Vector vortex solitons in nematic liquid crystals

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We analyze the existence and stability of two-component vector solitons in nematic liquid crystals for which one of the components carries angular momentum and describes a vortex beam. We demonstrate that the nonlocal, nonlinear response can dramatically enhance the field coupling leading to the stabilization of the vortex beam when the amplitude of the second beam exceeds some threshold value. We develop a variational approach to describe this effect analytically. © 2009 Optical Society of America

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Optical vortices are usually introduced as phase singularities in diffracting optical beams [1] and can be generated in both linear and nonlinear media. The well-known effect accompanying the propagation of such singular beams and vortex solitons in self-focusing, nonlinear media is vortex breakup into several fundamental solitons via a symmetry-breaking azimuthal instability [2]. However, recent numerical studies have revealed that spatially localized vortex solitons can be stabilized in highly nonlocal self-focusing nonlinear media [3,4]. This stabilization effect was later explained analytically [5] by employing a modulation theory for the vortex parameters based on an averaged Lagrangian.

Spatial optical-vector solitons can form when several beams propagate together, interacting parametrically or via the effect of cross-phase modulation [6]. The simplest vector solitons are known as shape-preserving self-localized solutions of coupled nonlinear evolution equations [6]. A class of vector solitons in the form of two-color spatial solitons in a highly nonlocal and anisotropic Kerr-like medium were predicted to exist in nematic liquid crystals [7–9]. The first experimental observations of anisotropic, nonlocal vector solitons in unbiased nematic liquid crystals were reported by Alberucci *et al.* [9], who investigated the interaction between two beams of different wavelengths and observed that two extraordinarily polarized beams of different wavelengths can nonlinearly couple, compensating for the beam walk-off, so creating a vector soliton.

The main purpose of this Letter is twofold. First, we introduce a class of vector solitons in nonlocal, nonlinear media, such as nematic liquid crystals, and study their properties. These vector solitons appear as two-color, self-trapped beams for which one of the components carries angular momentum and describes a vortex beam. Secondly, we demonstrate that the nonlocal, nonlinear response may dramatically enhance the field coupling, leading to the stabilization of the vortex for much-weaker nonlocality when the amplitude of the second beam exceeds some threshold value. We develop a variational approach to describe this effect analytically.

We consider the propagation of two light beams of different wavelengths in a cell filled with a nematic liquid crystal. The light propagates in the z direction, with the (x,y) plane orthogonal to this. The electric fields of the light beams are assumed to be polarized in the x direction. The system for the dimensionless complex field amplitudes u and v can be written in the form

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+2u\theta=0,$$

$$i\frac{\partial v}{\partial z}+\frac{1}{2}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+2v\theta=0,$$

$$\nu\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)-2\theta=-2(|u|^2+|v|^2), \tag{1}$$

where θ describes the change of the director angle from the pretilt state, which is related to the nonlinear correction to the optical refractive index. In Eqs. (1) the longitudinal (z) and transverse (x,y) coordinates are normalized to the diffraction length and the beam width, respectively. The parameter ν describes the degree of nonlocality of the nonlinear response. When $\nu\to 0$, Eqs. (1) reduce to the Manakov vector nonlinear equations. The system [Eq. (1)] conserves the energy flow $P=P_1+P_2=\int\int_{-\infty}^{+\infty}(|u|^2+|v|^2)\mathrm{d}x\mathrm{d}y$.

We are interested in a special class of vector solitons for which one of the components carries angular momentum and the other component describes a spatially localized mode in the form of a spatial bright beam. Solutions of this type have been discussed earlier for nonlinear systems with a local response [10,11], and they have been shown to be unstable in a large region of their existence domain [12]. For our system described by Eqs. (1) such solutions can be found in the form $u = w_1(r)\exp(ib_1 z)$ and $v = w_2(r)\exp(i\phi)\exp(ib_2 z)$, where $w_1(r)$ and $w_2(r)$ are real functions describing the beam envelopes, $b_{1,2}$ are real propagation constants, and $r = \sqrt{x^2+y^2}$ is the radial coordinate. The resulting system of equations obtained after substitution of these solution forms into Eqs. (1) is solved using a standard numerical relaxation method. Without loss of generality, we search for solutions with $b_2 \leqslant b_1$ and set $b_1 = 3$ to investigate the properties of vector vortex solitons by varying the propagation constant b_2 and the nonlocality parameter ν.

