

## Vector-boson pair production at the LHC: Electroweak corrections in HERWIG++

---

**Stefan Gieseke**

Karlsruhe Institute of Technology (KIT), Institut für Theoretische Physik,  
D-76128 Karlsruhe, Germany  
E-mail: Stefan.Gieseke@kit.edu

**Tobias Kasprzik\***

Karlsruhe Institute of Technology (KIT), Institut für Theoretische Teilchenphysik,  
D-76128 Karlsruhe, Germany  
E-mail: Tobias.Kasprzik@kit.edu

**Johann H. Kühn**

Karlsruhe Institute of Technology (KIT), Institut für Theoretische Teilchenphysik,  
D-76128 Karlsruhe, Germany  
E-mail: Johann.Kuehn@kit.edu

Vector-boson pair production is of great phenomenological importance at the LHC. These processes will help to validate the Standard Model at highest energies, and they may also open the door for the discovery of new physics potentially showing up in subtle modifications of the non-abelian structure of weak interactions. In this letter, we review the status of the corresponding theory predictions, focusing on the higher-order electroweak corrections. We present a NLO analysis of electroweak corrections to W-pair, W<sup>+</sup>Z and Z-pair production at the LHC, including all mass effects as well as leptonic decays. Contributions of photon-induced processes and massive-boson radiation are also discussed. The electroweak corrections are implemented in the HERWIG++ Monte Carlo generator, where they are combined with QCD corrections. We also propose a simple and straight-forward method allowing for an *a posteriori* implementation of electroweak corrections to vector-boson pair production in any Monte Carlo event sample.

TTP13-030  
KA-TP-30-2013  
LPN13-070  
SFB/CPP-13-72

EPS HEP,  
18-24 July 2013,  
Stockholm, Sweden

---

\*Speaker.

## 1. Introduction

As stated in the abstract, a profound understanding of vector-boson pair production processes at the LHC is desirable for various reasons. Consequently, great effort has been made during the last years to push the theory predictions for this process class to a new level, where, besides the dominating next-to-leading-order (NLO) QCD corrections (see, e.g., Ref. [1] and references therein), also electroweak (EW) effects have been studied extensively. In particular, the interplay of EW corrections and anomalous couplings has been investigated in Ref. [2] in the high-energy limit, including leptonic decays and off-shell effects. Very recently, EW corrections to W-pair production have been computed in the double-pole approximation (DPA), even taking into account the full mass dependence [3]. Leading two-loop effects at high transverse momenta were discussed in Ref. [4] for W-pairs. A detailed analysis of on-shell  $V$ -boson pair production ( $V = W^\pm, Z$ ) including EW corrections has been provided in Refs. [1, 5], consistently including all mass effects. A detailed review of NLO effects in pair production of massive bosons can also be found in Ref. [6], emphasizing the importance of photon-induced contributions. A brief discussion of the phenomenological implications of those effects will be given in section 2.1.

In addition to NLO corrections, first steps towards the computation of NNLO QCD corrections to massive  $V$ -pair production have been taken [7], and approximate NNLO results for  $W^+Z$  and  $WW$  production have been provided for high- $p_T$  observables [8], as well as for  $WW$  production in the threshold limit [9].

