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Vector Directional Filters—A New Class of Multichannel Image Processing Filters

P. E. Trahanias and A. N. Venetsanopoulos

Abstract—Vector directional filters (VDF) for multichannel image processing are introduced and studied in this paper. These filters separate the processing of vector-valued signals into directional processing and magnitude processing. This provides a link between single-channel image processing, where only magnitude processing is essentially performed, and multichannel image processing where both the direction and the magnitude of the image vectors play an important role in the resulting (processed) image.

VDF find applications in satellite image data processing, color image processing, and multispectral biomedical image processing. In this paper, results are presented for the case of color images, as an important example of multichannel image processing. It is shown that VDF can achieve very good filtering results for various noise source models.

I. INTRODUCTION

Although conventional approaches to multichannel image processing are based on processing the image channels separately, they fail to utilize the inherent correlation that is usually present in multichannel images. Consequently, *vector processing* of multichannel images is

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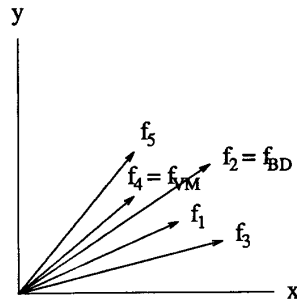


Fig. 1. A set of 2-dim vectors. The output of BVDF is always the middle vector (f_2); this is not necessarily the case for the VMF output (f_4).

desirable [1]. Recently, this has been adopted by many researchers [2]–[4]. An important case of vector image processing operators are the vector median filters (VMF) that have been introduced as extension of scalar median filters [5]. VMF can be derived 1) as maximum likelihood estimates when the underlying probability densities are double-exponential or 2) using vector order statistics [6]. In the latter case, the vector median of a population is defined as the minimal vector according to the aggregate ordering technique [6]. Based on vector order statistics, extensions or modifications of VMF have also been proposed [2], [7].

The operation of the above-mentioned filters can be described according to some distance criterion that is applied to the set of vectors inside the processing window. However, the features that uniquely characterize a vector, namely *direction* and *magnitude*, are not considered by such an operation and this may produce erroneous results in certain cases. Such an example is shown in Fig. 1, where VMF is applied to the set of vectors f_1, \dots, f_5 . The output produced is vector f_4 , although vector f_2 would be a better candidate to output.

This paper approaches the aforementioned problem by explicitly considering the vector features and separating the processing of vector-valued signals into two steps: *directional processing* and *magnitude processing*. A new class of filters is introduced, called *vector directional filters* (VDF). VDF perform the first step, namely directional processing. They operate on the direction of the image vectors aiming at eliminating vectors with *atypical* directions in the vector space. This is achieved by employing a novel vector ordering technique in which the *angle* between the image vectors serves as the ordering criterion. The term "directional processing" used here denotes the processing performed according to the vectors' direction in the vector space. This term has been adopted by other authors to denote processing in certain directions in the image plane [8]. Here it is used in the context of vector spaces and hence it should not bring any confusion. Similarly, the term "magnitude processing" denotes the processing of image data where only the vector magnitudes are taken into account.

The application of VDF results in the removal of vectors with atypical directions and a set containing vectors with approximately the same direction in the vector space is produced as the output set. Since the vectors in this set are approximately collinear, a magnitude processing operation (second step) can be applied to produce a single output vector at each image pixel. This step can be performed by any classical gray-level image processing filter.¹

¹This is obvious since gray-level image processing filters operate on the *magnitude* at each pixel location.

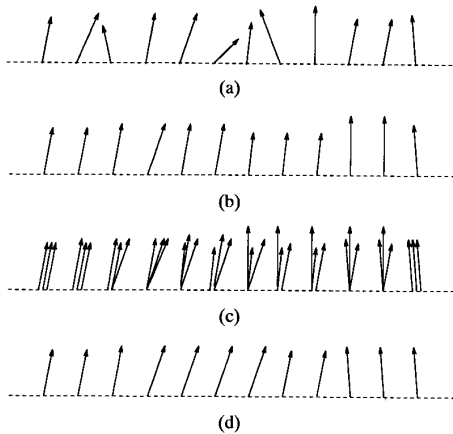


Fig. 2. (a) 1-D vector valued signal. (b) Application of the BVDF to the signal, $n = 5$. (c) Application of the GVDF to the signal, $n = 5$, $k = 3$. (d) Scalar median in cascade with the GVDF.

