Very Low Sensitivity FIR Filter Implementation Using "Structural Passivity" Concept

P. P. VAIDYANATHAN, MEMBER, IEEE, AND SANJIT K. MITRA, FELLOW, IEEE

Abstract —The concept of "structurally bounded" or "structurally passive" FIR filter implementation is introduced, as a means of achieving very low passband sensitivities. The resulting filter structures, called FIRBR structures, can easily be transformed into very low-sensitivity "passive" two-dimensional FIR filter structures. From a layout point of view, the new structures are not any more complicated than the well-known cascade form. The FIRBR structures do not depend, for synthesis, upon continuous-time filter circuits.

I. Introduction

The design of low-sensitivity digital filters has received considerable attention in the last few years [1]–[4]. Some of these design methods are based on continuous-time doubly terminated prototype *LC* filters [1], [4], whereas, some others are derived using an independent z-domain approach [2], [3]. The digital filter structures that are derived from continuous-time *LC* filter prototypes satisfy certain "passivity" properties in the digital domain. The structures described in [2], [3] are, however, not based on any passivity notions.

Recently a general theoretical framework for low-sensitivity digital filter structures has been reported [5]. The framework is independent of the continuous-time notions and is entirely based on z-domain concepts of "passivity." For any given stable digital filter transfer function, we develop in [5] a procedure in the z-domain for synthesizing structures that have low passband sensitivity. These structures are based on interconnections of digital two-pairs, where each two-pair is constrained to be lossless. In this manner, the overall implementation is structurally passive and leads to low-sensitivity realizations. One of the many advantages of taking an entirely z-domain point of view is the possibility of designing "passive" structures for finite impulse-response (FIR) filters, which is the purpose of this paper. Section II reviews the concepts of "structural passivity" and "structural boundedness" [5] of a digital filter implementation. A novel approach for the design of passive low-sensitivity FIR filter structures for linear phase transfer functions is presented in Section III. From a layout viewpoint, the resulting structure resembles the conventional cascade form. Computer simulation examples are presented in Section IV to demonstrate the low-sensitivity properties. The noise performance is studied in the following section. Finally, the design of "passive" two-dimensional FIR filters with very low passband sensitivity, by using the well-known McClellan transformation [7], is indicated in Section VI. Simulation results are included.

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P. P. Vaidyanathan is with the Department of Electrical Engineering, California Institute of Technology, Pasadena, CA 91125.
S. K. Mitra is with the Department of Electrical and Computer Engineering, University of California, Santa Barbara, CA 93106.

II. Preliminaries

Consider the transfer function of a FIR filter:

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + \cdots + h_N z^{-N} \tag{1}$$

where $\{h_0, h_1, \cdots, h_N\}$ represents the impulse response sequence. We wish to design a structure, with multiplier coefficients $m_0, m_1, \cdots, m_N$ (related to $h_0, h_1, \cdots, h_N$) such that the sensitivity of $|H(e^{j\omega})|$ with respect to each m_i is very small in the passband. For the well-known direct-form structure, $m_i = h_i$ for all i, and the sensitivity properties are not very acceptable [8]. For the cascade form structure, the stopband behavior is good, but the passband sensitivity is not particularly good [8].

Let us assume that the structure is such that, regardless of the actual values of the multipliers m_i, the quantity $|H(e^{j\omega})|$ is bounded above by a fixed constant, say unity:

$$|H(e^{j\omega})| \leqslant 1, \qquad \text{for all } \omega. \tag{2}$$

In other words, assume that the structure forces an upper bound on $|H(e^{j\omega})|$, regardless of what the values of the multipliers are, as long as the multipliers are within a certain range. We call such implementations "structurally passive" or "structurally bounded" [5]. Now consider a transfer function $H(z)$ with a typical frequency response magnitude, as shown in Fig. 1. At frequencies $\omega = \omega_k$, the magnitude $|H(e^{j\omega_k})|$ is assumed to be precisely equal to unity. Let $H(z)$ be implemented in a "structurally bounded" manner. If now the multiplier m_i is perturbed, the quantity $|H(e^{j\omega_k})|$ can only decrease, regardless of the sign of perturbation. Thus a plot of $|H(e^{j\omega_k})|$ with respect to m_i has zero slope at the nominal values of all parameters, and this is true for each m_i and each ω_k (see Fig. 2). In effect, we have zero first-order sensitivity at frequencies $\omega = \omega_k$:

$$\frac{\partial |H(e^{j\omega_k})|}{\partial m_i} = 0, \qquad \forall i, \forall k. \tag{3}$$

