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Vehicle Replacement Costing With Age and Budget Constraints

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1. Introduction

Vehicle replacement decisions have traditionally been based on life cycle costing methods, in which a comparison among investments with different life spans are made. Existing life cycle costing methods include the Machinery and Allied Products Institute (MAPI) Method (Schwan, 1963), the Net Present Value (NPV) Method (Copeland and Weston, 1979), and a number of discounted Weighted Average Cost (DWAC) Techniques (Simmons, 1982). These approaches are applicable to the determination of the service life of an individual vehicle. They are however not applicable to the decision on whether a group of vehicles should be replaced. While this practical problem was addressed by Bath (1989) by grouping vehicles in a fleet according to their age and completed kilometres, he considered the annualised equivalent cost as a function of age and running kilometres, without considering the residual value of an old vehicle upon replacement. Bath's bus cost model requires that the number of vehicles to be replaced be predetermined, which should actually be the result of vehicle replacement decision making. His model also requires that all vehicles in the fleet be of the same type, which is not valid in many applications.

At any renewal point in time, a vehicle can be replaced, be subject to major rebuilds, or continue in its current state. However, the decision on individual vehicle replacement should be subject to an operator-imposed budget constraint, and in some countries an external fleet average age constraint, which are imposed on the fleet as a whole. In the State of New South Wales (Australia), under the 1990 Passenger Transport Act, an accredited vehicle operator must not have a fleet whose average age exceeds 12 years. These types of practical constraints have not been addressed in the literature.

This paper proposes a 0-1 integer programming model to vehicle replacement, taking into account the age and budget constraints and being able to cope with different types of vehicles in a fleet. In calculating the annualised equivalent costs (AEC) which will be very important coefficients in the 0-1 programming model, the paper introduces a vehicle residual value as a function of used years and completed kilometres of a vehicle. The introduction of the residual value function eliminates the difficulty in determining the life span which is essential in the calculation of annualised equivalent costs. The AEC of a vehicle includes four types of major rebuilds as discussed in the next section.

2. Assumptions and Notation

The basic assumption underlying a vehicle replacement decision is that the existing vehicle fleet is sufficient to carry out all routine tasks. The decision on whether additional vehicles should be introduced into the existing fleet should be evaluated using other techniques such as cost-benefit analysis. We concentrate on the "replacement-rebuild" decision process, centred on a consideration of cost, emphasising the annual cost of continuing/upgrading a vehicle and the annualised cost of replacing a vehicle.

In the calculation of the annualised equivalent costs (AEC), four types of commonly practiced rebuilds are identified: engine rebuild, transmission rebuild, body refurbishment and frame rehabilitation. These types of rebuilds contribute significantly to the AEC and thus differ from the normal operational maintenance. Although the annualisation of the major rebuild costs are currently based on these four types of rebuilds, the basic idea can be extended to any number of rebuilds.

Once the AECs of rebuilds and replacement are derived, they are built into a 0-1 integer programming model with age and budget constraints. Ideally, a vehicle with a higher rebuild AEC compared to the AEC associated with replacement should be replaced. Given the budget limitations of fleet operators, however, it is not always possible to replace eligible vehicles. Furthermore, coexisting with the average fleet age constraint, it may not be possible for the 0-1 programming model to derive a feasible solution. In this case, the vehicle operator must try to allocate a higher budget for the replacement.

In the following sections, we present a residual value function of a vehicle (Section 3) and calculate the annualised equivalent costs of a vehicle incorporating the options of rebuild and replacement (Section 4). Finally, we develop a 0-1 integer programming model for the replacement problem for the whole vehicle fleet and propose a corresponding heuristic greedy algorithm for the optimisation (Section 5).

Vehicles in a fleet can be grouped according to vehicle types (car, bus, coach, etc). For each vehicle type, the following general information is required.

