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VEHICLE FLEET MANAGEMENT: A BAYESIAN APPROACH

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Abstract: This paper focuses on companies that have both a fleet to serve customers and depots for vehicle maintenance. When management of such a vehicle fleet is considered, one of the most important problems is the computation (estimation) of the reliability and the availability of the vehicles. What often makes this computation difficult is the insufficient data for statistical inference or total lack of data (such an aggravating circumstance arises, for example, whenever the fleet is renewed). Concentrating on such a case, this paper presents some analytical formulae based on the Bayesian approach to uncertainty that contribute to the solution of the problem.

Keywords: Vehicle fleet management, uncertainty, Bayesian approach, probability distributions.

1. INTRODUCTION

The fleets considered in this paper are maintained in depots that belong to the same company as the fleet. When the company organizes its current activities and plans future ones, it must make a long-term general plan for employing the vehicles and an executive work plan (EWP) for the present time and near future.

Along with making plans, the company must ensure that the necessary number of appropriate vehicles are in working condition when needed (according to the EWP), and that there are also stand-by vehicles available. This requires information about the size and structure of the vehicle fleet, the condition of each vehicle (which is subject to changes) and the maintenance facilities in the depots.

Information about changes in vehicle conditions and reliability data, in particular, constitute the basis for:

- determining the probability of fulfilling the tasks specified by the EWP [6],
- defining the concept of corrective maintenance and the system of preventive maintenance,
- defining the capacity of maintenance facilities and the organization of the maintenance system,

- computing the availability of the vehicles, and
- defining the necessary transport capacities (the number and type of vehicles) which correspond to the set of transport demands.

How is this to be done if there is a lack of appropriate information? In other words, how can we deal with the described uncertainty?

The objective of this paper is to help solve the problem of predicting the state of the vehicle fleet. The existence of uncertainty in vehicle fleet management and maintenance due to the lack of data for statistical inference is taken into consideration. Since no method has been elaborated to determine the reliability and availability of the vehicle fleet under uncertain conditions, this paper suggests a method based on the Bayesian treatment of uncertainty, which can include all specific features of the problem.

2. STATEMENT OF THE PROBLEM

A vehicle fleet is usually a heterogeneous set comprising vehicles of different structure and age. Vehicles of the same structure and age form homogeneous subsets, which can be called construction-operation groups or CO groups. Often a CO group comprises a small number of vehicles. A heterogeneous vehicle fleet, having more than one CO group, would have $N_i^{(j)}$ inventory vehicles in the j-th CO group. Our consideration can be confined to only one CO group since the procedure can be repeated for each CO group separately. So, hereinafter, index j will be omitted.

Each CO group of vehicles has transportation tasks to fulfill in a given time period, the tasks being defined by the EWP. By presenting the variables relevant to the EWP on a chart (time being the abscissa), for a CO group under consideration we can visualize the relation between the number of vehicles $N_n(t)$ necessary to fulfill the transportation tasks and the number of available vehicles (technically fit for operation). $N_n(t)$ is a non-random function of time. This function is often periodical, periods T being one day, one week, etc. An example is given in Fig. 1.

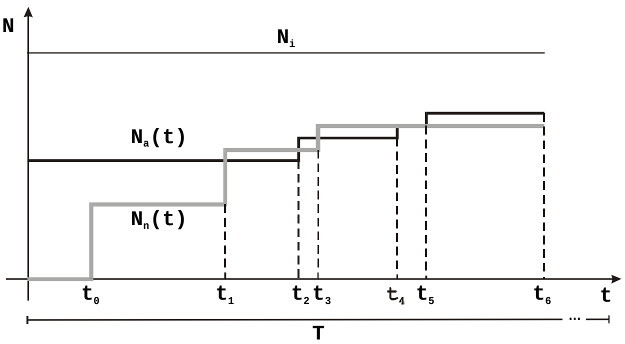

Figure 1.

The variables under consideration are:

N_i - number of vehicles

$N_a(t)$ - number of available vehicles (a random variable)

$N_m(t)$ - number of vehicles not fit for operation due to failure or in need of regular maintenance (a random variable)

$N_n(t)$ - number of vehicles necessary to fulfill the tasks

$N_o(t)$ - number of vehicles in operation (a non-random variable if $N_o(t) = N_n(t)$ and a random variable if $N_o(t) = N_a(t) < N_n(t)$)

$N_s(t)$ - number of stand-by vehicles fit for operation - vehicles in reserve (a random variable)

t_i - moments when either the number of necessary vehicles, $N_n(t)$, or the number of available vehicles, $N_a(t)$, changes.