Thus if we have a number of closely spaced points in the passband where $|H(e^{j\omega})| = 1$, then we can expect low passband sensitivity. Note that the above argument is

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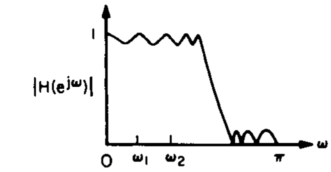

Fig. 1. A typical magnitude response.

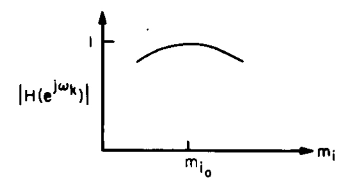

Fig. 2. Zero first-order sensitivity.

analogous to the well-known "Orchard's argument" [12], in classical filter theory.

It is therefore clear that the fundamental requirement for low passband sensitivity is structural boundedness. Let us turn our attention to the task of finding such "bounded" structures for FIR transfer functions. A stable transfer function $H(z)$, scaled such that the condition of (2) holds, is called a "bounded real (BR)" function (note that $H(z)$ is real for real values of z).[1] The "structurally bounded" FIR filter implementations will also be termed as "*FIRBR*" structures.

III. An Approach for Designing FIRBR Structures

Most of the FIR filters that are of interest in practice have linear phase. There are four types of linear phase FIR filters, as outlined in [9], and are listed below:

$$\text{Type 1:}\quad h_n = h_{N-n} \quad N \text{ even} \tag{4a}$$

$$\text{Type 2:}\quad h_n = h_{N-n} \quad N \text{ odd} \tag{4b}$$

$$\text{Type 3:}\quad h_n = -h_{N-n} \quad N \text{ even} \tag{4c}$$

$$\text{Type 4:}\quad h_n = -h_{N-n} \quad N \text{ odd.} \tag{4d}$$

Of these, Types 3 and 4 are used for the design of Hilbert-transformers and differentiators, and we do not consider them here. Type 2 filters have the restriction that $H(e^{j\pi}) = 0$, and therefore are less general than Type 1 filters. In this paper, we shall therefore consider only Type 1 linear phase FIR filters. The frequency response of such a filter can be written as

$$H(e^{j\omega}) = e^{-j\omega N/2} H_0(e^{j\omega}) \tag{5}$$

where $H_0(e^{j\omega})$ is a real function of ω. Let us now consider a "complementary transfer function" $G(z)$, defined as:

$$G(z) = z^{-N/2} - H(z). \tag{6}$$

Clearly,

$$G(e^{j\omega}) = e^{-j\omega N/2}\left[1 - H_0(e^{j\omega})\right] = e^{-j\omega N/2} G_0(e^{j\omega}) \tag{7}$$

and $G(z)$ is again a linear phase transfer function of Type 1. Fig. 3 shows typical plots of $H_0(e^{j\omega})$ and $G_0(e^{j\omega})$. Note that the function $G_0(e^{j\omega})$ has double zeros at the frequencies $\omega_1, \omega_2, \cdots, \omega_M$ where $|H(e^{j\omega})|$ is equal to unity. In other words, $G(z)$ has factors of the form

$$G_{1,k}(z) = \left(1 - 2z^{-1}\cos\omega_k + z^{-2}\right)^2 \tag{8}$$

and can, therefore, be written as

$$G(z) = \prod_{k=1}^{M} \left(1 - 2z^{-1}\cos\omega_k + z^{-2}\right)^2 G_2(z) = G_1(z)G_2(z). \tag{9}$$

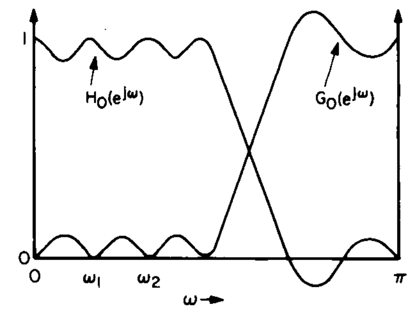

Fig. 3. Typical plots of $H_0(e^{j\omega})$ and $G_0(e^{j\omega})$.

