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Vendor-buyer ordering policy when demand is trapezoidal

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A B S T R A C T

A joint vendor-buyer strategy is analyzed which is beneficial to both the players in the supply chain. The demand is assumed to be trapezoidal. It is established numerically that the joint venture decreases the total cost of the supply chain when compared with the independent decision of the buyer. To entice the buyer to order more units, a permissible credit period is offered by the vendor to the buyer. A negotiation factor is incorporated to share the cost savings.

1. Introduction

Silver and Meal (1969), Silver(1979), Xu and Wang (1991), Chung and Ting (1993, 1994), Bose et al. (1995), Hariga (1995), Giri and Chaudhari (1997), Lin et al. (2000), etc. discussed optimal ordering policy when demand is linearly changing with respect to time which is superficial in the market of fashion good, Air seats, Smart phones etc. Mehta and Shah (2003, 2004) assumed the demand to be exponentially time varying, which is again unrealistic for newly launched product. Shah et al. (2008) introduced the quadratic demand, which is again not observed in the market for indefinite period. In order to have an alternative demand pattern, the trapezoidal demand is considered. This type of demand increases for some time, then gets constant up to some time and afterwards decreases exponentially with time.

Most of the models available in the literature assumed that the buyer is dominant player to make the decision for procurement. This strategy may not be economical for the vendor. An integrated vendor-buyer policy should be analyzed which is beneficial to the players of the supply chain. Clark and Scarf(1960) proposed a mathematical model for vendor-buyer integration. Banerjee (1986) discussed an economic lot-size model when production is finite. Goyal (1988) extended Banerjee's model by

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relaxing the assumption of the lot-for-lot production. Shah et al. (2008) analyzed joint decision when demand is quadratic.

In this study, a joint vendor–buyer inventory system is analyzed when demand is trapezoidal. A negotiation factor is incorporated to share the savings. The credit period is offered to the buyer to attract the buyer for placing larger order.

2. Notations and Assumptions

The proposed study uses following notations and assumptions.

2.1 Notations

K_v Buyer's ordering cost per order

A_v Vendor's ordering cost per order

C_b Buyer's purchase cost per unit

C_v Vendor's purchase cost per unit

I_b Inventory carrying charge fraction per unit per annum for the buyer

I_v Inventory carrying charge fraction per unit per annum for the vendor

$I_b(t)$ Buyer's inventory level at any instant of time t

$I_v(t)$ Vendor's inventory level at any instant of time t

n Number of orders during cycle time for the buyer (a decision variable)

K_b Buyer's total cost per unit time

K_v Vendor's total cost per unit time

K_{NJ} Total cost for vendor-buyer inventory System when they take independent decision

K_J Total cost for vendor-buyer inventory System when they take joint decision

T Vendor's cycle time (a decision variable)

T_b $(=T/n)$, Buyer's cycle time (a decision variable)

M Credit period offered by the vendor to the buyer (a decision variable)

r Continuous discounting rate

2.2 Assumptions

- A supply chain of single vendor and single buyer is considered.
- An inventory system deals with single item.
- The demand rate is trapezoidal. Its functional form is

$$R(t) = \begin{cases} f(t), 0 \le t \le u_1 \\ D_0, u_1 \le t \le u_2 \\ g(t), u_2 \le t \le T \end{cases}$$

where $f(t)$ is linear in t,

$D_0 = f(u_1) = g(u_2)$, and $g(t)$ is exponentially decreasing in t

$$R(t)=\begin{cases} a(1+b_1 t), 0 \le t \le u_1 \\ a(1+b_1 u_1), u_1 \le t \le u_2 \\ a(1+b_1 u_1)e^{-b_2(-u_2+t)}, u_2 \le t \le T \end{cases}$$

where a denotes scale demand, $0 < b_1, b_2 < 1$ denotes rates of change of demand. (See Fig. 1)

- The lead time is zero and shortages are not allowed.
- The credit period is offered for settling the accounts due against purchases to attract the buyer to opt a joint decision policy.

