2 - 1) \\
&= q_1^2 + 2q_2^2.
\end{aligned} \tag{16}
$$

In consideration of correlations among random variables, we apply principal component analysis (PCA) to transform the correlated variables into a set of independent variables.

4. NEW ORTHOGONAL POLYNOMIAL BASED EXTRACTION METHOD: STAT-CAP

In this section, we present our new spectral stochastic method based method, *statCap*, which uses the orthogonal polynomials to represent random variables starting from the geometry parameters. This is contrast to existing approach [18], where only the capacitances are presented by the orthogonal polynomials.

In our new method, we first represent the variation potential matrix P into a first-order form using Taylor expansion. We want to stress that our method can be further extended to the second-order based extraction.

4.1 Expansion of potential coefficient matrix

Specifically, each element in the potential coefficient matrix P can be expressed as:

$$
P_{ij} = \frac{1}{s_j} \int_{S_j} G(x_i, x_j) da_j \tag{17}
$$

where $G(x_i, x_j)$ is the free space Green function define in (3).

Notice that if panel i and panel j are far away (their distance is much larger than the panel area), we can have the following approximation [4]:

$$
P_{ij} \approx G(x_i, x_j) \quad i \neq j \tag{18}
$$

Suppose variation of panel i can be written as $\Delta n_i = \delta i \, \vec{n_i}$ where $\vec{n_i}$ is the unit normal vector of panel i and δi is the scalar variation. Then use Taylor expansion on the Green function,

$$
G(x_i + \Delta n_i, x_j + \Delta n_j) = \frac{1}{|x_i - x_j + \Delta n_i - \Delta n_j|} \tag{19}
$$

$$
= \frac{1}{|x_i - x_j|} + \nabla \frac{1}{|x_i - x_j|} \cdot (\Delta n_j - \Delta n_i) + O((\Delta n_i - \Delta n_j)^2) \tag{20}
$$

From free space Green function, we have

$$
\nabla G(x_i, x_j) = \nabla \frac{1}{|x_i - x_j|} = \nabla \frac{1}{|\vec{r}|} = \frac{\vec{r}}{|\vec{r}|^3} \tag{21}
$$

$$
\vec{r} = x_i - x_j \tag{22}
$$

In this paper, we ignore the second order terms to make the variation in the affine form. As a result, the potential coefficient matrix P can be written as

$$
P \approx P_0 + P_1 = \\
\begin{pmatrix}
G(x_1 + \Delta n_1, x_1 + \Delta n_1) & \dots & G(x_1 + \Delta n_1, x_n + \Delta n_n) \\
G(x_2 + \Delta n_2, x_1 + \Delta n_1) & \dots & G(x_2 + \Delta n_2, x_n + \Delta n_n) \\
\dots & \dots & \dots \\
G(x_n + \Delta n_n, x_1 + \Delta n_1) & \dots & G(x_n + \Delta n_n, x_n + \Delta n_n)
\end{pmatrix} \tag{23}
$$

where

$$
P_0 = \begin{pmatrix}
G(x_1, x_1) & G(x_1, x_2) & \dots & G(x_1, x_n) \\
G(x_2, x_1) & G(x_2, x_2) & \dots & G(x_2, x_n) \\
\dots & \dots & \dots & \dots \\
G(x_n, x_1) & G(x_n, x_2) & \dots & G(x_n, x_n)
\end{pmatrix}
$$

$$
P_1 = \begin{pmatrix}
0 & \dots & \nabla G(x_1, x_n) \cdot (\Delta n_n - \Delta n_1) \\
\nabla G(x_2, x_1) \cdot (\Delta n_1 - \Delta n_2) & \dots & \nabla G(x_2, x_n) \cdot (\Delta n_n - \Delta n_2) \\
\dots & \dots & \dots \\
\nabla G(x_n, x_1) \cdot (\Delta n_1 - \Delta n_n) & \dots & 0
\end{pmatrix}
$$