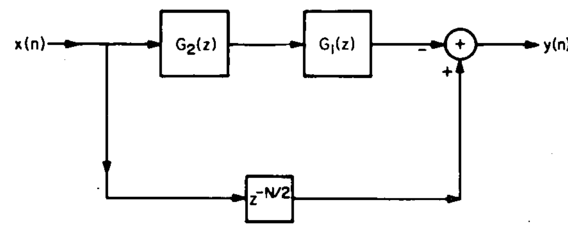

Fig. 4. The FIRBR implementation.

Let us now implement $H(z)$ in the following form:

$$H(z) = z^{-N/2} - G(z) \tag{10}$$

where $G(z)$ is implemented as in (9). Let us consider the effect of quantizing the multiplier coefficients "$2\cos\omega_k$" in (9). Clearly, the sign of $G_0(e^{j\omega})$ is not affected by the quantization, because the zeros of G_0 represented by (8) are all double. In other words, $G_0(e^{j\omega})$ cannot become negative in the passband of $H(z)$. Consequently, $H_0(e^{j\omega})$, which is defined to be

$$H_0(e^{j\omega}) = 1 - G_0(e^{j\omega}) \tag{11}$$

cannot exceed unity in the passband. Thus $H(z)$ is "structurally bounded" with respect to all the multiplier coefficients involved in the implementation of $G_1(z)$. Next, $G_2(z)$ has no zeros on the unit circle, and, therefore, quantization of multipliers in $G_2(z)$ cannot affect the sign of $G_0(e^{j\omega})$. In conclusion, therefore, the implementation shown in Fig. 4 is "structurally bounded" with respect to all the digital multipliers involved. We call these structures "FIRBR" structures.

For completeness of the theoretical argument, let us note that in (9), even though the zeros of $G_2(z)$ are not on the unit circle of the z-plane, they occur in reciprocal pairs, and thus if z_0 is a zero, then so is $1/z_0$. Under extreme

[1] Note that BR functions are well-known in continuous-time network theory [13].

conditions of multiplier quantization, a zero of $G_2(z)$, say, $z_0 = re^{j\theta}$, may move onto the unit circle. This is possible if r is very close to unity. Such zero pairs can then be implemented by combining the factors $(1-2z^{-1}r\cos\theta + r^2z^{-2})$ and $(r^2 - 2z^{-1}r\cos\theta + z^{-2})$ in such a manner that if z_0 moves on to the unit circle because of quantization, then so does $1/z_0$. Thus the zeros (of the quantized implementation) on the unit circle are still double zeros, hence $H(z)$ is still structurally bounded.

Note that $G_2(z)$ is a linear phase FIR filter of degree $N-4M$, and requires at most $N/2-2M+1$ multiplications. $G_1(z)$ itself requires $2M$ multiplications, leading to a total of $N/2+1$ multiplications. The total number of additions involved in implementing $G(z)$ is $4M+N-4M = N$. $H(z)$ is obtained from $G(z)$ by means of an extra addition, as in (10). In summary, the FIRBR implementation requires the same number of multipliers as the direct form, and one adder more than the direct form, which is a negligible overhead.

According to our arguments in Section II, the FIRBR structure shown in Fig. 4 is expected to have low sensitivity. We now proceed with an example to demonstrate this.