Figure 1 presents an example of vector vortex solitons for Eqs. (1) for which one component has the shape of a bright soliton [Fig. 1(a)] and the other component carries angular moment, so forming a vortex soliton [Figs. 1(b) and 1(d)]. Owing to the physical nature of the nonlocal response of the nematic liquid crystal, we notice that the refractive index change features a bell-shaped distribution [Fig. 1(c)], even though there is a singularity in the center of the vortex beam, this being crucial for the stabilization of vortex solitons. As shown in Fig. 2(a), for fixed propagation constant b_1 and nonlocality parameter ν the power of the vortex beam is a monotonically increasing function of the propagation constant b_2, whereas the power of the bright soliton decreases monotonically. It is important to note that vector vortex solitons exist in a finite band of the propagation constant b_2 [7,13]. At the lower band edge the vortex beam vanishes, and one obtains a scalar bright soliton. However, at the upper band edge the beam with a bell shape vanishes, so that the vector vortex soliton transforms into a scalar vortex soliton. We find that the existence domain of vector vortex solitons shrinks with increasing nonlocality parameter ν [Fig. 2(b)].

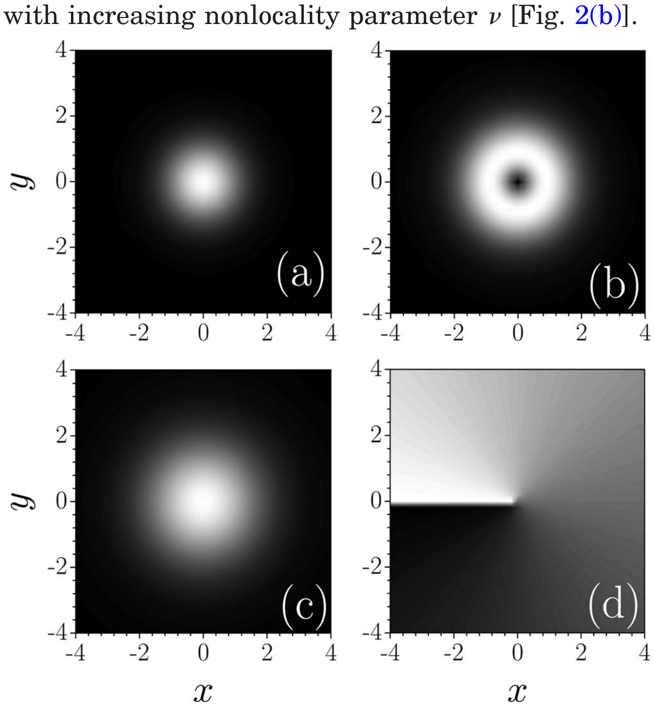

Fig. 1. Field distribution for (a) a bright soliton and (b) a vortex beam. (c) Nonlinear correction to the refractive index. (d) Phase structure of the corresponding vortex beam shown in (b). Here $\nu=1$, $b_1=3$, and $b_2=1.8$.

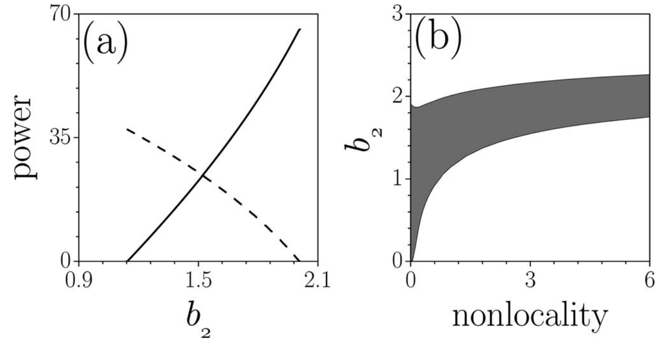

Fig. 2. (a) Power of bright (dashed) and vortex (solid) beams for $\nu=1$. (b) Existence domain (gray) of vector solitons as a function of ν (at $b_1=3$).

One of the central results we find is that a bright beam with a finite amplitude can stabilize an otherwise unstable vortex beam. To address this issue we performed extensive numerical simulations of Eqs. (1) using the beam-propagation method. First, we employed a stationary form of the vortex beam (in order to minimize radiation) as an input beam for the v component only (namely, there is no bright soliton as an input for the u component), noting that a vortex beam is unstable when propagating alone. Then we added a Gaussian pulse to the u component and studied the dynamics of a vortex beam propagating together with a Gaussian beam by varying the nonlocality parameter ν. Our main results are presented in Fig. 3, from which one can see that for low nonlocality, a higher amplitude of the bright beam is required for stabilization of the vortex beam, while for high-enough nonlocality, the vortex beam is observed to be stable, even when propagating alone. Figure 4 shows some illustrative examples. It is clearly seen that when the vortex beam propagates alone it becomes unstable and breaks up into two filaments [see Fig. 4(b)]. However, when we add a Gaussian beam with amplitude 0.9 above the threshold (namely, $a_u^T=0.9$), then the vortex beam copropagates with the Gaussian beam in a stable manner. Thus we draw the conclusion that a nonlocal, nonlinear response can dramatically enhance the field coupling, leading to the stabilization of the vortex soliton when the amplitude of the Gaussian beam exceeds some threshold value. It is interesting to mention that the vortex stabilization described above can be compared with the effect

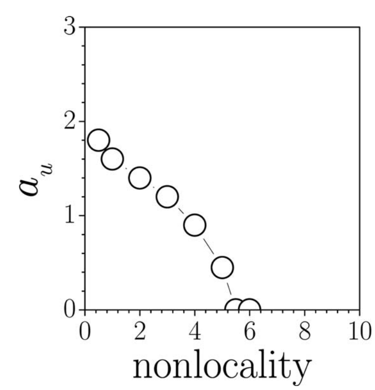

Fig. 3. Dependence of the critical value of the amplitude for a bright soliton on the nonlocality degree ν.

