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Sara Ifqir, Naima Ait Oufroukh, Dalil Ichalal, Said Mammar

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***Abstract*— A systematic design methodology for interval estimation of switched uncertain linear systems subject to uncertainties and unknown inputs is presented. The uncertainties under consideration are assumed to be unknown but bounded with a priori known bounds. The proposed observer is used to robustly estimate the vehicle yaw rate and lateral velocity using a vision system measurement. The road curvature is treated as an unknown input and a linear adaptive tire model is considered to take into account the changes of the road adhesion. Sufficient conditions allowing the design of such observer are derived using Multiple Quadratic ISS-Lyapunov function and an LMIs (Linear Matrix Inequalities) formulation is obtained. Performance of the algorithm is evaluated using vehicle real data, results show that the proposed estimation scheme succeeds to appropriately estimate the upper and lower bounds of vehicle lateral dynamics despite of the presence of unknown inputs.**

I. Introduction

A basic requirement for autonomous vehicle systems operating in unstructured environments is the ability to efficiently estimate the state in the presence of parameter uncertainties and disturbance inputs. Most existing methods for estimation of vehicle lateral dynamics state rely on a deterministic analysis that assumes accurate knowledge of the vehicle parameters [11], [12], [13] and [14]. However, in field conditions, vehicle parameters, such that, mass, location of the center of gravity and cornering stiffness at the front and rear tires, might have significant uncertainties due to vehicle motions, different load conditions, and road frictions. Note that, the change on road conditions is one of the most important factors which can significantly influence the estimation accuracy. In the literature, there is a few works that explicitly take into consideration the cornering stiffness uncertainties. For example, [15] uses a linear system identification while [16] use an adaptive observer to identify the cornering stiffness coefficients at the front and rear tires. Liu and Peng proposed in [17] an identification scheme to estimate simultaneously the states and the parameters. However, this approach shows slow convergence for non-nominal conditions.

The main particularity of this work is that the front and rear cornering stiffness are assumed to be unknown but bounded with a priori known bounds. A switched representation is also considered to take into account the variations of longitudinal velocity. An interval observer is then designed to estimate upper and lower bounds of the state vector under consideration of suitable intervals in which the true but unknown cornering stiffness parameters values are definitely included.

It should be recalled that, an interval observer is a pair of estimators whose dynamics are defined such that their trajectories characterize at any instant upper and lower bounds of the state values. They are appeared in the last decade as an alternative approach for robust estimation and they were originally developed in [18] for the estimation of biological systems subject to unknown uncertainties. There are various approaches to design interval observers for continuous times systems, see for instance [8], [21], [20], [19] and references therein. Let us recall that a few works exists on design of such observer for switched systems, see for example the recent results presented in [22] and [23] in which interval observers were designed for switched linear systems subject to exogenous disturbances and measurement noises. Note that, an interval observer for switched LPV (Linear Parameter Varying) systems was proposed in [5] with application to vehicle lateral dynamics estimation. Unfortunately, the proposed observer is not able to deal with unknown inputs.

The objective of this paper is to propose some new results on switched interval observers for switched uncertain systems subject to unknown inputs. Furthermore, it should be pointed out and to the best of the authors knowledge, a few results exist on design of interval observer for switched systems subject to parameter uncertainties and unknown inputs.

A solution to the problem of robust state estimation for a class of continuous uncertain switched linear systems under constrained switching signal is proposed. Taking into account the parameters uncertainties and unknown inputs, the main contribution of this paper is as follows: -1- The robust state estimation is achieved by means of a new structure of time-varying unknown inputs interval observer which incorporates the nominal state matrix. -2- Sufficient conditions for existence of the proposed observer is formulated in terms of Linear Matrices Inequalities using Multiple Quadratic ISS-Lyapunov Functions. -3- Design of switched interval observer for robust estimation of vehicle lateral dynamics. The effectiveness of the proposed approach is validated through experimental data.

The outline of the paper is as follows: section 2 introduces some preliminaries and definitions. Section 3, gives the vehicle lateral dynamics model and the problem statement. A design methodology of the proposed Switched Unknown Input Interval Observer is presented in section 4. The experimental validation using real data is presented in section 5. Section 6 concludes the paper.

All the authors are with the Laboratory of Computing, Integrative Biology and Complex Systems (IBISC), Univ Evry, Paris-Saclay University, Evry 91000, France Email: {sara.ifqir, naima.aitoufroukh, dalil.ichalal, said.mammar}@ibisc.univ-evry.fr.