This property of VDF (separation of processing) establishes a link between multichannel image processing and single-channel image processing. After directional processing a multichannel signal can *locally* be seen as a single-channel signal, since this processing results in vectors being approximately collinear. Consequently, the bulk of techniques developed for gray-level image processing can be employed in the step of magnitude processing. This is a major advantage of this approach, since it facilitates the use of many efficient image processing operators (order statistics, α -trimmed mean, morphological) to multichannel images.

II. VECTOR DIRECTIONAL FILTERS

Basic Vector Directional Filter

Let $f(x) : Z^l \rightarrow Z^m$, represent a multichannel signal and let $W \in Z^l$ be a window of finite size n (filter length). l represents the signal dimensions and m represents the number of signal channels. The pixels in W will be denoted as $x_i, i = 1, 2, \dots, n$ and $f(x_i)$ will be denoted as f_i . f_i are m -dimensional (m -D) vectors in the vector space defined by the m signal channels. A window size n is implied in all subsequent operations, if not stated otherwise.

The definition of the *basic vector directional filter* (BVDF) follows. This is a special case of VDF. However, it facilitates their introduction and mathematical treatment.

Definition 1: The output of the BVDF, for input $\{f_i, i = 1, 2, \dots, n\}$, is $f_{BD} = BVDF[f_1, f_2, \dots, f_n]$, such that

$$f_{BD} \in \{f_i, i = 1, 2, \dots, n\} \quad (1)$$

and

$$\sum_{i=1}^n \mathcal{A}(f_{BD}, f_i) \leq \sum_{i=1}^n \mathcal{A}(f_j, f_i), \quad \forall j = 1, 2, \dots, n \quad (2)$$

where $\mathcal{A}(f_i, f_j)$ denotes the angle between the vectors f_i and f_j , $0 \leq \mathcal{A}(f_i, f_j) \leq \pi$. \square

BVDF outputs the vector from the input set that minimizes the sum of the *angles* with the other vectors. In other words, it chooses the vector most centrally located, without considering magnitudes. The operation of BVDF can be parallelized with the operation of VMF which outputs the vector that minimizes the sum of the *distances* to the other vectors.

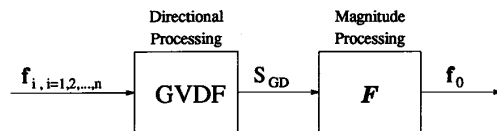


Fig. 3. Multichannel image processing using a cascade of directional processing and magnitude processing.

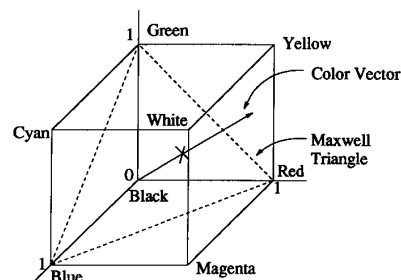


Fig. 4. Perspective representation of the color cube.

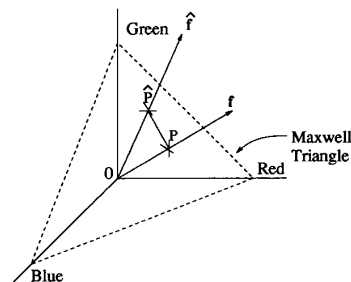


Fig. 5. Definition of the chromaticity error for two vectors f and \hat{f} .

The application of BVDF to a vector-valued 1-D signal is illustrated in Fig. 2. The BVDF output is shown in Fig. 2(b) for the input shown in Fig. 2(a). It should be explicitly noted that for the special case of BVDF processing, a single output vector is produced at each pixel and the next step, i.e. magnitude processing, is not applicable. In other words, BVDF filtering considers only directional information in producing the output vector signal.

For 2-D vectors, the angle of f_{BD} is the least error estimate of the angle location.