IV. An Example

A 34th-order wideband lowpass FIR filter with equiripple passband extending from 0 to 0.8π and equiripple stopband extending from 0.9π to π was designed using the McClellan–Parks algorithm [10]. The resulting transfer function $H(z)$ has maximum magnitude of unity at the following frequencies:

$$\omega_0 = 0.0, \omega_1 = 0.1182\pi, \omega_2 = 0.236\pi, \omega_3 = 0.354\pi,$$
$$\omega_4 = 0.472\pi, \omega_5 = 0.588\pi, \omega_6 = 0.698\pi, \omega_7 = 0.784\pi. \quad (12)$$

Thus $G_1(z)$ becomes

$$G_1(z) = (1-z^{-1})^2 \prod_{k=1}^{7} \left(1-2\cos\omega_k z^{-1} + z^{-2}\right)^2 \quad (13)$$

and has degree equal to 30. $G_2(z)$, therefore, is a fourth-order linear phase filter.

At this point, we have a number of choices available for implementing $G(z)$. Instead of implementing $G(z)$ with all factors of the form of (8) grouped together, we may pick a subset of these and group them together. For example, we could define $G_1(z)$ to be

$$G_1(z) = \left(1-2z^{-1}\cos\omega_1 + z^{-2}\right)^2\left(1-2z^{-1}\cos\omega_3 + z^{-2}\right)^2 \times\left(1-2z^{-1}\cos\omega_5 + z^{-2}\right)^2\left(1-2z^{-1}\cos\omega_7 + z^{-2}\right)^2 \quad (14)$$

instead of as in (13) and redefine $G_2(z)$ accordingly. This flexibility allows us to have a tradeoff between the passband and stopband sensitivities. In the example under consideration, we define G_1 to be as in (14) and implement $H(z)$ in the form shown in Fig. 4.

Fig. 5 shows the passband frequency response of the direct form and FIRBR implementations with 5 bits per multiplier, as compared to the ideal frequency response. In Fig. 6 the entire frequency response with 5 bits per multiplier is shown. It is clear from these implementations that the FIRBR structure has excellent passband sensitivity properties. According to the arguments of Section II, nothing can, in general, be claimed about the stopband sensitivity.

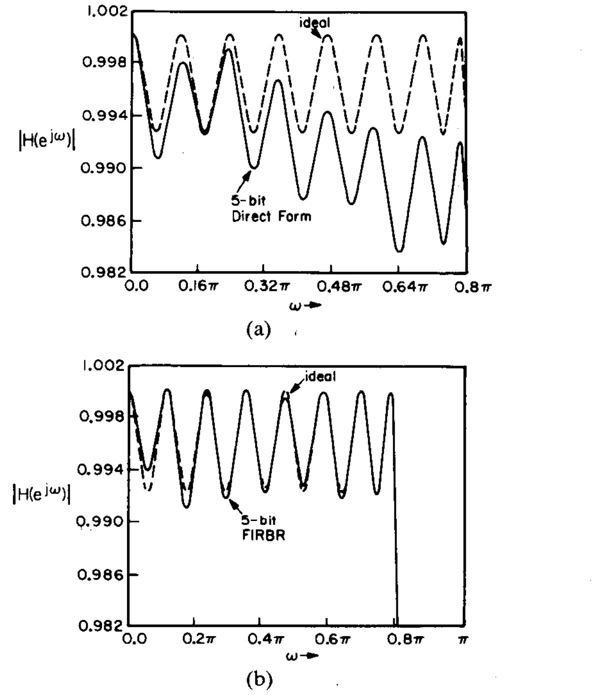

Fig. 5. Passband response of quantized implementations (5 bits per multiplier).

V. Roundoff Noise Properties of the FIRBR Implementation

From Fig. 4 it is clear that the roundoff noise generated by the section implementing $G_2(z)$ has a noise transfer function equal to $G_1(z)$. From the definition of $G_1(z)$ (equation (9)), it is clear that $G_1(z)$ has zeros in the passband of the transfer function $H(z)$. Similarly, the noise generated by the various sections $G_{1,k}(z)$ in the implementation of $G_1(z)$ have certain zeros in their respective noise transfer function, which fall in the passband of $H(z)$. Note that these zeros are precisely among the zeros of sensitivity (equation (3)) in the passband of $H(z)$. Intuitively, we can, therefore, expect the new structure to have low noise. The actual roundoff noise, however, depends upon the section ordering, because this affects the actual scaling and noise-transfer functions. Intuitively, it is advisable to have $G_2(z)$ to the left of $G_1(z)$ so that the zeros of $G_1(z)$ attenuate the noise due to $G_2(z)$. In the numerical example under consideration, there are 24 different orderings of the sections $G_{1,k}(z)$ within $G_1(z)$. There is one that gives the lowest output roundoff noise variance, and the results are as follows, for an l_2-scaling policy:

Structure	Roundoff Noise Gain	
	Quantization Scheme #1	Quantization Scheme #2
FIRBR Structure (lowest noise achievable)	3.94	1.68
Direct-Form Structure	18	1

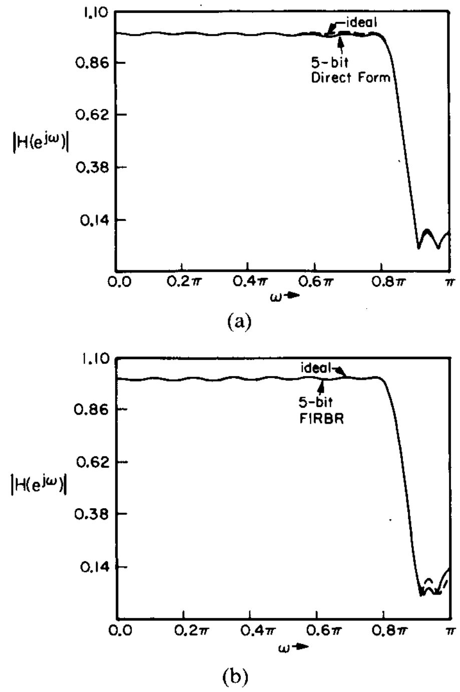

Fig. 6. Overall frequency response (magnitude) of quantized implementations (5 bits per multipler).

Here quantization Scheme #1 corresponds to quantizing after each multiplication. We have 18 multiplications for the direct form implementation (because of mirror-image symmetry in $h(n)$) and therefore under the usual assumptions, the noise gain is 18. The FIRBR structure clearly shows a marked improvement in noise performance. Quantization Scheme #2 corresponds, in the direct form case, to quantizing the signal after performing all additions. For the FIRBR case, quantization Scheme #2 corresponds to quantizing at the end of each section in the cascade of Fig. 4. Quantization Scheme #2 is somewhat expensive to implement, particularly for the direct form [3]. The direct form clearly has a noise gain of unity under such a situation, whereas the FIRBR structure has a noise gain slightly in excess of unity. Quantization Scheme #1 is less expensive to implement, and the FIRBR performance is clearly superior to direct form in this case.

VI. Two-Dimensional FIRBR Filter Implementations

Given a linear phase one-dimensional FIR filter transfer function, one can convert it into a two-dimensional filter by employing the mappings due to McClellan [7]. The mappings are such that each two-dimensional steady-state frequency pair (ω_1, ω_2) corresponds to a frequency ω of the one-dimensional prototype. Thus if the one-dimensional FIR filter is structurally bounded, then so is the two-dimensional version. Let us assume that the impulse response $h(n)$ has been shifted in time, so that the prototype one-dimensional filter is a zero-phase FIR filter. Then each one of the sections $G_{1,k}(z)$ and $G_2(z)$ in Fig. 4 is a zero-phase FIR filter, and we can write

$$G_{1,k}(z) = \left(z + z^{-1} - 2\cos\omega_k\right)^2 \tag{15}$$

whence,

$$G_{1,k}(e^{j\omega}) = 4(\cos\omega - \cos\omega_k)^2 \tag{16}$$

Similarly, $G_2(e^{j\omega})$ can be rewritten in terms of powers of $\cos\omega$. The quantity "$\cos\omega$" can then be replaced with

$$\cos\omega = \sum_{p=0}^{P}\sum_{q=0}^{Q} t(p,q)\cos p\omega_1 \cos q\omega_2 \tag{17}$$

where the choice of P, Q and $t(p,q)$ depends upon the type of two-dimensional filter desired. The transformation of (17) can be directly implemented by replacing the building blocks $(z+z^{-1})/2$ in the one-dimensional implementation with suitable functions $f(z_1, z_2)$ of z_1 and z_2 [11]. In our numerical example, we choose the following special case of (17),

$$\cos\omega = (\cos\omega_1 + \cos\omega_2 + \cos\omega_1\cos\omega_2 - 1)/2. \tag{18}$$

