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VERTEX OPERATORS AND QUANTUM HALL EFFECT

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ABSTRACT

It is shown that the F.V. vertex operator allows a consistent bosonization of fermions, bosons and anyons. It thus plays an essential role in the general theory of Fractional Quantum Hall Effect.

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1) Introduction

The similarity between the mathematical methods and the physical ideas of elementary particles and condensed matter physics has been emphasized in a previous note[1].

A strong convergence of point of views becomes apparent if we compare the two dimensional models used in high energy physics (like string theory) and two dimensional problems in condensed matter (like fractional quantum Hall effect).

In this paper I wish to show that the vertex operator first introduced in dual resonance models by G. Veneziano and the present author[2] plays a fundamental role in the theory of *F.Q.H.E.* since it represents in a general way two dimensional fermions and bosons and those objects of fractional spin and statistics which are called anyons[3].

It will be shown that the many electron wave functions [4] introduced by *R* Laughlin in the theory of *F.Q.H.E.* can be expressed as the zero expectation values of appropriate vertex operators, which allow a consistent "second quantization" of

different kinds of particles and quasi-particles. The parameters appearing in the definition of the ***FV*** vertex describe in a simple way spin, statistics and compositeness of the various objects appearing in the Laughlin theory of *F.Q.H.E.*

The present paper is addressed both to elementary particles and condensed matter physicists. It thus contains a certain amount of redundancy. Chapter 2 is mostly well known by condensed matter physicists who can happily skip it. In the same way the material in chapter 3 should be reasonably familiar to string experts. People who are competent in both fields can jump directly to chapters 4, 5 and 6 which contain the main results of this investigation.

2) The Landau levels

This section is dedicated to a brief review of some well known properties of Landau levels. Consider a charged particle subject to a constant magnetic field B directed along the 3 axis. All interesting physics happens in the (1,2) plane, so we adopt from the beginning a two dimensional notation.

The equations of motion are:

$$\frac{d\pi_i}{dt} - \omega\epsilon_{ij}\pi_j \qquad (1)$$

where

$$\omega = \frac{qB}{mc}$$

$$\epsilon_{11} = \epsilon_{22} = 0 \qquad \epsilon_{12} = -\epsilon_{21} = 1$$

and π_i is the kinetic momentum

$$\pi_i = m\frac{dx_i}{dt} \qquad (2)$$

It is easy to we that the vector

$$\beta_i = \pi_i - m\omega\epsilon_{ij}x_j \qquad (3)$$

is a constant of the motion

$$\frac{d\beta_i}{dt} = 0 \qquad (4)$$

It is well known that β_i are related to the coordinates of the centre of the circle which is the classical orbit of the particle.

In the framework of Hamiltonian formulation we have to introduce the electromagnetic field

$$A_i \;=\; -\frac{B}{2}\,\epsilon_{ij}\,x_j \;+\; \frac{\partial\phi}{\partial x_i} \tag{5}$$

where the scalar ϕ is an arbitrary function of x_i which does not affect the value of the field. Its choice determines the "gauge" in we are working, $\phi = 0$ corresponds to the symmetric gauge which is adopted in this paper

The canonical momentum p_i is related to π_i by

$$\pi_i \;=\; p_i - q\,A_i \tag{6}$$

This means that the fundamental vectors π_i and β_i are given by

$$\begin{pmatrix}\pi_i\\ \beta_i\end{pmatrix} \;=\; p_i - \frac{\partial\phi}{\partial x_i} \pm \frac{m\omega}{2}\epsilon_{ij}x_j \tag{7}$$

It is useful to perform a canonical transformation using as new phase space variables π_i and β_i instead of x_i and p_i; we have

$$\begin{aligned} x_i &= \frac{\epsilon_{ij}}{m\omega}\,(\beta_j - \pi_j) \\ p_i - q\frac{\partial\phi}{\partial x_i} &= \frac{\beta_i + \pi_i}{2} \end{aligned} \tag{8}$$

The fundamental action

$$I \;=\; \int (p_i\dot{x}_i - H)\; dt \tag{9}$$

can be reexpressed as (*)

$$I \;=\; \int \left[\frac{\pi_2\dot{\pi}_1 + \beta_1\dot{\beta}_2}{m\omega} - H\right]\; dt \tag{10}$$

We see that π_1 and β_2 can be considered as new coordinates. The corresponding momenta are $\pi_2/m\omega$ and $\beta_1/m\omega$.

