Vehicle Dynamics: Theory and Application

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Dedicated to
my son, *Kavosh,*
my daughter, *Vazan,*
and my wife, *Mojgan.*

Happiness is when you win a race against yourself.

Preface

This text is for engineering students. It introduces the fundamental knowledge used in *vehicle dynamics*. This knowledge can be utilized to develop computer programs for analyzing the ride, handling, and optimization of road vehicles.

Vehicle dynamics has been in the engineering curriculum for more than a hundred years. Books on the subject are available, but most of them are written for specialists and are not suitable for a classroom application. A new student, engineer, or researcher would not know where and how to start learning vehicle dynamics. So, there is a need for a textbook for beginners. This textbook presents the fundamentals with a perspective on future trends.

The study of classical vehicle dynamics has its roots in the work of great scientists of the past four centuries and creative engineers in the past century who established the methodology of dynamic systems. The development of vehicle dynamics has moved toward modeling, analysis, and optimization of multi-body dynamics supported by some compliant members. Therefore, merging dynamics with optimization theory was an expected development. The fast-growing capability of accurate positioning, sensing, and calculations, along with intelligent computer programming are the other important developments in vehicle dynamics. So, a textbook help the reader to make a computer model of vehicles, which this book does.

Level of the Book

This book has evolved from nearly a decade of research in nonlinear dynamic systems and teaching courses in vehicle dynamics. It is addressed primarily to the last year of undergraduate study and the first year graduate student in engineering. Hence, it is an intermediate textbook. It provides both fundamental and advanced topics. The whole book can be covered in two successive courses, however, it is possible to jump over some sections and cover the book in one course. Students are required to know the fundamentals of kinematics and dynamics, as well as a basic knowledge of numerical methods.

The contents of the book have been kept at a fairly theoretical-practical level. Many concepts are deeply explained and their application emphasized, and most of the related theories and formal proofs have been explained. The book places a strong emphasis on the physical meaning and applications of the concepts. Topics that have been selected are of high interest in the field. An attempt has been made to expose students to a broad range of topics and approaches.

There are four special chapters that are indirectly related to vehicle dynamics: *Applied Kinematics*, *Applied Mechanisms*, *Applied Dynamics*, and *Applied Vibrations*. These chapters provide the related background to understand vehicle dynamics and its subsystems.

Organization of the Book

The text is organized so it can be used for teaching or for self-study. Chapter 1 “Fundamentals,” contains general preliminaries about tire and rim with a brief review of road vehicle classifications.

Part *I* “One Dimensional Vehicle Dynamics,” presents forward vehicle dynamics, tire dynamics, and driveline dynamics. Forward dynamics refers to weight transfer, accelerating, braking, engine performance, and gear ratio design.

Part *II* “Vehicle Kinematics,” presents a detailed discussion of vehicle mechanical subsystems such as steering and suspensions.

Part *III* “Vehicle Dynamics,” employs Newton and Lagrange methods to develop the maneuvering dynamics of vehicles.

Part *IV* “Vehicle Vibrations,” presents a detailed discussion of vehicle vibrations. An attempt is made to review the basic approaches and demonstrate how a vehicle can be modeled as a vibrating multiple degree-of-freedom system. The concepts of the Newton-Euler dynamics and Lagrangian method are used equally for derivation of equations of motion. The RMS optimization technique for suspension design of vehicles is introduced and applied to vehicle suspensions. The outcome of the optimization technique is the optimal stiffness and damping for a car or suspended equipment.

Method of Presentation

This book uses a "*fact-reason-application*" structure. The "fact" is the main subject we introduce in each section. Then the reason is given as a "proof." The application of the fact is examined in some "examples." The "examples" are a very important part of the book because they show how to implement the "facts." They also cover some other facts that are needed to expand the subject.

Prerequisites

Since the book is written for senior undergraduate and first-year graduate-level students of engineering, the assumption is that users are familiar with matrix algebra as well as basic dynamics. Prerequisites are the fundamentals of kinematics, dynamics, vector analysis, and matrix theory. These basics are usually taught in the first three undergraduate years.

Unit System

The system of units adopted in this book is, unless otherwise stated, the international system of units (SI). The units of degree (deg) or radian (rad) are utilized for variables representing angular quantities.

Symbols

- Lowercase bold letters indicate a vector. Vectors may be expressed in an n dimensional Euclidian space. Example:

$$\begin{array}{ccccccccccc} \mathbf{r} & , & \mathbf{s} & , & \mathbf{d} & , & \mathbf{a} & , & \mathbf{b} & , & \mathbf{c} \\ \mathbf{p} & , & \mathbf{q} & , & \mathbf{v} & , & \mathbf{w} & , & \mathbf{y} & , & \mathbf{z} \\ \boldsymbol{\omega} & , & \boldsymbol{\alpha} & , & \boldsymbol{\epsilon} & , & \boldsymbol{\theta} & , & \boldsymbol{\delta} & , & \boldsymbol{\phi} \end{array}$$

- Uppercase bold letters indicate a dynamic vector or a dynamic matrix, such as force and moment. Example:

$$\mathbf{F} \quad , \quad \mathbf{M}$$

- Lowercase letters with a hat indicate a unit vector. Unit vectors are not bolded. Example:

$$\begin{array}{ccccccccccc} \hat{\imath} & , & \hat{\jmath} & , & \hat{k} & , & \hat{e} & , & \hat{u} & , & \hat{n} \\ \hat{I} & , & \hat{J} & , & \hat{K} & , & \hat{e}_\theta & , & \hat{e}_\varphi & , & \hat{e}_\psi \end{array}$$