Three types of intervals can be distinguished on the chart in Figure 1:

1. intervals such as

$$(t_0,t_1), (t_2,t_3) \text{ and } (t_5,t_6)$$

in which $N_n(t) < N_a(t)$ and hence $N_o(t) = N_n(t) < N_a(t)$ and $N_s(t) > 0$. These are the intervals in which all tasks are fulfilled and $N_s(t)$ vehicles are in reserve.

2. intervals such as

$$(t_4,t_5)$$

in which $N_n(t) = N_a(t)$, and hence $N_o(t) = N_n(t) = N_a(t)$, while $N_s(t) = 0$. In such intervals all tasks are fulfilled, but at a high risk, since there are no stand-by vehicles (no vehicles in reserve).

3. intervals such as

$$(t_1,t_2) \text{ and } (t_3,t_4)$$

in which $N_n(t) > N_a(t)$ and hence $N_o(t) = N_a(t) < N_n(t)$ with $N_s(t) = 0$. In such intervals some tasks cannot be fulfilled.

Since all tasks will be fulfilled only when $N_n(t) \le N_a(t)$, i.e. when $N_n(t) \le N_i - N_m(t)$, it follows that, for given N_i and $N_n(t)$, limitations are put on the number of vehicles not fit for operation, $N_m(t)$.

Vehicles are not fit for operation either due to regular maintenance activities or due to failures. Considering the influence of various kinds of failures on the process of transportation, three groups of failures can be distinguished:

A - "unimportant" failures - those which do not directly influence the basic functioning of a vehicle (and safe driving).
The failure is identified after the vehicle has completed its task and has returned to the depot. The number of vehicles with this kind of failure will be denoted by N_m^I.

B - "delay provoking" failures - those which obstruct the basic functioning of the vehicle (or safe driving) but can be corrected on the spot.
The vehicle may be repaired by the driver or by a group of repairmen, so that the vehicle completes its task with some delay. The number of vehicles with this kind of failure will be denoted by N_m^{II}.

C - "critical" failures - those which hamper the basic functioning of the vehicle (or safe driving) and are to be taken care of at the depot.
The number of vehicles with this kind of failure will be denoted by N_m^{III}.

By adopting this classification of failures, the state of the vehicles, for the time period under consideration, may be represented by the diagram shown in Figure 2.

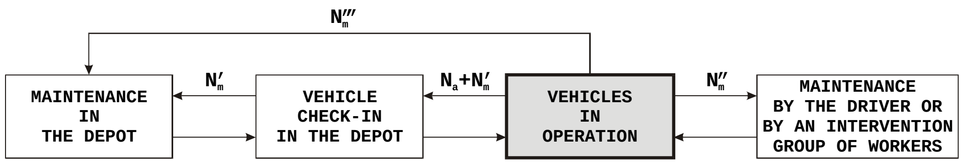

Figure 2.

The possibility of failures and the necessity of regular maintenance require the existence of stand-by vehicles, $N_s(t)$.

The influence of failures from group A on the size of the stand-by fleet can usually be neglected since $N_s(t)$ is rarely diminished due to these failures. The repair capacities at the depot are usually sufficient to correct these failures before the departure time of the vehicle, scheduled by EWP.

The influence of failures from group B is small since such failures require the use of a stand-by vehicle only if the delay overlaps the beginning of the next task assigned to the vehicle.

Failures of group C have a strong influence on the size of the "stand-by" group of vehicles. Due to these failures a vehicle from the stand-by fleet must be mobilized to continue the task. Often these failures require a tow vehicle to be mobilized as well.

The moment the vehicles are introduced into operation it is important to know the regularity of appearance of various kinds of failures and the dynamics of their elimination.

To our knowledge, attempts to solve this problem which incorporate the above kind of uncertainty are not found in the available literature.

The possibility of predicting changes in the state of the vehicle fleet, obtaining knowledge about the functioning of the maintenance system and having appropriate tools to quantify these processes, which is of ultimate concern here, would enable the determination of a stand-by fleet that satisfies real demands and reduces costs.

To reach this goal we started from the assumption that the time between consecutive failures is distributed exponentially for each vehicle. As it is known from reliability theory [3,4,8] and illustrated using experimental data and the simulation method [2], this assertion is valid under the following conditions:

- the system (vehicle) may be regarded as a complex system, structured into s components (assemblies) that are mutually independent in regard to possible failure;
- the number of components, i.e. the number of possible types of failures, s, is large (at least several dozen)[1];
- each component has its own distribution of the time between failures;
- any failure of any of the components results in the failure of the system, i.e. of the vehicle.

