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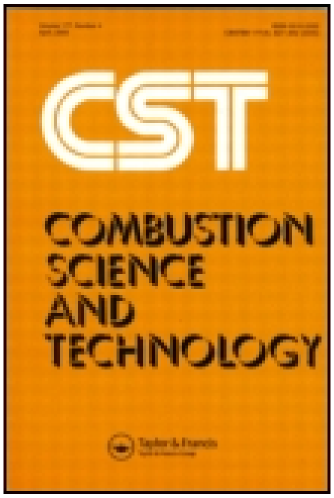
3 authors, including:

Vitaly Bychkov
Umeå University

Michael A. Liberman
Nordic Institute for Theoretical Physics



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VELOCITY OF TURBULENT FLAMELETS OF FINITE THICKNESS

VITALIY BYCHKOV [a] , MICHAEL A. LIBERMAN [b c] & RAYMOND REINMANN [d]
[a] Institute of Physics, Umeå University , Umeå, Sweden
[b] Department of Physics , Uppsala University , Uppsala, Sweden
[c] P. Kapitza Institute for Physical Problems , Moscow, Russia
[d] Saab Automobile AB , Södertälje, Sweden
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VELOCITY OF TURBULENT FLAMELETS OF FINITE THICKNESS

VITALIY BYCHKOV

Institute of Physics, Umeå University, Umeå, Sweden

MICHAEL A. LIBERMAN

Department of Physics, Uppsala University, Uppsala, Sweden and P. Kapitza Institute for Physical Problems, Moscow, Russia

RAYMOND REINMANN

Saab Automobile AB, Södertälje, Sweden

Propagation of turbulent premixed flamelets of finite thickness is considered using the model nonlinear equation proposed recently. The nonlinear equation takes into account both influence of external turbulence and intrinsic properties of a flame front caused by density variations across the front. The formula for the turbulent flame velocity is derived for the case of flamelets of small but finite thickness. The obtained results agree well with experimental data on flame propagation in a turbulent flow of moderate and high values of the Reynolds number. Known expressions for the flame speed found by Clavin-Williams and by Yakhot for the case of zero thermal expansion of the fuel mixture are recovered.

INTRODUCTION

The quantitatively correct numerical simulation of turbulent flames is currently one of the most challenging problems in combustion science being related to the practical needs in different branches of industry, such

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Address correspondence to V. Bychkov, Inst. of Physics, Umeå University, S-901 87,

as design of more efficient engines of low emission. A distinct feature of a turbulent flow is the broad spectrum of length and time scales ranging over several orders of magnitude, which can not be properly resolved in a numerical simulation. In order to overcome this difficulty a reliable theoretical sub-grid model of turbulent burning is needed. For most practical combustion systems, including internal-combustion engines and gas turbine combustors, the concept of flamelets suggested by Williams (1970) may be applied. The flamelet concept is applicable to a regime of turbulent combustion when chemistry is fast so that burning occurs in a symptotically thin layer embedded within the external turbulent flow field. In the flamelet regime a flame front is strongly distorted on large scales in comparison with the flame thickness L_f, while the inner flamelet structure remains similar to the structure of laminar flames (Williams, 1985; Griffiths and Barnard, 1995).

Previous theoretical studies of turbulent flamelets have been restricted to the peculiar case of equal densities of a fuel mixture ρ_f and burning products ρ_b, so that the expansion coefficient $\Theta \equiv \rho_f/\rho_b$ is equal to unity (Kerstein et al., 1988; Sivashinsky, 1988; Ronney and Yakhot, 1992). The assumption of zero thermal expansion implies that flame dynamics is effected by the turbulence, but there is no return influence of the flame on the turbulent flow. Many interesting results have been obtained under this approximation, such as calculation of the flamelet velocity in weak turbulence (Clavin and Williams, 1979) and generalization of this formula for the case of strong turbulence by use of the renormalization group theory (Yakhot, 1988). However, the assumption of zero thermal expansion is not realistic for laboratory and industrial flames, where expansion coefficients are typically of order $\Theta = 5 - 10$. Therefore one should expect considerable influence of a propagating flame on a turbulent flow, so that the results obtained under the assumption of $\Theta = 1$ may provide only qualitative picture of the turbulent burning. Besides, thermal expansion is related to the Darrieus-Landau (DL) instability of laminar flames bending initially planar flame front and increasing flame velocity (Zel'dovich et al., 1985; Bychkov and Liberman, 2000). It is reasonable to expect that the DL instability causes additional velocity increase in the case of turbulent flamelets, too. Numerical simulations of turbulent flames in the limit of small but non-zero thermal expansion $\Theta - 1 \ll 1$ have demonstrated noticeably larger flamelet velocity because of the DL instability (Joulin and Cambray, 1994; Denet, suppressed, one should expect considerable difference in turbulent flamelet velocity between the hypothetical case of $\Theta = 1$ and the case of realistic expansion coefficients $\Theta = 5 - 10$.