- Lowercase letters with a tilde indicate a 3×3 skew symmetric matrix associated to a vector. Example:

$$\tilde{a} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \quad , \quad \mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

- An arrow above two uppercase letters indicates the start and end points of a position vector. Example:

$$\overrightarrow{ON} = \text{a position vector from point } O \text{ to point } N$$

- The length of a vector is indicated by a non-bold lowercase letter. Example:

$$r = |\mathbf{r}| \quad , \quad a = |\mathbf{a}| \quad , \quad b = |\mathbf{b}| \quad , \quad s = |\mathbf{s}|$$

- Capital letter B is utilized to denote a body coordinate frame. Example:

$$B(oxyz) \quad , \quad B(Oxyz) \quad , \quad B_1(o_1 x_1 y_1 z_1)$$

- Capital letter G is utilized to denote a global, inertial, or fixed coordinate frame. Example:

$$G \quad , \quad G(XYZ) \quad , \quad G(OXYZ)$$

- Right subscript on a transformation matrix indicates the *departure* frames. Example:

$$R_B = \text{transformation matrix from frame } B(oxyz)$$

- Left superscript on a transformation matrix indicates the *destination* frame. Example:

$$\begin{aligned} {}^G R_B &= \text{transformation matrix from frame } B(oxyz) \\ &\quad \text{to frame } G(OXYZ) \end{aligned}$$

- Capital letter R indicates rotation or a transformation matrix, if it shows the beginning and destination coordinate frames. Example:

$${}^G R_B = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Whenever there is no sub or superscript, the matrices are shown in a bracket. Example:

$$[T] = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Left superscript on a vector denotes the frame in which the vector is expressed. That superscript indicates the frame that the vector belongs to; so the vector is expressed using the unit vectors of that frame. Example:

$${}^G\mathbf{r} = \text{position vector expressed in frame } G(OXYZ)$$

- Right subscript on a vector denotes the tip point that the vector is referred to. Example:

$$\begin{aligned} {}^G\mathbf{r}_P &= \text{position vector of point } P \\ &\quad \text{expressed in coordinate frame } G(OXYZ) \end{aligned}$$

- Right subscript on an angular velocity vector indicates the frame that the angular vector is referred to. Example:

$$\boldsymbol{\omega}_B = \text{angular velocity of the body coordinate frame } B(oxyz)$$

- Left subscript on an angular velocity vector indicates the frame that the angular vector is measured with respect to. Example:

$$
\begin{aligned}
{}_G\boldsymbol{\omega}_B \quad = \quad & \text{angular velocity of the body coordinate frame } B(oxyz) \\
& \text{with respect to the global coordinate frame } G(OXYZ)
\end{aligned}
$$

- Left superscript on an angular velocity vector denotes the frame in which the angular velocity is expressed. Example:

$$
\begin{aligned}
{}_G^{B_2}\boldsymbol{\omega}_{B_1} \quad = \quad & \text{angular velocity of the body coordinate frame } B_1 \\
& \text{with respect to the global coordinate frame } G, \\
& \text{and expressed in body coordinate frame } B_2
\end{aligned}
$$

  Whenever the subscript and superscript of an angular velocity are the same, we usually drop the left superscript. Example:

$$
{}_G\boldsymbol{\omega}_B \equiv {}_G^G\boldsymbol{\omega}_B
$$

  Also for position, velocity, and acceleration vectors, we drop the left subscripts if it is the same as the left superscript. Example:

$$
{}_B^B\mathbf{v}_P \equiv {}^B\mathbf{v}_P
$$

- Left superscript on derivative operators indicates the frame in which the derivative of a variable is taken. Example:

$$
\frac{{}^G d}{dt}\, x \quad , \quad \frac{{}^G d}{dt}\, {}^B\mathbf{r}_P \quad , \quad \frac{{}^B d}{dt}\, {}_B^G\mathbf{r}_P
$$

  If the variable is a vector function, and also the frame in which the vector is defined is the same frame in which a time derivative is taken, we may use the following short notation,

$$
\frac{{}^G d}{dt}\, {}^G\mathbf{r}_P = {}^G\dot{\mathbf{r}}_P \quad , \quad \frac{{}^B d}{dt}\, {}_o^B\mathbf{r}_P = {}_o^B\dot{\mathbf{r}}_P
$$

  and write equations simpler. Example:

$$
{}^G\mathbf{v} = \frac{{}^G d}{dt}\, {}^G\mathbf{r}(t) = {}^G\dot{\mathbf{r}}
$$

- If followed by angles, lowercase c and s denote *cos* and *sin* functions in mathematical equations. Example:

$$
c\alpha = \cos\alpha \quad , \quad s\varphi = \sin\varphi
$$

- Capital bold letter **I** indicates a unit matrix, which, depending on the dimension of the matrix equation, could be a 3 × 3 or a 4 × 4 unit matrix. $\mathbf{I}_3$ or $\mathbf{I}_4$ are also being used to clarify the dimension of **I**. Example:

$$\mathbf{I} = \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- An asterisk ★ indicates a more advanced subject or example that is not designed for undergraduate teaching and can be dropped in the first reading.

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