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Various Schemes of Neutrino Mixing

Tsutomu YANAGIDA and Motohiko YOSHIMURA*

Physics Department, College of General Education, Tohoku Univesity, Sendai 980

**National Laboratory for High Energy Physics (KEK), Ibaraki 305*

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We make a few comments that have to be kept in mind in analyzing phenomena of neutrino oscillation. Topics discussed include the most general oscillation scheme containing both Dirac- and Majorana-type of mass mixing, and effects of *CP*-violation.

Recently, there have been many discussions on possible neutrino oscillation both from purely theoretical[1)] and from phenomenological[2)] viewpoints. With forthcoming, new experiments in mind, it would be useful to base analysis of oscillation phenomena on schemes as general as possible. In this short note we shall focuss on a few aspects of neutrino oscillation which have so far escaped much attention. Topics discussed here include the most general oscillation scheme that incorporates both Dirac- and Majorana-type of mass mixing, and effects of CP-violation in neutrino oscillation.

Neutrino oscillation occurs only if two conditions are met; (1) that there are mass differences between mass eigenstates ν_i; (2) that the weak eigenstates ν_α (distinguished by Greek suffices) in charged current processes are mixtures of mass eigenstates through a unitary transformation, $\nu_\alpha = U_{\alpha i}\nu_i$. Depending on types of mass terms one can think of three mixing schemes: (A) Dirac-type; here (diagonal) neutrinos are described by massive four-component Dirac spinors, and the usual weak interactions as well as neutrino mass terms conserve the lepton number. This is the mixing scheme originally introduced by Maki, Nakagawa and Sakata.[3)] (B) Majorana-type; in this case two states of a neutrino for each mass are distinguished by their helicities, $h=\pm$, and there is no distinction between a neutrino and its antineutrino. (C) Mixed-type; both of Dirac-type and Majorana-type mass terms are introduced. There are four states for each family of neutrino unlike the scheme (B). This is the most general mixing scheme and was first introduced by Bilenky and Pontecorvo.[3)]

In principle, all the three schemes can be distinguished experimentally, but this distinction depends very much on how the lepton nonconservation is incorporated in the entire weak interaction scheme. Here, we take the view that ordinary, charged and neutral current processes are described by the standard gauge theory[4)] which is only modified by finite neutrino masses, and leave open various possibilities[1)] for the origin of neutrino mass terms without specifying any one of them. In this picture the lepton nonconservation, if it exists, is dominantly caused by neutrino mass terms. Hence its effects are significantly suppressed by small neutrino masses, which makes it virtually impossible to distinguish the first two schemes, (A) and (B). On the other hand, the scheme (C) can give entirely different description of oscillation phenomena, which we shall now explain briefly. Suppose that the neutrino mass terms are written as follows by using two independent fields of opposite chiralities, ν_L and ν_R,

$$L_m = \frac{1}{2}(\overline{\nu^c_{L\alpha}} m^L_{\alpha\beta} \nu_{L\beta} + \overline{\nu^c_{R\alpha}} m^R_{\alpha\beta} \nu_{R\beta}) + \bar{\nu}_{R\alpha} m_{\alpha\beta} \nu_{L\beta} + (\text{h.c.}). \quad (1)$$

Here $L, R = (1 \mp \gamma_5)/2$ are the usual chiral projections and the superscript c denotes the charge conjugation, $\nu_L{}^c = (\nu_L)^c$, etc. From the Fermi statistics both matrices of m^L and m^R are symmetric. Note that the first two terms in Eq. (1) are of Majorana-type (lepton number nonconserving) and the third term is of Dirac-type. In this general formalism the scheme (A) corresponds to a special case $m^L = m^R = 0$, and the scheme (B) corresponds to a case $m^R = m = 0$. Mass diagonalization mixes ν_L and ν_R in a delicate manner. In terms of $N_F \times N_F$ matrices $U^{(i)}$ (N_F=number of families), the weak eigenstates ν_α are related to the mass eigenstates ν_i by

$$\nu_{L\alpha} = U^{(1)}_{\alpha i} \nu_{Li} + U^{(2)}_{\alpha i} \nu^c_{Ri}, \quad (2)$$

$$\nu^c_{R\alpha} = U^{(3)}_{\alpha i} \nu_{Li} + U^{(4)}_{\alpha i} \nu^c_{Ri}. \quad (3)$$