The main point to be noticed is that the two-dimensional filter that results by transforming the FIRBR structure has factors of the form $[f(z_1,z_2) - \cos\omega_k]^2$, which implies that $G(e^{j\omega_1}, e^{j\omega_2})$ (which is the two-dimensional version of $G(e^{j\omega})$) cannot go negative for any (ω_1, ω_2), and, therefore, $H(e^{j\omega_1}, e^{j\omega_2})$, which is the two-dimensional version of $H(e^{j\omega})$ defined by

$$H(z_1,z_2) = 1 - G(z_1,z_2) \tag{19}$$

is implemented in a structurally bounded or passive manner. The low passband sensitivity properties of the one-dimensional FIRBR structure are, therefore, preserved in the two-dimensional version.

As an example of 2-D FIRBR realization, the one-dimensional FIRBR structure considered earlier for the study of sensitivity is transformed into a two-dimensional version by using (18). The same transformation is also applied to the direct form implementation. The resulting two-dimensional direct-form and two-dimensional FIRBR are then implemented with finite precision for multipliers.

Fig. 7 shows the frequency response magnitude for both the direct form and FIRBR structures for 12-bit parameter quantization level. (As pointed out in [11], the two-dimensional direct-form requires higher multiplier precision than the one-dimensional direct form, because of the new set of multiplier coefficients generated by using the "Chehyshev transformation.") The improved passband behavior of the FIRBR structures is clearly in evidence. The figure also shows the response for FIRBR implementation, with 12 bits per multiplier in $G_2(z)$ but only 6 bits for each multiplier in $G_1(z)$. Even with this additional quantization, the behavior of the FIRBR structures is excellent.

VII. Concluding Remarks

In this paper, we have outlined an approach for FIR filter implementations with "passive" or "bounded" structures. The resulting structures (FIRBR-structures) have very low passband sensitivity, and can easily be transformed into two-dimensional low-sensitivity structures. The total number of multiplications and additions in the FIRBR structure is essentially the same as that for the direct form. The FIRBR structure, like the direct form, preserves

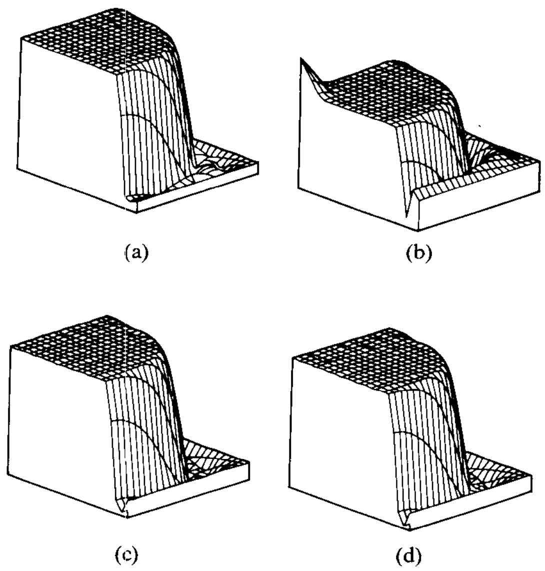

Fig. 7. Frequency response magnitudes for quantized two-dimensional filters.

linear-phase property in spite of the quantization. In addition, the structures have a resemblance to the well-known cascade form. Suitable ordering of the sections in an FIRBR structure leads to low-roundoff noise implementations.

Finally, even though the FIRBR structures are not derived from continuous-time prototype, the explanation for their low-sensitivity behavior (Section II) is analogous to the well-known Orchard's argument [12].

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