II. Preliminaries

A. Notations

Left and right endpoints of an interval $[x]$ will be denoted respectively by x^- and x^+ such as $[x] = [x^-, x^+]$. For any two vectors x_1, x_2 or matrices M_1, M_2 the inequalities $x_1 \leq x_2$, $x_1 \geq x_2$, $M_1 \leq M_2$ and $M_1 \geq M_2$ must be interpreted component-wise. $m_{\hat{i}\hat{j}}$ denotes the element on the $\hat{i}$th line and $\hat{j}$th column of the matrix M. $M > 0$ (resp. $M < 0$) denotes a matrix with positive (resp. negative) components and $M \succ 0$ (resp. $M \prec 0$) means that the matrix is positive (resp. negative) definite. $M^\dagger$ denotes the generalized inverse of the matrix M. M^T means the transpose of matrix M. $\mathbb{R}$ ($\mathbb{R}_+$) is the set of all real (positive) numbers. $\mathbb{R}^n$ ($\mathbb{R}^n_+$) is n-dimensional real (positive) vector space. We denote by $\mathcal{I}_n$ an identity matrix of dimension $n \times n$. The absolute value and euclidean norm of a vector $x \in \mathbb{R}^n$ is denoted respectively by $|x|$ and $\|x\|$. Let a vector $x \in \mathbb{R}^n$ and a matrix $M \in \mathbb{R}^{n\times n}$, we denotes, $\overline{x} = \max\{0, x\}$, $\underline{x} = \overline{x} - x$, $\overline{M} = max(0, M)$ and $\underline{M} = \overline{M} - M$. By $\mathcal{K}$, we will denote the class consisting of all functions $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ which are continuous, strictly increasing, and satisfy $\alpha(0) = 0$. By $\mathcal{K}_\infty$, we will denote the class of functions of class $\mathcal{K}$ and $\alpha(s) \to +\infty$ as $s \to +\infty$.

B. Interval Analysis and positive systems

Definition 1. [7] A real matrix M is called Metzler matrix if all its elements outside the main diagonal are positive: $m_{\hat{i}\hat{j}} \geq 0, \;\; \forall \hat{i} \neq \hat{j}$.

Lemma 1. [8] A matrix M is a Metzler if and only if there exist $\eta \in \mathbb{R}_+$ such that $M + \eta\mathcal{I}_n \geq 0$.

Lemma 2.[9] Consider a continuous time uncertain switched system of the following general form

$$\dot{x}(t) = A_{\sigma(t)}x(t) + \delta_{\sigma(t)}(t) \tag{1}$$

where $x \in \mathbb{R}^{n\times n}$ is the state, $\sigma(t) : \mathbb{R}^+ \to \mathcal{I} = \{1, 2, ..., N\}$ is the switching rule, $\delta \geq 0$ represents additive uncertainty. The system (1) is said to be positive, i.e. $x(t) \geq 0, \forall t \geq t_0$, if $A_{\sigma(t)}$ is Metzler matrix $\forall\sigma(t)$, $x(t_0) \geq 0$ and $\delta_{\sigma(t)}(t) \geq 0$.

Lemma 3. [6] Let the vector $x \in \mathbb{R}^n$ be a variable vector with given bounds $x^+ \, x^- \in \mathbb{R}^n$ such that $x^- \leq x \leq x^+$.

1) If $M \in \mathbb{R}^{n\times n}$ is a constant matrix, then

$$\overline{M}x^- - \underline{M}x^+ \leq Mx \leq \overline{M}x^+ - \underline{M}x^- \tag{2}$$

2) If $M \in \mathbb{R}^{n\times n}$ is a variable such that $M^- \leq M \leq M^+$ for some M^-, $M^+ \in \mathbb{R}^{n\times n}$, then

$$\begin{aligned} &\underline{M}^+\underline{x}^+ - \overline{M}^+\underline{x}^- - \underline{M}^-\overline{x}^+ + \overline{M}^-\overline{x}^- \leq Mx \leq \\ &\overline{M}^+\overline{x}^+ - \underline{M}^+\overline{x}^- - \overline{M}^-\underline{x}^+ + \underline{M}^-\underline{x}^- \end{aligned} \tag{3}$$

C. Input-to-state stability of switched systems

We will briefly review in this subsection the main idea of Multiple Input-to-State Stable-Lyapunov Functions as a tool for stability analysis of switched systems. To this end, consider the switched system (1), and, suppose that we can find a family of Quadratic Lyapunov functions $\{V_i : i \in \mathcal{I}\}$, associated with each subsystem $\dot{x} = A_ix + \delta_i$, $i \in \mathcal{I}$. Then, $V(x(t))$ is called a piecewise Quadratic ISS-Lyapunov function candidate if it can be written as $V(x(t)) = V_{\sigma(t)}(x(t))$, where $V_{\sigma(t)}(x(t))$ is switched among $V_i(x(t)) = x^T(t)Q_ix(t)$ in accordance with the piecewise constant switching signal $\sigma(t)$.

Definition 2. [5] For a switching signal $\sigma(t)$ and any $t_2 > t_1 > t_0$, let $N_\sigma(t_1, t_2)$ be the number of switching over the interval $[t_1, t_2)$. If the condition $N_\sigma(t_1, t_2) \leq N_0 + \frac{(t_2 - t_1)}{\tau_a}$ holds for $N_0 \geq 1$, $\tau_a > 0$, then N_0 and τ_a are called the chatter and the average dwell time bound respectively.