We thus have the quantum commutators

$$\begin{cases} \left[\pi_1\,,\,\pi_2\right] \;=\; i\hbar\, m\omega \\ \left[\beta_2\,,\,\beta_1\right] \;=\; i\hbar\, m\omega \\ \left[\pi_i\,,\,\beta_j\right] \;=\; 0 \end{cases} \tag{11}$$

(*) The scalar ϕ appears in an exact differential which is disregarded

The NR Hamiltonian is simply

$$H \;=\; \frac{\pi_1^2 + \pi_2^2}{2m} \tag{12}$$

We thus see that β_1, β_2 commute with Hamiltonian. The fact that β_1 and β_2 are non commuting constants of the motion is responsible for the degeneracy of the Landau levels.

It is of course useful to introduce the creation and destruction operators

$$\begin{aligned} \begin{pmatrix} a \\ a^+ \end{pmatrix} &= \frac{1}{(2\hbar m\omega)^{\frac{1}{2}}}\,(\pi_1 \pm i\pi_2) \\ \begin{pmatrix} b \\ b^+ \end{pmatrix} &= \frac{1}{(2\hbar m\omega)^{\frac{1}{2}}}\,(\beta_2 \pm i\beta_1) \end{aligned} \tag{13}$$

leading to the well known equations

$$\begin{aligned} H &= \hbar\omega\Big(a^+a + \frac{1}{2}\Big) \\ \left[a\,,\,a^+\right] &= \left[b\,,\,b^+\right] \;=\; 1 \end{aligned} \tag{14}$$

all other commutators being zero.

In the symmetric gauge we introduce the complex numbers

$$\begin{pmatrix} z \\ z^* \end{pmatrix} \;=\; \frac{1}{2\ell}(x_1 \pm ix_2) \tag{15}$$

where the magnetic length is

$$\ell \;=\; \sqrt{\frac{\hbar}{m\omega}} \;=\; \sqrt{\frac{\hbar c}{qB}}$$

The operators in eq. (15) are given by

$$\begin{aligned} a &= \frac{-i}{\sqrt{2}}\Big(z + \frac{\partial}{\partial z^*}\Big) & a^+ &= \frac{i}{\sqrt{2}}\Big(z^* - \frac{\partial}{\partial z}\Big) \\ b &= \frac{1}{\sqrt{2}}\Big(z^* + \frac{\partial}{\partial z}\Big) & b^+ &= \frac{1}{\sqrt{2}}\Big(z - \frac{\partial}{\partial z^*}\Big) \end{aligned} \tag{16}$$

The (degenerate) lowest Landau level obeys the equation

$$a\,|\,0> \;=\; 0$$

i.e.

$$\left(z + \frac{\partial}{\partial z^*}\right) \Psi_o(z, z^*) = 0 \tag{17}$$

from which it follows

$$\Psi_o(z, z^*) = f(z) e^{-zz^*} \tag{18}$$

The degeneracy of the lowest level is exhibited by the fact that $f(z)$ can be any analytic function of z.

It is useful to introduce a complete set of zero energy eigenfunctions

$$f_m(z) = (2m!)^{\frac{1}{2}} z^m \tag{19}$$

corresponding to all positive integer values of the angular momentum.

When the external magnetic field is very large, the transition rates between different Landau levels is very small. It is thus useful to develop a formalism that keeps us always at the lowest level.

The procedure which allows to project all states on the lowest level has been developed by Girvin and Jach(5). The most general wave function is

$$\Psi = e^{-z^* z} g(z^*, z) \tag{20}$$

The recipe is as follows: order $g(z^* z)$, first z^* and then z, then perform the substitution

$$z^* \Longrightarrow \frac{1}{2}\frac{\partial}{\partial z} \tag{21}$$

The projection of Ψ on the lowest level is given by

$$\Psi_o \longrightarrow e^{-z^* z} : g\left(\frac{1}{2}\frac{\partial}{\partial z}, z\right) : \tag{22}$$

Let us now study the wave function of many particles in the lowest level. The wave function is

$$f(z_1 \cdots z_n) e^{-\Sigma z_i z_i^*}$$

In the limit of weak coupling (quasifree particles) we introduce the state of minimum angular momentum which is given by the Slater (Vandermonde) determinant

$$f(z_1 \cdots z_n) = \begin{vmatrix} 1 & 1 & 1 & \cdots \\ z_1 & z_2 & z_3 & \cdots \\ z_1^2 & z_2^2 & z_3^2 & \\ \cdots & & & \end{vmatrix} = \prod_{i>j} (z_i - z_j) \tag{23}$$

The representation (23) is strongly reminiscent of the many body Veneziano form in dual models.