Recently, a model equation has been proposed in order to describe turbulent flamelets with realistically large thermal expansion (Bychkov, 2000). Relative intensity of external turbulence has been taken into account in the equation as an independent parameter. Solution to the model equation for the case of an infinitely thin flame front turned out to be in a good agreement with experimental results (Abdel-Gayed et al., 1987). In the present paper we develop the earlier studies and solve the model equation for the case of flamelets of small but finite thickness.

THE MODEL EQUATION

The model equation for weakly curved flames with realistic fuel expansion in an external turbulent flow incorporates the following rigorous theories:

1) The linear theory of the DL instability for a flame of finite thickness (Pelce and Clavin, 1982);
2) The nonlinear equation for curved flames resulting from the DL instability in the case of realistically large thermal expansion (Bychkov, 1998), and
3) The linear theory of a flame front in a weak external turbulent flow (Searby and Clavin, 1986).

Development of small perturbations at an initially planar flame front is described by the equation (Pelce and Clavin, 1982)

$$\frac{\Theta+1}{2\Theta}(1+C_1 L_f\hat{\Phi})\frac{\partial^2 F}{\partial t^2}+(1+C_2 L_f\hat{\Phi})U_f\hat{\Phi}\frac{\partial F}{\partial t} - \frac{\Theta-1}{2}\left(1-\frac{\lambda_c}{2\pi}\hat{\Phi}\right)U_f^2\hat{\Phi}^2 F = 0, \tag{1}$$

where $z = F(x, y, t)$ specifies position of the flame front, U_f stands for the laminar flame velocity, and the operator $\hat{\Phi}$ implies multiplication by absolute value of the wavenumber component along the flame surface in Fourier space

$$\hat{\Phi}F = \frac{1}{4\pi^2}\int |\mathbf{k}|F_k \exp(i\mathbf{k}\cdot\mathbf{x})d\mathbf{k}. \tag{2}$$

The numerical coefficients C_1, C_2, and the cut-off wavelength λ_c depend on the expansion factor Θ and on other fuel parameters according to Searby and Rochwerger (1991). Particularly, in the case of constant thermal conduction and unit Lewis number one has

$$C_1 = 0, \quad C_2 = \frac{\Theta \ln \Theta}{\Theta - 1}, \quad \lambda_c = \frac{2\pi L_f}{\Theta - 1}\left(\Theta \ln \Theta \frac{\Theta + 1}{\Theta - 1} + \Theta - 1\right). \tag{3}$$

The nonlinear equation for the velocity of curved stationary flames resulting from the DL instability takes the form (Bychkov, 1998):

$$\begin{aligned} 1 - U_w/U_f + \frac{\Theta}{2}(\nabla F)^2 + \frac{(\Theta - 1)^3}{16\Theta}[(\nabla F)^2 - (\hat{\Phi}F)^2] \\ - \frac{\Theta - 1}{2}\left(1 - \frac{\lambda_c}{2\pi}\hat{\Phi}\right)\hat{\Phi}F = 0, \end{aligned} \tag{4}$$

where U_w is the velocity of a curved flame front. It has been shown (Bychkov et al., 1999), that the time-dependent version of Eq. (4) also describes well the stability limits of curved stationary flames. Concerning validity of Eq. (4) one should remember that this equation does not involve expansion in powers of $(\Theta - 1) \ll 1$ with the accuracy of $(\Theta - 1)^3$. On the contrary, the nonlinear equation (4) has been derived for arbitrary expansion coefficients Θ (even large ones). The assumptions made in (Bychkov, 1998) in scope of the derivation are that flame thickness is small (which is usual in the theory of premixed flames (Pelce and Clavin, 1982; Williams, 1985) and the nonlinear effects are relatively weak. The latter assumption has been supported by direct numerical simulations of the DL instability at the nonlinear stage (Bychkov et al., 1996; Travnikov et al., 2000). One should point out excellent agreement of the analytical formula for the velocity of curved stationary flames obtained on the basis of Eq. (4) in (Bychkov, 1998) with the results of direct numerical simulations (Bychkov et al., 1996; Travnikov et al., 2000).

Finally, the linear equation for flame response to a weak external turbulent flow has been obtained by Searby and Clavin (1986), and in the case of an infinitely thin flame front it can be written in the following

$$\frac{\Theta+1}{2\Theta}\frac{\partial^2 F}{\partial t^2}+U_f\hat{\Phi}\frac{\partial F}{\partial t}-\frac{\Theta-1}{2}U_f^2\hat{\Phi}^2F-\left(\frac{\partial}{\partial t}+U_f\hat{\Phi}\right)u_{\mathrm{T}}=0, \tag{5}$$

where u_{T} is the turbulent velocity component in the direction of flame propagation.

