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# Velocity and Acceleration in Rotational Oblate Spheroidal Coordinates

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***Abstract:*** *we had established the velocity and Acceleration in some of orthogonal curvilinear coordinates such as oblate spheroidal, prolate spheroidal, parabolic cylindrical, parabolic and second torridal coordinates for application in Mechanics, in this paper we proceed to derive expression for the instantaneous velocity and acceleration in rotational oblate spheroidal coordinates*
***Keywords:*** *Rotational Oblate Spheroidal Coordinates, Velocity and Acceleration*

## I. Introduction

To increase the scope of classical mechanics, relativity mechanics and quantum mechanics, we had expressed the instantaneous velocity and acceleration in oblate spheroidal, prolate spheroidal, parabolic cylindrical, parabolic and second Torridal coordinates [1, 2. 3, 4, 5]. We therefore continue to derive the expression for instantaneous velocity and instantaneous acceleration in Rotation oblate spheroidal coordinates for application in mechanics. The Rotational Oblate Spheroidal Coordinates $(u, v, \omega)$are related to the Cartesian Coordinates $(x, y, z)$ as [6]:

$$x = \omega(u^2 + d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}} (1)$$

$$y = (u^2 + d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}}(1 - \omega^2)^{\frac{1}{2}} (2)$$

$$z = uv \qquad (3)$$

Consequently, be definition, the Rotational Oblate Spheroidal Metric Coefficients are given by:

$$h_u = \left(\frac{u^2 + v^2 d^2}{u^2 + d^2}\right)^{\frac{1}{2}} (4)$$

$$h_v = \left(\frac{u^2 + v^2 d^2}{1 - v^2}\right)^{\frac{1}{2}} (5)$$

$$h_\omega = \frac{(u^2 + d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}}}{1 - \omega^2} (6)$$

These metric coefficients define units vectors line element volume element, gradient, divergence, curl and Laplacian operations in Rotational Oblate Spheroidal Coordinates according to the theory of orthogonal curvilinear coordinates [7,8,9]. These quantities are necessary and sufficient for the derivation of the fields of all Rotational Oblate Spheroidal distribution of mass, charge and current. Therefore for the derivation of the equation of motion for test particles in these fields, we shall derive the expression for instantaneous velocity and acceleration in Rotational Oblate Spheroidal Coordinates.

## II. Mathematical Analysis

The Cartesian Unit vectors are related to the rotational oblate Spheroidal coordinates unit vector as:

$$\hat{u} = \frac{u\omega(1 - v^2)^{\frac{1}{2}}}{(u^2 + v^2 d^2)^{\frac{1}{2}}}\hat{\imath} + \frac{u(1 - v^2)^{\frac{1}{2}}(1 - \omega^2)^{\frac{1}{2}}}{(u^2 + v^2 d^2)^{\frac{1}{2}}}\hat{\jmath} + \frac{v(v^2 + d^2)^{\frac{1}{2}}}{(u^2 + v^2 d^2)^{\frac{1}{2}}}\hat{k} (7)$$

$$\hat{v} = -\frac{v\omega(u^2 + d^2)^{\frac{1}{2}}}{(u^2 + v^2 d^2)^{\frac{1}{2}}}\hat{\imath} - \frac{v(u^2 + d^2)^{\frac{1}{2}}(1 - \omega^2)^{\frac{1}{2}}}{(u^2 + v^2 d^2)^{\frac{1}{2}}}\hat{\jmath} + \frac{u(1 - v^2)^{\frac{1}{2}}}{(u^2 + v^2 d^2)^{\frac{1}{2}}}\hat{k} (8)$$

and

$$\hat{\omega} = (1 + \omega^2)^{\frac{1}{2}}\hat{\imath} - \omega\hat{\imath} (9)$$

The inversion is given by:

$$\hat{\imath} = \frac{u\omega(1-v^2)^{\frac{1}{2}}}{(u^2+v^2d^2)^{\frac{1}{2}}}\hat{u} - \frac{v\omega(u^2+d^2)^{\frac{1}{2}}}{(u^2+v^2d^2)^{\frac{1}{2}}}\hat{v} + (1-\omega^2)^{\frac{1}{2}}\widehat{\omega}(10)$$

$$\hat{\imath} = \frac{u\omega(1-v^2)^{\frac{1}{2}}}{(u^2+v^2d^2)^{\frac{1}{2}}}\hat{u} - \frac{v\omega(u^2+d^2)^{\frac{1}{2}}}{(u^2+v^2d^2)^{\frac{1}{2}}}\hat{v} + (1-\omega^2)^{\frac{1}{2}}\widehat{\omega}(10)$$