Respecting the foregoing conditions, one can simulate the failure of each component by assigning to each a certain distribution. These distributions can then be combined to give a superposed distribution which, after applying a statistical test, proves to be exponential. The CO groups may differ in the number of components and they usually differ in the distributions assigned to each component. Consequently, the CO groups usually differ in the parameters of the resultant exponential distributions and in failure rates.

Due to the assumption that the resultant distribution is an exponential distribution, the uncertainty can be incorporated through the unknown failure rate l . The conditional distribution of the number of failures in time period t therefore has a Poisson distribution $P(lt)$ and the Poisson distribution has a natural conjugate [1,4,9]. Now all the conditions necessary to use the Bayesian approach to uncertainty are fulfilled.

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[1] In detailed analysis this number can amount to several thousand.

3. SOLUTION OF THE PROBLEM

In order to solve the problem, the essential question to be answered is the following: how to obtain, in conditions of uncertainty, the distribution of the number of failures per unit time interval Δt . Unit time intervals, called sequences, are notions associated to sequential planning and record keeping. At the end of the i-th sequence, in order to plan for sequence $(i+1)$ (respecting the number of indispensable vehicles, the needed capacity of the maintenance depot and other elements of the logistic support), it is necessary to predict the number of failures in sequence $(i+1)$. The number of failures recorded in the i-th sequence is to be used at the beginning of sequence $(i+1)$ to correct the parameters of the failure rate distribution.

Let us now consider a homogeneous set of n vehicles, each having the same failure rate l^*, which is unknown. Failure rate is the average number of failures in a sequence. In accordance with the Bayesian approach, an unknown rate is treated as a random variable. In order to have compatible distributions [1, 4, 9] in this case a gamma distribution is chosen:

$$l^* \sim \Gamma(a,b)\,,\ 0 < a, b < \infty \tag{1}$$

At the beginning of the Bayesian treatment of uncertainty, i.e. at the beginning of the first sequence, the a priori values of parameters a and b in the gamma distribution must be determined. This is done either on the basis of available partial information or subjectively when data about the failure rate do not exist [4, 7].

Due to the additivity of the gamma distribution, the whole set of n vehicles, i.e. the whole CO group, is characterized by failure rate l , which is treated as a random variable with a gamma distribution:

$$l \sim \Gamma(\mathrm{n}a, b) \tag{2}$$

This distribution (the a priori distribution) is characterized by the density function:

$$\mathrm{f}(l) = \frac{b(bl)^{\mathrm{n}a-1}\mathrm{e}^{-bl}}{\Gamma(\mathrm{n}a)}, \quad l > 0 \tag{3}$$

The marginal distribution of the number of failures in the whole group of vehicles during the first sequence (unit time interval), denoted by $X_{\Delta t_1}$, can be obtained from the following equation:

$$\begin{aligned} P(X_{\Delta t_1} = k) &= \int_0^\infty P(X_{\Delta t_1} = k \mid l)\,\mathrm{f}(l)\,\mathrm{d}l = \\ &= \frac{\Gamma(\mathrm{n}a + k)}{k!\,\Gamma(\mathrm{n}a)} \cdot \frac{b^{\mathrm{n}a}}{(b+1)^{\mathrm{n}a}} \cdot \frac{1}{(b+1)^{k}} \end{aligned} \tag{4}$$

from which it follows that $X_{\Delta t_1}$ has a negative binomial distribution (N.B.):

$$X_{\Delta t_1} \sim \text{N.B. } (na, \frac{b}{b+1}) \tag{5}$$

When computing individual probabilities, according to (5), the following recurrent formula is used:

$$\begin{cases} P(X_{\Delta t_1} = 0) = (\frac{b}{b+1})^{na} \\ P(X_{\Delta t_1} = k) = \frac{na + k - 1}{k(b+1)} P(X_{\Delta t_1} = k-1), \quad k = 1,2,... \end{cases} \tag{6}$$

This formula is used to predict the number of failures in a homogeneous group of n vehicles during the first sequence Δt_1. If during the first N consecutive sequences (unit time intervals), $\Delta t_1, \Delta t_2, ..., \Delta t_N$, we have registered $x_1, x_2, ..., x_N$ failures for the whole CE group under consideration, the *a posteriori* distribution of l will be:

$$\begin{aligned} f(l|x_1, x_2, ..., x_N) &= \frac{\prod_{i=1}^{N} P(X_{\Delta t_i} = x_i | l) f(l)}{\int_0^\infty \prod_{i=1}^{N} P(X_{\Delta t_i} = x_i | l) f(l) dl} = \\ &= \frac{(b+N)[(b+N)l]^{na + \sum_{i=1}^{N} x_i - 1} e^{-(b+N)l}}{\Gamma(na + \sum_{i=1}^{N} x_i)} \end{aligned} \tag{7}$$