Diagonal masses are of Majorana-type, namely,

$$L_m = \frac{1}{2}(\overline{\nu^c_{Li}} m^L_{Di} \nu_{Li} + \overline{\nu^c_{Ri}} m^R_{Di} \nu_{Ri}) + (\text{h.c.})$$

and given by

$$\begin{pmatrix} m_D{}^L & 0 \\ 0 & m_D{}^{R\dagger} \end{pmatrix} = \begin{pmatrix} U^{(1)} & U^{(2)} \\ U^{(3)} & U^{(4)} \end{pmatrix} \begin{pmatrix} m^L & m^T \\ m & m^{R\dagger} \end{pmatrix} \begin{pmatrix} U^{(1)} & U^{(2)} \\ U^{(3)} & U^{(4)} \end{pmatrix}. \quad (4)$$

Unitarity means that $U^{(1)\dagger} U^{(1)} + U^{(2)\dagger} U^{(2)} = 1$, etc.

An important feature of this mixing scheme is that only a half of the neutrino states $\nu_{L\alpha}$ (hereafter called active components) participates in the usual weak interactions and another half of them $\nu_{R\alpha}$ is sterile. In the ordinary oscillation experiments production and detection of neutrinos are "measured" by the usual weak processes, thus at a superficial level the unitarity, namely conservation of probabilities, appears to be violated. For example, the total probability of observing all active neutrinos at a distance D from a neutrino source is not unity in general since

$$\sum_\beta |\langle \nu_{L\beta}; D | \nu_{L\alpha} \rangle|^2 = 1 - \sum_\beta |\langle \nu^c_{R\beta}; D | \nu_{L\alpha} \rangle|^2 \neq 1. \quad (5)$$

This possibility calls for a special caution to be taken in performing a model-independent analysis of oscillation phenomena. The idea[5] of using neutral current (N.C.) processes for a flux normalization is not justified on these general grounds, because event rate induced by the neutral current is given by Eq. (5). Only in the first two cases of the oscillation schemes (A) and (B) the neutral current is diagonal and complete in the unitarity sum, and the number of total events induced by N.C. does not oscillate as the distance D is varied. Instead, in the general case (C) this number of N. C. events can change when the source-detector distance is varied. Thus, the measurement of the number of N. C. events vs. D can be used to probe whether there are indeed sterile neutrinos communicating with active components. In practice, it may turn out very difficult to analyze oscillation data with sterile components taken into account. Nevertheless, we want to emphasize the importance of this sterile component because, if confirmed, its existence would immediately exclude the simple $SU(5)$ model[6] without any doubt. So far we assumed that the sterile component ν_R has vanishing weak isospin, but it is straightforward to extend to the case of a finite weak isospin.

The next subject we wish to discuss is effect of CP-violation on neutrino oscillation. We believe that there are at least two effects of CP-violation already observed in nature: One is the CP-violation in the K_L-K_S complex, and the other is the cosmological baryon asymmetry which may be explained[7] by baryon nonconserving and CP-violating processes in the very early universe. Relation of these two types of CP-violation is not illuminated so far, but one thing is clear; the phase factor characterizing the CP-violation in baryon nonconserving processes must be sizable, namely of $O(1)$, in order to explain[7] the amount of the observed baryon excess, N_B/N_γ of order 10^{-9}. It is perfectly possible that CP-violation to be expected in neutrino oscillation is also sizable, thus it becomes important to find out good ways of detecting effects of CP-violation is various kinds of oscillation experiments. Most direct way of checking the CP-violation would be to measure the transition rates,

$$P(\alpha\to\beta, D)\equiv|\langle\nu_{L\beta};\, D|\nu_{L\alpha}\rangle|^2 \tag{6}$$

at various distances D from a source. For illustration, suppose that there is no sterile component and that a hierarchy of mass differences among three neutrinos,

$$|\varDelta m^2_{23}|\sim|\varDelta m^2_{31}|\gg|\varDelta m^2_{12}| \tag{7}$$

exists ($\varDelta m^2_{ij}\equiv m_i{}^2-m_j{}^2$). Then, one can separate two regions, $D\sim D_1\sim D_2$ and $D\sim D_3$, where

$$D_i\equiv\frac{1}{2}\,\varepsilon_{ijk}\frac{4\pi E_\nu}{|\varDelta m^2_{jk}|}\,. \tag{8}$$