$$\hat{\jmath} = \frac{u(1-v^2)^{\frac{1}{2}}(1-\omega^2)^{\frac{1}{2}}}{(u^2+v^2d^2)^{\frac{1}{2}}}\hat{u} - \frac{v(1-\omega^2)^{\frac{1}{2}}(u^2+d^2)^{\frac{1}{2}}}{(u^2+v^2d^2)^{\frac{1}{2}}}\hat{v} - \omega\widehat{\omega}(11)$$

and

$$\hat{k} = \frac{v(u^2-d^2)^{\frac{1}{2}}}{(u^2+v^2d^2)^{\frac{1}{2}}}\hat{u} + \frac{u(1-v^2)^{\frac{1}{2}}}{(u^2+v^2d^2)^{\frac{1}{2}}}\hat{v}(12)$$

Hence denoting one time differentiating by a dot, it follows from (7), (8) and (9) and some manipulation that;

$$\hat{u} = \frac{1}{(u^2+v^2d^2)^{\frac{1}{2}}}\left[-\frac{vd^2(1-v^2)^{\frac{1}{2}}}{(u^2+d^2)^{\frac{1}{2}}}\dot{u} + \frac{u(u^2+d^2)^{\frac{1}{2}}}{(1-v^2)^{\frac{1}{2}}}\dot{v}\right]\hat{v} + \frac{u(1-v^2)^{\frac{1}{2}}}{(1+\omega^2)^{\frac{1}{2}}(u^2+v^2d^2)^{\frac{1}{2}}}\dot{\omega}\widehat{\omega}(13)$$

Similarly, it follows form (8), (7) and (9) that:

$$\hat{v} = \frac{1}{(u^2+v^2d^2)}\left[\frac{vd^2(1-v^2)^{\frac{1}{2}}}{(u^2+d^2)^{\frac{1}{2}}}\dot{u} - \frac{u(u^2-d^2)^{\frac{1}{2}}}{(1-v^2)^{\frac{1}{2}}}\dot{v}\right]\hat{u} + \frac{v(1-v^2)^{\frac{1}{2}}}{(u^2+v^2d^2)^{\frac{1}{2}}(1+\omega^2)^{\frac{1}{2}}}\dot{\omega}\widehat{\omega}(14)$$

and consequently, from (9), (10) and (11):

$$\widehat{\omega} = \frac{1}{(1-\omega^2)^{\frac{1}{2}}(u^2+v^2d^2)^{\frac{1}{2}}}\left[u(1-v^2)^{\frac{1}{2}}\hat{u} + u(u^2-d^2)^{\frac{1}{2}}\hat{v}\right]\dot{\omega}(15)$$

Now it follows from definition of instantaneous position vector $\underline{r}$ as:

$$\underline{r} = xi + yj + z\dot{k}(16)$$

And (1) – (3) and (10) – (12) that the instantaneous position vector may be expressed entirely in terms of rotational oblate speroidal coordinates as:

$$\underline{r} = \frac{u(u^2-d^2)^{\frac{1}{2}}}{(u^2+v^2d^2)^{\frac{1}{2}}}\hat{u} - \frac{vd^2(1-v^2)^{\frac{1}{2}}}{(u^2+v^2d^2)^{\frac{1}{2}}}\hat{v}\widehat{\omega}(17)$$

It now follows from definition of instantaneous velocity vector $\underline{u}$ as:

$$\underline{u} = \underline{\dot{r}}(18)$$

and (17), (13) and (12) that the instantaneous velocity vector may be expressed entirely in term of rotational oblate speroidal coordinates as:

$$\underline{u} = u_u\hat{u} + u_u\hat{v} + u_\omega\widehat{\omega}(19)$$

where

$$u_u = \left(\frac{u^2+v^2d^2}{u^2+d^2}\right)^{\frac{1}{2}}(20)$$

$$u_v = \left(\frac{u^2+v^2d^2}{1+v^2}\right)^{\frac{1}{2}}(21)$$

and

$$u_\omega = \left(\frac{(u^2+d^2)(1-v^2)}{(1-\omega^2)}\right)^{\frac{1}{2}}(22)$$