One starts thus seeing some analogy between Laughlin theory and dual model, which is the main object of this paper.

In the next section I shall discuss some well known properties of the vertex operator of dual models which will allow a deeper understanding of this relation.

3) The F.V. vertex operator

The vertex operator was first introduced by G. Veneziano and the present author[2] in the study of the dual resonance model. In the last 20 years it has played a key role in many important investigations in the string theory and in conformal field theories. As a consequence I shall just sketch some of the main properties which are relevant to this investigation.

One introduces an infinite set of creation and destruction operators A_μ^n , $A_\mu^{n\dagger}$ whose commutators are

$$\left[A_\mu^n \,,\, A_{\mu'}^{n'\dagger}\right] = \frac{1}{n}\delta_{n\,n'}\,\delta_{\mu\,\mu'} \tag{24}$$

the zero mode operators p_μ and q_μ

$$[p_\mu, q_\nu] = -i\delta_{\mu\nu} \tag{25}$$

Now define

$$Q_\mu(z) = q_\mu + i p_\mu \log z + Q_\mu^+(z) + Q_\mu^-(z) \tag{26}$$

$$\begin{aligned} Q_\mu^+(z) &= -i \sum A_\mu^{n+} z^n \\ Q_\mu^-(z) &= i \sum A_\mu^n z^{-n} \end{aligned} \tag{27}$$

The vertex operator is defined by

$$U_\alpha(z) = :\, e^{i\alpha Q(z)} : \tag{28}$$

where the ordering means

$$U_\alpha(z) = e^{i\alpha Q^+(z)} e^{i\alpha q} e^{i\alpha p \log z} e^{i\alpha Q^-(z)} \tag{29}$$

The vertex operator (28) has been introduced in order to "factorize" products like the one appearing in the r. h. s. of eq. (23).

Defining the "zero state" $|0>$ by

$$A^n_\mu\, |0> = 0 \qquad p_\mu\, |0> = 0 \tag{30}$$

we have the well known identity

$$< 0|\, U_{\alpha_1}(z_1)\, \cdots\, U_{\alpha_n}(z_n)\, |0> = \prod_{i>j}(z_i - z_j)^{\vec{\alpha}_i \cdot \vec{\alpha}_j} \tag{31}$$

which is valid where

$$\sum \vec{\alpha}_i = 0 \tag{32}$$

We need to generalize eq. (31) to the case when

$$\sum \vec{\alpha}_i = -\vec{A} \neq 0$$

This can be done by a simple device. Introduce the extra operator $U_{+A}(y)$ and use eq. (31) in order to write

$$\langle 0 |\, U_{+A}(y)\, U_{\alpha_i}(z_i)\, \cdots\, U_{\alpha_n}(z_n)\,|0\rangle = \\ = \prod(y - z_i)^{-(\vec{A}\vec{\alpha}_i)} \prod (z_i - z_j)^{\vec{\alpha}_i \vec{\alpha}_j}$$

Setting $y \to \infty$, we simply get

$$\lim_{y\to\infty} y^{-A^2} \langle 0 |\, U_{+A}(y)\, U_{\alpha_i}(z_i)\, \cdots\, U_{\alpha_n}(z_n)\,|0\rangle \\ = \prod (z_i - z_j)^{\vec{\alpha}_i \vec{\alpha}_j} \tag{33}$$

It is easy to compute the limit indicated in eq. 33

$$\langle -A |\, U_{\alpha_i}(z_i)\, \cdots\, U_{\alpha_n}(z_n)\,|0\rangle = \prod (z_i - z_j)^{\vec{\alpha}_i \vec{\alpha}_j} \tag{34}$$

where $\langle -A|$ is defined by

$$\langle -A\,|\, A^{n\,\dagger}_\mu = 0 \qquad \langle -A\,|\,p = \langle A\,| - A \tag{35}$$

Equation (34), which will be written simply as

$$\langle |\, U_\alpha(z_1)\, \cdots\, U_{\alpha_n}(z_n)\,| \rangle = \prod (z_i - z_j)^{\vec{\alpha}_i \vec{\alpha}_j},$$

will provide the fundamental link between the dual formalism and Laughlin theory of Hall effect. It will be important to discuss some well known important properties of the vertex operator.