Lemma 4. Consider the switched system (1), and let $\varepsilon > 0$. Suppose that there exist smooth functions $V_{\sigma(t)} : \mathbb{R}^n \to \mathbb{R}$, $\sigma(t)$, $\mathcal{K}$-function γ, two $\mathcal{K}_\infty$ functions $\beta > \alpha > 0$ such that for each $\sigma(t) = i$, the following conditions hold:

$$\alpha(\|x(t)\|) \leq V_i(x(t)) \leq \beta(\|x(t)\|) \tag{4}$$

$$\dot{V}_i(x(t)) < -\varepsilon V_i(x(t)) + \gamma(\|\delta_{\sigma(t)}\|) \tag{5}$$

then the system (1) is Input-to-State Stable with respect of the additive term $\delta_{\sigma(t)}$ for any switching signal with Average Dwell Time

$$\tau_a \geq \tau_a^* = \frac{ln(\mu)}{\varepsilon} \tag{6}$$

where $\mu = \frac{\beta}{\alpha}$.

Remark 1. Note that, $\mu = 1$, corresponds to the case when the system is Uniformly Input-to-State Stable. This implies the existence of a common ISS-Lyapunov function for the switched system (1), and thus the system is ISS-Stable under arbitrary switching. It should be noticed that the ADT method proposed in [10], [5] needs the conditions (4)-(5) and an additional condition as

$$V_i(x(t)) \leq \mu V_j(x(t)), \; \mu \geq 1, \; i \neq j, \; i, j \in \mathcal{I} \tag{7}$$

which place restrictions on the switching instant to guarantee stability of the overall switched system. Moreover, Lemma 4 needs fewer conditions and it is easy to demonstrate that using (4) and (5) leads to condition (7) for an appropriate choice of α and β ensuring the existence of $\mu \geq 1$.

Proof. Due to space limitations, the proof is omitted.

III. Vehicle Model

In this section, we will first describe the vehicle lateral dynamics based on the well known bicycle model, then the model of the vision system measurement based on the lateral displacement will be presented.

A. Vehicle Lateral Dynamics

In this paper, a simple model known as the bicycle model (Figure 1) is used for Interval Observer design. This model describes the vehicle yaw and lateral motions [1], and largely simplifies the equations of motion of the vehicle and reduces the implementation complexity of equations. The two-dimensional model describing the vehicle lateral behavior can be represented by the following differential equations:

$$\begin{cases} m\dot{v}_y + mv_xr = F_{yf} + F_{yr} \\ I_z\dot{r} = l_fF_{yf} - l_rF_{yr} \end{cases} \tag{8}$$

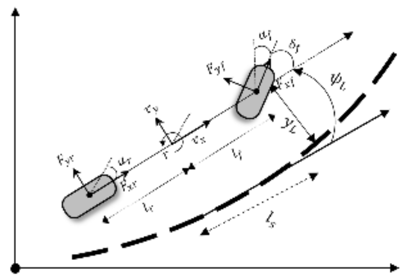

Fig. 1. Bicycle model and vision system measurement .

where m, I_z, are the mass and the yaw moment, v_x and v_y are lateral and longitudinal velocities, r is the yaw rate, l_f, l_r are distances from front and rear axle to the CG, while F_{yf} and F_{yr} are lateral tire force of front and rear tires.

The lateral forces F_{yf} and F_{yr} are highly nonlinear and usually functions of the wheel sideslip angle and wheel longitudinal slip ([3], [2]). Using the so-called Pacejka magic formula [3], and under assumption of small sideslip angle variation, lateral forces are taken to be linear and given as:

$$F_{yf} = c_f(\delta_f - \frac{v_y}{v_x} - \frac{l_f}{v_x}r) \ \ , \ \ F_{yr} = c_r(-\frac{v_y}{v_x} + \frac{l_r}{v_x}r) \quad (9)$$

where c_f, c_r are the cornering stiffness of front and rear tires.

In the proposed model, it is assumed that the available measurements are yaw rate r, longitudinal velocity v_x and front steering angle δ_f. Gathering equations (8) and (9) and chosen v_y and r, as state variables, leads to the following state equations:

$$\begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{-(c_f+c_r)}{mv_x} & \frac{c_r l_r - c_f l_f}{mv_x} - v_x \\ \frac{c_r l_r - c_f l_f}{I_z v_x} & -\frac{c_r l_r^2 + c_f l_f^2}{I_z v_x} \end{bmatrix} \begin{bmatrix} v_y \\ r \end{bmatrix} + \begin{bmatrix} \frac{c_f}{m} \\ \frac{c_f l_f}{Iz} \end{bmatrix} \delta_f \quad (10)$$

B. Vision system measurement

The vision system model describes the evolution of the angular and lateral displacements of the vehicle from the centerline at a particular look ahead distance l_s (Figure 1). These measurements are extracted from images obtained with a suitable vision system, taking into consideration the motion of car and changes in the road geometry.