Proof: With respect to Fig. 1, we observe that the angles of the vectors with the x -axis form a sequence of numbers with median the angle of f_{BD} ($= f_2$). It is well known that the median is the least absolute error estimate of the location of a set of numbers, which, since the angles are positive, proves the above statement. Q.E.D.

In 3-D the above statement holds approximately for small angles between the vectors.

Proof: Without loss of generality we consider a set of vectors $\{f_i, i = 1, 2, \dots, n\}$ of equal length. Then, the least error estimate of the location of the vectors $\{f_i\}$ (constrained to the set f_i) is the vector median f_{VM} , since

$$\sum_i \|f_i - f_{VM}\| \leq \sum_i \|f_i - f_j\|, \quad \forall j = 1, 2, \dots, n \quad (3)$$

i.e. the minimum of $\sum_i \|f_i - f_c\|$ is obtained for $f_c = f_{VM}$. Since the vectors are of equal length, say $l (> 0)$, $\|f_i - f_{VM}\|$ can be

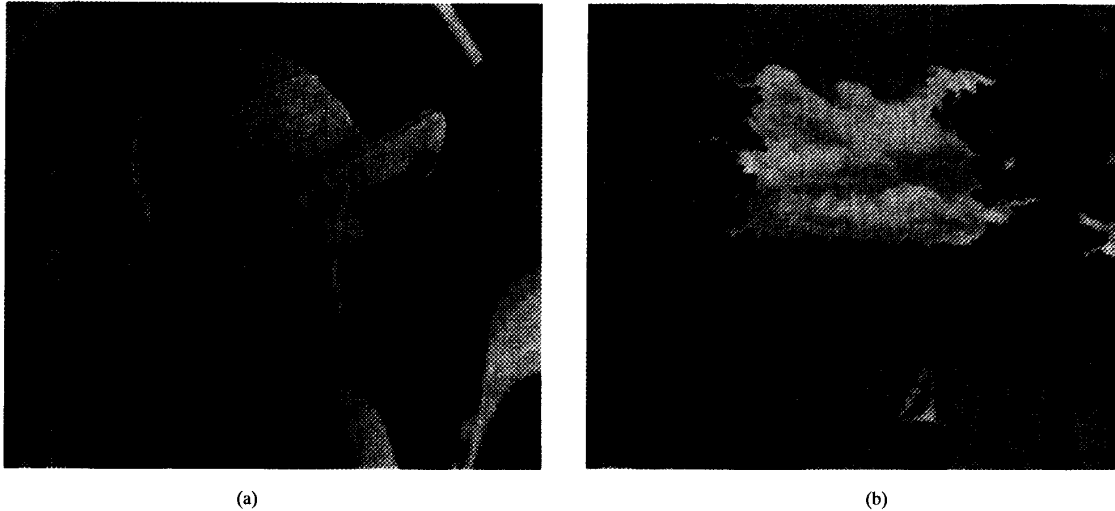


Fig. 6. Color images. (a) Lena. (b) Lake.

expressed as

$$\|f_i - f_{VM}\| = 2l \sin \frac{\phi_i}{2} \quad (4)$$

where, $\phi_i = \mathcal{A}(f_i, f_{VM})$. Since $\sum_i \|f_i - f_{VM}\|$ is a minimum, $\sum_i \sin \frac{\phi_i}{2}$ is a minimum. By assumption, the angles ϕ_i are small and the $\sin \frac{\phi_i}{2}$ can be substituted with the angles $\frac{\phi_i}{2}$ (expressed in radians). Q.E.D.

From the above proof it is evident that the outputs of VMF and BVDF are the same for equal-length vectors. However, this is not the case for vectors of unequal length. In the latter case, the BVDF output is still the vector that would have resulted if the vectors were of equal length. The VMF, however, would generally produce different results. This is demonstrated in Fig. 1 where the BVDF output is vector f_2 whereas the VMF output is vector f_4 .

Generalized Vector Directional Filters

BVDF may perform well when the vector magnitudes are of no importance and the direction of the vectors is the major issue. However, this may not be the case in some multichannel signal processing applications and the magnitudes of the vectors should also be considered. This can be accomplished by generalizing BVDF so that the output at each pixel is not a vector $f_j \in W$ but instead a set of vectors $f_i \in W$, $i = 1, 2, \dots, k$, $k < n$.