4) Transformation properties

I wish now to discuss the transformation properties of the vertex operators under translations and rotations.

Translations and rotation are two particular cases of the full conformal algebra represented by the celebrated Virasoro operators

$$L_n = \frac{1}{2ki} \oint\, : \left(\frac{dQ_\mu}{dz}\right)^2 : z^{n+1} dz \tag{36}$$

The commutator between L_n and the vertex operator are well known

$$\left[L_n, U_\alpha(z)\right] = z^n \left[z \frac{d}{dz} + \frac{1}{2}\vec{\alpha}^2 (n+1)\right] U_\alpha(z) \tag{37}$$

where the term in $\vec{\alpha}^2$ in the r. h. s. is due to an anomaly generated by the ordering in the definition of the vertex (28). This anomaly will play a key role in the further treatment. We now specialize our discussion to the generator of translations

$$L_{-1} = A^{1\dagger}_\mu p_\mu + \sum A^{(1+i)\dagger}_\mu A^i_\mu \tag{38}$$

and that of rotations

$$L_o = p_\mu p_\mu + \sum A^{i\dagger}_\mu A^i_\mu \tag{39}$$

The commutations relations (37) lead to

$$\left[L_{-1}, U_\alpha(z)\right] = \frac{dU_\alpha}{dz} \tag{40}$$

and

$$\left[L_o, U_\alpha(z)\right] = z\frac{dU_\alpha}{dz} + \frac{1}{2}\vec{\alpha}^2 U_\alpha(z) \tag{41}$$

The commutator (40) shows that U_α behaves regularly under translations. Since

$$\begin{cases} L_{-1}\,|0\rangle = 0 \\ \langle -A\,|\,L_{-1} = 0 \end{cases}$$

we can check that the expectation value (33) is indeed translation invariant and thus depends only on the differences $(z_i - z_j)$ of the coordinates.

Let us now concentrate on the commutator (41) where the anomalous term shows that $U_\alpha(z)$ possesses an internal spin $\frac{1}{2}\,\vec{\alpha}^2$ which is not necessarily integer or seminteger.

In the case of a finite rotation eq. (41) can be written as

$$e^{iL_0\theta}U_\alpha(z)e^{-iL_0\theta} \;=\; U_\alpha(ze^{i\theta})e^{i\frac{\alpha^2}{2}\theta} \tag{42}$$

And for a 2π rotation

$$e^{i2\pi L_0}U_\alpha(z)e^{-i2\pi L_0} \;=\; U_\alpha(z)e^{i\pi\vec{\alpha}^2} \tag{43}$$

We see that for a rotation of 360° degrees
for $\vec{\alpha}^2$ **even** $U_K(z)$ transforms into itself $\Longrightarrow$ **boson**
for $\vec{\alpha}^2$ **odd** $U_K(z)$ changes sign $\Longrightarrow$ **fermion**
for **any** $\vec{\alpha}^2$ **non integer**, $U_K(z)$ acquires a phase $\pi\vec{\alpha}^2$ $\Longrightarrow$ **anyon.**

The F V vertex, depending on the value of $\vec{\alpha}^2$, bosonizes fermions, bosons and anyon. It is thus the most natural candidate for obtaining second quantization for the three kind of objects in a unified manner.

The correspondence between spin and statistics is given by the well known algebraic relation

$$U_\alpha(z_1)U_\alpha(z_2) \;=\; U_\alpha(z_2)U_\alpha(z_1)e^{i\pi\vec{\alpha}^2\epsilon(\theta)} \tag{44}$$

where

$$\epsilon(\theta)\begin{cases} +1 & \theta > 0 \\ -1 & \theta < 0 \end{cases}$$

θ being the relative phase between z_1 and z_2.

Eq. (44) confirms that we have Bose statistics for $\vec{\alpha}^2$ even, Fermi statistics for $\vec{\alpha}^2$ odd and fractional statistics when $\vec{\alpha}^2$ is a fraction.

5) Application to Hall effect

I will now discuss some specific properties of the most important vertex operators and see their usefulness in the theory of the Fractional Quantum Hall effects.

