



Vertical wall breakwater stability

Živko Vuković

*Faculty of Civil Engineering, University of Zagreb,
Kačićeva 26, 10000 Zagreb, Croatia*

E-mail: kuspa@master.grad.hr

Abstract

The calculation of the minimum required widths of a concrete vertical wall breakwater with respect to the stability criteria of overturning, sliding and settling is presented in the paper. It is assumed that the vertical wall breakwater is built on a foundation and located in the conditions for non-overtopping and overtopping by incident waves. In the analysis of wave forces on the vertical wall breakwater a distinction is made between the action of nonbreaking, breaking and broken waves. The disturbing nonbreaking wave forces are calculated using Sainflou's and Molitor's methods. The breaking wave forces are calculated according to the method developed by Minikin, and the broken wave forces according to the method proposed by the CERC (Coastal Engineering Research Center). Using the derived expressions and appropriate values of the safety factor for overturning and sliding, and allowable normal stress of the foundation for settling, and applying the assumed environmental conditions, it is possible to determine directly the minimum required width of a vertical wall breakwater. A numerical example in which the minimum required widths of a vertical wall breakwater in different wave load conditions are calculated according to the analyzed stability criteria is also given in the paper.

1 Introduction

Vertical wall breakwater stability analysis involves the consideration of overturning, sliding and settling. These movements are a consequence of the active force due to pressure produced on the breakwater's vertical face exposed to the open sea, which is in turn counteracted by the resistive passive force provided by the structure's self-weight.



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The aim of this paper is to present, on the basis of the theoretical and experimental research done so far, the load parameters for the overtopping and non-overtopping case of a vertical wall breakwater exposed to the action of nonbreaking, breaking and broken waves, and to determine its minimum required width according to the overturning, sliding and settling criteria.

The nonbreaking wave forces are determined using Sainflou's and Molitor's methods, the breaking wave forces according to Minikin's method, and the broken wave forces according to the method proposed by the CERC.

2 Assumptions

- (a) A vertical wall breakwater is constructed on a foundation.
- (b) The wave approach is at right angle to the breakwater.
- (c) In the cases when the crown of the breakwater is not high enough to prevent overtopping of waves, it is assumed that the overtopping is not too severe, so that the resulting pressure distribution on the wall is the same as in the non-overtopped case.
- (d) In the analysis of nonbreaking wave forces on the structure, it is assumed that the complete standing waves (clapotis) will be developed at the front of the sea side face of the breakwater, and that the wave height of the clapotis is twice the wave height of the incoming (unreflected) wave.
- (e) In the analysis of broken wave forces on the breakwater, it is assumed that, immediately after breaking, the water mass in a wave moves forward with the velocity of propagation attained before breaking; that is, upon breaking, the water particle motion changes from oscillatory to translatory motion.

3 Wave forces on vertical wall breakwater

3.1 Nonbreaking wave forces

3.1.1 Wave forces according to Sainflou

One of the generally accepted nonbreaking wave pressure diagrams is due to Sainflou.¹ However, according to recent investigations, the loads calculated from this diagram are too large if the waves are steep.²

The Sainflou pressure diagram, Fig. 1, is theoretically based on the trochoidal motion of standing waves. A limiting condition of the theory is that the sea depth, d , as measured from the still water level (SWL), must be twice the wave height, H .³

The symbols in Fig. 1 mean: H = incoming wave height; L = wave length, T = wave period; d = water depth; d_1 = breakwater height above the SWL; d_2 = breakwater height from the base of the breakwater to the SWL; b = breakwater

width; $\Delta H_c = \pi H^2/L \coth kd$ = height above the SWL of the oscillation center (clapotis); $k = 2\pi/L$ = wave number; p_1 = wave pressure at the SWL; p_2 = wave pressure on the base of the breakwater; p_c = wave pressure on the crown of the breakwater where the crest of the design clapotis is above the crown of the breakwater; F_{HW} and F_{VW} = resultant horizontal and vertical wave forces against the breakwater, respectively; and F_{VS} = vertical force from the breakwater self-weight.