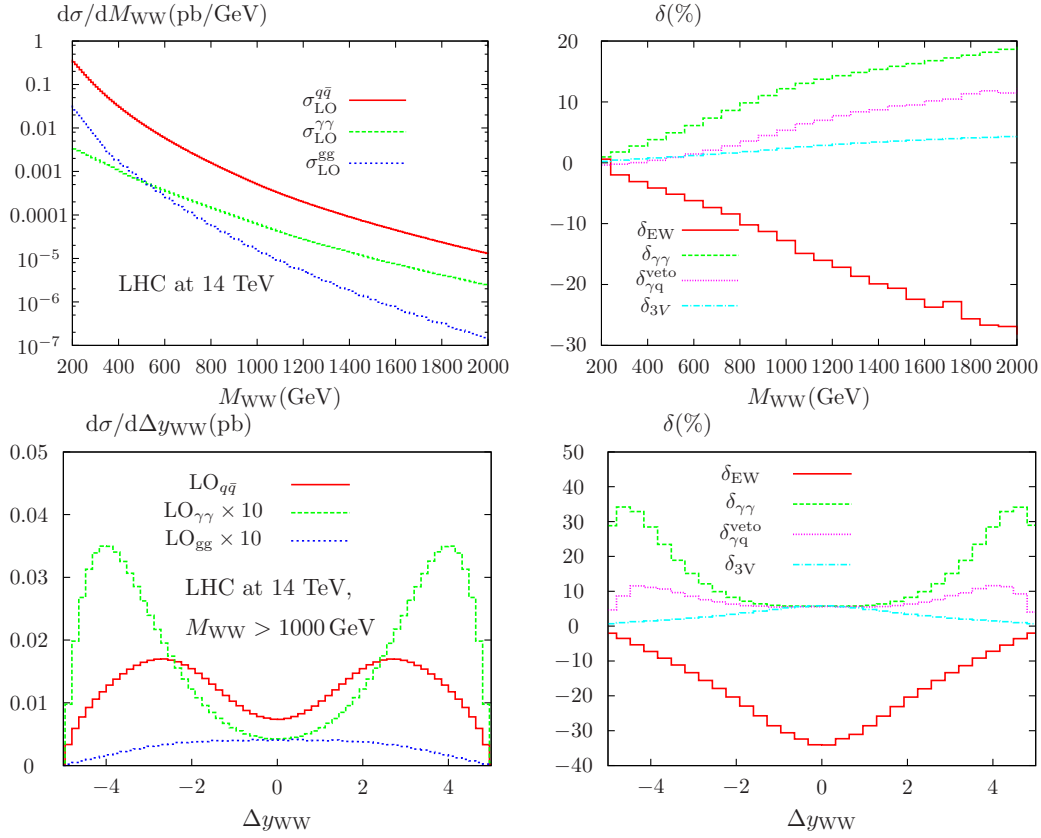
Nevertheless, a phenomenological analysis of EW corrections in  $V$ -pair production at the LHC, including leptonic decays *and* mass effects, is still missing for the  $ZZ$  and  $WZ$  channels. This gap is partly closed in section 2.2, where first results of resonant four-lepton production at NLO EW accuracy are presented. To facilitate the phenomenological analysis of data including EW corrections, a straight forward implementation of EW corrections in the HERWIG++ [10] setup is proposed in section 3, combining the flexibility of a state-of-the-art Monte Carlo (MC) generator with EW precision.

## 2. Electroweak Corrections at NLO

### 2.1 On-shell gauge-boson pair production

Let us briefly recapitulate the combination of different electroweak effects in on-shell  $V$ -pair production [1, 5, 6]. In W-pair production, the invariant-mass distribution (Fig. 1 (top)) receives logarithmically enhanced negative EW corrections ( $\delta_{EW}$ ). Positive contributions arise from the partonic subprocess  $\gamma\gamma \rightarrow WW$  ( $\delta_{\gamma\gamma}$ ) and the photon-quark induced processes ( $\delta_{\gamma q}^{\text{veto}}$ ), which have been evaluated applying the jet veto defined in Ref. [1]. The effect of massive-boson radiation ( $\delta_{3V}$ ) is moderate, however, it strongly depends on the event selection [5].

The above picture significantly changes if angular distributions of the W-pair are studied at high invariant masses. This can be seen in Fig. 1 (bottom) where distributions of the rapidity gap of the two Ws are shown for  $M_{WW} > 1000$  GeV. While the genuine EW corrections drastically reduce the differential cross sections at high  $p_{T,W}$ , corresponding to small rapidity gap, the photon-induced contributions significantly increase the rates at small scattering angles, corresponding to large rapidity gap. As a result, a dramatic distortion of angular distributions is visible.