The operator $Q_\mu(\mu = 1\cdots N)$ lives in general in an N dimensional space; of course the most elementary operators can be described in only one dimension.

The operator corresponding to $\alpha = 1$(i.e. $\alpha^2 = 1$) represents an elementary fermion and has been discussed in detail by Goddard and Olive[6] who recognized an example of the "bosonization" first introduced by Skyrme[7].

We shall thus have

$$U_F(z) \;=\; U_1(z) \;= :\exp iQ(z): \tag{45}$$

It it easy to see that the Slater Vandermonde determinant (22) is given by

$$\langle n\,|\,U_F(z_i)\cdots U_F(z_n)|\,0\,\rangle \;=\; \prod_{z_i>z_j}(z_i - z_j) \tag{46}$$

confirming that U_F is indeed bosonizing a fermion.

The most elementary boson is obtained by taking $\alpha^2 = 2$ i.e. $\alpha = \sqrt{2}$. We shall thus have

$$U_B(z) \;=\; U_{\sqrt{2}}(z) \;= :\exp i\sqrt{2}Q(z): \tag{47}$$

We can construct excited fermion and boson operators by multiplied elementary operators corresponding to **independent** scalar fields $Q_i(z)$.

It is easy to check that two fermions make up a boson by writing

$$U_F^{(1)}(z)U_F^{(2)} \;= :\exp iQ_1(z)::\exp iQ_2(z): = :\exp iQ_i(z)\alpha_i: \tag{48}$$

where the two dimensional vector u_α is given by

$$\vec{\alpha} \;=\; \begin{pmatrix}1\\1\end{pmatrix} \tag{49}$$

Changing bases and orienting our new horizontal axis I at 45° from the old one, we get

$$U_F^{(1)}(z)U_F^{(2)} \;= :e^{i\sqrt{2}Q_I(z)}: \tag{50}$$

which agrees with the definition (47) of the boson operator.

We can now consider Laughlin wave function as due to the presence of the interaction between the electrons which can be represented by the exchange of a boson. The two body wave function can be pictured as

boson

Fermion

We shall thus have the product

$$U_L = U_F U_B \tag{51}$$

at each vertex.

It is now easy to see that

$$U_L(z) = :\exp i(Q_i(z)\alpha_i^L): \tag{52}$$

where

$$\vec{\alpha}_L = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} \tag{53}$$

Since $\vec{\alpha}_L^2 = 3$ we can say that $U_L(z)$ corresponds to a **superfermion** and can write the expectation value

$$\langle | U_L(z_1) \cdots U_L(z_n) | \rangle = \\ = \prod_{i>j}(z_i - z_j)^3 = f_L(z_1 \cdots z_n) \tag{54}$$

which is indeed the lowest Laughlin wave function.

The higher Laughlin exponents $5, 7 \cdots$ are obtained by exchanging more than one boson, this leads to vectors

$$\alpha_L' = \begin{pmatrix} 1 \\ \sqrt{2} \\ \sqrt{2} \\ \cdots \end{pmatrix}$$

with square moduli 5, 7, 9 etc.

An amusing application is obtained when one has to deal with electrons of different spin orientations.

Labelling the up and down coordinates as z_i^u and z_i^d we have the vertex operators

$$\begin{aligned} U_L^u(z^u) &= U_F^u(z^u)U_B(z^u) = :\exp i\, Q_i(z^u)\alpha_i^u : \\ U_L^d(z^d) &= U_F^d(z^d)U_B(z^d) = :\exp i : Q_i(z^d)\alpha_i^d : \end{aligned} \tag{55}$$

where

$$\vec{\alpha}^u = \begin{pmatrix} 1 \\ 0 \\ \sqrt{2} \end{pmatrix} \qquad \vec{\alpha}^d = \begin{pmatrix} 0 \\ 1 \\ \sqrt{2} \end{pmatrix}$$

$$\begin{aligned} \vec{\alpha}^u \cdot \vec{\alpha}^u &= \vec{\alpha}^d \cdot \vec{\alpha}^d = 3 \\ \vec{\alpha}^u \cdot \vec{\alpha}^d &= 2 \end{aligned} \tag{56}$$

taking the matrix element

$$f_{ud}(z^u, z^d) = \langle | U_L^u(z^u) \cdots U_L^d(z^d) | \rangle \tag{57}$$

one gets the well known formula: (8)

$$f_{ud} = \prod (z_i^u - z_j^u)^3 (z_i^d - z_j^d)^3 (z_i^u - z_j^d)^2 \tag{58}$$

The result of this section is that the vertex operator formalism reproduces beautifully the fundamental wave functions appearing in *F.Q.H.E.*

In the next section it will be seen that the same formalism can be used to express the excitations around Laughlin ground states.