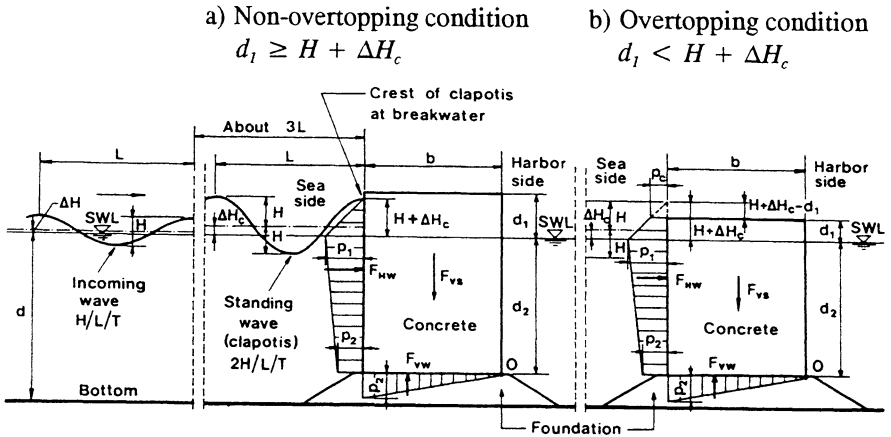


Figure 1: Wave forces against the vertical wall breakwater according to the Sainflou diagram.

The expressions for pressures p_1 , p_2 and p_c , and the resultant horizontal force, $F_H = F_{HW}$, and the resultant vertical force, $F_V = F_{VS} - F_{VW}$, are given in Table 1, where g = acceleration due to gravity; ρ = water density; and ρ_c = concrete breakwater density.

3.1.2 Wave forces according to Molitor

The empirical wave pressure diagram on a vertical breakwater proposed by Molitor⁴ is shown in Fig. 2, where p_1 = maximum wave pressure at a height h_1 above the SWL; p_2 = wave pressure at halfway between the trough elevation and the point of maximum pressure; and p_c = wave pressure on the crown of the breakwater in the overtopped case.

The expressions for pressures p_1 , p_2 and p_c , and the resultant horizontal force, $F_H = F_{HW}$, and the resultant vertical force, $F_V = F_{VS}$, are given in Table 1, where k = a coefficient, between 1.3 and 1.8 (usually 1.7); c = velocity of wave propagation = L/T ; and v_{max} = maximum orbital velocity = $HgT/2L$ for $0.04 \leq d/L \leq 0.5$ or $H/2 \sqrt{g/d}$ for $d/L < 0.04$.

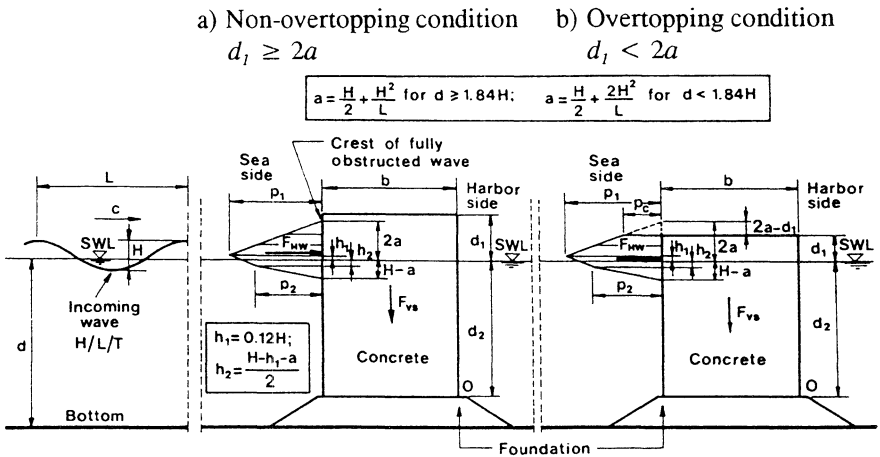


Figure 2: Wave forces against the vertical wall breakwater according to the Molitor diagram.

3.2 Breaking wave forces

According to Minikin,⁵ the total pressure on the breakwater due to the action of breaking waves is a combination of dynamic and hydrostatic pressures, Fig. 3, where p_1 = maximum dynamic pressure occurring at the SWL; p_2 = resultant hydrostatic pressure; p_c = resultant hydrostatic pressure on the crown of the breakwater in the overtopped case; H_b = height of the wave just breaking on the breakwater; d_w = water depth one wave length in front of the structure; and L = wave length in water of depth d_w .