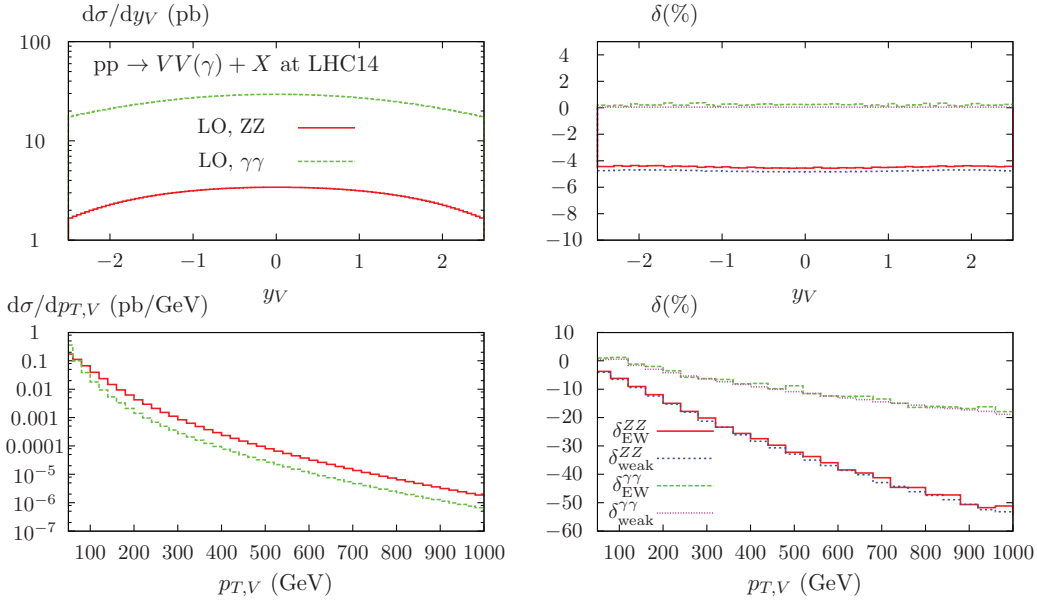
**Figure 1:** Left: Differential LO cross sections to W-pair production at LHC14. Right: various EW corrections relative to the quark-induced LO process. Top: invariant-mass distribution; Bottom: WW rapidity gap for  $M_{WW} > 1$  TeV. The results presented here are obtained in the default setup of Ref. [1].

The photon-induced corrections presented above suffer a large systematic error stemming from our ignorance of the photon content of the proton. This becomes obvious from Fig. 25 of Ref. [11], where the NNPDF2.3QED [11] set has been used to estimate the error on the  $\gamma\gamma$ -induced W-pair cross section. A relative error of  $\pm 50\%$  on the leading-order (LO) cross section at  $M_{WW} = 1000$  GeV is solely induced by the photon PDF error. This indicates that a significant improvement in the determination of the photon PDFs is mandatory to reliably predict the W-pair production cross section at high energies. Turning to WZ production, the situation is qualitatively similar to WW production, though here the  $\gamma\gamma$  process is absent and the genuine EW corrections are smaller. In Z-pair production, however, the  $\gamma q$ -induced contributions are negligible, and particularly large negative corrections dramatically affect Z-pair production at high transverse momenta.

## 2.2 Polarization and decays

Realistic predictions for massive vector-boson pair production require the inclusion of leptonic decays in the event simulation. Accordingly, corresponding radiative corrections must be computed for polarized cross sections to properly include spin correlations.

The purely weak corrections for ZZ (and also  $\gamma\gamma$ ) production are well defined and the QED



**Figure 2:** Left: Differential LO cross sections to Z- and  $\gamma$ -pair production at LHC14. Right: Full EW corrections and purely weak corrections defined relative to the quark-induced LO process. Top: vector-boson rapidity; Bottom: vector-boson transverse momentum. The results presented here are obtained in the default setup of Ref. [5].

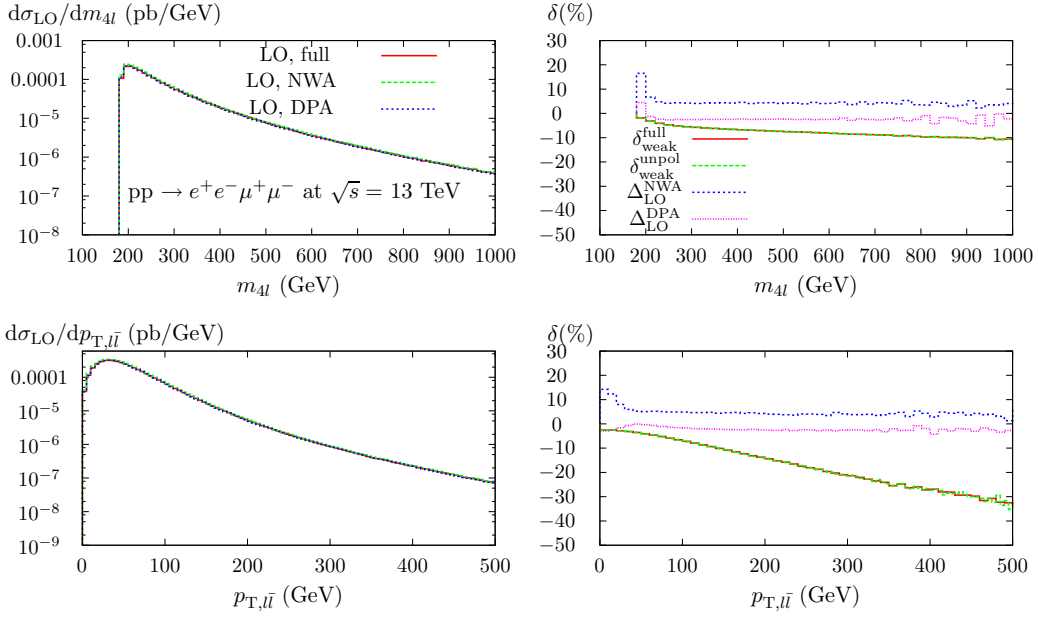
contributions can be separated in a gauge-invariant and infrared-safe way [5], and the numerical effect of QED corrections in general is below 1% in all regions of phase space (see Fig. 2). Consequently, it is well motivated to neglect the QED part in the computation of EW corrections to Z-pair production.