- i real interest or discount rate
- C new capital cost of a vehicle
- n useful economic life of a vehicle
- v_r residual value of a vehicle at the end of the useful life

Com	- vehicle operations and maintenance cost per kilometre
K _m	- annual kilometres per vehicle
C _{re}	- average cost of an engine rebuild
K _{me}	_ average kilometres between rebuilds
C _{rt}	_ average cost of transmission rebuild
K _{mt}	_ average kilometres between transmission rebuilds
C _{rb}	- average cost of major body refurbishment
K _{mb}	- average kilometres between body refurbishment
C _{rf}	- average cost of major frame rehabilitation
K _{mf}	- average kilometres between frame rehabilitation

In addition, for each individual vehicle, the following information is required:

t	- type of vehicle
u k _u	 age of vehicle completed kilometres by the vehicle
n _e	- number of completed engine rebuilds
n _t	- number of completed transmission rebuilds
n _b	- number of completed body refurbishments
n _f	- number of completed frame rehabilitations

Overall, an operator's budget for rebuilds and replacements and the average fleet age limit are denoted as BL_r and U respectively.

3. The Residual Value of a Vehicle

A vehicle is normally replaced after its useful economic life. However, it may be replaced before its useful life if the overall costs can be shown to be minimised. When the vehicle is replaced before its useful life, the residual value must be greater than the residual value at the end of its useful life. Likewise, the residual value can be less than the residual value at the end of its useful life or even zero if the vehicle is too old and has accumulated too many kilometres. The residual value is predominantly a function of the number of elapsed years and total vehicle kilometres. For simplicity, let us assume that the function is linear with respect to the vehicle age and vehicle kilometres within its positive range. That is,

$$v(x,y) = \max \{ 0, ax + by + c \}$$
 (1)

where x and y represent the elapsed vehicle age and vehicle kilometres respectively.

Although different sophisticated forms of the residual value function can be investigated based on the different types of vehicles and their patterns of use, this simple form can at least reflect the following aspects of the residual value from where the coefficients a, b, and c are empirically determined.

(1) Initially the value of the vehicle is its purchase cost C, i.e.,

$$v(0,0) = c = C.$$
 (2)

(2) If the vehicle is not used over its useful life, then the value of the vehicle should be $C/(1+i)^n$, that is, $v(n,0) = an + c = C/(1+i)^n$. Therefore,

$$a = C \left(\frac{1}{(1+i)^n} - 1\right)/n.$$
(3)

(3) If the value of y is the expected number of kilometres, nk_m , at the end of the vehicle's useful life, then the residual value of the vehicle should be v_r . That is, if x = n and $y = nK_m$, then $v(n,nK_m) = an + bnK_m + c = v_r$. Therefore,

$$b = \left(v_r - \frac{C}{(1+i)^n}\right) / (nK_m).$$
(4)

The positive portion of v(x,y) can be depicted in the following figure (Figure 1).

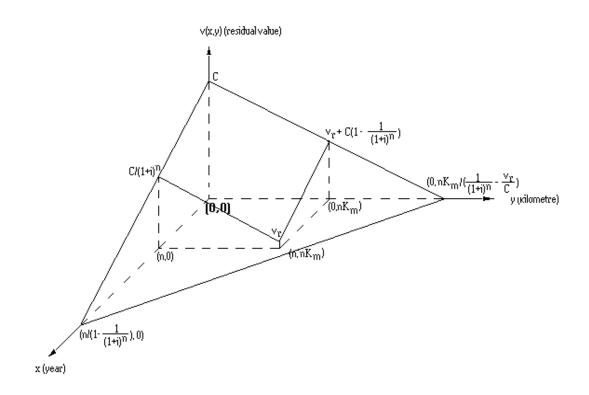


Figure 1 Positive portion of the residual value of a vehicle

4. Annualised Equivalent Costs of a Vehicle

To calculate the AEC of a vehicle, we need to know whether the vehicle will be subject to certain types of major rebuilds. This can be determined based on the assumption that the fleet operator should be able to carry out a rebuild of type j every K_{mj} kilometres (j=e, t, b, f). Therefore, if $k_u/K_{mj} - 1 > n_j$, then rebuild type j should be undertaken. Otherwise, no rebuild is necessary. j represents the type of rebuild, e.g. j = e, t, b and f, representing engine rebuild, transmission rebuild, body refurbishment, and frame rehabilitation, respectively.

Whether the vehicle undergoes major rebuilds or continues in its current state, we will be able to calculate the additional optimal kilometres and years to be served by the vehicle before undergoing another rebuild. Let us denote

$$d_{j} = \begin{cases} 1 & \text{ if } K_{u}/K_{mj} - 1 > n_{j} \\ 0 & \text{ if } K_{u}/K_{mj} - 1 & n_{j} \end{cases} (j=e,\,t,\,b,\,f) \quad (5)$$

 $d_j = 1$ indicates that the rebuild type j should be carried out. Therefore, K_{mj} (j=e, t, b, f) additional kilometres can be served after rebuild type j is made and ideally additional k_{mj}/K_m years can be served.