Definition 2: The output of the generalized vector directional filter (GVDF), for input $\{f_i, i = 1, 2, \dots, n\}$, is the set $S_{GD} = GVDF[f_1, f_2, \dots, f_n]$, where

$$S_{GD} = \{f^{(1)}, f^{(2)}, \dots, f^{(k)}\}, f^{(i)} \in \{f_j, j = 1, 2, \dots, n\} \quad (5)$$

$$\forall i = 1, 2, \dots, k$$

Let α_i correspond to f_i and be defined as

$$\alpha_i = \sum_{j=1}^n \mathcal{A}(f_i, f_j), i = 1, 2, \dots, n. \quad (6)$$

An ordering of the α_i s

$$\alpha_{(1)} \leq \alpha_{(2)} \leq \dots \leq \alpha_{(k)} \leq \dots \leq \alpha_{(n)} \quad (7)$$

implies the same ordering to the corresponding f_i s

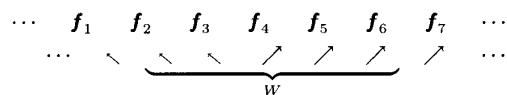
$$f^{(1)} \leq f^{(2)} \leq \dots \leq f^{(k)} \leq \dots \leq f^{(n)} \quad (8)$$

The first k terms of the ordered sequence $f^{(i)}$ constitute the output of the GVDF. \square

We note here that the first term in (8), $f^{(1)}$, is the BVDF output. GVDF generalizes BVDF in the sense that its output is a superset of the (single) BVDF output. GVDF outputs the set of vectors whose angle from all the other vectors is *small* as opposed to the BVDF which outputs the vector whose angle from all the other vectors is a minimum. In other words, the output set of the GVDF consists of vectors centrally located in the population with approximately the same direction in the vector space. The application of the GVDF to the signal of Fig. 2(a) is illustrated in Fig. 2(c) where a set of vectors is output at each sample point.

After the application of GVDF (directional processing) the output produced is a set of k vectors with approximately the same direction and the magnitude processing step should be applied to produce a single output vector at each pixel. The whole operation is illustrated in Fig. 3, where the GVDF is cascaded with the \mathcal{F} module (filter). Since \mathcal{F} processes the vectors using only magnitude information, it can be any gray-scale image processing filter [9], [10]. This is illustrated in Figs. 2(c), (d). The GVDF output (Fig. 2(c)) has been passed through a gray-scale median filter (\mathcal{F}) to produce the final output shown in Fig. 2(d).

Some properties that constitute VDF appropriate for image processing have been studied elsewhere [11]. We mention here the preservation of step edges, invariance under scaling and rotation, and existence of and convergence to root signals.² The behavior of VDF in the case of vector edges can be illustrated by the following 1-D example, where the window W of size five is centered over vector f_4 and f_2, \dots, f_6 are inside W .



²A proof of this property has been derived for the BVDF of length three only. However, experimentation with larger filter lengths has demonstrated this property.

TABLE I
NMSE ($\times 10^{-2}$) FOR THE "LENA" IMAGE

noise model	VMF		BVDF		GVDF ¹		GVDF ²		GVDF ³	
	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5
Gaussian	1.60	1.17	3.28	3.41	1.46	1.08*	2.48	4.04	2.89	2.56
Gaussian/impulsive	1.70	1.24	3.52	3.67	1.53	1.13*	2.51	4.05	3.18	2.74
double-exponential	0.41	0.60	0.61	0.89	0.36*	0.56	0.73	0.97	0.52	0.37
impulsive	0.33	0.58	0.44	0.77	0.30	0.54	0.62	0.91	0.43	0.29*