The equations describing the vision system model are given by the following state representation form:

$$\begin{bmatrix} \dot{\psi}_L \\ \dot{y}_L \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & v_x \end{bmatrix} \begin{bmatrix} \psi_L \\ y_L \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & l_s \end{bmatrix} \begin{bmatrix} v_y \\ r \end{bmatrix} + \begin{bmatrix} -v_x \\ -l_s v_x \end{bmatrix} \rho \quad (11)$$

where y_L and ψ_L are the offset and angular displacements at a look ahead distance l_s, however, ρ represents the road curvature.

C. Problem formulation

Combining the vehicle lateral dynamics (10) and the vision system model (11) leads to a single dynamical system subject to the road curvature as an unknown input and describing as:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \\ y(t) = Cx(t) \end{cases} \quad (12)$$

with the state vector $x = \begin{bmatrix} v_y & r & \psi_L & y_L \end{bmatrix}^T$, the control input $u(t) = \delta_f$, the unknown input $d(t) = \rho$ and the matrices A, B, and C defined at the top of the next page.

Note that the model (12) describing the vehicle lateral dynamics is subject to several variations and uncertainties. When road friction changes or when the nonlinear tire domain is reached, the tire forces F_{yf} and F_{yr} are no longer linearly proportional to slip angles due to the tire saturation property.

Taking into account this variation, the linear tire model (9) could correct the cornering stiffness by adding two uncertain terms Δc_f and Δc_r as:

$$c_f = c_{f_0} + \Delta c_f \ \ , \ \ c_r = c_{r_0} + \Delta c_r \quad (14)$$

where the linear part, denoted by c_{i0}, i $\in \{r, f\}$, presents a known nominal value and the uncertainty term, denoted by Δc_i, i $\in \{r, f\}$, is assumed to be unknown but bounded with a priori known bounds. Moreover, to deal with longitudinal velocity variations, a switched representation of the vehicle model is used and v_x is assumed to be piecewise constant.

Considering the cornering stiffness uncertainties and adopting a switched representation depending on the measured longitudinal speed, the system (12) is transformed into a Switched Uncertain System given as follows:

$$\begin{cases} \dot{x}(t) = (A_{0,\sigma(t)} + \Delta A_{\sigma(t)}(\xi(t)))x(t) + \\ (B_0 + \Delta B(\xi(t)))u(t) + E_{\sigma(t)}d(t) \\ y(t) = Cx(t) \end{cases} \quad (15)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$, $d(t) \in \mathbb{R}^q$ represent respectively the state, the control input, the output vector and the unknown input. $\xi(t) = [\Delta c_f \ \ \Delta c_r]^T$ is the vector of parameters uncertainty. $\sigma : \mathbb{R}^+ \to \mathcal{I} = \{1, 2, ..., N\}$ is the switching signal. $A_{\sigma(t)} \in \{A_1, A_2, ..., A_N\}$ and ΔB are bounded time-varying matrices. N is the number of subsystems known a priori. For technical reasons, let define $\delta(t) = \Delta B(\xi)u(t)$, the system (15) becomes

$$\begin{cases} \dot{x}(t) = (A_{0,\sigma(t)} + \Delta A_{\sigma(t)}(\xi(t)))x(t) + \\ B_0 u(t) + E_{\sigma(t)}d(t) + \delta(t) \\ y(t) = Cx(t) \end{cases} \quad (16)$$

IV. SUIIO DESIGN AND LMI SYNTHESIS

In this section, we will first present the structure of the proposed Switched Unknown Input Interval Observer (SUIIO), then derive sufficient conditions in term of Linear Matrix Inequalities for the existence of the SUIIO. For this purpose, the following assumptions are made.

Assumption 1. We assume that there exist constants $\mathcal{X} \geq 0$ and $\mathcal{U} \geq 0$ such that $\|x\| \leq \mathcal{X}$ and $\|u\| \leq \mathcal{U}$.

Remark 1. The assumption 1 is not restrictive since for vehicle dynamics, these variables evolve in a bounded region.

Assumption 2. Assume that the input vector $u(t)$ is bounded with an a priori known bound ζ. Then,

$$\begin{aligned} u^-(t) &= u(t) - \zeta \\ u^+(t) &= u(t) + \zeta \end{aligned} \quad (17)$$

Assumption 3. There exist known constants matrices A_i^+, A_i^-, ΔB^+, ΔB^- $\forall i \in \mathcal{I}$ such that:

$$A = \begin{bmatrix} -\frac{c_f+c_r}{mv_x} & \frac{c_r l_r - c_f l_f}{mv_x} - v_x & 0 & 0 \\ \frac{c_r l_r - c_f l_f}{I_z v_x} & -\frac{c_r l_r^2 + c_f l_f^2}{I_z v_x} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & l_s & v_x & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{c_f}{m} \\ \frac{c_f l_f}{I_z} \\ 0 \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ 0 \\ -v_x \\ -l_s v_x \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1) $A_i^- \leq A_0 + \Delta A_i \leq A_i^+$
2) $\Delta B^- \leq \Delta B \leq \Delta B^+$

Assumption 4. $rank(CE_i) = rank(E_i),\ q < p$.