As usual, we have assumed that the neutrino energy $E_\nu\gg|m_i|$. Effects of CP-violation in neutrino oscillation are all related to the phase factors defined by

$$\phi_{\alpha i\beta j}=\arg(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}), \tag{9}$$

which is nonvanishing only for $\alpha\neq\beta$. A simple way of extracting one of the individual oscillation terms would be to take a difference between the transition rate at $D\sim D_3$ and an averaged (either over the energy or distance) rate at $D\gg D_3$, which is given by

$$P(\alpha\to\beta, D)-\langle P(\alpha\to\beta, D)\rangle_3$$
$$=2|U_{\alpha1}U_{\beta2}U_{\alpha2}U_{\beta1}|$$
$$\times\cos\left(2\pi\frac{D}{D_3}\mp\phi_{\alpha1\beta2}\right)\,. \tag{10}$$

By collecting a large amount of data it should be able to determine both the amplitude and the phase of this formula (10). In practice, things may become more complicated. If all mass differences are comparable unlike Eq. (7), the kind of analysis suggested here is not valid, and one needs a more elaborate analysis.

Mixed neutrino beams also give valuable information on CP-violation in the neutrino sector. For instance, neutrino beams in beam dump experiments may contain equal mixtures of ν_e, ν_μ, $\bar\nu_e$ and $\bar\nu_\mu$ originating from charm particle decays. (We ignore a possible contamination of τ-neutrino.) Integrating out all momenta of detected charged leptons at a single distance D, one finds relative yields of charged leptons given by

$$w(e^-)=P_{ee}+P_{\mu e},\ w(e^+)=\frac{1}{3}(P_{\bar e\bar e}+P_{\bar\mu\bar e}),$$
$$w(\mu^-)=P_{\mu\mu}+P_{e\mu},\ w(\mu^+)=\frac{1}{3}(P_{\bar\mu\bar\mu}+P_{\bar e\bar\mu}), \tag{11}$$

where $P_{\alpha\beta}=P(\alpha\to\beta, D)$ with α and β active components alone, and the factor 1/3 is due to the ratio of antineutrino to neutrino cross sections. This formula (11) and subsequent ones are valid for all schemes of (A) $\sim$ (C). By the CPT theorem one finds that

$$P_{\bar\alpha\bar\alpha}=P_{\alpha\alpha},\quad P_{\bar\alpha\bar\beta}=P_{\beta\alpha}\,, \tag{12}$$

hence

$$w(e^-)-3w(e^+)=3w(\mu^+)-w(\mu^-)\,(CPT)\,. \tag{13}$$

Similarly, CP-invariance or T-invariance implies that

$$P_{\alpha\beta}=P_{\bar{\alpha}\bar{\beta}},\quad P_{\alpha\beta}=P_{\beta\alpha} \tag{14}$$

which means that

$$w(e^-)-3w(e^+)=w(\mu^-)-3w(\mu^+)=0\ (CP). \tag{15}$$

Relevant formula with CP-violation incorporated is given by

$$P_{\alpha\beta}-P_{\bar{\alpha}\bar{\beta}}=\pm 4\sum_{i<j}|U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j}| \times\sin\left(2\pi\frac{D}{D_k}\right)\sin(\phi_{\alpha i\beta j}) \tag{16}$$

with $\varepsilon_{ijk}=1$, which is valid even if the sterile component is present. Test of the relations (13) and (15) and observation of any departure from them would then provide much information on parameters of CP-violation. Similar effects are presumably observable for mixed neutrino beams from stored muons, and perhaps from other sources.

In summary, we have discussed effects of sterile components and of CP-violation in phenomena of neutrino oscillation. We wish to stress that the neutrino oscillation, if experimentally confirmed, opens up a new area of particle physics and is promised to give important clues towards a complete unified theory of particle interactions.

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