In Fig. 3 selected LO distributions and corresponding weak corrections for the process

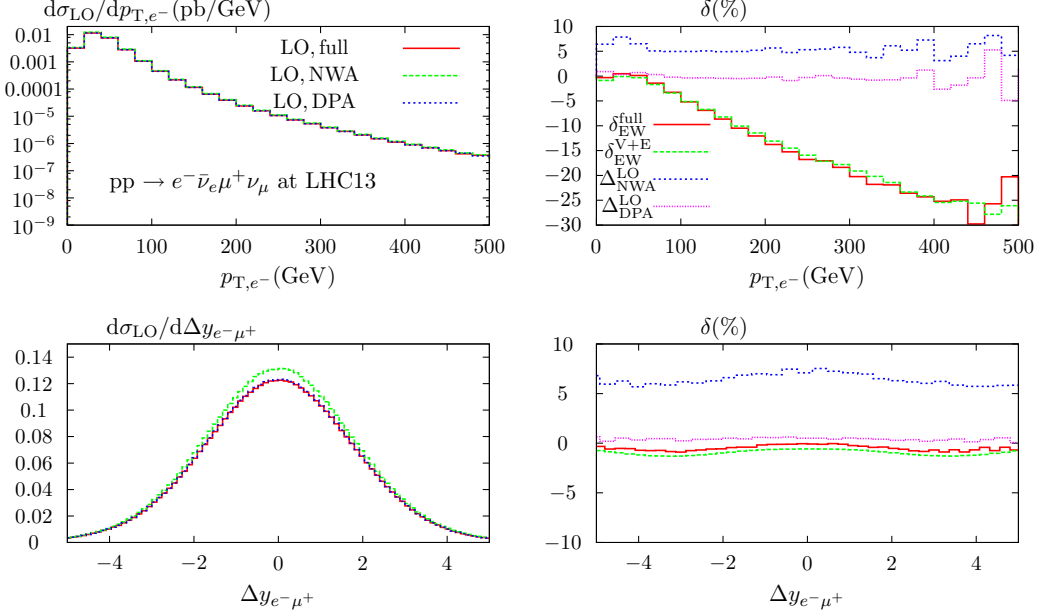
$$pp \rightarrow (Z/\gamma^*)(Z/\gamma^*) \rightarrow e^+e^-\mu^+\mu^- \quad (2.1)$$

are presented, where  $p_{T,l} > 10$  GeV and  $|y_l| < 5$  is required for the charged leptons. Additionally, the two-lepton invariant masses are constrained to  $|M_{l\bar{l}} - M_V| < 25$  GeV to reduce the admixture of non-resonant contributions and improve the validity of the pole approximation applied below. The full LO results obtained in the complex-mass scheme [13] (including also non-resonant and singly-resonant contributions) are well approximated by the DPA, while in the narrow-width approximation (NWA) a 10% discrepancy from the full result is observed. The relative weak corrections are consistently computed in the NWA, assuming full factorization of the corrections to production and decay, respectively. The full spin correlations are accounted for in the NLO result  $\delta_{weak}^{full}$ . Final-state photon radiation from the leptons, which potentially leads to drastic distortions of leptonic observables, is not included in these results and will be taken care of in the MC implementation to be discussed in the next section.