GVDF¹: GVDF followed by α -trimmed mean

GVDF²: GVDF followed by morphological open-close

GVDF³: GVDF followed by multistage max/median

TABLE II
NMSE ($\times 10^{-2}$) FOR THE "LAKE" IMAGE

noise model	VMF		BVDF		GVDF ¹		GVDF ²		GVDF ³	
	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5
Gaussian	1.54	1.20	3.86	4.20	1.52	1.16*	2.56	3.66	2.45	2.41
Gaussian/impulsive	1.64	1.29*	4.04	4.53	1.64	1.33	2.76	3.12	2.58	2.51
double-exponential	0.49*	0.93	1.20	2.13	0.53	0.95	1.45	2.64	0.71	0.51
impulsive	0.66	1.14	1.35	2.37	0.63	1.38	1.47	1.85	0.93	0.48*

GVDF¹: GVDF followed by α -trimmed mean

GVDF²: GVDF followed by morphological open-close

GVDF³: GVDF followed by multistage max/median

TABLE III
MCRE FOR THE "LENA" IMAGE

noise model	VMF		BVDF		GVDF ¹		GVDF ²		GVDF ³	
	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5
Gaussian	7.48	5.71	5.96	4.45	6.05	4.20*	6.55	6.13	8.32	7.64
Gaussian/impulsive	7.64	5.80	6.30	5.04	6.22	4.37*	6.80	6.38	8.59	8.15
double-exponential	2.98	2.90	2.81	2.73	2.82	2.66*	3.06	3.47	2.95	2.74
impulsive	2.14	2.48	1.97	2.46	2.02	2.39	2.44	2.94	1.18*	1.51

GVDF¹: GVDF followed by α -trimmed mean

GVDF²: GVDF followed by morphological open-close

GVDF³: GVDF followed by multistage max/median

Ordering of f_2, \dots, f_6 will result in $f_4 = f_5 = f_6 < f_2 = f_3$. It is evident that the vectors in the high ranks (f_2, f_3) will be eliminated after VDF processing and will not affect the vector edge.

III. MOTIVATION AND INTUITIVE ISSUES

The introduction of VDF for multichannel image processing has been motivated by a number of factors mostly related to color imaging. In this section we attempt to introduce the reader to these concepts. As already explained, VDF attempt to process vector images by exploiting the vector features, direction and magnitude. They separate the processing of multichannel images into directional processing and magnitude processing. This allows the bridging of scalar image processing and vector image processing and also enables the use of all the successful scalar image processing techniques in multichannel images.

As a special (and important) case of multichannel images, we may consider the case of color images. Color image processing can be regarded from the perspective of the color cube, as shown in Fig. 4. The three axes that define the color cube represent the three primaries (R,G,B). A particular color may be described as a vector in the color space. Such a color vector is shown in Fig. 4; the point marked with a "x" denotes the intersection point of the color vector with the *Maxwell triangle* (the triangle drawn between the three primaries, R,G,B). It is well known that the intersection point of a color vector with the Maxwell triangle gives an indication of the *chromaticity* of the color (*hue* and *saturation*) in terms of

the distances of the point from the vertices of the triangle [12]. It is obvious that this point depends only on the direction of a color vector and not on its magnitude. Therefore, the operation of VDF can be described in terms of the color chromaticity. Since BVDF results in the least error estimate of the angle location, directional filtering renders the color vector f_{BD} , from the set of input vectors, with the least chromaticity error. In the case of GVDF, a set of vectors with similar chromaticities with the chromaticity of f_{BD} is rendered. In other words, VDF operate on the chromaticity components of a color by filtering out color vectors with large chromaticity errors. This is very important in color image processing where chromaticity is actually the attribute which determines the color perception [13], [14, p. 264–67] and luminance (proportional to the vector magnitude) is the attribute which determines the brightness of a given color.

It should be clear at this point that for the case of color image processing, VDF operate actually as *chromaticity filters*. The significance of this stems from the fact that color is generally perceived first in terms of the hue component [15, p. 31] and then in other terms such as brightness, richness, purity, and saturation. On the other hand, luminance filtering can be efficiently performed subsequently using classical gray-level image processing filters.