Assuming weak nonlinear effects, weak turbulence, and a thin flame front Eqs. (1), (4), (5) can be combined into one model equation

$$\begin{aligned}&\frac{\Theta+1}{2\Theta}(1+C_1L_f\hat{\Phi})\frac{\hat{\Phi}^{-1}}{U_f^2}\frac{\partial^2 F}{\partial t^2}+(1+C_2L_f\hat{\Phi})\frac{1}{U_f}\frac{\partial F}{\partial t}\\&+1-U_w/U_f+\frac{\Theta}{2}(\nabla F)^2+\frac{(\Theta-1)^3}{16\Theta}[(\nabla F)^2-(\hat{\Phi}F)^2]\\&-\frac{\Theta-1}{2}\left(1-\frac{\lambda_c}{2\pi}\hat{\Phi}\right)\hat{\Phi}F-\left(1+\frac{\hat{\Phi}^{-1}}{U_f}\frac{\partial}{\partial t}\right)\frac{u_{\mathrm{T}}}{U_f}=0.\end{aligned} \tag{6}$$

Since the model equation (6) is written in the reference frame of the average position of the turbulent flame front, then the value U_w in Eq. (6) plays the role of the average velocity of the turbulent flamelet. Without the last term related to the external turbulent flow the model equation describes development of the DL instability at the linear and nonlinear stages. Together with the last term, the equation determines flame response to the external turbulence. Unlike the popular eikonal equation (Kerstein et al., 1988; Sivashinsky, 1988; Yakhot, 1988) the proposed model equation does include influence of the flame on the external turbulent flow, since all three components of the model equation Eqs. (1), (4), (5) take into account this influence. Equations (1) and (5) reproduce linear effect of the DL flame instability on the underlying flow-field. Equation (4) describes even nonlinear modifications of the gas flow by a corrugated flame including vorticity generation behind a curved flame front.

Similar to Denet (1997) we accept the following model for the turbulent term

$$u_{\mathrm{T}}=\sum U_i\cos(k_iz'\pm\Omega_it+\varphi_{iz})\cos(k_ix+\varphi_{ix})\cos(k_iy+\varphi_{iy}), \tag{7}$$

where φ_{iz}, φ_{ix}, φ_{iy} are random phases and z' is the coordinate normal to coordinate in the reference frame of the flame front is $z = z' - U_w t$. The amplitudes U_i are determined by the Kolmogorov spectrum $U_i \propto k_i^{-5/6}$ and the rms-turbulent velocity in one direction in this model is given by the formula

$$U_{\mathrm{rms}}^2 = \sum U_i^2/8. \tag{8}$$

Direct influence of time-dependent terms in the representation (7) on the turbulent flame velocity has been discussed in several papers (Denet, 1999a; Ashurst, 2000). Still in the majority of theoretical studies of turbulent flames the Taylor hypothesis has been adopted, which neglects explicit time dependence of the turbulent flow. The Taylor hypothesis has been popular both in the classical papers on turbulent flames (Clavin and Williams, 1979; Yakhot, 1988) and in recent papers (Dold et al., 1995; Aldredge, 1996; Helenbrook and Law, 1999; Denet, 1999b; Kagan and Sivashinsky, 2000). In scope of the model (7) the Taylor hypothesis implies that turbulent oscillations are negligible in comparison with time variations due to the flame propagation, so that $\Omega_i \ll U_w k_i$ and Eq. (7) can be presented in the form

$$u_{\mathrm{T}} = \sum U_i \cos(k_i z + U_w k_i t + \varphi_{iz}) \cos(k_i x + \varphi_{ix}) \cos(k_i y + \varphi_{iy}). \tag{9}$$

We will also use the Taylor hypothesis below leaving the question about direct influence of time-dependent turbulent pulsations for later work.

In general, solution to the model equation (6) should reflect wrinkling of the flame front both by external turbulence and by the DL instability. The role of the DL instability in the dynamics of turbulent flamelets is not quite clear yet. Results of many model numerical simulations (Joulin and Cambray, 1994; Denet, 1997; Helenbrook and Law, 1999) indicate that influence of the DL instability may be significant if the integral turbulence length scale exceeds the cut-off wavelength λ_c considerably. However, in many experiments the instability turns out to be of minor importance. For example, in recent experiments (Aldredge et al., 1998) a methane-air flame front propagated in a turbulent flow created by two coaxial cylinders with the gap between the cylinders equal 1 cm. Such a small distance is comparable by order of magnitude to the cut-off wavelength λ_c of the DL instability, which implies that the instability has present study we consider a particular solution to Eq. (6) related to the external turbulence only with no direct influence of the DL instability. Solution to the problem including both effects of external turbulence and the DL instability will be considered elsewhere.