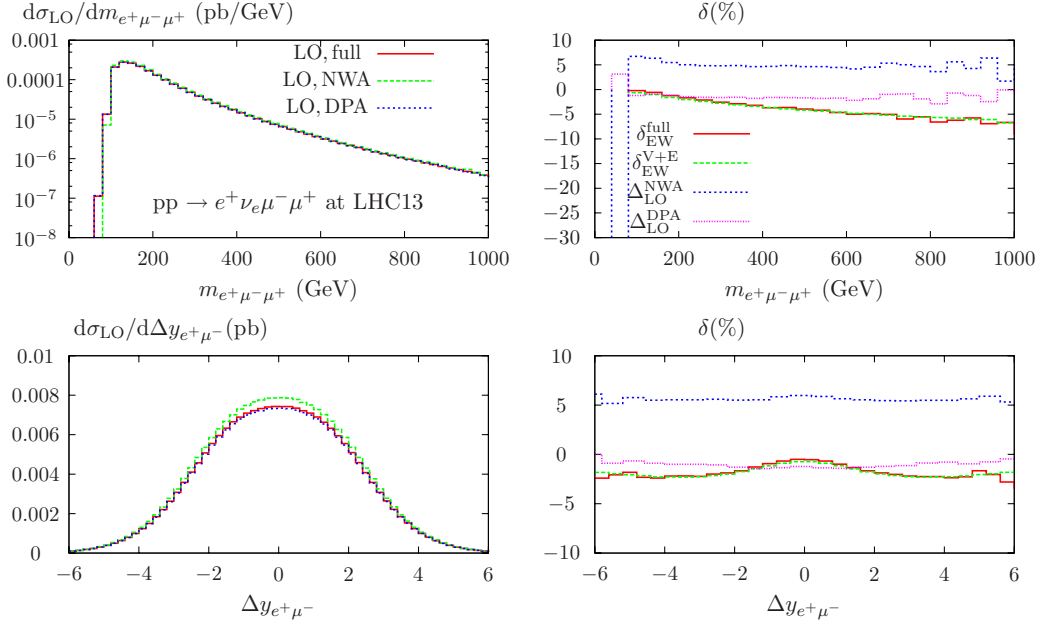
In addition to the full EW corrections we also present corrections obtained in a simple  $K$ -factor approach ( $\delta_{weak}^{unpol}$ ), where the unpolarized relative weak corrections  $K_{q\bar{q}}^{ZZ}(\hat{s}, \hat{t})$  to the partonic subprocesses  $q\bar{q} \rightarrow ZZ$  ( $q = u, d, s, c, b$ ) have simply been multiplied with the tree-level result. We



**Figure 3:** Left: Selected differential LO cross sections for process (2.1) at LHC13. Right: relative weak corrections with ( $\delta_{\text{weak}}^{\text{full}}$ ) and without ( $\delta_{\text{weak}}^{\text{unpol}}$ ) spin correlations and relative deviations of the NWA and DPA results from the full LO, respectively. Top: four-lepton invariant mass; bottom: Z-boson transverse momentum. The results presented have been obtained with the CT10 PDF set [12].



**Figure 4:** Left: Selected differential LO cross sections for process (2.2, left) at LHC13. Right: Full EW corrections, EW corrections in the V+E approximation and relative deviations of the LO NWA and LO DPA results from the full LO, respectively. Top:  $e^-$  transverse momentum; bottom: charged-lepton rapidity gap. The results presented have been obtained with the CT10 PDF set [12].



**Figure 5:** Left: Selected differential LO cross sections for process (2.2, right) at LHC13. Right: Full EW corrections, EW corrections in the V+E approximation and relative deviations of the LO NWA and LO DPA results from the full LO, respectively. Top: charged-lepton invariant mass; bottom: charged-lepton rapidity gap. The results presented have been obtained with the CT10 PDF set [12].

observe nearly perfect agreement with the full result, indicating that spin correlations are hardly affected by weak corrections [5].