IV. SIMULATION RESULTS

VDF have been evaluated, and their performance has been compared against the performance of VMF, using RGB color images as test data. Simulations have been conducted for both BVDF and

TABLE IV
MCRE FOR THE "LAKE" IMAGE

noise model	VMF		BVDF		GVDF ¹		GVDF ²		GVDF ³	
	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5	3 × 3	5 × 5
Gaussian	10.60	8.75	8.65	7.38	8.80	6.97*	9.71	9.53	11.09	10.58
Gaussian/impulsive	10.84	8.89	8.99	8.37	8.95	7.09*	9.22	9.82	11.32	11.09
double-exponential	5.12	5.57	4.92	5.42	4.96	5.32	5.62	6.41	3.72*	4.03
impulsive	4.77	5.44	4.94	6.35	4.74	5.43	5.50	6.63	2.87*	3.31

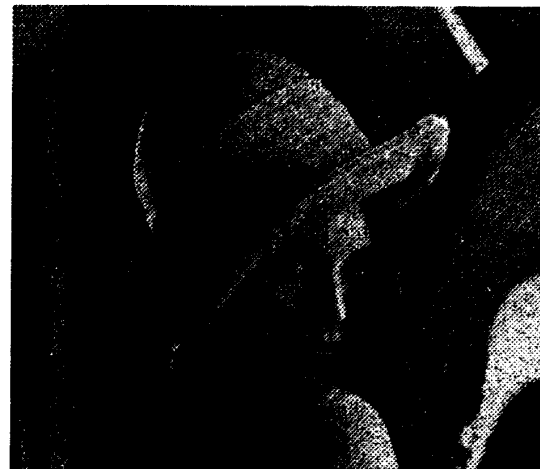
GVDF¹: GVDF followed by α -trimmed mean

GVDF²: GVDF followed by morphological open-close

GVDF³: GVDF followed by multistage max/median



(a)



(b)



(c)



(d)

Fig. 7. (a) "Lena" corrupted with Gaussian noise ($\sigma = 30$). (b) "Lena" corrupted with 4% impulsive noise. (c), (d) 3×3 VMF of (a) and (b), respectively.

GVDF. In the case of GVDF, three filters have been used for the step of magnitude processing: the α -trimmed mean [9], the morphological open-close [16], and the multistage max/median [8]. Two quantitative measures have been employed. The first is the *normalized mean squared error* (NMSE) which is a standard measure also used by

other authors in evaluation experiments [2], [8]:

$$NMSE = \frac{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} \|f(i, j) - \hat{f}(i, j)\|^2}{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} \|f(i, j)\|^2} \quad (9)$$