This means that the a posteriori distribution of l is again a gamma distribution:

$$l \mid x_1, x_2, ..., x_N \sim \Gamma(na + \sum_{i=1}^{N} x_i, \; b + N) \tag{8}$$

A comparison of the a posteriori distribution (8) to the a priori distribution (2) reveals how the number of registered failures $x_1, x_2, ..., x_N$ and the number of past sequences N are used to correct the parameters of the distribution. Hence, in sequence $(N+1)$ (unit time interval), the distribution of the number of failures which will serve for planning purposes is obtained from the following equation:

$$\begin{aligned} P(X_{\Delta t_{N+1}} = k) &= \int_0^\infty P(X_{\Delta t_{N+1}} = k \mid l) f(l \mid x_1, x_2, ..., x_N) dl = \\ &= \frac{\Gamma(na + \sum_{i=1}^{N} x_i + k)}{k! \Gamma(na + \sum_{i=1}^{N} x_i)} (\frac{b+N}{b+N+1})^{na + \sum_{i=1}^{N} x_i} (\frac{1}{b+N+1})^k \end{aligned} \tag{9}$$

Thus, from (9) it follows that the distribution is again a negative binomial distribution:

$$X_{\Delta t_{N+1}} \sim \text{N.B.}\ (na + \sum_{i=1}^{N} x_i, \frac{b+N}{b+N+1}) \tag{10}$$

The expected number of failures and the corresponding variance, respectively, are given by:

$$E(X_{\Delta t_{N+1}}) = \frac{na + \sum_{i=1}^{N} x_i}{b+N}$$

and (11)

$$V(X_{\Delta t_{N+1}}) = \frac{(na + \sum_{i=1}^{N} x_i)(b+N+1)}{(b+N)^2}$$

The set of failures for which the former calculations were performed was the set comprising all failures: "unimportant" failures (type A), "delay provoking" failures (type B) and "critical" failures (type C). Now, calculations will be performed, first, for the subset comprising only B and C failures (the union of subsets B and C) and then for the subset comprising only C failures.

Let us look for the distribution of the number of failures of type B or C. (These are failures which, unlike those of type A, do influence the fulfillment of the tasks). In order to do this we will modify equation (10) using a suitable theorem [1].

If the probability that the failure is of type A is denoted by p then the probability that it is of type B or C is $1 - p = q$.

After N sequences in which the number of failures was registered, the number of failures during the sequence Δt_{N+1}, $X_{\Delta t_{N+1}}$, has a negative binomial distribution (10) with parameters $na + \sum_{i=1}^{N} x_i$ and $\frac{b+N}{b+N+1}$. According to the theorem [1] it follows that the number of failures of type B or C, $Y_{\Delta t_{N+1}}$, also has a negative binomial distribution, given by:

$$Y_{\Delta t_{N+1}} \sim \text{N.B.}\ (na + \sum_{i=1}^{N} x_i, \frac{b+N}{b+N+q}) \tag{12}$$

We shall now proceed to calculations for the subset comprising C failures alone. Failures of type C are a subset of the set of all failures. These "critical" failures always require a stand-by vehicle to be mobilized. (From the viewpoint of vehicle fleet management, it is convenient that these failures occur rarely.) In order to treat these failures, the following model is introduced.

Let n be the number of vehicles in the CO group under consideration. Let us suppose that the number per sequence of type C failures is a random variable denoted by Z with a conditional binomial distribution $\mathbf{B}(n, p)$, where parameter p is unknown. Thus:

$$P(Z = k \mid p) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0,1,\ldots,n \tag{13}$$

Parameter *p* represents the probability per sequence of a type C failure. The Bayesian learning algorithm treats p as a random variable with a beta distribution. The density function $g(p)$, with the a priori determined values of parameters a and b, is given by:

$$g(p) = \frac{p^{a-1}(1-p)^{b-1}}{B(a,b)}, \quad 0 < p < 1, \quad a > 0, \quad b > 0. \tag{14}$$

After N sequences in which $z_1, z_2, \ldots, z_N$ failures of type C are registered, using the Bayes formula we can obtain the a posteriori distribution which is defined by the density function:

$$g(p \mid z_1, z_2, \ldots, z_N) = \frac{p^{a+\sum_{i=1}^{N} z_i - 1}(1-p)^{b+nN+\sum_{i=1}^{N} z_i - 1}}{B(a+\sum_{i=1}^{N} z_i \,,\; b+nN-\sum_{i=1}^{N} z_i)} \tag{15}$$