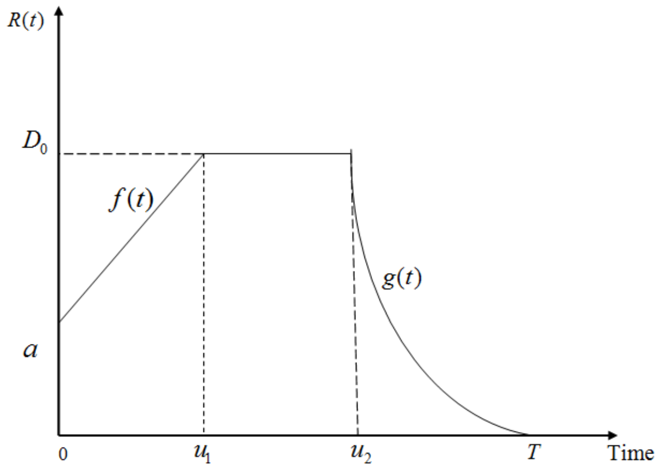

Fig. 1. Demand function

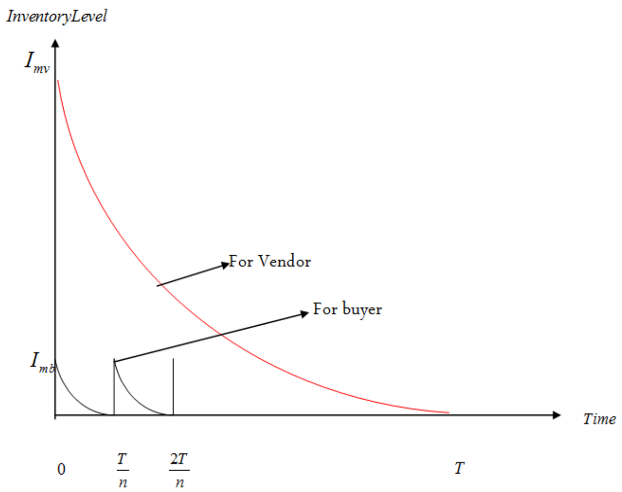

Fig. 2 Vendor –Buyer Inventory Status

3. Mathematical Model

Fig. 2 depicts the time-varying inventory status for the vendor and the buyer. The inventory changes due to trapezoidal demand for both vendor and buyer. The rate of change of inventory for both the players is governed by the differential equations:

$$\frac{dI_b(t)}{dt} = -R(t), 0 \le t \le T_b \tag{1}$$

$$\frac{dI_v(t)}{dt} = -R(t), 0 \le t \le T \tag{2}$$

with the boundary conditions $I_b(T_b)=0$, $I_v(T)=0$ and initial conditions $I_b(0)=I_{mb}$, $I_v(0)=I_{mv}$. The solutions of the differential equations are

$$I_b(t)=\begin{cases} a\left[u_1+\frac{b_1u_1^2}{2}-t-\frac{b_1t^2}{2}\right]+a(1+b_1u_1)(u_2-u_1)+\frac{a(1+b_1u_1)e^{b_2u_2}}{b_2}\left[e^{-b_2u_2}-e^{-\frac{b_2T}{n}}\right], 0\le t\le u_1 \\ a(1+b_1u_1)(u_2-t)+\frac{a(1+b_1u_1)e^{b_2u_2}}{b_2}\left[e^{-b_2u_2}-e^{-\frac{b_2T}{n}}\right], u_1\le t\le u_2 \\ \frac{a(1+b_1u_1)e^{b_2u_2}}{b_2}\left[e^{-b_2t}-e^{-\frac{b_2T}{n}}\right], u_2\le t\le T_b \end{cases} \quad (3)$$

and

$$I_v(t)=\begin{cases} a\left[u_1+\frac{b_1u_1^2}{2}-t-\frac{b_1t^2}{2}\right]+a(1+b_1u_1)(u_2-u_1)+\frac{a(1+b_1u_1)e^{b_2u_2}}{b_2}\left[e^{-b_2u_2}-e^{-b_2T}\right], 0\le t\le u_1 \\ a(1+b_1u_1)(u_2-t)+\frac{a(1+b_1u_1)e^{b_2u_2}}{b_2}\left[e^{-b_2u_2}-e^{-b_2T}\right], u_1\le t\le u_2 \\ \frac{a(1+b_1u_1)e^{b_2u_2}}{b_2}\left[e^{-b_2t}-e^{-b_2T}\right], u_2\le t\le T \end{cases} \quad (4)$$