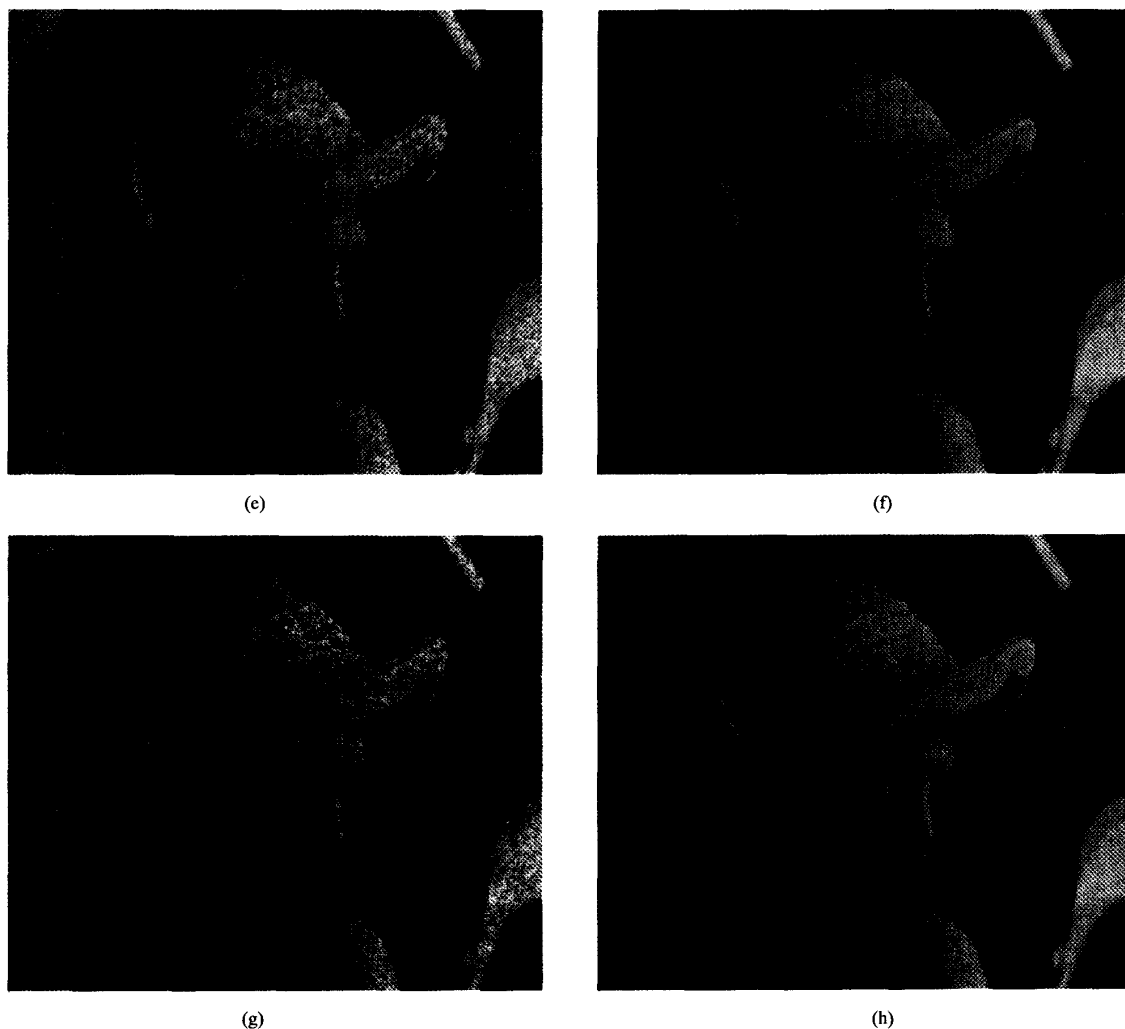


Fig. 7 (continued). (e), (f) 3×3 GVDF/ α -trimmed mean of (a) and (b), respectively. (g), (h) 5×5 GVDF/multistage max/median of (a) and (b), respectively.

where N_1 and N_2 are the image dimensions, and $\mathbf{f}(i, j)$ and $\hat{\mathbf{f}}(i, j)$ denote the original and the estimated image vector at pixel (i, j) , respectively. The second measure is related to the color chromaticity and is referred to as the *mean chromaticity error* (MCRE). We have introduced this measure since VDF operate as chromaticity filters and, consequently, their performance in terms of chromaticity error should be evaluated. MCRE is defined as

$$MCRE = \frac{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} C[\mathbf{f}(i, j), \hat{\mathbf{f}}(i, j)]}{N_1 N_2} \quad (10)$$

where N_1 , N_2 , $\mathbf{f}(i, j)$ and $\hat{\mathbf{f}}(i, j)$ are as in (9) and $C[\mathbf{f}(i, j), \hat{\mathbf{f}}(i, j)]$ is the chromaticity error between vectors $\mathbf{f}(i, j)$ and $\hat{\mathbf{f}}(i, j)$. It is defined as the distance PP' between the two points P and P' , which are the intersection points of $\mathbf{f}(i, j)$ and $\hat{\mathbf{f}}(i, j)$ with the Maxwell triangle, respectively. This is shown graphically in Fig. 5.

We have applied VDF to a number of color images ranging from detailed indoor and outdoor scenes to human faces. We present here the results for two images, "Lena" and "Lake," shown in Fig. 6. The test images have been contaminated using the following noise

distributions: (a) Gaussian ($\sigma = 30$), (b) Gaussian ($\sigma = 30$) contaminated with 2% of impulses, (c) double-exponential ($\sigma = 40$), and (d) impulsive (4%). The impulsive noise has been simulated in two steps. In the first step each image channel is corrupted independently with 4% impulsive noise. In the second step, a correlation factor $\rho = 0.5$ is used to further determine the corruption of pixel (i, j) in channel C , if the same pixel (i, j) is corrupted in any of the two other channels. The second step simulates the channel correlation in multichannel images. It is noted that, although vector order statistics based filters are not optimal for Gaussian noise, this type of noise has been used in our experiments since, for a suitable selection of the magnitude processing filter (e.g., α -trimmed mean), VDF are expected to perform efficiently in this case, too.