It follows that p has an a posteriori distribution which is again a beta distribution:

$$p \mid z_1, z_2, \ldots, z_N \sim B(a+\sum_{i=1}^{N} z_i \,,\; b+nN-\sum_{i=1}^{N} z_i) \tag{16}$$

If we denote by Z_{N+1} the number of failures in sequence $(N+1)$, then the distribution of Z_{N+1} is determined by the equation:

$$P(Z_{N+1} = k) = \frac{\binom{n}{k} B(a+\sum_{i=1}^{N} z_i + k \,,\; b+n(N+1)-\sum_{i=1}^{N} z_i - k)}{B(a+\sum_{i=1}^{N} z_i \,,\; b+nN-\sum_{i=1}^{N} z_i)} \tag{17}$$

This distribution is used to forecast the number of type C failures in the sequence $(N+1)$. With these forecasts at hand the decision-maker should be able to do better planning.

To summarize: The distribution of all failures (types A, B and C) is given by (10), the distribution of "critical" failures (type C) by (17) and the distribution of "influential" failures (failures of type B or C) by (12). Thereby, we have implicitly considered failures of type A, B and C.

4. NUMERICAL EXAMPLE

In order to illustrate the validity of the derived formulae and show what, based on them, can be further computed and utilized in the decision process, we will now present a real-life example [5]. The data were collected for a construction-operation group (CO) of 13 cistern vehicles, model FAP1314, during the period from 1 July 1987 till 31 December 1990. All these vehicles were put into operation in the Public Company "Gradska ^isto}a" Belgrade as brand new at the beginning of that period. Common characteristics for the whole group are: nearly identical, difficult working conditions for all vehicles (vehicles are used to wash the streets manually, therefore they work in the 1st gear during their operation time), and approximately the same mileage during the corresponding time periods. Failures were recorded during the 1 July 1987 - 31 December 1990 time period and classified according to type A, B, and C failures. Table 1 shows the number of failures in the defined time periods.

Table 1: Number of failures

Time period / Type of failure	1 July 1987 - 31 Dec. 1987	1 January 1988 - 31 Dec. 1988	1 January 1989 - 31 Dec. 1989	1 January 1990 - 31 Dec. 1990
A + B + C	321	754	792	794
B + C	27	69	117	123
C	10	21	34	23

Let us consider that we are at the beginning of the observation period (1 July 1987) and that the unit time interval is a calendar month. Since the vehicles are brand new, and we do not have any record of failures of our own, we can forecast l^*, the failure rate per vehicle (which is the same for each of the 13 observed vehicles), either using other customers' data, or the manufacturer's forecasts. In order to show how the choice of a priori forecast of the average number of failures during one month of vehicle operation affects the solution, we made a sensitivity analysis of outcomes, given the choice of a priori values of distribution parameters, observing the obtained results in four time points (which coincide with the end of the calendar year), and in three different alternatives.

The first two of these alternatives are similar in that the a priori estimate of the average number of failures per vehicle per month (regardless of the failure type, therefore belonging to class A+B+C) is 5. The difference between them is in the degree of our confidence in that value, which is different, so that in the first (more optimistic) alternative we trust that estimate more, while in the second one we trust it less. In the first alternative, that assumption led to the smaller variance of estimates of gamma distribution parameters in formula (1). In the third alternative, we assumed that on average there are 4 failures (regardless of the failure type) per vehicle per month, and that we have an optimistic attitude regarding the ratio between variance and mean value. Distributions and corresponding values of distribution parameters (in accordance with formulae (1), (2), and (5)), as well as distributions of the forecast number of failures for the following month and corresponding values of means and variances of the number of failures (in accordance with formulae (10) and (11)), are presented in Table 2. The results are given for four time points (end of calendar year), and for three alternatives, described above.

Table 2.