Using $I_b(0)=I_{mb}$, $I_v(0)=I_{mv}$, the maximum procurement quantities for the buyer and the vendor are

$$I_{mb}=a\left[u_1+\frac{b_1u_1^2}{2}\right]+a(1+b_1u_1)(u_2-u_1)+\frac{a(1+b_1u_1)e^{b_2u_2}}{b_2}\left[e^{-b_2u_2}-e^{-\frac{b_2T}{n}}\right]$$

$$I_{mv}=a\left[u_1+\frac{b_1u_1^2}{2}\right]+a(1+b_1u_1)(u_2-u_1)+\frac{a(1+b_1u_1)e^{b_2u_2}}{b_2}\left[e^{-b_2u_2}-e^{-b_2T}\right]$$

respectively. During the cycle time $[0,T]$, the buyer's

- Purchase Cost ; $PC_b=nC_bI_{mb}$
- Holding Cost ; $HC_b=nC_bI_b\int_0^{T_b}I_b(t)dt$
- Ordering Cost ; $OC_b=nA_b$

Hence, the buyer's total cost; K_b per unit time is

$$K_b=\frac{1}{T}\left[PC_b+HC_b+OC_b\right]. \quad (5)$$

The vendor's inventory is the difference between the vendor-buyer combined inventory and the buyer's inventory during n-orders. This is known as the joint two-echelon inventory model. The vendor's

- Purchase Cost ; $PC_v=nC_vI_{mv}$
- Holding Cost ; $HC_v=C_vI_v\left[\int_0^{T}I_v(t)dt-n\int_0^{T_b}I_b(t)dt\right]$
- Ordering Cost ; $OC_v=A_v$

Hence, the vendor's total cost; K_v per unit time is

$$K_v = \frac{1}{T}\left[PC_v + HC_v + OC_v\right] \tag{6}$$

The joint total cost K is the sum of K_b and K_v where $T_b = \frac{T}{n}$. Thus K is the function of discrete variable n and continuous variable.

4. Computational Procedure

There are two cases to analyze *T*.

Case 1: When the vendor and the buyer take decision independently.

For given value of n, differentiate K_b with respect to T_b (equivalently, T) and solve $\frac{dK_b}{dT_b} = 0$. This n and T_b minimizes K_v provided

$$K_v(n-1) \geq K_v(n) \leq K_v(n+1) \tag{7}$$

satisfies. Here, the total cost per unit time with independent decision; K_{NJ} is given by

$$K_{NJ} = \min_n[\min_n K_b + K_v] \tag{8}$$

Case 2: When vendor and buyer make decision jointly.

The optimum value of T and n must satisfy the following conditions simultaneously:

$$\frac{\partial K}{\partial T} = 0 \text{ and } K(n-1) \geq K(n) \leq K(n+1) \tag{9}$$

Thus, the total joint cost is

$$K_J = \min_{n,T}[K_b + K_v] \tag{10}$$

It is obvious that $K_J \leq K_{NJ}$. Hence, total cost savings Sav_J is defined as $Sav_J = K_{NJ} - K_J$. Now define buyer's cost saving, Sav_b as $Sav_b = \alpha \ Sav_J$, where $0 \leq \alpha \leq 1$ is the negotiation factor. When negotiation factor equals to one, all saving goes to buyer; when it is equal to zero, all saving is in the vendor's pocket. When negotiation factor is 0.5, the total cost savings is equally distributed between the vendor and the buyer. The present value of unit after a time interval M is e^{-rM}, where r is discounting rate. Solving the following equation