Tables I and II show the NMSE results for the two test images and for filter windows 3×3 and 5×5 . A "*" in a table entry indicates the best filter performance in the corresponding row (noise distribution). From these results it can be concluded that VDF perform at least as good and in most cases better than VMF. It can be observed that the GVDF followed by an α -trimmed mean filter has very good performance in short tailed noise. Similarly, with a

multistage max/median filter as the magnitude processing filter, very good performance in long tailed noise is obtained. Tables III and IV show the MCRE results for the same images. As can be observed, VDF result always in a better chrominance estimate than VMF and this justifies their employment in color image processing.

In addition to the quantitative evaluation presented above, a qualitative evaluation seems worthwhile since the topic is *image processing* and the visual assessment of the processed images is, ultimately, the best subjective measure of the efficacy of any method. Therefore, we present sample processing results in Fig. 7. Fig. 7(a) shows the "Lena" image corrupted with Gaussian noise ($\sigma = 30$) and Fig. 7(b) shows the same image corrupted with 4% impulsive noise. Figs. 7(c) and (d) show the VMF results for Figs. 7(a) and (b), respectively, and Figs. 7(e), (f), (g), and (h) refer to the VDF processing results. Figs. 7(e) and (f) show the GVDF/ α -trimmed mean results and Figs. 7(g) and (h) show the GVDF/multistage max/median results. The superiority of the GVDF/ α -trimmed mean in the case of Gaussian noise and the GVDF/multistage max/median in the case of impulsive noise is illustrated in this example. In effect, the properties of the gray-scale processing filters that make them appropriate for monochrome image processing have also been retained in the case of color images. In other words, the α -trimmed mean is still efficient in low pass filtering the Gaussian noise, whereas the multistage max/median possesses the detail preserving property. This demonstrates the advantage of combining the directional filters with efficient gray-scale filters. The former filter out vectors with "atypical" direction producing thus a set of vectors with almost the same direction; the latter can, subsequently, perform on this output set as if it has been produced by a single-channel source.

It should be noted that the correlated noise has no effect on VDF (and VMF) since they operate directly on the image vectors and not on the individual channels of a multichannel image. This can also be verified from the results obtained.

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Recursive High-Resolution Reconstruction of Blurred Multiframe Images

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Abstract—An approach to obtain high-resolution image reconstructions from low-resolution, blurred, and noisy multiple-input frames is presented. A recursive least-square approach with iterative regularization is developed in the DFT domain.

When the input frames are processed recursively, the reconstruction does not converge in general due to the measurement noise and ill-conditioned nature of the deblurring. Through the iterative update of the regularization function and the proper choice of the regularization parameter, good high-resolution reconstructions are obtained. This provides an unified approach for deblurring, noise removal, and high resolution reconstruction of low-resolution, blurred, and noisy input frames. The proposed recursive algorithm minimizes the required computations and provides the parallel computation structure since the reconstruction is done independently for each DFT element. Computer simulations demonstrate the performance of the algorithm.

I. INTRODUCTION

In the applications such as satellite remote sensing and computer vision applications, it is required to reconstruct a high-resolution image from multiframe of undersampled low-resolution images from a scene. In many practical cases, input images are degraded by both blurs and noises. A DFT-based algorithm for reconstruction of high-resolution images from multiple input frames corrupted by noises has been developed in [1]. However, this result can not be directly used if the input frames have blur distortions. In this correspondence, the previous result in [1] is advanced for blurred input frames. In [2] and [3], some multi-channel image restoration schemes were presented for blurred images, and in [4] and [5], restorations from multiple measurements were studied. The issues on increasing the resolution of the input images, however, have not been addressed.

In the high-resolution reconstruction problem where blur distortions are included, it is desirable to include the deblurring computation into the reconstruction process since the deblurring of input frames separately would introduce phase and high-frequency distortions in the input frames, which is not desirable for high

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