	Alternatives		
	I	II	III
Distribution of the failure rate l^*	$l^* \sim \Gamma(10, 2)$	$l^* \sim \Gamma(1, 0.2)$	$l^* \sim \Gamma(8, 2)$
Distribution of the failure rate l of the whole CE group	$l \sim \Gamma(130, 2)$	$l \sim \Gamma(13, 0.2)$	$l \sim \Gamma(104, 2)$
Marginal distrib. of the number of all types (A+B+C) of failures $X_{\Delta t_1}$ in the first month (Δt_1) and numerical characteristics	$X_{\Delta t_1} \sim$N.B. (130, 2/3) $E(X_{\Delta t_1}) = 65$ $V(X_{\Delta t_1}) = 97.5$	$X_{\Delta t_1} \sim$N.B. (13, 1/6) $E(X_{\Delta t_1}) = 65$ $V(X_{\Delta t_1}) = 390$	$X_{\Delta t_1} \sim$N.B. (104, 2/3) $E(X_{\Delta t_1}) = 52$ $V(X_{\Delta t_1}) = 78$
After $N_1 = 6$ unit time intervals (months) have passed (31 Dec. 1987)			
Number of registered failures (A+B+C) $\sum_{i=1}^{6} x_i = 321$	$X_{\Delta t_7} \sim$N.B. (451, 8/9) $E(X_{\Delta t_7}) = 56.375$ $V(X_{\Delta t_7}) = 63.42$	$X_{\Delta t_7} \sim$N.B. (334, 0.861) $E(X_{\Delta t_7}) = 53.92$ $V(X_{\Delta t_7}) = 62.62$	$X_{\Delta t_7} \sim$N.B. (425, 8/9) $E(X_{\Delta t_7}) = 53.125$ $V(X_{\Delta t_7}) = 59.77$
After $N_1 = 18$ months have passed (31 Dec. 1988)			
Number of registered failures $\sum_{i=1}^{18} x_i = 1075$	$X_{\Delta t_{19}} \sim$N.B. (1206,20/21) $E(X_{\Delta t_{19}}) = 60.3$ $V(X_{\Delta t_{19}}) = 63.315$	$X_{\Delta t_{19}} \sim$N.B. (1089,0.9479) $E(X_{\Delta t_{19}}) = 59.796$ $V(X_{\Delta t_{19}}) = 63.083$	$X_{\Delta t_{19}} \sim$N.B. (1180,20/21) $E(X_{\Delta t_{19}}) = 59$ $V(X_{\Delta t_{19}}) = 61.95$
After $N_1 = 30$ months have passed (31 Dec. 1989)			
$\sum_{i=1}^{30} x_i = 1867$	$X_{\Delta t_{31}} \sim$N.B. (1997,32/33) $E(X_{\Delta t_{31}}) = 62.406$ $V(X_{\Delta t_{31}}) = 64.356$	$X_{\Delta t_{31}} \sim$N.B. (1880,0.968) $E(X_{\Delta t_{31}}) = 62.04$ $V(X_{\Delta t_{31}}) = 64.098$	$X_{\Delta t_{31}} \sim$N.B. (1971,32/33) $E(X_{\Delta t_{31}}) = 61.594$ $V(X_{\Delta t_{31}}) = 63.519$
After $N_1 = 42$ months have passed (31 Dec. 1990)			
$\sum_{i=1}^{42} x_i = 2661$	$X_{\Delta t_{43}} \sim$N.B. (2791,44/45) $E(X_{\Delta t_{43}}) = 63.432$ $V(X_{\Delta t_{43}}) = 64.874$	$X_{\Delta t_{43}} \sim$N.B. (2674,0.977) $E(X_{\Delta t_{43}}) = 63.384$ $V(X_{\Delta t_{43}}) = 64.098$	$X_{\Delta t_{43}} \sim$N.B. (2765,44/45) $E(X_{\Delta t_{43}}) = 62.841$ $V(X_{\Delta t_{43}}) = 64.269$

Comparing the results of different alternatives, some interesting facts might be noticed. Since the first two alternatives deal with a well chosen estimate of the average number of failures per vehicle per month $(l^* = 5)$, it can be seen that after a small number of periods the differences between forecast (expected) values are minimal, and the differences between alternatives are small. The most important fact is that negative binomial distribution, which is used to describe the forecast number of failures, converges faster to the Poisson distribution, which is the basic assumption of the model. This phenomenon could be proved easily observing the ratio between variance and mean value which, compared to the starting value (1.5 in the 1st variant, and 6 in the 2nd), decreased to the value of just above 1. It is also interesting to note that the much worse estimate of average number of failures per vehicle per month $(l^* = 4)$ in the 3rd alternative, despite our pretty large a priori faith in estimation capability (expressed through the relation $V(X_{\Delta t_1}) = 1.5E(X_{\Delta t_1})$, quickly adapted through the incorporation of already recorded data into parameters of the a posteriori distribution. This can be observed through the fact that the expected values of the number of forecast failures (regardless of the failure type) in this alternative differ by less than 1% (and standard deviations by less than 0.5%) from values in other alternatives after 42 unit time intervals (months), and only slightly more after just 18 months of data collecting.