CALCULATION OF THE FLAMELET VELOCITY

Keeping in mind the validity conditions of Eq. (6) we start with the case of weak turbulence $U_w - U_f \ll U_f$. Besides, unlike the calculations (Bychkov, 2000) we take into account finite flame thickness L_f and the dependence of the turbulent flame velocity on the ratio of the integral turbulent length scale and the flame thickness L_T/L_f. In the case of weak turbulence and weak nonlinearity the solution to Eq. (6) takes the form

$$F = \sum \cos(k_i x + \varphi_{ix}) \cos(k_i y + \varphi_{iy})[F_{ci} \cos(\omega_i t + \varphi_{iz}) + F_{si} \sin(\omega_i t + \varphi_{iz})], \tag{10}$$

where $\omega_i = U_w k_i \pm \Omega_i$. Substituting (10) into (6) we find with the accuracy of the linear terms the set of equations for F_{ci} and F_{si}

$$\begin{aligned} - \left[\frac{\Theta + 1}{2\Theta}\omega_i^2(1 + \sqrt{2}C_1 k_i L_f) + \frac{\Theta - 1}{2} 2U_f^2 k_i^2 \left(1 - \sqrt{2}\frac{k_i \lambda_c}{2\pi}\right)\right] F_{ci} \\ + \sqrt{2}\omega_i U_f k_i (1 + \sqrt{2}C_2 k_i L_f) F_{si} = \sqrt{2} U_f k_i U_i, \end{aligned} \tag{11}$$

$$\begin{aligned} \left[\frac{\Theta + 1}{2\Theta}\omega_i^2(1 + \sqrt{2}C_1 k_i L_f) + \frac{\Theta - 1}{2} 2U_f^2 k_i^2 \left(1 - \sqrt{2}\frac{k_i \lambda_c}{2\pi}\right)\right] F_{si} \\ + \sqrt{2}\omega_i U_f k_i (1 + \sqrt{2}C_2 k_i L_f) F_{ci} = \omega_i U_i. \end{aligned} \tag{12}$$

Introduction designations

$$\alpha_i = \frac{\Theta + 1}{2\Theta}\omega_i^2(1 + \sqrt{2}C_1 k_i L_f) + (\Theta - 1) U_f^2 k_i^2 \left(1 - \sqrt{2}\frac{k_i \lambda_c}{2\pi}\right) \tag{13}$$

and

$$\beta_i = \sqrt{2}\omega_i U_f k_i (1 + \sqrt{2}C_2 k_i L_f) \tag{14}$$

we find expressions for the amplitudes F_{ci}, F_{si}

$$F_{si} = \frac{\alpha_i \omega_i + \sqrt{2}\beta_i U_f k_i}{\alpha_i^2 + \beta_i^2} U_i, \tag{15}$$

$$F_{ci} = \frac{\beta_i \omega_i - \sqrt{2}\alpha_i U_f k_i}{\alpha_i^2 + \beta_i^2} U_i. \tag{16}$$

The average turbulent flame velocity is related to the nonlinear terms of Eq. (6)

$$U_w/U_f - 1 = \left\langle \frac{\Theta}{2}(\nabla F)^2 + \frac{(\Theta - 1)^3}{16\Theta}[(\nabla F)^2 - (\hat{\Phi} F)^2] \right\rangle, \tag{17}$$

where $\langle \ldots \rangle$ denotes time and space averaging.

The last two terms in the right-hand side of Eq. (17) give zero after averaging, and after substituting representation Eq. (10) with the amplitudes (15), (16) into Eq. (17) we find

$$U_w/U_f - 1 = \left\langle \frac{\Theta}{2}(\nabla F)^2 \right\rangle = \Theta \sum \frac{k_i^2}{8}(F_{ci}^2 + F_{si}^2), \tag{18}$$

or

$$U_w/U_f - 1 = \Theta \sum \frac{U_i^2 k_i^2}{8} \frac{\omega_i^2 + 2U_f^2 k_i^2}{\alpha_i^2 + \beta_i^2}. \tag{19}$$