 $d_j = 0$ indicates that rebuild type j is not required. Since the vehicle has consumed $k_u - n_j K_{mj}$ kilometres since its last rebuild, the additional kilometres to be served by the vehicle is $K_{mj} - (K_u - n_j K_{mj}) = (n_j + 1)K_{mj} - K_u$ and hence ideally, an additional $((n_j + 1)K_{mj} - K_u)/K_m$ years can be served.

Therefore, the additional kilometres to be served is

$$k'_{u} = \min \{ d_{j}K_{mj} + (1-d_{j})((n_{j}+1)K_{mj} - k_{u}) \}$$
(6)
j=e,t,b,f

and an additional $u' = k'_u/K_m$ years can be served after the rebuilds.

With the additional kilometres and years, the annualised equivalent cost can be derived by the sum of capital AEC (AEC_c), operation and maintenance AEC (AEC_{om}), and major rebuild AEC (AEC_r), which are calculated as follows.

Capital AEC (AEC_c)

$$AEC_{c} = [C - v(u+u',k_{u}+k'_{u})]F(i,u+u')$$
(7)

where

$$F(i,u) = \frac{1}{1 - 1/(1+i)^{u}}$$
(8)

is the amortisation factor with real interest i and year u.

Operation and Maintenance AEC (AEC_{om})

There are two cases. In the first case, we assume that the operation and maintenance (OM) cost incurred is constant in each year of its useful life, which is given by

$$AEC_{om} = K_m C_{om}$$
⁽⁹⁾

In the second case, we assume that the OM cost increases with cumulative kilometres over the life of the vehicle, which is given by

$$AEC_{om} = K_m (C_{om} + C_{lom})/2$$
⁽¹⁰⁾

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where

$$C_{lom}$$
 = last year OM cost per kilometre
= $C_{om} + C_u(k_u + k'_u)$

where, C_u is the additional operating and maintenance cost coefficient per completed kilometre.

The OM cost in this case is referred to as an escalating operating and maintenance cost. We will use the escalating OM cost in the calculation since it appears to be more realistic.

Major Rebuild AEC (AEC_r)

The major rebuild costs of the vehicle consist of the costs of engine rebuild, transmission rebuild, body refurbishment, and frame rehabilitation. The costs of each rebuild are

$$C_{rj}(n_j + d_j)$$
 (j = e, t, b, f) (11)

Thus, the total major rebuild cost is

$$C_{r} = \sum_{(j=e,t,b,f)} C_{rj}(n_{j} + d_{j})$$
(12)

and the total annualised cost of rebuilds $AEC_r = C_r/(u+u')$.

In summary, the total AEC of the vehicle

$$AEC_{total} = AEC_{c} + AEC_{om} + AEC_{r}$$
 (13)

The above AEC of the vehicle is the annualised equivalent cost without considering replacement. When the replacement alternative is considered, the AEC is actually the AEC of operating a new vehicle. Therefore, there are three similar components of the total AEC:

Capital AEC over the vehicle's useful life (AEC'_c)

$$AEC'_{c} = (C - v(u,k_{u}))F(i,n)$$
(14)

Operation and Maintenance AEC (AEC'_{om})

Two cases of AEC'om: a constant annualised OM cost of its useful life

$$AEC'_{om} = K_m C_{om}$$
(15)

and an escalating annualised OM cost over the life of the vehicle

$$AEC'_{om} = K_m (C_{om} + C_{lom})/2$$
(16)

where

$$C_{lom} = C_{om} + C_u n K_m$$

We will again use an escalating operating and maintenance cost in the calculation.

Major Rebuild AEC (AEC'_r)

The major rebuild cost of the vehicle consists of the costs of engine rebuild, transmission rebuild, body refurbishment and frame rehabilitation. The total cost of each rebuild is

$$C_{rj}(nK_m/K_{mj} - 1)$$
 (j = e, t, b, f) (17)

Thus, the total major rebuild cost is

$$C'_{r} = \sum_{(j=e,t,b,f)} C_{rj} (nK_{m}/K_{mj} - 1)$$
 (18)

and AEC'_r = C'_r/n.