We pointed out earlier that some types of failures (B+C) lead to disturbances in the execution of the EWP (executive work plan), therefore we will discuss them separately. We propose to make forecasts using formula (12). In the three alternatives described earlier we again observe time points at the end of the calendar year. (In fact, we could have presented the results for each month, if the complete data were displayed.) In formula (12), next to the a priori values of parameters a and b, N_i - number of past time units (i - ordinal number of observed time point), and $\sum x_i$ - total number of recorded failures (of types A+B+C), we introduced q - probability that the failure is of type B or C. At the beginning of the calculating process shown in Table 3 the value $q = 0.1$ was taken. Then it was adjusted, at first to $q' = 0.15$, and then to $q'' = 0.16$, because the increasing tendency of these types of failures was observed.

The correction from $q = 0.1$ to $q' = 0.15$ was made after 30 months of vehicle fleet operation, because it was observed that the expected number of failures of types B or C for the whole fleet per year in the first case was 75, and in the second case, depending on the alternative, 113, 112, and 111, respectively. Following the increase in the number of failures of type B or C after 42 months of operation, the probability increased to $q'' = 0.16$ which corresponds to the level of approximately 122 failures per year per entire fleet of 13 vehicles.

Table 3.

	Alternatives		
	I	II	III
Marginal distrib. of the number of failures (type B or C) $Y_{\Delta t_1}$ in the first month and numerical characteris. $q = 0.1$	$Y_{\Delta t_1} \sim N.B.(130, 0.9524)$ $E(Y_{\Delta t_1}) = 6.5$ $V(Y_{\Delta t_1}) = 6.825$	$Y_{\Delta t_1} \sim N.B.(13, 0.667)$ $E(Y_{\Delta t_1}) = 6.5$ $V(Y_{\Delta t_1}) = 9.75$	$Y_{\Delta t_1} \sim N.B.(104, 0.9524)$ $E(Y_{\Delta t_1}) = 5.2$ $V(Y_{\Delta t_1}) = 5.46$
After $N_1 = 6$ unit time intervals (months) have passed (31 Dec. 1987)			
$\sum_{i=1}^{6} x_i = 321$ $q = 0.1$	$Y_{\Delta t_7} \sim N.B.(451, 0.9877)$ $E(Y_{\Delta t_7}) = 5.638$ $V(Y_{\Delta t_7}) = 5.708$	$Y_{\Delta t_7} \sim N.B.(334, 0.9841)$ $E(Y_{\Delta t_7}) = 5.387$ $V(Y_{\Delta t_7}) = 5.474$	$Y_{\Delta t_7} \sim N.B.(425, 0.9877)$ $E(Y_{\Delta t_7}) = 5.312$ $V(Y_{\Delta t_7}) = 5.379$
After $N_1 = 18$ months have passed (31 Dec. 1988)			
$\sum_{i=1}^{18} x_i = 1075$ $q = 0.1$	$Y_{\Delta t_{19}} \sim N.B.(1205, 0.995)$ $E(Y_{\Delta t_{19}}) = 6.03$ $V(Y_{\Delta t_{19}}) = 6.06$	$Y_{\Delta t_{19}} \sim N.B.(1088, 0.9945)$ $E(Y_{\Delta t_{19}}) = 5.984$ $V(Y_{\Delta t_{19}}) = 6.016$	$Y_{\Delta t_{19}} \sim N.B.(1179, 0.995)$ $E(Y_{\Delta t_{19}}) = 5.944$ $V(Y_{\Delta t_{19}}) = 6.018$
After $N_1 = 30$ months have passed (31 Dec. 1989)			
$\sum_{i=1}^{30} x_i = 1867$; $q = 0.1$	$Y_{\Delta t_{31}} \sim N.B.(1997, 0.9969)$ $E(Y_{\Delta t_{31}}) = 6.241$* $V(Y_{\Delta t_{31}}) = 6.260$	$Y_{\Delta t_{31}} \sim N.B.(1880, 0.9967)$ $E(Y_{\Delta t_{31}}) = 6.225$* $V(Y_{\Delta t_{31}}) = 6.246$	$Y_{\Delta t_{31}} \sim N.B.(1971, 0.9969)$ $E(Y_{\Delta t_{31}}) = 6.159$* $V(Y_{\Delta t_{31}}) = 6.179$
$q' = 0.15$	$Y'_{\Delta t_{31}} \sim N.B.(1997, 0.9953)$ $E(Y'_{\Delta t_{31}}) = 9.361$** $V(Y'_{\Delta t_{31}}) = 9.405$	$Y'_{\Delta t_{31}} \sim N.B.(1880, 0.9967)$ $E(Y'_{\Delta t_{31}}) = 9.338$** $V(Y'_{\Delta t_{31}}) = 9.384$	$Y'_{\Delta t_{31}} \sim N.B.(1971, 0.9953)$ $E(Y'_{\Delta t_{31}}) = 9.239$** $V(Y'_{\Delta t_{31}}) = 9.282$
After $N_1 = 42$ months have passed (31 Dec. 1990)			
$\sum_{i=1}^{42} x_i = 2661$ $q'' = 0.16$	$Y''_{\Delta t_{43}} \sim N.B.(2791, 0.9964)$ $E(Y''_{\Delta t_{43}}) = 10.149$ $V(Y''_{\Delta t_{43}}) = 10.186$	$Y''_{\Delta t_{43}} \sim N.B.(2674, 0.9962)$ $E(Y''_{\Delta t_{43}}) = 10.138$ $V(Y''_{\Delta t_{43}}) = 10.177$	$Y''_{\Delta t_{43}} \sim N.B.(2765, 0.9964)$ $E(Y''_{\Delta t_{43}}) = 10.055$ $V(Y''_{\Delta t_{43}}) = 10.091$