Assumption 5. $rank\left(\begin{bmatrix} s\mathcal{I}_n - A_i & E_i \\ C & 0 \end{bmatrix}\right) = n + q$, holds for all complex number s with $\mathcal{R}e(s) \geq 0$.

A. SUIIO Design

Given the system (16), consider the interval observer structure given (19) in the top of the next page, where $x^+(t),\ x^-(t) \in \mathbb{R}^n$ are upper and lower bounds of the state vector $x(t)$. $N_{\sigma(t)}, K_{\sigma(t)}, G_{\sigma(t)}, P_{\sigma(t)}$ and $H_{\sigma(t)}$ are matrices to be designed for achieving boundedness of the state vector and unknown input decoupling. $\delta^+(t)$ and $\delta^-(t)$ are the upper and lower bound of the additive term $\delta(t) = \Delta B u(t)$, using Lemma 3 and Assumptions 2 and 3, it can be bounded as follows $\delta^-(t) \leq \delta(t) \leq \delta^+(t)$ such that

$$\begin{aligned} \delta^-(t) &= \underline{\Delta B}^+ \underline{x}^+ - \overline{\Delta B}^+ \underline{x}^- - \underline{\Delta B}^- \overline{x}^+ + \overline{\Delta B}^- \overline{x}^- \\ \delta^+(t) &= \overline{\Delta B}^+ \overline{x}^+ - \underline{\Delta B}^+ \overline{x}^- - \overline{\Delta B}^- \underline{x}^+ + \underline{\Delta B}^- \underline{x}^- \end{aligned} \tag{18}$$

The following theorem provides the conditions which should be verified to ensure an interval estimation of the state $x(t)$ despite the presence of unknown inputs using the observer structure described previously.

Theorem 1. Consider the SUIIO (19). Let $H_{\sigma(t)}$ be chosen such that $P_{\sigma(t)}$ is positive element-wise, and let $K_{\sigma(t)}$ be chosen such that $(P_{\sigma(t)} A^+_{\sigma(t)} - K_{\sigma(t)} C)$ is Hurwitz and $(P_{\sigma(t)} A^-_{\sigma(t)} - K_{\sigma(t)} C)$ is Metzler $\forall \sigma(t)$. Then, for all $x^-(t_0) \leq x(t_0) \leq x^+(t_0)$, the solutions of the system (16) and (19) satisfy: $x^-(t) \leq x(t) \leq x^+(t),\ \forall t \geq t_0,\ \forall \sigma(t)$.

The proof of Theorem 1 consists of two parts. The upper and lower interval estimation errors, i.e. $e^+(t) = x^+(t) - x(t)$ and $e^-(t) = x(t) - x^-(t)$ are positive which guarantees that, at each instant, the true state, solution of the switched system (16) lie inside the interval defined by the upper and lower estimates $x^+(t)$ and $x^-(t)$. On the other hand, the proposed interval observer is Input-to-State Stable ensuring that estimated bounds remains bounded.

Proof. When the observer (19) is applied to the system (16), the upper and lower estimation errors $e^+(t)$ and $e^-(t)$ are governed by the following equations:

$$\left\{ \begin{array}{l} \dot{e}^+(t) = N_{0,\sigma(t)} e^+(t) + (G_{\sigma(t)} - P_{\sigma(t)} B_0) u(t) - \\ P_{\sigma(t)} E_{\sigma(t)} d(t) + \overline{P}_{\sigma(t)} \delta^+(t) - \underline{P}_{\sigma(t)} \delta^-(t) \\ -P_{\sigma(t)} \delta(t) + (N^+_{\sigma(t)} - N_{0,\sigma(t)}) |x^+(t)| \\ \dot{e}^-(t) = N_{0,\sigma(t)} e^-(t) + (P_{\sigma(t)} B_0 - G_{\sigma(t)}) u(t) + \\ P_{\sigma(t)} E_{\sigma(t)} d(t) + P_{\sigma(t)} \delta(t) - \overline{P}_{\sigma(t)} \delta^-(t) \\ +\underline{P}_{\sigma(t)} \delta^+(t) + (N^+_{\sigma(t)} - N_{0,\sigma(t)}) |x^-(t)| \end{array} \right.$$

where $P_{\sigma(t)} = H_{\sigma(t)} C + \mathcal{I}_n$. If one can make the following conditions hold $\forall i \in \mathcal{I}$:

$$N_i^+ = P_i A_i^+ - K_i C \tag{20a}$$

$$N_i^- = P_i A_i^- - K_i C \tag{20b}$$

$$N_{0,i} = P_i A_{0,i} - K_i C \tag{20c}$$

$$G_i - P_i B_0 = 0 \tag{20d}$$

$$P_i E_i = 0 \tag{20e}$$

The upper and lower estimation errors will then be:

$$\left\{ \begin{array}{l} \dot{e}^+(t) = N_{0,\sigma(t)} e^+(t) + \Delta^+_{\sigma(t)}(t) \\ \dot{e}^-(t) = N_{0,\sigma(t)} e^-(t) + \Delta^-_{\sigma(t)}(t) \end{array} \right. \tag{21}$$

where

$$\begin{array}{l} \Delta^+_{\sigma(t)}(t) = \overline{P}_{\sigma(t)} \delta^+(t) - \underline{P}_{\sigma(t)} \delta^-(t) - P_{\sigma(t)} \delta(t) + \\ (N^+_{\sigma(t)} - N_{0,\sigma(t)}) |x^+(t)| \\ \Delta^-_{\sigma(t)}(t) = P_{\sigma(t)} \delta(t) - \overline{P}_{\sigma(t)} \delta^-(t) + \underline{P}_{\sigma(t)} \delta^+(t) + \\ (N^+_{\sigma(t)} - N_{0,\sigma(t)}) |x^-(t)| \end{array} \tag{22}$$

It's clear that, if $P_{\sigma(t)}$ is positive element-wise, and, $N^-_{\sigma(t)}$ is Metzler, then, $N_{\sigma(t)}$ is also Metzler for any $A_{\sigma(t)}$ in the interval: $A^-_{\sigma(t)} \leq A_{\sigma(t)} \leq A^+_{\sigma(t)}$. According to Lemma 2, if $N^-_{\sigma(t)}$ is Metzler matrix, since, $\Delta^+_{\sigma(t)}(t)$ and $\Delta^-_{\sigma(t)}(t)$ are positive by construction (easy to check, using (18) and Lemma 3), $e^+(t_0) \geq 0$ and $e^-(t_0) \geq 0$, then, $e^-(t) \geq 0$ and $e^+(t) \geq 0$ for all $t \geq t_0$ implies that $x^-(t) \leq x(t) \leq x^+(t)$. In order to derive convergence conditions of the proposed interval observer, we define the interval estimation error $e(t) = x^+(t) - x^-(t)$, then

$$\dot{e}(t) = N^+_{\sigma(t)} e(t) + \Delta_{\sigma(t)}(t) \tag{23}$$

where $\Delta_{\sigma(t)}(t) = \Delta^+_{\sigma(t)}(t) - \Delta^-_{\sigma(t)}(t)$. Furthermore, if $\Delta_{\sigma(t)}(t) = 0$, the interval error (23) exponentially converges to zero. But when $\Delta_{\sigma(t)}(t) \neq 0$ $e(t)$ is positive bounded.

The interval state estimation problem is then reduced to determine the observer gains matrices such that the upper and lower estimate errors $e^+(t)$ and $e^-(t)$ evolves in the positive orthant and the total error $e(t)$ governed by equation (23) achieves Input-to-state property with respect to uncertainty represented by $\Delta_{\sigma(t)}$.

Therefore the problem of designing the Switched Interval Observer with unknown inputs is reduced to find the positive matrices P_i satisfying (20e), equivalently the matrices H_i satisfying,

$$(\mathcal{I}_n + H_i C) E_i = 0 \tag{24a}$$

$$\mathcal{I}_n + H_i C \geq 0 \tag{24b}$$

and gains matrices K_i such that conditions of Theorem 1 holds. The general solution of (24a), $\forall i \in \mathcal{I}$, is given by

$$H_i = -E_i (CE_i)^\dagger - Y_i (\mathcal{I}_n - (CE_i)(CE_i)^\dagger) \tag{25}$$

$$\begin{cases} \dot{x}^+(t) = N^+_{\sigma(t)}x^+(t) + K_{\sigma(t)}y + G_{\sigma(t)}u(t) - H_{\sigma(t)}\dot{y} + (N^+_{\sigma(t)} - N_{0,\sigma(t)})(|x^+(t)| - x^+(t)) + \overline{P}_{\sigma(t)}\delta^+(t) - \underline{P}_{\sigma(t)}\delta^-(t) \\ \dot{x}^-(t) = N_{\sigma(t)}x^-(t) + K_{\sigma(t)}y + G_{\sigma(t)}u(t) - H_{\sigma(t)}\dot{y} - (N^+_{\sigma(t)} - N_{0,\sigma(t)})(|x^-(t)| + x^-(t)) + \overline{P}_{\sigma(t)}\delta^-(t) - \underline{P}_{\sigma(t)}\delta^+(t) \end{cases} \tag{19}$$

where (CE_i) is the generalized inverse matrix of CE_i, given by $(CE_i)^\dagger = ((CE_i)^T(CE_i))^{-1}(CE_i)$ and Y_i is an arbitrary matrix of appropriate dimension chosen to satisfy (24b).