This will be done introducing the anyon operators described in the last section.

6) Excitations as anyons

According to *R* Laughlin there are two kind of excitations:

1) quasi-holes whose wave function is:

$$(1) \qquad f_{\text{q.hole}}(y\ z_1 \cdots z_n) = \prod (y - z_j) f_L(z_1 \cdots z_n)$$

2) quasi-particles with wave function

$$f_{\text{q.particle}}(y^*\ z_1 \cdots z_n) = \prod \left(y^* - \frac{1}{2}\frac{\partial}{\partial z_j} \right) f_L(z_1 \cdots z_n)$$

We discuss here the, easier problem of quasi-holes; quasi-particles will be hopefully discussed in a next paper.

Let us attribute a vertex operator

$$U_H(y) = :\exp i\alpha_i^H Q_i(y): \tag{59}$$

Equating $f_{\text{q.hole}}$ to the expectation value

$$\langle | U_H(y) U_L(z_1) \cdots U_L(z_n) | \rangle \tag{60}$$

one obtains

$$\vec{\alpha}_H \cdot \vec{\alpha}_L = 1 \tag{61}$$

Since, for physical reasons, $\vec{\alpha}_H$ should be parallel to $\vec{\alpha}_L$ we find

$$\vec{\alpha}_L = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} \qquad \vec{\alpha}_H = \frac{1}{3}\begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} \tag{62}$$

We see that

$$\begin{aligned} \vec{\alpha}_L \cdot \vec{\alpha}_L &= 3 \qquad \vec{\alpha}_L \cdot \vec{\alpha}_H = 1 \\ \vec{\alpha}_H \cdot \vec{\alpha}_H &= \frac{1}{3} \end{aligned} \tag{63}$$

One thus sees that the quasi-hole is represented by a vertex with fractional angular momentum and statistics.

This confirms the beautiful result obtained by Arovas[9] Schrieffer Wickzek using the Berry phase.

If we consider many quasi-holes of coordinates $y_1\, y_2 \cdots y_n$ it is possible that, as an effect of interaction, and extra boson is exchanged. We thus have **superanyous** corresponding to the vector

$$\alpha'_H = \begin{pmatrix} \frac{1}{3} \\ \frac{\sqrt{2}}{3} \\ \sqrt{2} \end{pmatrix} \qquad \left(\vec{\alpha}'_H\right)^2 = 2 + \frac{1}{3} = \frac{7}{3}$$

In the products we see that $\vec{\alpha}_L \vec{\alpha}_L$ and $\vec{\alpha}_L \vec{\alpha}'_H$ are unchanged whereas we have products like $(y_i - y_j)^{7/3}$. We see the beginning of the hierarchical model in the form due to Halperin[10]. We notice however that before having the whole hierarchy we need to obtain a satisfactory theory of quasi-particles.

Another amusing case is the one of semions (i.e. half fermions) which seem to play a role in high T_c superconductivity.

Their vertex operator is

$$U_s(z) = :\exp i \frac{1}{\sqrt{2}} Q(z): \tag{64}$$

corresponding to quasi-holes in a boson background (see eq. 47).

Conclusions

The main result of this paper is that the formalism which has been introduced in dual resonant models and, more recently, two dimensional conformal theory is

useful in providing a simple mathematical formulation of the modern theory of Fractional Quantum Hall Effect.

The vertex operator of dual models provides a general second quantization for fermions, bosons and anyons appearing in that context. The parameters appearing its definition define clearly the physical properties of the "particle" at hand. The dimensionality of $\vec{\alpha}$ describes compositeness whereas the modulus $\vec{\alpha}^2$ describes spin and statistics.

In conclusion I strongly believe that interdisciplinarity is fundamental in the progress of our science. Already in the past cross fertilization between two fields so different as high energy and condensed matter have been extremely fruitful. Now a bridge has been established between two fields that span ultrahigh and ultralow energies. I hope that the results discussed here will be of some usefulness to research in both fields.

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