Similarly, it follows form definition of instantaneous acceleration vector, $\underline{a}$ as:

$$\underline{a} = \underline{u}(23)$$

and (19), (13) – (15) that the instantaneous acceleration may be expressed entirely in term of rotational oblate speroidal coordinates as:

$$\underline{a} = a_u\hat{u} + a_u\hat{v} + a_\omega\widehat{\omega}(24)$$

where

$$a_u = \frac{(u^2+v^2d^2)}{(u^2+d^2)^{\frac{1}{2}}}\left[\ddot{u}\frac{ud^2(1-v^2)}{(u^2+d^2)(u^2+v^2d^2)}\dot{u}^2 + \frac{2vd^2}{(u^2+v^2d^2)}\dot{u}\dot{v} - \frac{u(u^2+d^2)}{(1+v^2)(u^2+v^2d^2)}\dot{v} + \frac{u(u^2+d^2)(1-v^2)}{(1+\omega^2)(u^2+v^2d^2)}\dot{\omega}^2\right](25)$$

$$a_u = \frac{(u^2+v^2d^2)^{\frac{1}{2}}}{(u^2+d^2)^{\frac{1}{2}}}\left[\ddot{v} + \frac{u(u^2+d^2)}{(1+v^2)(u^2+v^2d^2)}\dot{v}^2 + \frac{2u}{(u^2+v^2d^2)}\dot{u}\dot{v} - \frac{vd^2(1+v^2)}{(u^2+d^2)(u^2+v^2d^2)}\dot{u}^2 + \frac{v(u^2+d^2)(1-v^2)}{(1+\omega^2)(u^2+v^2d^2)}\dot{\omega}^2\right] (26)$$

and

$$a_\omega = \frac{(u^2+v^2d^2)^{\frac{1}{2}}(1+v^2)^{\frac{1}{2}}}{(1+\omega^2)^{\frac{1}{2}}}\left[\ddot{\omega} + \frac{\omega}{(1+\omega^2)}\dot{\omega}^2 + \frac{2u}{u^2+d^2}\dot{u}\dot{\omega} - \frac{2v}{(1+v^2)}\dot{v}\dot{\omega}\right] (27)$$

This is the completion of the instantaneous velocity and instantaneous and instantaneous acceleration in rotational oblate spheroidal coordinates system.

## III. Results and Discussion

In this paper we derived the component of velocity and acceleration in Rotational oblate sphereiodal coordinates as (19) – (22) and (24) – (27), are necessary and sufficient for expressing all mechanical quantities (Kubear momentum, kinetic energy, Langrangian and Hamiltonian) in terms of Rotational Oblate Spheroidal coordinates.

## IV. Conclusion

The velocity and acceleration component (20), (21), (22), (25), (26), (29) obtained in this paper paves a way for expressing all dynamical laws of motion (Newton's law, Lagrange's Law, Hamiltonian's Law, Einstein's Special Relativities Law of Motion and Schrödinger's Law Quantum Mechanics) entirely in terms of Rotational Oblate Spheroidal Coordinates.

## Reference


[1]. S.X.K. Howusu, F.J.N Omaghali and F.F.Musongong, Velocity and Acceleration in Oblate Spheroidal Coordinates, Science Forum; Journal of Applied Sciences [Abubakar Tafewa Balawa Universit] Volume 7, Number 1 2004.

[2]. J.F. Omonile, D.J. Koffa and S.X.K Howusu, Velocity and Acceleration in Prolate Spheroidal Coordinates, Archives of Physics Reearch, 5(1), 2014, 56-59.

[3]. D.J. Koffa, J.F. Omonile and S.X.K Howusu, Velocity and Acceleration in Parabolic Coordinates, IOSR Journal of Applied Physics, Volume 6 Issue 2 Ver I (Mar-Apr. 2014), PP32-33.

[4]. J.F. Omonile, B.B Ogunwale and S.X.K Howusu, Velocity and Acceleration in Parabolic Cylindrical Coordinates Palagia Research Library Advances in Applied Science Research 6(5), 2015

[5]. J.F. Omonile, J.Ahmen and S.K.K Howusu, Velocity and Acceleration in Second Torridal Coordinates, IOSR Journal of Applied Physics, Volume 7, Issue 3 Ver. IV (May-Jun, 2015), 01-02.

[6]. S.X.K. Howusu, Vector and Tensor Analysis (Jos: Jos University Press Ltd, 2003)

[7]. F.B Hilderbrand, Advance Calculus for Applications (Prentice Hall: Engle wood-cliff, 1962) 298-306

[8]. Morce.H. Freshback, Method of Theoretical Physics (New-York MC Graw-Hill, 1953), 21-54

[9]. B. Garfinkel Problem of Quadraturer Astron J. 63, 88, 1958.