We can further write the P_1 as the following form:

$$
P_1 = V_1 \cdot N_1 \cdot J_1 - J_1 \cdot N_1 \cdot V_1 \tag{24}
$$

$$
J_1 = \begin{pmatrix}
0 & \nabla G(x_1, x_2) & \dots & \nabla G(x_1, x_n) \\
\nabla G(x_2, x_1) & 0 & \dots & \nabla G(x_2, x_n) \\
\dots & \dots & \dots & \dots \\
\nabla G(x_n, x_1) & \dots & \nabla G(x_n, x_{n-1}) & 0
\end{pmatrix}
$$

where

$$
N_1 = \begin{pmatrix}
\vec{n_1} & 0 & \dots \\
0 & \vec{n_2} & \dots \\
\dots & \dots & \dots \\
0 & \dots & \vec{n_n}
\end{pmatrix}
$$

$$
V_1 = \begin{pmatrix}
\delta n_1 & 0 & \dots \\
0 & \delta n_2 & \dots \\
\dots & \dots & \dots \\
0 & \dots & \delta n_n
\end{pmatrix}
$$

J_1 and N_1 are vector matrices and V_1 is a diagonal matrix.

To deal with spatial correlation, P_1 can be further expressed as linear combination of the dominate and independent variables

$$
\xi = [\xi_1, \xi_2, \dots, \xi_p] \tag{25}
$$

through the principal component analysis (PCA) operation. As a result, J_1 can be further expressed as

$$
\begin{pmatrix}
a_{11}\xi_1 + \dots + a_{1p}\xi_p & 0 & \dots \\
0 & a_{21}\xi_1 + \dots + a_{2p}\xi_p & \dots \\
\dots & \dots & \dots \\
\dots & 0 & a_{n1}\xi_1 + \dots + a_{np}\xi_p
\end{pmatrix}
$$

Finally we can represent the P_1 as

$$
P_1 = \sum P_{1i}\xi_i \tag{26}
$$

where

$$
P_{1i} = A_i \cdot N_1 \cdot J_1 - J_1 \cdot N_1 \cdot A_i \tag{27}
$$

and

$$
A_i = \begin{pmatrix}
a_{1i} & 0 & \dots & 0 \\
0 & a_{2i} & \dots & 0 \\
\dots & \dots & \dots & \dots \\
0 & \dots & 0 & a_{ni}
\end{pmatrix} \tag{28}
$$

4.2 Formulation of the augmented system

Once the potential coefficient matrix is represented in the affine form as shown in (26), we are ready to solve for the coefficients P_{1i} by using the Galerkin method, which will result in a larger system with augmented matrices and variables.

Specifically, for p independent Gaussian random variables $\xi = [\xi_1, \ldots, \xi_p]$, there are $K = 2p + p(p-1)$ first and second order Hermite polynomials. $H_i(\xi)$ $i = 1, \ldots, K$ represents each Hermite polynomial and $H_1 = \xi_1, \ldots, H_p = \xi_p$. So for the vector of variational potential variables $q(\xi)$, it can be written as:

$$q(\xi) = q_0 + \sum_{i=1}^{K} q_i H_i(\xi) \tag{29}$$

where each q_i is a vector associated with one polynomial. So the random linear equation can be written as:

$$Pq = (P_0 + \sum_{i=1}^{p} P_{1i} H_i)(q_0 + \sum_{i=1}^{K} q_i H_i) = v \tag{30}$$