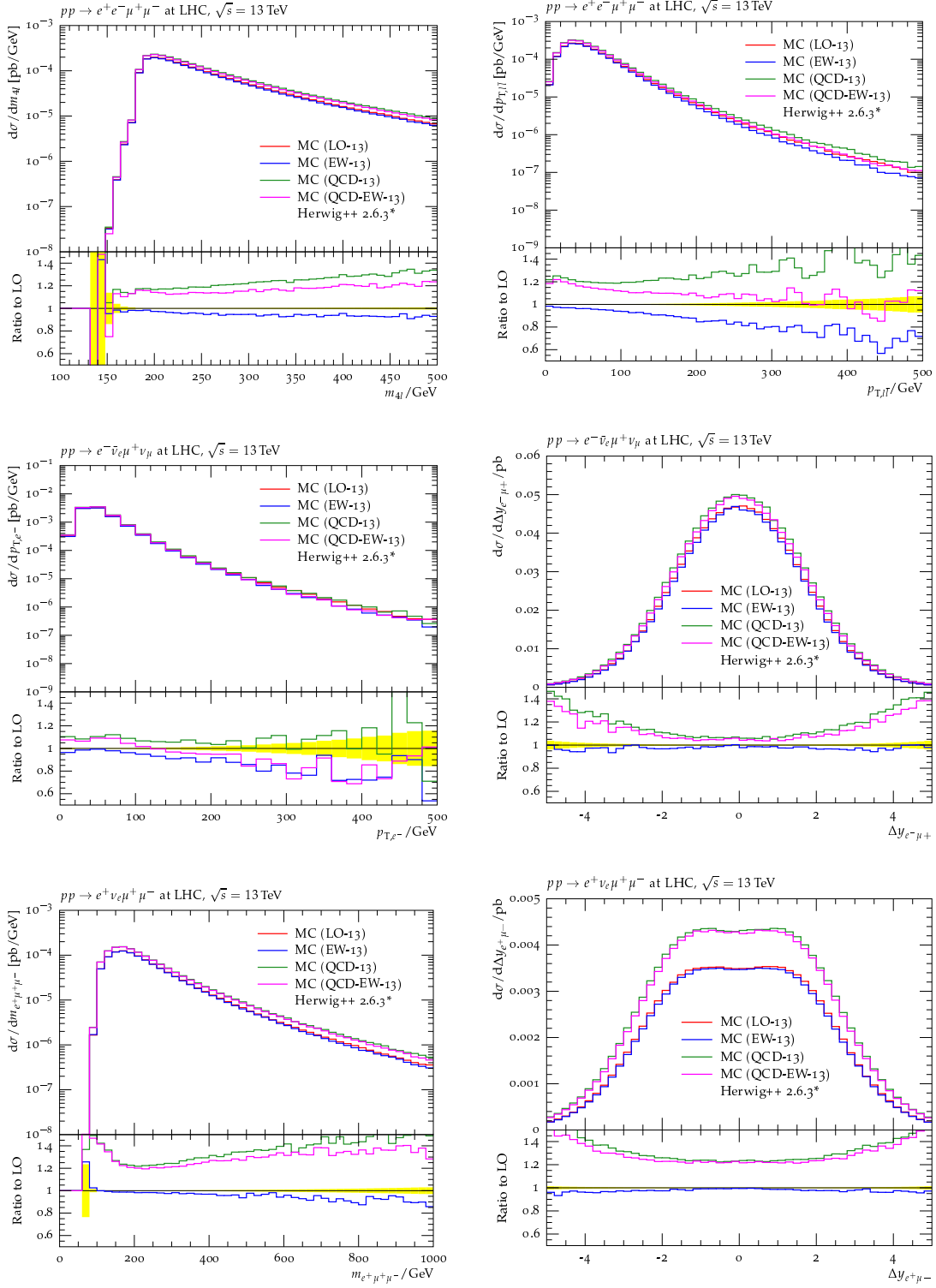
In contrast to the case of Z-pair production, the situation is more involved in WW or WZ production, where it is not possible to simply separate the (presumably small) QED corrections. However, the infrared singularities in the virtual corrections related to photon exchange can be eliminated by the endpoint contribution as defined in the dipole subtraction procedure [14]. This virtual+endpoint (V+E) approximation gives a surprisingly good approximation at the level of a few percent, as clearly visible in Figs. 4 and 5, where selected differential distributions for the processes

$$pp \rightarrow W^-W^+ \rightarrow e^- \bar{\nu}_e \mu^+ \nu_\mu \quad \text{and} \quad pp \rightarrow W^+(Z/\gamma^*) \rightarrow e^+ \nu_e \mu^- \mu^+ \quad (2.2)$$

are presented obtained in the setup defined above, additionally requiring a minimal missing transverse momentum of 25 GeV. Here, unpolarized partonic  $K$ -factors have been used to compute the corrections  $\delta_{EW}^{V+E}$  in the NWA using the V+E approximation; again, good agreement to the full result  $\delta_{EW}^{full}$  is observed. As in the ZZ case one observes that spin correlations are hardly modified by NLO corrections.

### 3. Implementation in HERWIG++

Following the philosophy described in the previous section, our MC implementation of EW corrections is based on partonic unpolarized  $K$ -factors  $K(\hat{s}, \hat{t})$  obtained in the V+E approximation.