Since the original dispersion relation (1) has been derived in the limit of a thin flame front $kL_f \ll 1$, we can use the same approximation keeping only terms of the first order in kL_f. Then Eq. (19) may be reduced to

$$U_w/U_f - 1 = \Theta \sum \frac{U_i^2 k_i^2}{8} \frac{\omega_i^2 + 2U_f^2 k_i^2}{A_i^2 + B_i^2}(1 + C_3 k_i L_f), \tag{20}$$

$$C_3 = \frac{2\sqrt{2}}{A_i^2 + B_i^2}\left(A_i U_f^2 k_i^2 (\Theta - 1)\frac{\lambda_c}{2\pi L_f} - A_i \omega_i^2 C_1 \frac{\Theta + 1}{2\Theta} - C_2 B_i^2\right), \tag{21}$$

$$A_i = \frac{\Theta + 1}{2\Theta}\omega_i^2 + \frac{\Theta - 1}{2} 2U_f^2 k_i^2, \quad B_i = \sqrt{2}\omega_i U_f k_i. \tag{22}$$

It has been shown in (Bychkov, 2000) that for $\Theta = 1$ the velocity increase becomes

$$U_w/U_f - 1 = \sum \frac{U_i^2 k_i^2}{8\omega_i^2}. \tag{23}$$

This expression coincides with the well-known Clavin-Williams formula (Clavin and Williams, 1979) written for the turbulence model (7). With help of the Taylor hypothesis for week turbulence $U_w - U_f \ll U_f$ Eq. (23) takes the form

$$U_w/U_f - 1 = \frac{1}{U_f^2}\sum \frac{U_i^2}{8} = \frac{U_{\rm rms}^2}{U_f^2}. \tag{24}$$

Using the Taylor hypothesis in the case of arbitrary expansion coefficient Θ and weak turbulence one finds for a flame with $Le = 1$ and constant transport coefficients

$$A_i = \left(\frac{\Theta + 1}{2\Theta} + \Theta - 1\right) U_f^2 k_i^2, \quad B_i = \sqrt{2} U_f^2 k_i^2, \tag{25}$$

$$C_3 = \frac{4\sqrt{2}\Theta^2(\Theta - 1)}{(2\Theta^2 - \Theta + 1)^2 + 8\Theta^2}\left(\frac{2\Theta^2 + 3\Theta - 1}{\Theta - 1}\ln\Theta + \frac{2\Theta^2 - \Theta + 1}{\Theta}\right). \tag{26}$$

Amplification of the flame velocity in the case of weak turbulence can be found by calculating the sum in the expression (20). In the case of negligible flame thickness we find the velocity amplification

$$U_w/U_f - 1 = C_\Theta^2 \frac{U_{\rm rms}^2}{\ldots}, \tag{27}$$

with the coefficient C_Θ depending on fuel expansion as

$$C_\Theta^2 = \frac{12\Theta^3}{(2\Theta^2 - \Theta + 1)^2 + 8\Theta^2}. \tag{28}$$

The coefficient C_Θ varies slightly with variations of thermal expansion being somewhat lower than unity in the case of realistic expansion factors $\Theta = 5 - 10$. For example, for $\Theta = 5 - 6$ characterizing propane flames the coefficient is $C_\Theta = 0.7 - 0.8$. If we are interested in influence of the finite flame thickness, then it is more convenient to go over to integral instead of the sum in (20) with $U_i^2 \propto k_i^{-5/3}$ for the Kolmogorov turbulence. Then Eq. (20) takes the form

$$U_w/U_f - 1 = C_\Theta^2 \frac{\Lambda}{U_f^2} \int\limits_{1/L_\mathrm{T}}^{1/L_k} k^{-5/3}(1 + C_3 k L_f) dk, \tag{29}$$

where the integral turbulent length scale L_T is much larger than the Kolmogorov length scale of turbulent dissipations $L_\mathrm{T} \gg L_K$ and the scaling factor Λ calculated according to Eq. (8)

$$U_{\mathrm{rms}}^2 = \Lambda \int\limits_{1/L_\mathrm{T}}^{1/L_k} k^{-5/3} dk \approx \frac{3}{2} \Lambda L_\mathrm{T}^{2/3}, \tag{30}$$

can be presented as

$$\Lambda = \frac{2}{3} \frac{U_{\mathrm{rms}}^2}{L_\mathrm{T}^{2/3}}, \tag{31}$$

Then the velocity amplification obtained from Eq. (29) is

$$U_w/U_f - 1 = C_\Theta^2 \frac{U_{\mathrm{rms}}^2}{U_\mathrm{f}^2} \left(1 + 2C_3 \frac{L_f}{L_\mathrm{T}^{2/3} L_K^{1/3}}\right). \tag{32}$$

In the case of Kolmogorov turbulence the ratio of the integral and Kolmogorov length scales is related to the turbulent Reynolds number the kinematic viscosity, and the formula for the velocity amplification becomes

$$U_w/U_f - 1 = C_\Theta^2 \frac{U_{\mathrm{rms}}^2}{U_f^2}\left(1 + 2C_3 \,\mathrm{Re}^{1/4} \frac{L_f}{L_\mathrm{T}}\right). \tag{33}$$