$$R(t)C_b(1-e^{-rM}) = Sav_b \tag{11}$$

the buyer's credit period is given by

$$M = \frac{1}{r}\ln\left[\frac{C_b R(t)}{C_b R(t) - Sav_b}\right] \tag{12}$$

5. Numerical Example and Sensitivity Analysis

Consider following inventory parameters values in proper units:

$$[a\ b_1\ b_2\ A_b\ A_v\ C_b\ C_v\ I_b\ I_v\ r\ ] = [40000\ \ 0.04\ \ 0.02\ \ 600\ \ 3000\ \ \ 10\ \ 6\ \ 0.\ 11\ 0.10\ \ \ 0.06]$$

Let $\alpha = 0.5$.The optimal solution is listed in Table 1 for independent and joint decisions.

Table 1
Optimal solution for independent and joint decisions

Case	*n*	T_b	*T*	K_b	K_v	*K*	*PJCR*	*M(days)*
1(Independent Decision)	3	0.179004	0.537012	407422	249129	656551	-	-
2(Joint Decision)	2	0.288339	0.576678	408163	247815	655978	0.08735049	7.5864

The buyer's cost and cycle time increases injoint decisions. The vendor gains $1314 and the buyer loses $741. This hinders the buyer to agree for joint decision. To entice the buyer to joint decision, the vendor offers the buyer a credit period of days with equal sharing of cost savings. This reduces the joint total cost PJCR by 0.08735049 %, where PJCR is defined as $\frac{K_{NJ} - K_J}{K_J} \times 100$. The convexity of total integrated cost and independent costs are shown in Fig. 3.

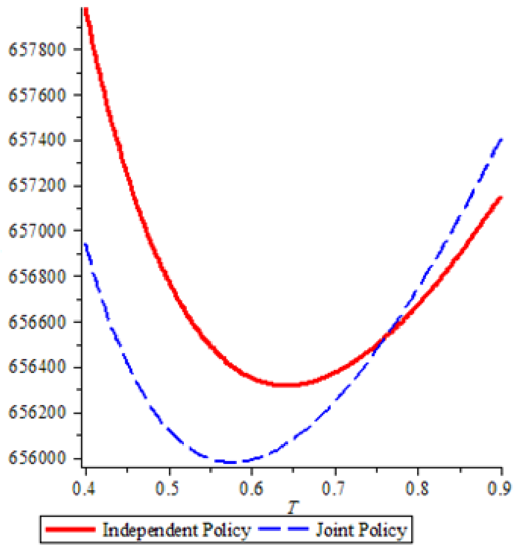

Fig. 3. Total cost for independent vs joint decision

Table 2
Sensitivity Analysis of Demand Rate

a	24000	32000	40000	48000	56000
K_{NJ}	396567	526669	656551	786277	915884
K_J	396123	526156	655978	785649	915205
PJCR	0.112086397	0.097499601	0.08735049	0.079933915	0.074191028
$M\,(days)$	7.560363512	7.57972378	7.586401505	7.603471479	7.624091074

Observations

- Increase in fixed demand a, decreases percentage of cost reduction and increases delay period. (See Fig. 4 and 5)

Table 3
Sensitivity Analysis of Linear Rate of Change of Demand

b_1	0.024	0.032	0.04	0.048	0.056
K_{NJ}	656278	656414	656551	656688	656825
K_J	655705	655842	655978	656115	656251
$PJCR$	0.087386858	0.087216128	0.08735049	0.087332251	0.087466533
$M\,(days)$	7.582111052	7.570846658	7.586401505	7.588383352	7.603621229

Observations

- Increase in linear rate of change of demand b_1, may increase or decrease percentage of cost reduction and delay period.(See Fig. 4 and 5)

Table 4
Sensitivity Analysis of Exponential Rate of Change of Demand

b_2	0.012	0.016	0.02	0.024	0.028
K_{NJ}	657145	656854	656551	656236	655909
K_J	656660	656327	655978	655611	655224
$PJCR$	0.073858618	0.080295341	0.08735049	0.09533092	0.1045444
$M\,(days)$	6.848732823	7.214091565	7.586401505	7.981409057	8.410917618