Since type C failures, "critical failures", are the worst regarding fulfillment of the EWP (because the engagement of a replacement vehicle is needed), forecasting their number is of great importance from the planner's point of view. Since Z, the conditional distribution of the number of failures, includes the unknown probability of a type C failure appearing in any of the vehicles, Table 4 presents the changes caused by recording type C failures in the time points that coincide to the end of the calendar year. Again, we observe three alternatives according to a priori chosen parameter values. These changes influence the distribution parameters in formulae (16) and the expected value of p.

Table 4.

N_i	ALTERNATIVES			
	cumulative no. of observed type C failures	I(p ~ B(1.5, 10))	II(p ~B(15, 100))	III(p ~B(4, 20))
6	10	p ~ B (11.5, 78) $E(p)=0.128$	p ~ B(25, 168) $E(p)=0.130$	p ~ B(14, 88) $E(p)=0.137$
18	31	p ~ B(32.5, 213) $E(p)=0.132$	p ~ B(46, 303) $E(p)=0.132$	p ~ B(35, 223) $E(p)=0.136$
30	65	p ~ B(66.5, 335) $E(p)=0.166$	p ~ B(80, 435) $E(p)=0.155$	p ~ B(69,345) $E(p)=0.167$
42	88	p ~ B(89.5, 468) $E(p)=0.161$	p ~ B(103,560) $E(p)=0.155$	p ~ B(92,478) $E(p)=0.161$

At the end, we will illustrate how formula (17) can be concretely utilized at the time point of 31 December 1990 to forecast the number of type C failures for the entire fleet for the following month. For alternative II (a=15, b=100), we calculated probabilities $P(Z_{43}=k)$, for different values $k=0,1,2,...$, taking into account that $\sum Z_i = 88$. The following results were obtained:

$P(Z_{43} = 0) = 0.11296$
$P(Z_{43} = 1) = 0.26536$
$P(Z_{43} = 2) = 0.29101$
$P(Z_{43} = 3) = 0.19725$
$P(Z_{43} = 4) = 0.09219$
$P(Z_{43} = 5) = 0.03137$
$P(Z_{43} = 6) = 0.00799$
$P(Z_{43} = 7) = 0.00155$
$P(Z_{43} = 8) = 0.00022$
$P(Z_{43} = 9) = 0.00002$
$P(Z_{43} \geq 10) \approx 0$

Using these results the planner can forecast that, with a probability greater than 0.95, the number of type C failures during the 43rd month since the beginning of observations will be at most 4.

5. CONCLUSION

According to the present state of the art in the field of vehicle fleet management, we are without analytical planning tools in case we lack data on vehicle failures, or the available data are not sufficient for statistical inference. Under the conditions of such uncertainty the distributions of the number of failures are not known. As an alternative to omitting analytical planning while waiting for the data to accumulate for statistical inference, we proposed a procedure for determining the distributions, which are subjected to consecutive corrections as the recording of failures, in consecutive sequences, advances. The Bayesian learning algorithm proved to be a useful tool to obtain these distributions according to which planning can be made. Planning here involves planning the number of vehicles as well as planning the maintenance facilities since the companies considered in this paper have a maintenance depot along with their vehicle fleet.

We find the problem considered in this paper important enough to be tackled by more than one approach. We expect the technique we proposed, as well as other modern techniques, to be used in the future.

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