In order to go over from the case of weak turbulence to the case of strongly turbulent flamelets we will use the method proposed by Yakhot (1988). According to Yakhot's idea the turbulent velocity field can be split into components corresponding to narrow, almost monochromatic bands in the wave-number space with small amplitude of each band. Then turbulent flame velocity comes as integral action of all bands. The first band provides the increase of flame velocity given by Eq. (19)

$$U_{w_1} - U_f = U_f d_1 \Sigma, \tag{34}$$

where $d_1\Sigma$ denotes the sum

$$d_1\Sigma = \Theta \sum_1 \frac{U_i^2 k_i^2}{8} \frac{\omega_i^2 + 2U_f^2 k_i^2}{\alpha_i^2 + \beta_i^2} \tag{35}$$

corresponding to the first band. The second band corresponds to similar velocity amplification, but with U_{w_1} playing the role of flame velocity

$$U_{w2} - U_{w_1} = U_{w_1} d_2 \Sigma. \tag{36}$$

Because of narrow width of every band one can write a continuous equation

$$dU_w = U_w d\Sigma, \tag{37}$$

which can be easily solved as

$$U_w = U_f \exp \Sigma, \tag{38}$$

or

$$U_w = U_f \exp\left(\Theta \sum \frac{U_i^2 k_i^2}{8} \frac{\omega_i^2 + 2U_f^2 k_i^2}{\alpha_i^2 + \beta_i^2}\right) \tag{39}$$

with the sum taken over the whole turbulent spectrum. Though the Yakhot method has not been proven rigorously yet, it seems to be quite reasonable and may provide understanding of the propagation velocity of turbulent flamelets.

In the limit of small flame thickness Eq. (39) becomes

$$U_w = U_f \exp\left(\Theta \sum \frac{U_i^2 k_i^2}{8} \frac{\omega_i^2 + 2U_f^2 k_i^2}{A_i^2 + B_i^2}(1 + C_3 k_i L_f)\right). \tag{40}$$

Using the Taylor hypothesis in the case of strong turbulence, $\omega_i \approx U_w k_i$, one finds

$$A_i = U_w^2 k_i^2 \left(\frac{\Theta + 1}{2\Theta} + (\Theta - 1)\frac{U_f^2}{U_w^2}\right), \quad B_i = \sqrt{2} U_f U_w k_i^2, \tag{41}$$

$$\omega_i^2 + 2U_f^2 k_i^2 = U_w^2 k_i^2 \left(1 + 2\frac{U_f^2}{U_w^2}\right), \tag{42}$$

$$C_3 = \frac{4\sqrt{2}\Theta}{[\Theta + 1 + 2\Theta(\Theta - 1)U_f^2/U_w^2]^2 + 8\Theta^2 U_f^2/U_w^2} \times \left[\left(\Theta + 1 + 2\Theta(\Theta - 1)\frac{U_f^2}{U_w^2}\right)\left((\Theta - 1)\frac{\lambda_c}{2\pi L_f}\frac{U_f^2}{U_w^2} - C_1\frac{\Theta + 1}{2\Theta}\right) - 4\Theta C_2 \frac{U_f^2}{U_w^2}\right]. \tag{43}$$

In that case the factor C_3 is not a constant any more, but a function of the scaled turbulent flame velocity U_w/U_f.

In the case of negligible flame thickness we can reduce Eq. (40) to

$$\ln(U_w/U_f) = f_\Theta^2 \frac{U_{\mathrm{rms}}^2}{U_w^2}, \tag{44}$$

where the factor f_Θ depends on the expansion coefficient Θ and on the ratio of the turbulent flame velocity to the laminar velocity U_w/U_f as

$$f_\Theta^2 = \frac{4\Theta^3(1 + 2U_f^2/U_w^2)}{[\Theta + 1 + 2\Theta(\Theta - 1)U_f^2/U_w^2]^2 + 8\Theta^2 U_f^2/U_w^2}. \tag{45}$$

In the particular case of $\Theta = 1$ one has $f_\Theta = 1$ and Eq. (44) goes over to the Yakhot result

$$\ln(U_w/U_f) = \frac{U_{\rm rms}^2}{U_w^2} \tag{46}$$

In the case of weak turbulence $U_w - U_f \ll U_f$ the factor f_Θ goes over to the coefficient C_Θ according to Eq. (28). Finally, taking into account finite flame thickness we obtain expression for the velocity amplification of a turbulent flame

$$\ln(U_w/U_f) = f_\Theta^2 \frac{U_{\rm rms}^2}{U_w^2}\left(1 + 2C_3 {\rm Re}^{1/4}\frac{L_f}{L_{\rm T}}\right) \tag{47}$$

with C_3 determined by Eq. (43).