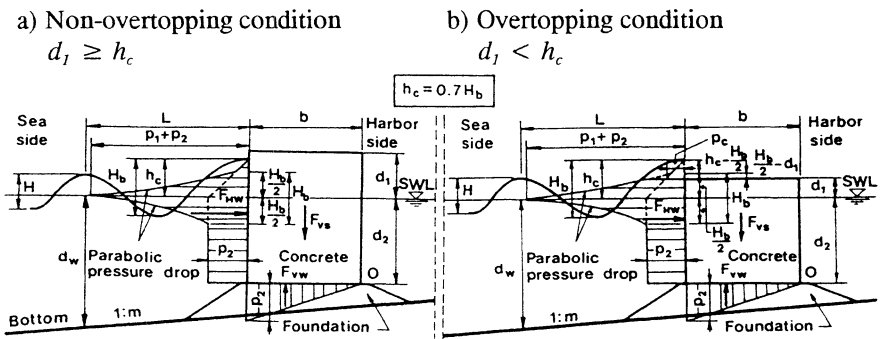


Figure 3: Wave forces against the vertical wall breakwater according to the Minikin diagram.

The assumption is made here that the maximum dynamic pressure occurs at the SWL and that it decreases parabolically to zero at the elevation of the wave crest. Minikin's method can give the wave forces that are extremely high, as much as 5 to 18 times those calculated for nonbreaking waves.⁶

The expressions for pressures p_1 , p_2 and p_c , and the resultant horizontal force, $F_H = F_{HW}$, and the resultant vertical force, $F_V = F_{VS} - F_{VW}$, are given in Table 2, where r_m = dynamic force reduction factor, Fig. 4a, for the case when the crown of the breakwater is lower than the crest of the breaking wave.

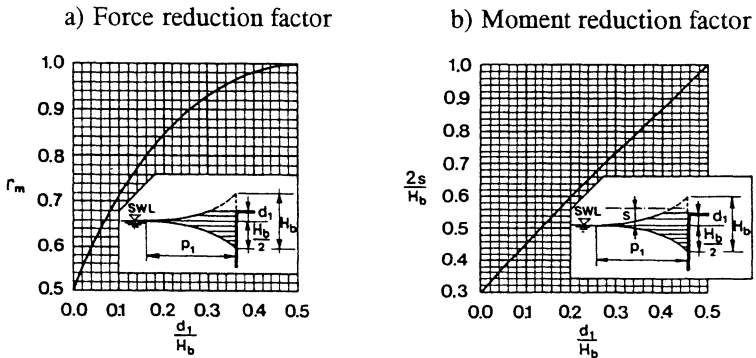


Figure 4: Minikin's dynamic reduction factors⁵

3.3 Broken wave forces

An approximate determination of the forces of a broken wave is possible according to the CERC.⁶

a) Non-overtopping condition
 $d_1 \geq h_c$

b) Overtopping condition
 $d_1 < h_c$

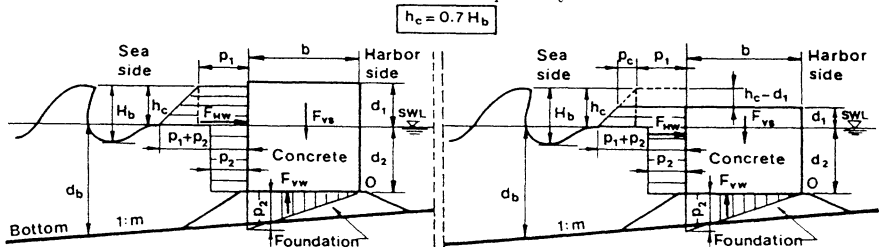


Figure 5: Wave forces against the vertical wall breakwater seaward of the SWL according to the CERC diagram.

The breakwaters located seaward of the SWL are subjected to wave pressures that are partly dynamic and partly hydrostatic, Fig. 5, where H_b = wave height at the moment of breaking; d_b = water depth at the point of breaking; p_1 = dynamic

pressure uniformly distributed from the SWL to a height h_c above the SWL; and p_2 = resultant hydrostatic pressure.

