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Velocity-Density Relations[1]

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In 1961 *Birch* [1961] found an empirical linear relationship between the density and the compressional velocity in rocks. *Anderson* [1967] subsequently showed, on theoretical grounds, that the density should be proportional to some power of the bulk sound velocity C where

$$C^2 = V_p^2 - (4/3)V_s^2 = \Phi = K_s/\rho = (\partial P/\partial \rho)_s$$

where V_p is the compressional velocity, V_s the shear velocity, ρ the density, K_s the adiabatic bulk modulus, Φ the seismic parameter and P is the pressure. The parameters, a and b, in the relationship

$$\rho/\langle M\rangle = a\Phi^b \qquad (1)$$

where $\langle M\rangle$ is the mean atomic weight were found by use of ultrasonic, static compression, and shock-wave data. At the same time equations of the following form were fitted to the same data

$$\rho/\langle M\rangle = a + b\Phi \qquad (2)$$

and

$$\rho/\langle M\rangle = a + b\Phi^{1/2} = a + bC \qquad (3)$$

Equation 2 is completely arbitrary, and equation 3 is the analog of Birch's relationship, with C replacing V_p. The results of the latter calculations were not presented because of their strictly empirical nature. However, *Wang* [1968] has recently used equation 3 in a discussion of the composition of the mantle, so that it is appropriate at this time to present the parameters found by fitting equations 2 and 3 to the data sets used in *Anderson* [1967].

TABLE 1. Parameters of Least-Square Solutions to Three Forms of the Velocity-Density Relation for Various Sets of Data

Data	Sample Size	a	b	Standard Deviation, g/cm^3	Per Cent Deviation
1) $\rho = \langle M\rangle(a + b\Phi)$					
$18.5 < \langle M\rangle < 90$	116	0.110	0.001	0.41	9.0
$18.5 < \langle M\rangle < 90$	29	0.115	0.001	0.49	8.3
$18.5 < \langle M\rangle < 88$	56	0.112	0.001	0.49	11.1
$18.6 < \langle M\rangle < 33.1$	31	0.105	0.001	0.14	4.2
(2) $\rho = \langle M\rangle(a + b\Phi^{1/2}) = \langle M\rangle(a + bC)$					
$18.5 < \langle M\rangle < 90$	116	0.064	0.015	0.38	8.8
$18.5 < \langle M\rangle < 90$	29	0.079	0.012	0.44	8.0
$18.5 < \langle M\rangle < 88$	56	0.066	0.015	0.47	10.9
$18.6 < \langle M\rangle < 33.1$	31	0.053	0.017	0.11	3.5
(3) $\rho = \langle M\rangle(a\Phi^b)$					
$18.5 < \langle M\rangle < 90$	116	0.056	0.281	0.38	8.9
$18.5 < \langle M\rangle < 90$	29	0.064	0.240	0.41	7.8
$18.5 < \langle M\rangle < 88$	56	0.059	0.274	0.48	11.1
$18.6 < \langle M\rangle < 33.1$	31	0.048	0.323	0.12	3.6

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The results are given in Table 1. The last four rows are the results previously published for the power law relation between ρ and Φ. Equations 1 and 3 give slightly better fits than equation 2. For practical purposes there is no difference, for any of the data sets, in the fits obtained with 1 and 3.

The errors involved in the use of equations 1, 2, and 3 for estimating the bulk sound speed, the bulk modulus or the seismic parameter Φ are unacceptably large for most applications, although they do account for the general trend of the data. This indicates that the mean atomic weight is not the only parameter controlling the relation between density and velocity. A variety of crystal structures, cation valences, cation radii, and porosities are represented by the data. *Anderson* [1969] and D. L. Anderson and O. L. Anderson (in preparation) have systematically investigated the effects of parameters other than the mean atomic weight.

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