Expanding the equation and performing inner product with H_i on both side, we can derive a new linear system equation:

$$(W_0 \otimes P_0 + \sum_{i=1}^{p} W_i \otimes P_{1i})Q = V \tag{31}$$

where $\otimes$ is the tensor product and

$$Q = \begin{pmatrix} q_0 \\ q_1 \\ \ldots \\ q_K \end{pmatrix}; \; V = \begin{pmatrix} v \\ 0 \\ \ldots \\ 0 \end{pmatrix} \tag{32}$$

and

$$W_i = \begin{pmatrix} <H_iH_0H_0> & <H_iH_0H_1> & \ldots & <H_iH_0H_K> \\ <H_iH_1H_0> & <H_iH_1H_1> & \ldots & <H_iH_1H_K> \\ \ldots & \ldots & <H_iH_lH_m> & \ldots \\ <H_iH_KH_0> & <H_iH_KH_1> & \ldots & <H_iH_KH_K> \end{pmatrix} \tag{33}$$

where $<H_iH_0H_0>$ represent the inner product of three Hermite polynomial H_i, H_0, H_0. The matrix $(W_0 \otimes P_0 + \sum_{i=1}^{p} W_i \otimes P_{1i})$ in (31) is called the *augmented potential coefficient* matrix. Since H_i are at most second order polynomials, we can quickly calculate every element in W_i with a lookup table for any number of random variables.

We remark that matrices W_i are very sparse due to the nature of the inner product. As a result, their tenor products with P_{1i} will also lead to the very sparse augmented matrix in (31). As a result, we have the following observations regarding the structure of the W_i and the augmented matrix.

1. Observation 1: W_0 is a diagonal matrix.

2. Observation 2: For W_i matrices, $i \neq 0$, all the diagonal elements are zero.

3. Observation 3: All W_i are symmetric and the resulting augmented matrix $W_0 \otimes P_0 + \sum_{i=1}^{p} W_i \otimes P_{1i}$ is also symmetric.

4. Observation 4: If one element at position (l, m) in W_i is not zero, i.e. $W_i(l, m) \neq 0$, then elements at the same position (l, m) of W_j, $j \neq i$, must be zero. In other words,

$$W_i(l, m) \cdot W_j(l, m) = 0 \quad \text{when } i \neq j$$
$$\forall \, i, j = 1, \ldots, P \text{ and } l, m = 1, \ldots, K$$

Such sparse property can help save the memory significantly as we do not need to actually perform the tensor product as shown in (31). Instead, we can add all W_i together and expand each element in the resulting matrix by some specific P_{1i} during the solving process, as there is no overlap among W_i for any element position.

As the original potential coefficient matrix is low rank, the augmented matrix is also low rank. As a result, the sparsity, low rank and symmetric properties can be exploited by iterative solvers to speed up the extraction process as shown in the experimental results. In our implementation, the Minimum Residue Conjugate Gradient method [13] is used as the solver since the augmented system is symmetric.

5. EXPERIMENTAL RESULTS

In this section, we compare the results of the proposed *statCap* method against the Monte Carlo method and the SSCM method [18], which is based on the spectral stochastic collocation method. The proposed method *statCap* has been implemented in Matlab 7.0. We use Minimum Residue Conjugate Gradient method as the iterative solver. We also implement the SSCM method in Matlab using the sparse grid package [6, 5]. We do not use any hierarchical algorithm to accelerate the calculation of the potential coefficient matrix for both *statCap* and SSCM. Instead, we use analytic formula in [15] to compute the potential coefficient matrices.

All the experimental results are carried out in a Linux system with dual Intel Xeon CPUs with 3.06Ghz and 2GB memory.

We test our algorithm on three test cases. The three examples are 1×1 bus with 28 panels, 2×2 bus with 352 panels, which is shown in Fig. 1, and three-layer metal plane capacitance with 75 panels shown in Fig. 2. In all experiments, we set standard deviation as 10% of the wire width and the η, the correlation length, as 200% of the wire width.
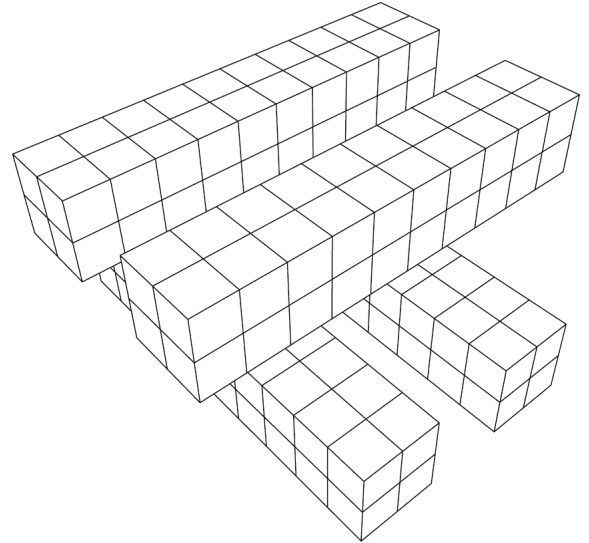