B. LMI Formulation

In this subsection, sufficient conditions using Input to State Stability of interval error (23) are established in terms of linear matrix inequalities (LMIs). Theorem 2 provides sufficient conditions for the existence of such observer.

Theorem 2. For the switched system (23), suppose that there exists a Piecewise Quadratic ISS-Lyapunov function $V_{\sigma(t)}(e(t))$ where $V_i(e(t)) = e^T(t)Q_ie(t)$. If there exist a positive diagonal matrices Q_i, matrices K_i, $\beta > \alpha > 0$, $\gamma > 0$ for a given $\eta \geq 0$, $\varepsilon > 0$, such that for all $i \in \mathcal{I}$,

$$\min_{Q_i, W_i, Y_i} \quad \gamma$$

$$\alpha\,\mathcal{I}_n \preceq Q_i \preceq \beta\,\mathcal{I}_n \tag{26}$$

$$\begin{bmatrix} {A_i^+}^T P_i^T Q_i^T - C^T W_i^T + Q_i P_i A_i^+ - W_i C + \varepsilon Q_i & Q_i \\ Q_i & -\gamma \mathcal{I}_n \end{bmatrix} \prec 0 \tag{27}$$

$$Q_i P_i A_i^- - W_i C + \eta Q_i \geq 0 \tag{28}$$

holds, then the system (23) can estimate the lower and upper bounds of the state vector $x(t)$, where $K_i = Q_i^{-1}W_i$ and $P_i = \mathcal{I}_n + (-E_i(CE_i)^\dagger - Y_i(\mathcal{I}_n - (CE_i)(CE_i)^\dagger))C$.
Furthermore the interval error (23) is Input-to-State Stable with respect to $\Delta_i(t)$, $\forall i \in \mathcal{I}$, then, if $\sup\|\Delta_i(t)\|_\infty \leq \Delta_{max}$, then, (23) satisfies

$$\lim_{t\to\infty} \|e\|_2 \leq \sqrt{\frac{\gamma}{\alpha\varepsilon}}\Delta_{max} \tag{29}$$

Proof. The Piecewise Quadratic ISS-Lyapunov Function is chosen as

$$V_{\sigma(t)}(e(t)) = e^T(t)Q_{\sigma(t)}e(t) \tag{30}$$

Taking the derivative of the Lyapunov function (30) along the trajectory of the interval error dynamics in each mode i,

$$\begin{aligned} \dot{V}_i(e(t)) &= e^T(t)\left({N_i^+}^T Q_i + Q_i N_i^+\right)e(t) + \\ &\Delta_i^T(t)Q_ie(t) + e^T(t)Q_i\Delta_i(t) \end{aligned} \tag{31}$$

By adding and subtracting the terms $\varepsilon e^T(t)Q_ie(t)$ and $-\gamma\,\Delta_i^T(t)\,\Delta_i(t)$, replacing N_i^+ by (20a) and denote $W_i = Q_iK_i$ (31) becomes

$$\begin{aligned} \dot{V}_i(e(t)) &= \begin{bmatrix} e^T(t) & \Delta_i^T(t) \end{bmatrix}^T \Lambda_i \begin{bmatrix} e(t) & \Delta_i(t) \end{bmatrix}^T - \\ &\varepsilon V_i(e(t)) + \gamma \Delta_i^T(t)\Delta(t) \end{aligned} \tag{32}$$

with

$$\Lambda_i = \begin{bmatrix} {A_i^+}^T P_i^T Q_i^T - C^T W_i^T + Q_i P_i A_i^+ - W_i C + \varepsilon Q_i & Q_i \\ Q_i & -\gamma \mathcal{I}_n \end{bmatrix}$$

then, satisfying (27) leads to

$$\dot{V}_i(e(t)) < -\varepsilon V_i(e(t)) + \gamma\Delta_i^T(t)\Delta_i(t) \tag{33}$$

integrating the inequality (33) over the interval $[t_k, t_{k+1})$ implies that

$$\begin{aligned} V_i(e(t)) &< \mathrm{e}^{-\varepsilon(t-t_k)}V_i(e(t_k)) + \\ &\gamma\int_{t_k}^{t} \mathrm{e}^{-\varepsilon((t-t_k)-s)}\|\Delta_i(s)\|_2^2\,\mathrm{d}s \end{aligned} \tag{34}$$

Using (26), we obtain

$$\|e(t)\|_2 < \frac{1}{\sqrt{\alpha}}\left(\mathrm{e}^{-\varepsilon(t-t_k)}V_i(e(t_k)) + \frac{\gamma}{\varepsilon}\|\Delta_i(t)\|_\infty^2\right)^{\frac{1}{2}}$$