**Figure 6:** Numerical results for processes (2.1), (2.2, left) and (2.2, right) obtained with the HERWIG++ generator (v2.6.3) for resonant ZZ (top), WW (center) and W<sup>+</sup>Z (bottom) production at LHC13. The results directly correspond to those presented in Figs. 3, 4 and 5. In addition, the requirement  $|\sum_i \mathbf{p}_{T,i}^i| < 0.3 \sum_i |\mathbf{p}_{T,i}^i|$  for balanced vector-boson pairs is included in the HERWIG++ predictions.

The resulting EW correction factor is combined with the respective partonic cross section provided by the generator in an event-by-event fashion. The  $\hat{s}$  and  $\hat{t}$  values are either provided by the HERWIG++ generator or computed from the final-state lepton momenta.

Since the MC sample generated by HERWIG++ provides, in addition to pure LO predictions, NLO QCD corrections matched to parton showers as well as hadronization effects, it is not obvious how to combine the MC cross sections with partonic EW  $K$ -factors which are defined relative to a naive LO. First, it is not clear whether to simply add the EW corrections on top of the QCD corrected cross section, or to use a multiplicative ansatz, assuming a factorization of EW and QCD effects. Second, in the presence of parton showers or NLO QCD corrections, the simple two-body phase space of the hard  $q\bar{q}' \rightarrow V_1 V_2$  process, defined by  $\hat{s}$  and  $\hat{t}$ , is distorted by additional parton radiation, and a proper mapping has to be defined to restore the two-by-two kinematics.

In our approach, we assume complete factorization of QCD and EW corrections. For each event the contribution to the differential partonic cross sections is defined by

$$d\hat{\sigma}_{\text{EW} \times \text{QCD}} = K_{q\bar{q}'}^{V_1 V_2}(\hat{s}, \hat{t}) \times d\hat{\sigma}_{\text{QCD}}, \quad (3.1)$$

where  $d\hat{\sigma}_{\text{QCD}}$  denotes the QCD prediction for  $V$ -pair production provided by the default HERWIG++ setup [15]. The values for  $\hat{s}$  and  $\hat{t}$  are either directly provided by HERWIG++ or may be computed from the final-state lepton momenta according to the following prescription: The squared center-of-mass (CM) energy  $\hat{s}'$  is calculated from the four-lepton final state via  $\hat{s}' = M_{4l}^2$ . The momenta are boosted into the four-lepton CM frame (denoted by  $\Sigma^*$ ). In this frame the directions of initial-state hadrons shall be denoted by  $\mathbf{e}_i^* = \frac{\mathbf{p}_i^*}{|\mathbf{p}_i^*|}$ ,  $i = 1, 2$ . The direction of the effective scattering axis in  $\Sigma^*$  is now defined by  $\hat{\mathbf{e}}^* = \frac{\mathbf{e}_1^* - \mathbf{e}_2^*}{|\mathbf{e}_1^* - \mathbf{e}_2^*|}$ , and the effective scattering angle is, correspondingly, given by  $\cos \theta^* = \mathbf{v}_1^* \cdot \hat{\mathbf{e}}^*$ , where the  $\mathbf{v}_1^*$  denotes the momentum direction of vector boson  $V_1$ . The Mandelstam variable  $\hat{t}'$  is then computed from  $\theta^*$  assuming on-shell kinematics. We point out that this prescription allows for an *a posteriori* implementation of EW corrections in any MC event sample, provided that the information on the lepton four-momenta and the IS quark generations are accessible.

Note that the above prescription, as well as the assumption of factorization of QCD and EW corrections, are only expected to work reliably if the LO signature of "balanced" (i.e. back-to-back) vector bosons is not distorted dramatically by additional high- $p_T$  QCD radiation. Therefore, we propose the restriction  $\left| \sum_i \mathbf{p}_{T,i}^i \right| < 0.3 \sum_i |\mathbf{p}_{T,i}^i|$  on the  $p_T$  of the visible charged leptons  $i$  in the final states to enforce back-to-back vector bosons and, at the same time, avoid giant QCD  $K$ -factors typically occurring at high  $p_T$  [1, 8].