Taking into account that logarithm is a slow function and the flame thickness is small compared to the turbulent length scale we can rewrite formula (47) as

$$U_w/U_{\rm rms} = f_\Theta \ln^{-1/2}(U_w/U_f)\left(1 + C_3 {\rm Re}^{1/4}\frac{L_f}{L_{\rm T}}\right). \tag{48}$$

Formulas (47) and (48) give in implicit form the average propagation velocity U_w of a turbulent flamelet with finite thickness L_f and laminar flame velocity U_f in a turbulent flow characterized by the turbulent length scale $L_{\rm T}$ and by the turbulent intensity $U_{\rm rms}$, which can be also combined into the Reynolds number ${\rm Re} = U_{\rm rms}L_{\rm T}/\nu$. The factors f_Θ and C_3 in Eq. (48) are determined by Eq. (45) and (43), respectively.

To illustrate dependence of the turbulent flame velocity on the turbulent intensity we consider a flame with $Le = 1$ and constant transport coefficients. In this case the coefficient C_3 becomes

$$C_3 = \frac{4\sqrt{2}\Theta^2(\Theta-1)}{[\Theta+1+2\Theta(\Theta-1)U_f^2/U_w^2]^2 + 8\Theta^2 U_f^2/U_w^2} \times \left[\ln\Theta\left(1+2\Theta\frac{\Theta+1}{\Theta-1}\frac{U_f^2}{U_w^2}\right) + \frac{\Theta+1}{\Theta} + 2(\Theta-1)\frac{U_f^2}{U_w^2}\right]\frac{U_f^2}{U_w^2}. \tag{49}$$

Then in the case of strong turbulence $U_f/U_w \ll 1$ the formula for the velocity of a turbulent flamelet in a fuel with constant transport coefficients may be reduced to

$$U_w/U_{rms} = \frac{2\Theta^{3/2}}{\Theta+1}\ln^{-1/2}(U_w/U_f)\left(1 + C_3 \mathrm{Re}^{1/4}\frac{L_f}{L_T}\right) \tag{50}$$

with

$$C_3 = 4\sqrt{2}\Theta\frac{\Theta-1}{\Theta+1}\left(1+\frac{\Theta\ln\Theta}{\Theta+1}\right)\frac{U_f^2}{U_w^2}. \tag{51}$$

According to Eqs. (50), (51) the effect of finite flame thickness makes the flamelet velocity larger for small and moderate values of turbulent intensity, when $U_f/U_w \propto 1$, while in the case of strong turbulence $U_f/U_w \ll 1$ the effect of finite flame thickness becomes negligible. One can observe this tendency in Figure 1, where the average velocity of a turbulent flamelet of the expansion coefficient $\Theta = 8$ with constant transport coefficients and $Le = 1$ is plotted for the cases of both zero (dashed line) and non-zero flame thickness (solid line). Similar to the experimental configuration of Aldredge et al. (1998) we have taken $L_T/L_f = 30$. In the experiments (Aldredge et al., 1998) the Reynolds number of the flow varied within the limits $70 < \mathrm{Re} < 375$. However, direct influence of the variations of the Reynolds number in Eq. (50) is not so strong, since it comes with a small power exponent $\mathrm{Re}^{1/4}$. As a consequence, variations of the Reynolds number become important only as variations of the turbulence intensity U_{rms}/U_f for the fixed turbulent length scale. Both theoretical curves of Figure 1 agree well with the experimental data obtained in Aldredge et al. (1998), which are shown in the figure by markers. Still, the curve for the turbulent flamelet velocity with account of finite flame thickness comes closer to the experimental results. The physical reason of the velocity increase because of the finite

