Mathematical
Inequalities
& Applications
Volume 18, Number 2 (2015), 769–786 doi:10.7153/mia-18-57

VECTOR MAJORIZATION AND SCHUR–CONCAVITY OF SOME SUMS GENERATED BY THE JENSEN AND JENSEN–MERCER FUNCTIONALS

MAREK NIEZGODA

Abstract. In this paper we study a vector majorization ordering for comparing two m-tuples of vectors of a real linear space. This extends the classical approach of (scalar) majorization theory for comparing m-tuples of scalars in $\mathbf{R}$. We prove a Sherman type inequality for a vector-valued $\leqslant_C$-convex function f, where $\leqslant_C$ is a cone ordering. In consequence, we obtain a Hardy-Littlewood-Pólya-Karamata type inequality generated by m-tuples of vectors in a vector space. As applications, we present majorization generalizations of the superadditivity properties of the Jensen and Jensen-Mercer functionals generated by a convex function f. In addition, we show that some sums generated by the Jensen and Jensen-Mercer functionals are Schur-concave with respect to their weight vectors. We also give interpretations of the obtained results for tridiagonal doubly stochastic matrices and doubly stochastic circular matrices.

Mathematics subject classification (2010): 52A40, 06F20, 26B25, 15A39.

Keywords and phrases: Convex function, Jensen functional, Jensen-Mercer functional, sub-/super-additive function, vector majorization, cone ordering, Schur-convex/concave function, column stochastic matrix, doubly stochastic matrix, doubly stochastic circular matrix, tridiagonal doubly stochastic matrix.

REFERENCES

[1] J. BORCEA, *Equilibrium points of logarithmic potentials*, Trans. Amer. Math. Soc., **359**, (2007), 3209–3237.

[2] L. BOUGOFFA, *New inequalities about convex functions*, J. Inequal. Pure Appl. Math., **7**, 4 (2006), Art. 148.

[3] W. W. BRECKNER AND T. TRIF, *Convex Functions and Related Functional Equations: Selected Topics*, Cluj University Press, Cluj, 2008.

[4] A.-M. BURTEA, *Two examples of weighted majorization*, Annals of the University of Craiova, Mathematics and Computer Science Series, **32**, 2 (2010), 92–99.

[5] G. DAHL, *Matrix majorization*, Linear Algebra Appl., **288**, (1999), 53–73.

[6] S. S. DRAGOMIR, *Bounds for the normalised Jensen functional*, Bull. Austral. Math. Soc., **74**, (2006), 471–478.

[7] S. S. DRAGOMIR, *A refinement of Jensen's inequality with applications for f-divergence measures*, Taiwanese J. Math., **14**, 1 (2010), 153–164.

[8] S. S. DRAGOMIR, J. E. PEČARIĆ AND L. E. PERSSON, *Properties of some functionals related to Jensen's inequality*, Acta Math. Hung., **70**, 1–2 (1996), 129–143.

[9] M. KRNIĆ, N. LOVRIČEVIĆ AND J. PEČARIĆ, *On some properties of Jensen-Mercer's functional*, J. Math. Inequal., **6**, 1 (2012), 125–139.

[10] A. W. MARSHALL, I. OLKIN AND B. C. ARNOLD, *Inequalities: Theory of Majorization and Its Applications*, Second Edition, Springer, New York, 2011.

[11] A. MATKOVIĆ AND J. PEČARIĆ, *A variant of the Jensen-Mercer inequality for operators*, J. Math. Inequal., **2**, 3 (2008), 299–307.

[12] A. MCD. MERCER, *A variant of Jensen's inequality*, J. Ineq. Pure & Appl. Math., **4**, 4 (2003), Article 73.

[13] M. NIEZGODA, *A generalization of Mercer's result on convex functions*, Nonlinear Anal.: Theory Methods Appl., **71**, (2009), 2771–2779.

Paper MIA-18-57

[14] M. NIEZGODA, *Inequalities for convex functions and doubly stochastic matrices*, Math. Inequal. Appl., **16**, 1 (2013), 221–232.

[15] M. NIEZGODA, *On $(k,h;m)$-convex mappings and applications*, Math. Inequal. Appl., **17**, 2 (2014), 665–678.

[16] M. NIEZGODA, *Remarks on Sherman like inequalities for (α,β)-convex functions*, Math. Inequal. Appl., **17**, 4 (2014), 1579–1590.

[17] S. SHERMAN, *On a theorem of Hardy, Littlewood, Pólya, and Blackwell*, Proc. Nat. Acad. Sci. USA, **37**, 1 (1957), 826–831.