Observations

- Increase in exponential rate of change of demand b_2, increases percentage of cost reduction and delay period.(See Fig. 4 and 5)

Table 5
Sensitivity Analysis of Buyer's Ordering Cost

A_b	360	480	600	720	840
K_{NJ}	656058	656208	656551	656979	657444
K_J	655120	655555	655978	656388	656787
$PJCR$	0.143179875	0.099610254	0.08735049	0.090038209	0.100032431
$M\,(days)$	13.21695433	8.909292382	7.586401505	7.606669428	8.233058524

Observations

- Increase in Buyer's Ordering Cost A_b, may increase or decreases percentage of cost reduction and delay period.(See Fig. 4 and Fig. 5)

Table 6
Sensitivity Analysis of Vendor's Ordering Cost

A_v	1800	2400	3000	3600	4200
K_{NJ}	654317	655434	656551	657668	658786
K_J	653718	654897	655978	656983	657926
$PJCR$	0.091629724	0.081997627	0.08735049	0.104264494	0.130713788
$M\,(days)$	9.417741873	7.690606444	7.586401505	8.474548917	10.02331229

Observations

- Increase in Vendor's Ordering Cost A_v, may increase or decrease percentage of cost reduction and delay period.(See Fig. 4 and 5)

Table 7
Sensitivity Analysis of Buyer's Purchase Cost

C_b	6	8	10	12	14
K_{NJ}	494342	575451	656551	737623	818661
K_J	493364	574720	655978	737158	818275
$PJCR$	0.198230921	0.12719237	0.08735049	0.0630801	0.047172405
$M\,(days)$	18.172696	11.1807315	7.586401505	5.493492176	4.152027608

Observations

- Increase in buyer's purchase cost C_b, decreases percentage of cost reduction and delay period significantly.(See Fig. 4 and 5)

Table 8
Sensitivity Analysis of Vendor's Purchase Cost

C_v	3.6	4.8	6	7.2	8.4
K_{NJ}	559134	607843	656551	705260	753969
K_J	558908	607449	655978	704495	753001
$PJCR$	0.040435993	0.064861412	0.08735049	0.108588421	0.128552286
$M\,(days)$	2.815971583	5.064913311	7.586401505	10.41579493	13.53480563

Observations

- Increase in vendor's purchase cost C_v, increases percentage of cost reduction and delay period significantly.(See Fig. 4 and 5)

Table 9
Sensitivity Analysis of Inventory Carrying Charge Fraction of Buyer

I_b	0.066	0.088	0.11	0.132	0.154
K_{NJ}	654297	655423	656551	657634	658665
K_J	653169	654649	655978	657193	658321
$PJCR$	0.1726965	0.118231296	0.08735049	0.067103575	0.052254143
$M\,(days)$	12.01339335	9.300320019	7.586401505	6.331728601	5.295638308

Observations

- Increase in Buyer's inventory carrying charge fraction I_b, decreases percentage of cost reduction and delay period significantly.(See Fig. 4 and 5)

Table 10
Sensitivity Analysis of Inventory Carrying Charge Fraction of Vendor

I_v	0.06	0.08	0.1	0.12	0.14
K_{NJ}	654845	655698	656551	657405	658258
K_J	654535	655275	655978	656648	657291
PJCR	0.047361868	0.06455305	0.08735049	0.115282465	0.147119008
$M\,(days)$	3.694344159	5.327546336	7.586401505	10.4889503	13.9710132

Observations

- Increase in Vendor's inventory carrying charge fraction I_v, increases percentage of cost reduction and delay period significantly.(See Fig. 4 and 5)