The expressions for pressures p_1 , p_2 and p_c , and the resultant horizontal force, $F_H = F_{HW}$, and the resultant vertical force, $F_V = F_{VS} - F_{VW}$, are given in Table 2.

4 Stability analysis

4.1 Overturning stability

The resisting moment is due to the force from the breakwater self-weight, and the driving moment due to the pressure forces. The overturning safety factor, n_o , which shows how secure the structure is against overturning, is given as:

$$n_o = \frac{M_{SO}}{M_{PO}} \quad (1)$$

where M_{SO} = total stabilizing moment of vertical force, F_{VS} , about the point O (see Figs. 1 through 5); and M_{PO} = total overturning moment of horizontal pressure force, F_{HW} , and vertical pressure force, F_{VW} , about the point O.

When $n_o \geq 1.5$, the structure can be considered secure as it meets or exceeds the minimum accepted safety standard.

The expressions for moments M_{SO} and M_{PO} are given in Tables 1 and 2, where s = Minikin's dynamic moment reduction factor, Fig. 4b, for the breakwater of low height.

Substituting the expressions for moments M_{SO} and M_{PO} into Eqn. (1), obtained are the expressions by means of which, with an appropriate value of safety factor, n_o , it is possible to determine directly the overturning minimum required breakwater width (MRBW), b_{oi} , Tables 1 and 2. In the first subscript, o , identify overturning, while in the second subscript, $i = s, m$ or c , identify the analysis of Sainflou, Molitor or Minikin and CERC, respectively.

4.2 Sliding stability

The resistive frictional force is defined as the product of the resultant vertical force, F_V , and the relevant angle of friction, θ , while the resultant horizontal driving force, F_H , is obtained from the wave pressure.

The sliding safety factor, n_s , which shows how secure the breakwater is against sliding, is given as:

$$n_s = \frac{F_V \tan \theta}{F_H} \quad (2)$$

When $n_s \geq 1.5$, the structure can be considered secure.

Substituting the expressions for forces F_V and F_H from Tables 1 and 2 into Eqn. (2), obtained are the expressions by means of which, with an appropriate value of