Total AEC of the vehicle upon replacement is thus the sum of capital AEC, operation and maintenance AEC, and major rebuild AEC:

$$AEC'_{total} = AEC'_{c} + AEC'_{om} + AEC'_{r}$$
(19)

Ideally, a vehicle with higher total AEC associated with the rebuild alternative than the total AEC upon replacement should be replaced. This is not always possible, however, due to the financial ability of the vehicle operator. Considering the replacement for all vehicles in a fleet

as a whole, the decision problem is built into a 0-1 integer programming model in the next section, where the average fleet age constraint is also introduced.

5. The 0-1 Integer Programming Model for the Vehicle Replacement

Suppose there are N vehicles in the fleet, each of which is associated with replacement cost A_j and rebuild cost B_j (if the vehicle does not require any rebuild, $B_j = 0$). Furthermore, the total AEC upon replacement and the total AEC of rebuild alternative of the jth vehicle calculated from the preceding section are denoted as a_j and b_j respectively (j=1,2, ..., N). The decision problem of vehicle replacement can be formulated as the following 0-1 integer model:

minimise
$$\sum_{j=1}^{N} a_j x_j + b_j (1-x_j)$$
(20)

Subject to

$$\sum_{j=1}^{N} A_{j}x_{j} + B_{j}(1-x_{j}) \le BL_{r}$$

$$\sum_{j=1}^{N} u_{j}(1-x_{j}) \le N U$$

$$x_{j} \{0, 1\}$$
(21)
(21)
(21)
(22)

In this formulation, x_i are decision variables. That is,

$$x_{j} = \begin{cases} 1 & \text{if the vehicle is replaced} \\ 0 & \text{otherwise} \end{cases}$$

 u_j is the current age of the jth vehicle and d_{bj} is the body refurbishment indicator (i.e. $d_{bj}=1$ if body refurbishment is required and $d_{bj}=0$ otherwise) of the jth vehicle.

The objective of the model is to minimise the total annualised equivalent cost of the vehicle fleet. The first constraint of the model restricts rebuild and replacement costs within the total rebuild and replacement budget BL_r . The second constraint imposes a limit on the average age

of the fleet. It indicates that if the vehicle is replaced, its age will be set to zero and thus will not appear in the age constraint.

Rearranging the model, we can obtain the following 0-1 integer programming model:

maximise
$$\sum_{j=1}^{N} (b_j - a_j) x_j$$
(23)

Subject to

$$\sum_{j=1}^{N} (A_{j} - B_{j})x_{j} \leq BL_{r} - \sum_{j=1}^{N} B_{j}$$

$$\sum_{j=1}^{N} u_{j}x_{j} \leq N \ U - \sum_{j=1}^{N} u_{j}$$

$$x_{j} \{0, 1\}$$

$$(24)$$

$$(25)$$

This model is the Cargo Loading Problem with $x_j \{0, 1\}$ (Kovacs, 1980). When $b_j - a_j > 0$ and $A_j - B_j > 0$ are all positive integers, the model becomes 0-1 Multidimensional Knapsack Problem (Systo, 1983). From a computational point of view, the problem is difficult to solve. No polynomial-time algorithm has been available for this problem, and it is very unlikely that such an algorithm exists (Gilmore and Gomory, 1966). Exact methods such as branch and bound, cutting plans and dynamic programming, can only deal with a small number of vehicles and is computationally inefficient. The existing heuristic approaches emphasise solving the Knapsack Problem and Cargo Loading Problem only for non-negative integer coefficients.

A heuristic algorithm, which is greedy in nature, is proposed to derive the solution of model (20) - (22). The algorithm takes into account the decision behaviour of management in vehicle replacement while incorporating a cost minimisation strategy. The idea of the algorithm is to replace the vehicle with the largest negative gap between the AECs of the replacement and rebuilds, and to rebuild the vehicle with the highest positive gap between the replacement and rebuild AECs, as much as possible, subject to the age and budget constraints.