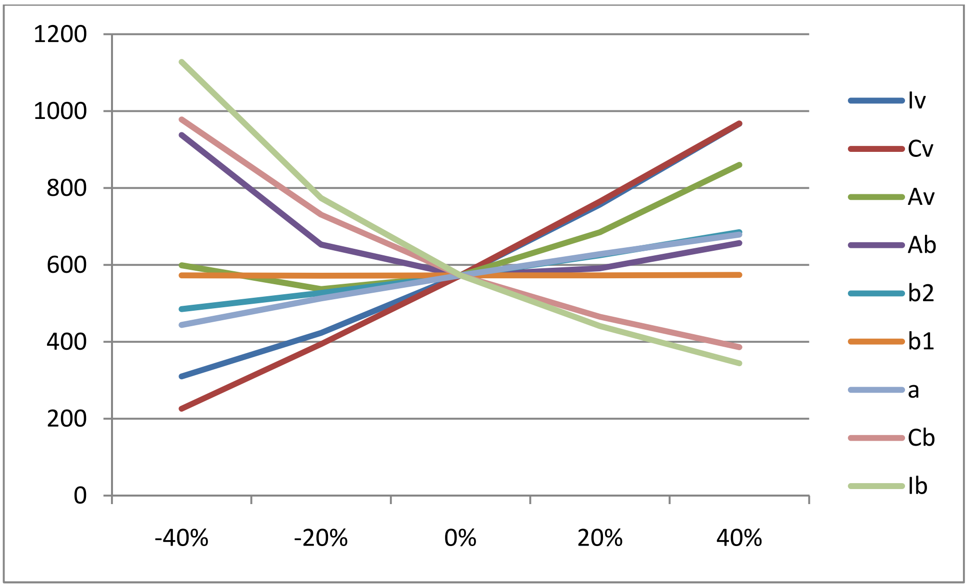

Fig. 4. Total savings Vs. percentage of changes in affecting parameters

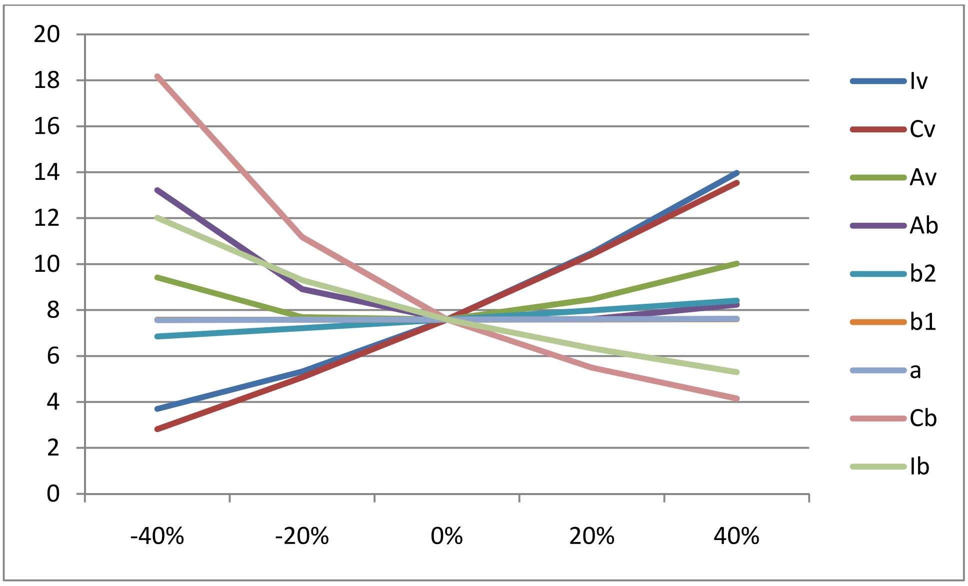

Fig. 5. Delayed time in days Vs. percentage of changes in affecting parameters

6. Conclusions

In this paper, a mathematical model is developed to analyze an optimal ordering policy for a supply chain comprising of vendor-buyer inventory system when demand is trapezoidal. It is established that the joint decision lowers the total cost of an inventory system, even though the buyer's cost increases significantly. To attract the buyer for a joint decision, a credit period has been offered by the vendor to the buyer to settle the account.

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