Figure 1: A 2×2 bus.

First, we compare the CPU times of the three methods. The results are summarized in Table 1, where MC(1000) means that 1000 runs used in MC method. The more specific running parameters, number of panels and number of random variables for each testing cases are summerized in Table 2 to Table 7. In those tables, *statCap* refers to our proposed method. All the capacitances are in picofarad.

It can be seen that *statCap* is much faster than both SSCM and Monte Carlo method. For three cases, *statCap* can deliver about two orders of magnitude speedup over the SSCM and three orders of magnitude speedup over Monte Carlo method. Notice that both SSCM and *statCap* use the same random variables after PCA reduction.
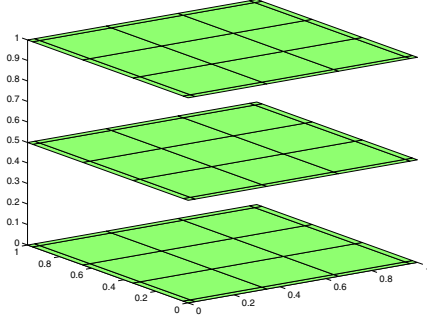
Figure 2: 3-layer metal planes.

Table 1: CPU Runtime (in seconds) comparison among MC, SSCM and *statCap*.

1×1 bus, MC (10000)					
	MC	SSCM	*statCap*	SP(MC)	SP(SSCM)
time(s)	9414	118	3.2	2942	37
2×2 bus, MC(6000)					
	MC	SSCM	*statCap*	SP(MC)	SP(SSCM)
time(s)	286620	10588	268	1069	39
3-layer metal plane, MC(2000)					
	MC	SSCM	*statCap*	SP(MC)	SP(SSCM)
time(s)	5479	387	7.6	721	51

We notice that both Monte Carlo and SSCM need to compute the potential coefficient matrices each time the geometry changes. This computation can be significant compared to the CPU time of solving potential coefficient equations. This is one of the reasons that SSCM and MC are much slower than *statCap*, in which the augmented system only needs to be setup once.

Also SSCM uses the sparse grid scheme to reduce the collocation points in order to derive the orthogonal polynomial coefficients. But the number of collocation points are still $O(m^2)$ for second-order Hermit polynomials, where m is the number of variables. Thus it requires $O(m^2)$ solutions to the changed geometry. In our algorithm, we only need to solve the augmented system once. The solving process can be further improved by using some advanced solver or acceleration techniques.

Next, we perform the accuracy comparison. We test the three algorithms on 1×1 bus and 2×2 bus examples. The statistics for 1×1 bus case from the three algorithms are summarized in Table 2 and Table 3 for the mean and variance values respectively. In both tables, *PCA R.V.* stands for the number of random variables after the PCA operations. We make sure that both *statCap* and SSCM use the same number of random variables.

Table 2: Capacitance mean values for the 1×1 bus.

Panel num=28, $\sigma = 0.1$, $\eta = 2$			
PCA R.V. num=9			
	MC	SSCM	*statCap*
C11	135.93	135.99	136.58
C12	-57.12	-57.26	-57.49
C21	-57.12	-57.13	-57.49
C22	135.97	135.94	136.58

Table 3: Variance values for the 1×1 bus.