		a) Non-overtopping condition Sainflou: $d_1 \geq H + \Delta H_c$ (Fig. 1a) Molitor: $d_1 \geq 2a$ (Fig. 2a)	b) Overtopping condition Sainflou: $d_1 < H + \Delta H_c$ (Fig. 1b) Molitor: $h_1 \leq d_1 < 2a$ (Fig. 2b)
Sainflou	p_1	$(p_2 + \rho g d_2) \frac{H + \Delta H_c}{S_0}$	
	p_2	$\rho g H \frac{\cosh[k(d-d_2)]}{\cosh kd}$	
	p_c	—	$p_1 \frac{S_1}{H + \Delta H_c}$
	F_H	$\frac{1}{2}[(p_2 + \rho g d_2)S_0 - \rho g d_2^2]$	$\frac{1}{2}[(p_2 + \rho g d_2)S_0 - \rho g d_2^2 - p_c S_1]$
	F_V	$g b D_0 - \frac{1}{2} p_2 b$	
	M_{S0}	$\frac{1}{2} g D_0 b^2$	
	M_{P0}	$\frac{1}{6}[(p_2 + \rho g d_2)S_0^2 - \rho g d_2^2 + 2 p_2 b^2]$	$\frac{1}{6}[(p_2 + \rho g d_2)S_0^2 - \rho g d_2^2 - 3 p_c S_2 + 2 p_2 b^2]$
	b_{os}	$\sqrt{\frac{(p_2 + \rho g d_2)S_0^2 - \rho g d_2^2}{3 g D_0 / n_s - 2 p_2}}$	$\sqrt{\frac{(p_2 + \rho g d_2)S_0^2 - \rho g d_2^2 - 3 p_c S_2}{3 g D_0 / n_s - 2 p_2}}$
b_{sz}	$\frac{n_s [(p_2 + \rho g d_2)S_0 - \rho g d_2^2]}{(2 g D_0 - p_2) \tan \theta}$	$\frac{n_s [(p_2 + \rho g d_2)S_0 - \rho g d_2^2 - p_c S_1]}{(2 g D_0 - p_2) \tan \theta}$	
b_{sz}	$\sqrt{\frac{(p_2 + \rho g d_2)S_0^2 - \rho g d_2^2}{\sigma_{all} - g D_0 - 3/2 p_2}}$	$\sqrt{\frac{(p_2 + \rho g d_2)S_0^2 - \rho g d_2^2 - 3 p_c S_2}{\sigma_{all} - g D_0 - 3/2 p_2}}$	
Molitor	p_1	$\frac{1}{2} k \rho (c + v_{max})^2$	
	p_2	$0.72 p_1$	
	p_c	—	$p_1 \frac{M_0}{M_1}$
	F_H	$\frac{1}{2} \{ p_1 M_1 + [2 p_2 + (p_1 - p_2)] M_2 + p_2 M_3 \}$	$\frac{1}{2} \{ p_1 M_1 - p_c M_0 + [2 p_2 + (p_1 - p_2)] M_2 + p_2 M_3 \}$
	F_V	$g b D_0$	
	M_{S0}	$\frac{1}{2} g D_0 b^2$	
	M_{P0}	$\frac{1}{2} [p_1 M_9 + 2 p_2 M_{10} + (p_1 - p_2) M_{11} + p_2 M_{12}]$	$\frac{1}{2} [p_1 M_9 - p_c M_{13} + 2 p_2 M_{10} + (p_1 - p_2) M_{11} + p_2 M_{12}]$
	b_{os}	$\sqrt{\frac{n_s [p_1 M_9 + 2 p_2 M_{10} + (p_1 - p_2) M_{11} + p_2 M_{12}]}{g D_0}}$	$\sqrt{\frac{n_s [p_1 M_9 - p_c M_{13} + 2 p_2 M_{10} + (p_1 - p_2) M_{11} + p_2 M_{12}]}{g D_0}}$
b_{sz}	$\frac{n_s \{ p_1 M_1 + [2 p_2 + (p_1 - p_2)] M_2 + p_2 M_3 \}}{2 g D_0 \tan \theta}$	$\frac{n_s \{ p_1 M_1 - p_c M_0 + [2 p_2 + (p_1 - p_2)] M_2 + p_2 M_3 \}}{2 g D_0 \tan \theta}$	
b_{sz}	$\sqrt{\frac{3 [p_1 M_9 + 2 p_2 M_{10} + (p_1 - p_2) M_{11} + p_2 M_{12}]}{\sigma_{all} - g D_0}}$	$\sqrt{\frac{3 [p_1 M_9 - p_c M_{13} + 2 p_2 M_{10} + (p_1 - p_2) M_{11} + p_2 M_{12}]}{\sigma_{all} - g D_0}}$	
$D_0 = [\rho_c d_1 + (\rho_c - \rho) d_2]; M_2 = h_1 + h_2; M_7 = d_2 - h_2 - \frac{1}{3} M_3; M_{12} = M_3 M_7;$ $S_0 = d_2 + H + \Delta H_c; M_3 = H - a - h_2; M_8 = d_1 + h_2 + \frac{1}{3} M_0; M_{13} = M_0 M_8$ $S_1 = H + \Delta H_c - d_1; M_4 = d_2 + h_1 + \frac{1}{3} M_1; M_9 = d_1 + h_2 + \frac{1}{3} M_0;$ $S_2 = S_1 \left(d_2 + d_1 + \frac{1}{3} S_1 \right); M_5 = d_2 + h_1 - \frac{1}{2} M_2; M_{10} = M_2 M_5;$ $M_0 = 2a - d_1; M_6 = d_2 + h_1 - \frac{1}{3} M_2; M_{11} = M_2 M_6;$ $M_1 = 2a - h_1;$			

Table 1: Stability parameters of vertical wall breakwater to the action of nonbreaking waves according to Sainflou and Molitor.