Exploiting the above ideas, a simple add-on for EW corrections in the HERWIG++ setup has been constructed. Numerical results (directly corresponding to Figs. 3, 4 and 5) are shown in Fig. 6 for resonant ZZ, WW and  $W^+Z$  production, respectively. The results presented are based on samples of 10M events obtained by the HERWIG++ generator (v2.6.3) [16]. The relative corrections (QCD, EW, EW $\times$ QCD) are normalized to the LO prediction, which includes parton showers and hadronization effects. In addition, all predictions also include final-state photon radiation which is implemented by default in the HERWIG++ setup [17]. One observes good agreement between the HERWIG++ ratios and the relative corrections from Figs. 3, 4 and 5, giving a proof of principle that our implementation works reliably.



## 4. Conclusions

An analysis of EW corrections to vector-boson pair production at the LHC is presented, including the full mass dependence as well as leptonic decays. Hence, our predictions are valid in the whole kinematic regime accessible at the LHC. Combining EW corrections and photon-induced contributions, drastic distortions of angular distributions at high energies are observed which could easily be misinterpreted as signals of anomalous couplings. For this reason, and in view of the future high-luminosity run of the LHC at a CM energy of 13 TeV, the inclusion of EW corrections in the analysis of experimental data by means of MC techniques is indispensable. Thus we have proposed a simple method to combine EW corrections with any multi-purpose MC generator. To demonstrate the practicability of the method, phenomenological results are presented on the basis of a HERWIG++ event sample, combining EW corrections with state-of-the-art QCD predictions.

## Acknowledgements

Supported by SFB TR9 “Computational and Particle Physics” and BMBF Contract 05HT4VKATI3.

## References

- [1] A. Bierweiler, T. Kasprzik, H. Kühn and S. Uccirati, *JHEP* **1211** (2012) 093 [arXiv:1208.3147].
- [2] E. Accomando and A. Kaiser, *Phys. Rev. D* **73** (2006) 093006 [hep-ph/0511088].
- [3] M. Billoni, S. Dittmaier, B. Jäger and C. Speckner, arXiv:1310.1564 [hep-ph].
- [4] J. H. Kühn, F. Metzler, A. A. Penin and S. Uccirati, *JHEP* **1106** (2011) 143 [arXiv:1101.2563].
- [5] A. Bierweiler, T. Kasprzik and J. H. Kühn, arXiv:1305.5402 [hep-ph].
- [6] J. Baglio, L. D. Ninh and M. M. Weber, arXiv:1307.4331 [hep-ph].
- [7] T. Gehrmann, L. Tancredi and E. Weihs, *JHEP* **1308** (2013) 070 [arXiv:1306.6344 [hep-ph]]; G. Chachamis, M. Czakon and D. Eiras, *JHEP* **0812** (2008) 003 [arXiv:0802.4028 [hep-ph]].
- [8] F. Campanario and S. Sapeta, *Phys. Lett. B* **718** (2012) 100 [arXiv:1209.4595 [hep-ph]]; F. Campanario, M. Rauch and S. Sapeta, arXiv:1309.7293 [hep-ph].
- [9] S. Dawson, I. M. Lewis and M. Zeng, arXiv:1307.3249 [hep-ph].
- [10] M. Bahr, S. Gieseke, M. A. Gigg, D. Grellscheid, K. Hamilton, O. Latunde-Dada, S. Plätzer and P. Richardson *et al.*, *Eur. Phys. J. C* **58** (2008) 639 [arXiv:0803.0883 [hep-ph]].
- [11] R. D. Ball *et al.* [The NNPDF Collaboration], arXiv:1308.0598 [hep-ph].
- [12] H. -L. Lai, M. Guzzi, J. Huston, Z. Li, P. M. Nadolsky, J. Pumplin and C. -P. Yuan, *Phys. Rev. D* **82** (2010) 074024 [arXiv:1007.2241 [hep-ph]].
- [13] A. Denner, S. Dittmaier, M. Roth and L. H. Wieders, *Nucl. Phys. B* **724** (2005) 247 [hep-ph/0505042]; A. Denner and S. Dittmaier, *Nucl. Phys. Proc. Suppl.* **160** (2006) 22 [hep-ph/0605312].
- [14] S. Dittmaier, *Nucl. Phys. B* **565** (2000) 69 [hep-ph/9904440].
- [15] K. Hamilton, *JHEP* **1101** (2011) 009 [arXiv:1009.5391 [hep-ph]].
- [16] K. Arnold, L. d’Errico, S. Gieseke, D. Grellscheid, K. Hamilton, A. Papaefstathiou, S. Plätzer and P. Richardson *et al.*, arXiv:1205.4902 [hep-ph].
- [17] K. Hamilton and P. Richardson, *JHEP* **0607** (2006) 010 [hep-ph/0603034].