Hence, when $t \to \infty$ the exponential converge to zero, and, knowing that $\sup\|\Delta_i(t)\|_\infty \leq \Delta_{max}$, (29) is obtained.
In order to optimize the ISS-gain given in (23), the gain γ is minimized for a given α and ϵ. On the other hand, according to Lemma 1, N_i^- defined in (20b) is Metzler if $N_i^- + \eta I_n \geq 0,\ \forall i \in \mathcal{I}$, multiplying in the left side by Q_i and using (20b) together with the change of coordinates $W_i = Q_iK_i$, (28) is obtained and the proof is complete.
Remark 2. We stress that the present work rests on analysis result in Theorem 1 which consists in finding a gain matrix K_i ensuring simultaneously the stability of N_i^+ and Metzler property of the matrix N_i^-, $\forall i \in \mathcal{I}$. However, the aforementioned requirements can be relaxed by using a time-varying or time-invariant change of coordinates [6].

V. Experimental results

In this section, the proposed SUIIO is applied to experimental data acquired using a prototype vehicle. Several sensors are implemented on the vehicle: The yaw rate r is measured using an inertial unit, the steering angle δ_f is measured by an absolute optical encoder while an odometer provides the vehicle longitudinal speed. Finally, a high precision Correvit sensor provides a measure of the sideslip angle. This measure is not used for observer design. It serves only for estimation evaluation. For our purpose, we assume that the cornering stiffness parameters are affected by 10% uncertainty in their nominal value. Furthermore, and as mentioned above, the switching law $\sigma(t)$ depends on the varying parameter v_x which is accessible in real time, such that:

$$\sigma(t) = \begin{cases} 1 & if \;\; v_x^1 \in [v_x^1 - \Delta V,\, v_x^1 + \Delta V] \\ 2 & if \;\; v_x^2 \in [v_x^2 - \Delta V,\, v_x^2 + \Delta V] \\ 3 & if \;\; v_x^3 \in [v_x^3 - \Delta V,\, v_x^3 + \Delta V] \end{cases} \tag{35}$$

The steering angle, longitudinal velocity and switching law are shown in Figure 2. The yaw rate , angular and offset displacements profiles are appeared in Figure 3. Using Matlab optimization tools (Yalmip or Sedumi), the set of LMIs given in Theorem 2 are solved minimizing γ. The observer gain matrices are omitted due to the lack of space. The simulation results are presented in Figure 3, the simulated lateral velocity and measured variables are shown with the corresponding estimated bounds. The inclusion property is verified and the real data are within the estimated interval showing the good estimate ability of the proposed observer in spite of the presence of unknown inputs.

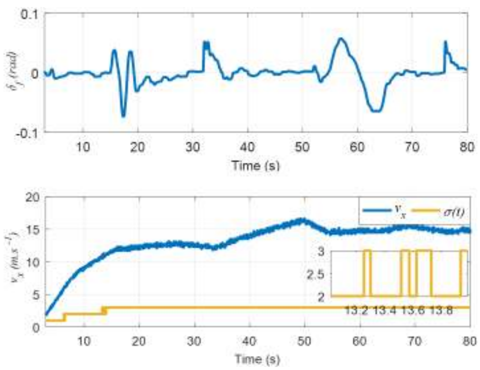

Fig. 2. Measurements of: Steering angle δ_f and longitudinal velocity v_x.

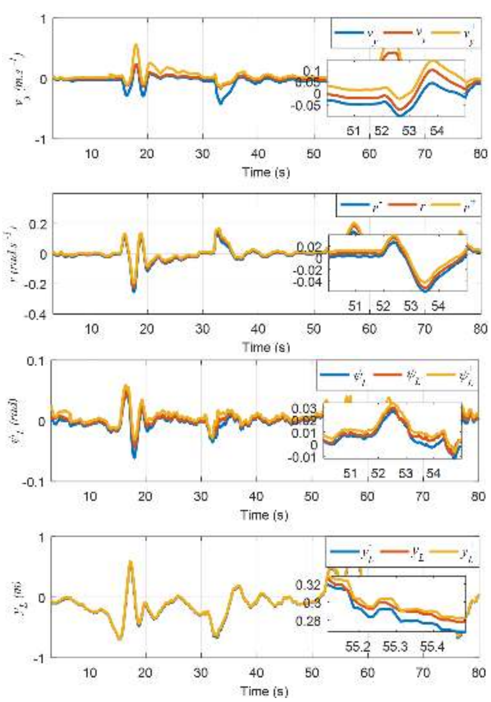

Fig. 3. Interval Estimation of: Lateral velocity v_y, yaw rate r, angular displacement ψ_L and offset displacement y_L.

VI. Conclusion

A methodology to cope with uncertainties in the vehicle lateral dynamics model have been addressed. A new approach for robust estimation of lateral velocity and yaw rate is proposed, where the classical observers are replaced with the interval one. Applicability conditions of the interval observers are expressed in terms of linear matrix inequalities. The proposed approach is illustrated through simulation using real data. Finally, an appealing direction for future works is the unknown inputs estimation, fault detection and isolation observers for systems with faulty inputs.

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