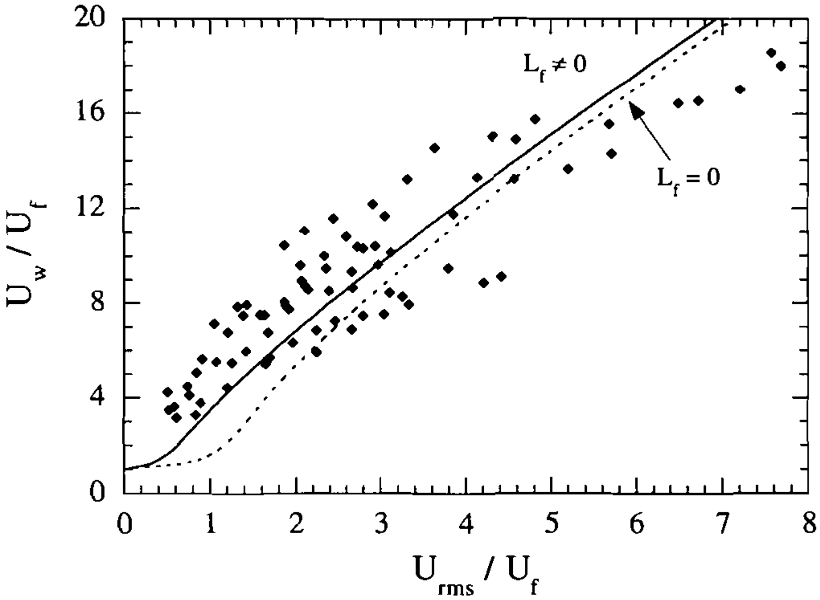

Figure 1. Scaled velocity U_w/U_f of a turbulent flame of finite thickness $L_f \neq 0$ (solid line) and zero thickness $L_f = 0$ (dashed line) for $\Theta = 8$ versus the scaled velocity of external turbulent flow field U_{rms}/U_f. The markers show the experimental results by Aldredge et al.

flame thickness L_f is the "smooth" resonance at the turbulence mode with a wave-number equal to the cut-off wave-number of the DL instability $k = k_c \equiv 2\pi/\lambda_c$. Possibility of the resonance has been pointed out already by Searby and Clavin (1986). Still, influence of the resonance becomes smaller at high turbulent intensity, since in this limit the third term of Eq. (1) responsible for the DL instability is negligible in comparison with the first term reflecting high-frequency oscillations in the turbulent flow. Figure 2 shows velocity of turbulent flamelets of finite thickness with different expansion factors $\Theta = 5 - 10$. As one can see, the flamelet velocity increases noticeably with the increase of the thermal expansion and differs considerably from the particular case of zero thermal expansion, $\Theta = 1$, studied by Yakhot (1988).

To conclude, analysis of the present paper shows that several parameters play a significant role in the turbulent flame speed: the scaled rms-turbulent velocity U_{rms}/U_f, the expansion coefficient of the fuel mixture Θ, the scaled integral turbulent length L_T/L_f (which may be expressed by use of the Reynolds or Karlovitz number), and others. In

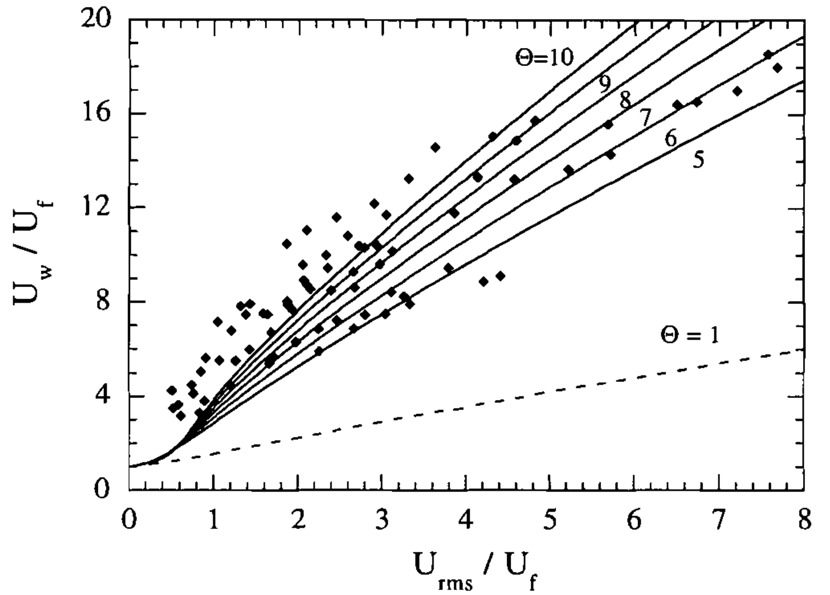

Figure 2. Scaled velocity U_w/U_f of a turbulent flame of finite thickness versus the scaled velocity of external turbulence U_{rms}/U_f for different expansion coefficients $\Theta = 5$–10 according to Eq. (48) (solid lines). The dashed line shows the Yakhot result for $\Theta = 1$. The markers show the experimental results by Aldredge et al. (1998).

the present paper we have discussed the situation of a unit Lewis number and constant transport coefficients of the flame, but, in general, parameters characterizing internal flame structure (the Lewis number, the Markstein number, the Prandtl number, etc.) may also influence the turbulent flamelet velocity. Thus the resulting expression for the flamelet velocity is much more complicated and depends much stronger on conditions of a particular experiment than it is supposed by the "universal" formulas obtained on the basis of the eikonal equation with $\Theta = 1$.

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