The necessary assumption for the algorithm is that $A_j > B_j$ for j=1,2, ..., n. This assumption ensures the cost of rebuilds to be less than the cost of replacement. Otherwise, the vehicle can be replaced before including it into the model since a_j can be shown to be less than or equal to

bj. In this case, the consequent budget should be deducted by A_j in the first constraint of the model, i.e. $BL_r = BL_r - A_j$, before the model is formulated. Further, we can assume $\sum_{j=1}^{N} B_j \leq BL_r$. Otherwise, the problem is infeasible. In this case, the budget should be increased such

that
$$\sum_{j=1}^{N} B_j \leq BL_r$$
.

The greedy algorithm is described as follows.

Arrange such that $\frac{a_1 - b_1}{u_1} \epsilon \frac{a_2 - b_2}{u_2} \epsilon \dots \epsilon \frac{a_N - b_N}{u_N}$.

Initialisation: Set initial age and budget to zero: a = 0, b = 0.

(1) For j = 1, 2, ..., N, if $a_j > b_j$ and $a + u_j = N$ U, then $x_j = 0$ (rebuild or do nothing) and set

$$\mathbf{a}:=\mathbf{a}+\mathbf{u}_{\mathbf{j}},\,\mathbf{b}:=\mathbf{b}+\mathbf{B}_{\mathbf{j}}.$$

Repeat the process for j = 1, 2, ..., N until $a_j \ b_j$ or $a + u_j > N$ U. The index upon termination is denoted as J_1 .

(2) Set b' = 0 and BL' = $BL_r - \sum_{j=1}^N B_j$. For j = N, N-1, ..., J₁, if a_j b j and b' + A_j - B_j BL', then $x_j = 1$ (replace) and set

$$BL': = BL' - (A_j - B_j)$$
$$b': = b' + (A_j - B_j)$$
$$b: = b + A_j$$

Repeat the process for $j = N, N-1, ..., J_1$ until $a_j > b_j$ or $b' + (A_j - B_j) > BL'$. The index upon termination is denoted as J_2 .

(3) If $a + u_{J1}$ N U, the age constraint has not been violated yet then go to step (4). Otherwise, do the following

For
$$j = J_1$$
, J_1+1 , ..., J_2 , set $x_j = 1$ (replace) and $b := b + A_j$.

Go to Step (6).

(4) For $j = J_1, J_1+1, ..., J_2$, if $a + u_j$ N U, set

 $x_i = 0$ (rebuild or do nothing), $a := a + u_i$, $b := b + B_i$.

Repeat the process for $j = J_1, J_1+1, ..., J_2$ until $a + u_j > N$ U (age constraint violated). The index is denoted as J.

- (5) For j = J, J+1, ..., J_2 , set $x_j = 1$ (replace), and $b := b + A_j$.
- (6) If $b > BL_r$, budget is not sufficient and is recommended increasing to b, i.e. $BL_r = b$.

Stop.

6. A Case Study

A microcomputer program called Vehicle Replacement System (VRS) has been developed to implement the method. The program is written in C running on any PC-based microcomputer. It has the flexibility in data entry for different types of vehicles. All vehicle types are input in the main menu of the program while the data on individual vehicles are entered in the Vehicle Data Spreadsheet similar to the data entry in any typical spreadsheet program. The program is easy to use so that all key operations in the program are kept at a minimum. With very few key operations, the user will be able to view solutions with displayed annualised equivalent costs for both rebuilds and replacement. The age and budget constraints are highly respected in this program. In addition, the model for the vehicle replacement is written in a selfcontained module so that it can be easily embedded into other user interface programs.

A case study with a fleet of 21 vehicles has been performed with the developed program. For simplicity, all vehicles are assumed to be the same type. The data for the unique vehicle type are summarised in Table 1:

ITEM	VALUE
Annual kilometre per vehicle:	40000.00
OM cost per kilometre (\$/km):	0.60
Increase factor for OM cost per kilometre:	0.00253000
Capital cost of new vehicle (purchase price \$):	200000.00

Table 1	Vehicle	Type	Data
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Useful life of a vehicle (year):	15.00	
Residual value of a vehicle (\$):	10000.00	
Real interest rate:	0.08	
Average cost of engine rebuild (\$):	4000.00	
Average kilometres between engine rebuilds:	200000.00	
Average cost of transmission rebuild (\$):	7000.00	
Average kilometres between transmission rebuilds:	310000.00	
Average cost of major body refurbishment (\$):	8000.00	
Average kilometer between body refurbishments:	320000.00	
Average cost of major frame rehabilitation (\$):	8000.00	
Average kilometer between frame rehabilitations:	300000.00	

The budget for the current decision year is \$500,000 and the average year limit is 12 years. By performing the method developed in this paper, the result in Table 2 is obtained.