		a) Non-overtopping condition $d_t \geq h_c$ (Fig. 3a and Fig. 5a)	b) Overtopping condition $d_t < h_c$ (Fig. 3b and Fig. 5b)
Minikin	p_1	$100\rho g \frac{H_b}{L} \frac{d_2}{d_w} (d_2 + d_w)$	
	p_2	$\rho g h_c$	
	p_c	—	$p_2 \left(1 - \frac{d_1}{h_c}\right)$
	F_H	$\frac{p_1 H_b}{3} + p_2 M_0$	$\frac{r_m p_1 H_b}{3} + p_2 M_0 - \frac{1}{2} p_c M_1$
	F_V	$g b D_0 - \frac{1}{2} p_2 b$	
	M_{SO}	$\frac{1}{2} g D_0 b^2$	
	M_{PO}	$\frac{1}{3} (p_1 H_b d_2 + p_2 b^2) + \frac{1}{2} p_2 M_3$	$\frac{1}{3} (p_1 H_b M_3 + p_2 b^2) + \frac{1}{2} (p_2 M_3 - p_c M_4)$
	b_{om}	$\sqrt{\frac{2/3 p_1 H_b d_2 + p_2 M_3}{g D_0 / n_o - 2/3 p_2}}$	$\sqrt{\frac{2/3 p_1 H_b M_3 + p_2 M_3 - p_c M_4}{g D_0 / n_o - 2/3 p_2}}$
	b_{em}	$\frac{n_s (1/3 p_1 H_b + p_2 M_0)}{(g D_0 - 1/2 p_2) \tan \theta}$	$\frac{n_s (1/3 r_m p_1 H_b + p_2 M_0 - 1/2 p_c M_1)}{(g D_0 - 1/2 p_2) \tan \theta}$
	b_{lm}	$\sqrt{\frac{2 p_1 H_b d_2 + 3 p_2 M_3}{\sigma_{all} - g D_0 - 3/2 p_2}}$	$\sqrt{\frac{2 p_1 H_b M_3 + 3(p_2 M_3 - p_c M_4)}{\sigma_{all} - g D_0 - 3/2 p_2}}$
CERC	p_1	$\frac{1}{2} \rho g d_b$	
	p_2	$\rho g h_c$	
	p_c	—	$p_2 \left(1 - \frac{d_1}{h_c}\right)$
	F_H	$p_1 h_c + p_2 M_0$	$p_1 d_1 + p_2 M_0 - \frac{1}{2} p_c M_1$
	F_V	$g b D_0 - \frac{1}{2} p_2 b$	
	M_{SO}	$\frac{1}{2} g D_0 b^2$	
	M_{PO}	$p_1 h_c M_0 + \frac{1}{2} p_2 M_3 + \frac{1}{3} p_2 b^2$	$p_1 d_1 C_0 + \frac{1}{2} (p_2 M_3 - p_c M_4) + \frac{1}{3} p_2 b^2$
	b_{oc}	$\sqrt{\frac{2 p_1 h_c M_0 + p_2 M_3}{g D_0 / n_o - 2/3 p_2}}$	$\sqrt{\frac{2 p_1 d_1 C_0 + p_2 M_3 - p_c M_4}{g D_0 / n_o - 2/3 p_2}}$
	b_{xc}	$\frac{n_s (p_1 h_c + p_2 M_0)}{(g D_0 - 1/2 p_2) \tan \theta}$	$\frac{n_s (p_1 d_1 + p_2 M_0 - 1/2 p_c M_1)}{(g D_0 - 1/2 p_2) \tan \theta}$
	b_{xc}	$\sqrt{\frac{3(2 p_1 h_c M_0 + p_2 M_3)}{\sigma_{all} - g D_0 - 3/2 p_2}}$	$\sqrt{\frac{3(2 p_1 d_1 C_0 + p_2 M_3 - p_c M_4)}{\sigma_{all} - g D_0 - 3/2 p_2}}$
$D_0 = [\rho_c d_1 + (\rho_c - \rho) d_2]; M_1 = h_c - d_1; M_3 = [r_m (d_2 + s) - s]; M_5 = h_c M_2 + d_2^2;$ $M_0 = d_2 + \frac{1}{2} h_c; M_2 = d_2 + \frac{1}{3} h_c; M_4 = M_1 \left(d_2 + d_1 + \frac{1}{3} M_1\right); C_0 = d_2 + \frac{1}{2} d_1$			

Table 2: Stability parameters of vertical wall breakwater to the action of breaking waves according to Minikin, and broken waves according to CERC.

safety factor, n_s , it is possible to determine directly the sliding minimum required breakwater width, b_{st} , (Tables 1 and 2). In the first subscript, s , identify sliding, while in the second subscript, $i = s, m$ or c , identify the analysis of Sainflou, Molitor or Minikin and CERC, respectively.