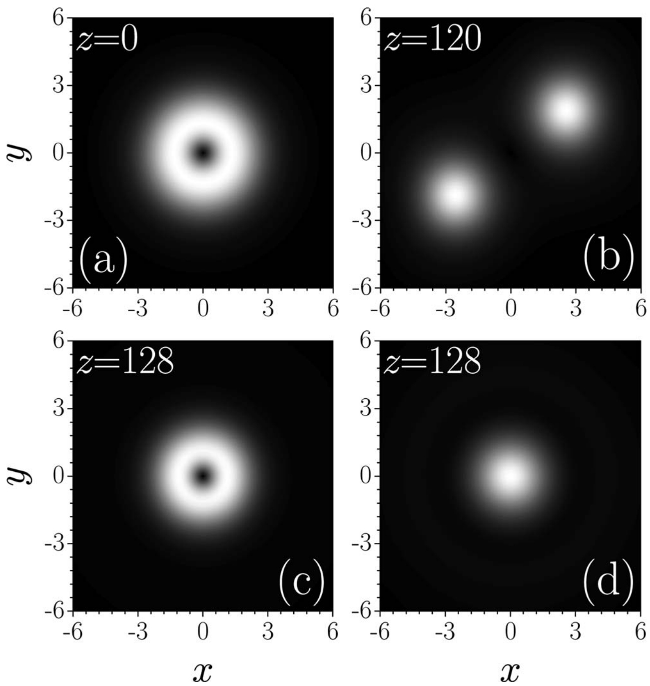

Fig. 4. (a), (b) Input and output of the intensity for an unstable vortex beam propagating alone in the medium. (c), (d) Stable propagation of the vortex beam coupled to the bright component. Here $\nu=4$ and $a_u^T=0.9$.

of instability suppression by partially incoherent light [14], where the light incoherence introduces effective bright components that provide the vortex stabilization [15].

For a deeper insight into the effect of the vortex stabilization, we apply a modulation theory analysis that quantitatively explains the results shown in Fig. 3. Using the two-component equations rewritten in a Lagrangian formulation [16], we employ the trial functions for the vortex, the bright component, and the director angle,

$$u = a_u \operatorname{sech}(r/w_u)e^{i\sigma_u z},$$

$$v = a_v r e^{-r/w_v} e^{i\phi + i\sigma_v z} + ige^{i\phi+i\sigma_v z},$$

$$\theta = \alpha_u \operatorname{sech}^2(r/\beta_u). \tag{2}$$

These functions are substituted into the Lagrangian, which is then averaged by integrating in r and ϕ over the plane. This procedure gives the averaged Lagrangian $\mathcal{L}=\mathcal{L}_u+\mathcal{L}_v+\mathcal{L}_{uv}$, where $\mathcal{L}_u$ is given in [17] and $\mathcal{L}_v$ is given in [5]. The shelf g in the vortex is decomposed into a modal expansion, with the $l=2$ mode found to be the most unstable [5]. The important interaction Lagrangian between the vortex and the soliton is $\mathcal{L}_{uv}=(a_u^2 w_u^2)(2\sqrt{2q\nu})^{-1}a_v^2 w_v^2$.

The vortex is stabilized as its width decreases and its amplitude increases [5]. Therefore, we just need to show that the vortex width decreases as the beam amplitude in the other component increases. If $A_v = a_v w_v e^{-1}$ is the vortex amplitude, then from [18], $(e^2A_v^2/8\sqrt{\nu})w_v^2+(a_u w_u)^2 w_v - 3/2=0$. Using the vortex width determined by this expression in the stability threshold of [5], we find that the vortex is stable provided $(405/128\nu)A_v^2 w_v^4 < 14.4$. Combining this criterion with the amplitude-width relation, we obtain that the vortex stabilizes for lower values of the nonlocality parameter ν as the amplitude a_u of the bright beam increases, which explains the results shown in Figs. 3 and 4.

In conclusion, we have described theoretically a type of stable vector vortex soliton in nonlocal nonlinear media such as nematic liquid crystals. These solitons appear in the form of two-color self-trapped beams for which one of the components carries angular momentum being stabilized by the nonlocality-enhanced coupling with the other localized beam. We have studied the effect of stabilization numerically and have also developed a variational approach to describe it analytically.

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