Panel num=28, $\sigma = 0.1$, $\eta = 2$			
PCA R.V. num=9			
	MC	SSCM	*statCap*
C11	2.42	2.49	3.13
C12	1.71	1.74	2.02
C21	1.72	1.71	2.02
C22	2.51	2.52	3.19

From the tables, we can see that both SSCM and *statCap* give similar results compared with the MC method.

For 2×2 bus case, we carry out 6000 times runs for Monte Carlo simulation. The results are summarized in Table 4 and Table 5.

Table 4: Capacitance mean values for the 2×2 bus.

2x2 Bus			
Panel num=352, $\sigma = 0.1$, $\eta = 2$			
PCA R.V. num=10			
	MC	SSCM	*statCap*
C11	242.54	242.41	242.63
C12	-82.46	-82.48	-82.60
C13	-47.48	-47.34	-47.34
C14	-47.42	-47.33	-47.35
C22	242.34	242.07	242.65
C23	-47.31	-47.24	-47.36
C24	-47.25	-47.15	-47.36
C33	242.52	241.99	242.64
C34	-82.50	-82.25	-82.60
C44	242.47	242.02	242.64

Table 5: Variance values for the 2×2 bus.

Panel num=352, $\sigma = 0.1$, $\eta = 2$			
PCA R.V. num=10			
	MC	SSCM	*statCap*
C11	3.15	2.72	3.36
C12	3.36	3.03	3.39
C13	2.06	1.94	2.17
C14	1.69	1.61	1.74
C22	4.09	3.69	4.41
C23	2.05	2.04	2.24
C24	1.81	1.76	1.99
C33	3.35	2.79	3.58
C34	3.21	2.89	3.32
C44	4.18	3.74	4.58

From Table 4 and Table 5, we can see that both *statCap* and SSCM give the similar results for both mean and variance also.

The similar results of the 3-layer metal layers are summarized in Table 6 and Table 7, for mean and variance values. The MC takes 2000 runs for this case.

6. CONCLUSION

In this paper, we have proposed a novel statistical capacitance extraction method, called *statCap*, for three-dimensional interconnects considering process variations. The new method is based on the spectral stochastic method where orthogonal polynomials are used to represent the variational geometrical parameters in a deterministic way. The new method avoids the sampling operations

Table 6: The mean values of the 3-layer plane.

Panel num=75, $\sigma = 0.1$, $\eta = 1$ PCA R.V. num=8			
	MC	SSCM	*statCap*
C11	53.53	53.47	52.76
C12	-23.52	-23.47	-22.91
C13	-5.24	-5.25	-5.14
C22	63.56	63.40	62.42

Table 7: The variance values of the 3-layer plane.

Panel num=75, $\sigma = 0.1$, $\eta = 1$ PCA R.V. num=8			
	MC	SSCM	*statCap*
C11	2.19	2.0	2.02
C12	2.52	2.34	2.42
C13	0.21	0.21	0.18
C22	3.34	3.02	3.13

in the existing spectral stochastic method. It solves an enlarged potential coefficient system to obtain the coefficients of orthogonal polynomails for capacitances. *statCap* only needs to set up the augmented equation once and can exploit the sparsity and low-rank property to speedup the extraction process. Experimental results show that our method is two orders of magnitude faster than the recently proposed statistical capacitance extraction method based on the spectral stochastic collocation method and many orders of magnitude faster than the Monte Carlo method for several practical interconnect structures.

7. REFERENCES

[1] J. Fan, N. Mi, S. X.-D. Tan, Y. Cai, and X. Hong, "Statistical model order reduction for interconnect circuits considering spatial correlations," in *Proc. European Design and Test Conf. (DATE)*, 2007, pp. 1508–1513.

[2] R. Ghanem, "The nonlinear Gaussian spectrum of log-normal stochastic processes and variables," *Journal of Applied Mechanics*, vol. 66, pp. 964–973, December 1999.