In Table 2, vehicles are listed with their name, type, age, kilometres, and number of rebuilds: engine rebuilds (Ne), tranmission rebuilds (Nt), body refurbishment (Nb) and frame rehabilitation (Nf). The results are represented in AEC of replacement option (AEC-RP), AEC of rebuild option (AEC-RB), and solution: replace or carry out one or some of the four rebuilds: RB-e, or t, or b, or f. The total AEC of replacement and rebuilds for the fleet (i.e. the objective of the model) is \$855,054. The model actually incurrs \$482,922 for the replacements and rebuilds for the whole vehicle fleet in the current decision year. The average age of the fleet from the program is 12 years, which saturates the upper average age limit.

From Table 2, we will observe that it is not necessary that oldest vehicle will be given the replacement option because of the lower AEC of rebuild than of replacement. The age limit is not crucial in vehicle replacement decision making.

Vehicle	Name	Туре	Year	Kilometre	Ne	Nt	Nb	Nf	Cost-RB	Cost-RP	Solution
0	Benz	0	20	900000	4	2	2	2	46931.54	48640.42	RB-f
1	Benz	0	15	700000	3	2	2	2	45001.54	43272.05	RB-
2	Benz	0	15	650000	3	2	2	2	45429.71	43432.52	Replace
3	Benz	0	14	600000	3	1	1	2	42236.89	42526.34	RB-
4	Benz	0	13	630000	3	2	1	2	42070.96	41092.61	RB-
5	Benz	0	4	160000	0	0	0	0	29264.59	32053.37	RB-
6	Benz	0	2	80000	0	0	0	0	22514.59	29770.63	RB-
7	Benz	0	1	5000	0	0	0	0	7612.25	28606.21	RB-
8	Hino	0	20	800000	3	2	2	2	47401.39	48961.37	RB-e
9	Hino	0	15	750000	3	2	2	2	44600.37	43788.52	RB-
10	Hino	0	9	500000	2	1	1	1	39765.23	36837.01	Replace
11	Hino	0	6	300000	1	1	1	1	37282.23	33601.98	Replace
12	Hino	0	4	160000	0	0	0	0	29264.59	32053.37	RB-
13	Hino	0	4	160000	0	0	0	0	29264.59	32053.37	RB-
14	Holden	0	20	800000	3	2	2	2	47401.39	48961.37	RB-e
15	Holden	0	18	800000	3	2	2	2	46474.05	46828.04	RB-e
16	Holden	0	15	700000	3	2	2	2	45001.54	43272.05	RB-
17	Holden	0	9	490000	2	1	1	1	39582.84	37410.66	Replace

 Table 2 Vehicle Data and Replacement Results

18	Suzuki	0	20	960000	4	3	2	3	48720.77	48583.23	RB-b
19	Suzuki	0	18	900000	4	2	2	2	46070.91	46507.09	RB-f
20	Suzuki	0	17	800000	3	2	2	2	46036.39	45761.37	RB-e
21	Suzuki	0	10	500000	2	1	1	1	40220.00	37903.68	Replace

7. Conclusions

Vehicle replacement has traditionally relied on the life cycle costing method which involves comparisons between investments with different life spans. These methods are not able to cope with different vehicle types in a vehicle fleet, and budget and age constraints. This paper resolves the limitation of the traditional vehicle replacement methods by introducing the concept of a residual value function of which the positive portion is the linear function of used years and completed kilometres. The introduction of the residual value function eliminates the difficulty in determining the life spans for calculating the annualised equivalent costs, resulting in the new way of calculating annualised costs of a vehicle. Hence comparison can be easily handled on a yearly basis for any life span. The budget and age constraints are incorporated into a 0-1 integer programming model whose optimal solution determines the decision on vehicle replacement for a whole vehicle fleet.

A microcomputer program called Vehicle Replacement System (VRS) has been developed to implement the proposed method. The program has the flexibility in data entry for different types of vehicles. The age and budget constraints are highly respected in this program.

This paper assumes that the positive portion of the residual value of a vehicle is the linear function of used years and completed kilometres. Although the linear form is the simplest solution, continuing studies are investigating other functional forms to describe the residual value.

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