4.3 Settling stability

The settling stability depends upon the applied normal stress, σ , which acts on the breakwater foundation, and the allowable normal stress, σ_{all} , of the foundation. Should $\sigma_{all} < \sigma$, there is a danger that the foundation may settle. The calculation of the maximum stress, σ , depends upon the eccentricity of the point of action on the breakwater-foundation interface of the resultant of the horizontal, F_H , and vertical forces, F_V . When the point of action is within the middle-third area of the concrete breakwater base, b , and the core is off-center towards the harbor side, the diagram of stress is a trapezium with the minimum stress at the sea side and the maximum stress at the harbor side, which is:

$$\sigma = \frac{F_V}{b} + \frac{M_{PO}}{W} = \frac{F_V}{b} + \frac{6M_{PO}}{b^2} \quad (3)$$

where W = moment of resistance.

Substituting in expressions for F_V and M_{PO} from Tables 1 and 2 and substituting in $\sigma = \sigma_{all}$, obtained are the expressions by means of which, with an appropriate value of allowable normal stress of the foundation, σ_{all} , it is possible to determine directly the settling minimum required breakwater width, b_{ki} , (Tables 1 and 2). In the first subscript, k , identify settling, while in the second subscript, $i = s, m$ or c , identify the analysis of Sainflou, Molitor or Minikin and CERC, respectively.

5 Numerical example

In order to illustrate the previous analysis, a numerical example of the calculation of overturning, sliding and settling minimum required breakwater widths for hypothetical environmental conditions is presented in the text below.

It is assumed that water density, $\rho = 1025$ [kgm^{-3}]; concrete density, $\rho_c = 2200$ [kgm^{-3}]; acceleration of gravity, $g = 9.81$ [ms^{-2}]; safety factors, $n_o = n_s = 1.5$; allowable normal stress of the foundation, $\sigma_{all} = 3 \cdot 10^5$ [Nm^{-2}]; angle of friction, $\theta = 45$ [$^\circ$]; and coefficient, $k = 1.7$.

The values of other parameters and the calculation results are given in Table 3.

It can be seen from Table 3 that for the considered range of parameters, the MRBW produced from the sliding analysis is always less than that of overturning, which is in turn less than the settling case. Consequently, settling presents the most probable failure mode of a vertical wall breakwater.

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		Nonbreaking waves				Breaking waves		Broken waves	
		Sainflou		Molitor		Minikin		CERC	
		$H/L/T = 4/100/10.7$ $d = 10$ [m]; $d_2 = 8$ [m]				$H_b = 5$ [m]; $L = 85$ [m] $d_2 = 4$ [m]; $d_w = 7$ [m]		$H_b = 5$ [m]; $d_b = 5.5$ [m] $d_2 = 3$ [m]	
		NOC $d_i = 5$ [m]	OC $d_i = 3.5$ [m]	NOC $d_i = 5$ [m]	OC $d_i = 3.5$ [m]	NOC $d_i = 4$ [m]	OC $d_i = 3$ [m]	NOC $d_i = 4$ [m]	OC $d_i = 3$ [m]
MRBW	Overturning	6.5	7.0	7.1	7.7	9.7	11.0	5.5	6.5
	Sliding	3.4	4.0	2.9	3.3	10.7	13.2	3.8	4.6
	Settling	16.7	12.7	14.2	12.3	12.7	11.6	6.4	5.6

Table 3: Overturning, sliding and settling minimum required breakwater widths, MRBW [m], for hypothetical environmental conditions.

NOC = non-overtopping condition; OC = overtopping condition

6 Conclusions

In the seemingly similar conditions, a vertical wall breakwater can be exposed to rather different hydrodynamic loads, and accordingly to different minimum required widths.

The performed analysis and the numerical example results indicate that according to the overturning and sliding criteria, the MRBW in the overtopping case of a vertical wall breakwater is greater than in the non-overtopping case, whereas according to the settling criterion, the situation is reverse.

The greatest MRBW value is obtained according to the settling criterion, except in the case of the vertical wall breakwater overtopping by broken waves, where the overturning criterion is relevant.

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