[3] R. G. Ghanem and P. D. Spanos, *Stochastic Finite Elements: A Spectral Approach*. Dover Publications, 2003.

[4] R. Jiang, W. Fu, J. M. Wang, V. Lin, and C. C.-P. Chen, "Efficient statistical capacitance variability modeling with orthogonal principle factor analysis," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, 2005, pp. 683–690.

[5] A. Klimke, "Sparse Grid Interpolation Toolbox – user's guide," University of Stuttgart, Tech. Rep. IANS report 2006/001, 2006.

[6] A. Klimke and B. Wohlmuth, "Algorithm 847: spinterp: Piecewise multilinear hierarchical sparse grid interpolation in MATLAB," *ACM Transactions on Mathematical Software*, vol. 31, no. 4, 2005.

[7] A. Labun, "Rapid method to account for process variation in full-chip capacitance extraction," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 23, pp. 941–951, June 2004.

[8] N. Mi, J. Fan, S. X.-D. Tan, Y. Cai, and X. Hong, "Statistical analysis of on-chip power delivery networks considering lognormal leakage current variations with spatial correlations," *IEEE Trans. on Circuits and Systems I: Fundamental Theory and Applications*, 2008, in press.

[9] N. Mi, S. X.-D. Tan, P. Liu, J. Cui, Y. Cai, and X. Hong, "Stochastic extended Krylov subspace method for variational analysis of on-chip power grid networks," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, 2007, pp. 48–53.

[10] K. Narbos and J. White, "FastCap: a multipole accelerated 3D capacitance extraction program," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 10, no. 11, pp. 1447–1459, 1991.

[11] S. Nassif, "Delay variability: sources, impact and trends," in *Proc. IEEE Int. Solid-State Circuits Conf.*, San Francisco, CA, Feb 2000, pp. 368–369.

[12] ——, "Design for variability in DSM technologies," in *Proc. Int. Symposium. on Quality Electronic Design (ISQED)*, San Jose, CA, Mar 2000, pp. 451–454.

[13] C. C. Paige and M. A. Saunders, "Solution of sparse indefinite systems of linear equations," *SIAM J. on Numerical Analysis*, no. 4, pp. 617–629, September 1975.

[14] S. Vrudhula, J. M. Wang, , and P. Ghanta, "Hermite polynomial based interconnect analysis in the presence of process variations," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 25, no. 10, 2006.

[15] D. Wilton, S. Rao, A. Glisson, D. Schaubert, O. Al-Bundak, and C. Butler, "Potential integrals for uniform and linear source distributions on polygonal and polyhedral domains," *IEEE Trans. on Antennas and Propagation*, vol. AP-32, no. 3, pp. 276–281, March 1984.

[16] D. Xiu and G.Karniadakis, "Modeling uncertainty in flow simulations via generalized polynomial chaos," *J. of Computational Physics*, no. 187, pp. 137–167, 2003.

[17] Y. Zhou, Z. Li, Y. Tian, W. Shi, and F. Liu, "A new methodology for interconnect parasitics extraction considering photo-lithography effects," in *Proc. Asia South Pacific Design Automation Conf. (ASPDAC)*, Jan. 2007, pp. 450–455.

[18] H. Zhu, X. Zeng, W. Cai, J. Xue, and D. Zhou, "A sparse grid based spectral stochastic collocation method for variations-aware capacitance extraction of interconnects under nanometer process technology," in *Proc. European Design and Test Conf. (DATE)*, 2007, pp. 1514–1519.

[19] Z. Zhu and J. White, "FastSies: a fast stochastic integral equation solver for modeling the rough surface effect," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, 2005, pp. 675–682.

[20] Z. Zhu, J. White, and A. Demir, "A stochastic integral equation method for modeling the rough surface effect on interconnect capacitance," in *ICCAD '04: Proceedings of the 2004 IEEE/ACM International conference on Computer-aided design*. Washington, DC, USA: IEEE Computer Society, 2004, pp. 887–891.