

Research Article

Vibration Suppression of a Large Beam Structure Using Tuned Mass Damper and Eddy Current Damping

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For a few decades, various methods to suppress the vibrations of structures have been proposed and exploited. These include passive methods using constrained layer damping (CLD) and active methods using smart materials. However, applying these methods to large structures may not be practical because of weight, material, and actuator constraints. The objective of the present study is to propose and exploit an effective method to suppress the vibration of a large and heavy beam structure with a minimum increase in mass or volume of material. Traditional tuned mass dampers (TMD) are very effective for attenuating structural vibrations; however, they often add substantial mass. Eddy current damping is relatively simple and has excellent performance but is force limited. The proposed method is to apply relatively light-weight TMD to attenuate the vibration of a large beam structure and increase its performance by applying eddy current damping to a TMD. The results show that the present method is simple but effective in suppressing the vibration of a large beam structure without a substantial weight increase.

1. Introduction

The suppression of mechanical and structural vibration has significant applications in engineering fields such as machine tool industries and civil, automotive, and aerospace structures. Over the past few decades much research effort has been applied to vibration suppression of engineering structures and machines. Traditionally, passive methods have been used to attenuate mechanical vibration. The recent advances in digital signal processing and sensors/actuators technology have resulted in substantial effort in using active methods [1]. In addition semiactive methods have filled the gap between passive and active methods. A popular method of passive vibration suppression is the use of constrained layer damping (CLD) treatments using viscoelastic material. CLD can significantly increase the damping of a structure and is a readily available commercial product. The vibration of a beam can also be suppressed by active methods using smart materials like piezoelectric materials. Many researchers have applied these methods to lightweight flexible beam structures. To suppress the vibration of a beam structure, which is large and heavy, very large actuation force is required and these methods may not be available. Active methods using piezoelectric materials may not be successful due to the limitation of actuation force. CLD solutions may require the addition of too much mass. Consequently the

ECD TMD

FIGURE 1: Schematic of magnetically tuned mass damper [19].



FIGURE 2: Schematic of a TMD.

FIGURE 3: Large beam structure used in the present study.



FIGURE 4: Normalized magnitude of the primary structure for various β and r.



FIGURE 5: TMD at tip of beam.

weight and the control cost increase considerably to attenuate the vibration of a large beam structure.

Eddy currents are generated when a moving conductor intersects a stationary magnetic field, or vice versa. The relative motion between the conductor and the magnetic field induces the circulation of the eddy current within the conductor. These circulating eddy currents induce their own magnetic field with the opposite polarity of the applied field that causes a resistive force. These currents dissipate due to the electrical resistance and this force will eventually disappear. Hence, the energy of the oscillating system will be dissipated. Since the resistive force induced by eddy currents is proportional to the relative velocity, the conductor and the magnet can be allowed to function as a form of viscous damping. This eddy current damping may not be much while it was very effective to suppress the vibration of a light flexible beam [2–6].

Sodano and Bae [7] have already presented the good literature review. There have been some applications utilizing eddy currents for vibration suppression [8-19]. Karnopp [8] introduced that linear electrodynamic motors consisting of a copper wire with permanent magnets can be used as an electromechanical damper. Takagi et al. [9] developed numerical analysis method for dynamic characteristics of an elastic thin plate with eddy current damping effect and Lee [10] studied the dynamic stability of conducting beam plates in a transverse magnetic field. Kienholtz et al. [11] introduced an adaptive passive damping system with remotely tunable eddy-current tuned mass dampers for the low-order modes of spacecraft large solar arrays. Larose et al. [12] studied the effectiveness of external means for reducing the oscillations of a full-bridge aeroelastic model using a tuned mass damper (TMD). To reduce the oscillation, they used a TMD that has the adjustable inherent damping provided by an eddy current mechanism. Teshima et al. [13] investigated the effects of an eddy current damper on vibrations associated with superconducting levitation. They showed that the damping in vertical vibrations was about 100 times improved by eddy current dampers when the eddy current damping was employed.

In addition there have been several studies that have investigated the effects of magnetic fields on vibration in cantilever beams. Matsuzaki et al. [14, 15] proposed the concept



FIGURE 6: Natural modes of TMD.

of a new vibration control system in which the vibration of a partially magnetized beam is suppressed by using electromagnetic forces and performed an experimental study to show the effectiveness of their concept. Kwak et al. [2] investigated the effects of an eddy current damper (ECD) on a cantilever beam. Their experimental results showed that an ECD can be an effective device for the vibration suppression of a cantilever beam. Bae et al. [3] developed a theoretical model of an ECD constructed by Kwak et al. [2]. Using this theoretical model, they investigated the damping characteristics of an ECD and simulated the vibration suppression of a cantilever beam with Kwak's ECD. Sodano et al. [4-6] proposed a new concept using the eddy currents induced in a conductive plate to suppress the vibration of a cantilevered beam. Cheng and Oh [16, 17] have studied the multimode vibration suppression using a permanent magnet and the coil with a shunt circuit for a semiactive control. Jung et al. [18] proposed the electromagnetic synchronized switching scheme to enhance the damping characteristics of flexible beam structures subject to dynamic loads. Recently, Bae et al. [19] introduced the concept of magnetically tuned mass damper (mTMD) shown in Figure 1 to improve the damping performance of a conventional TMD by using an eddy

current damping (ECD). They showed that their method could significantly increase the damping effect of the TMD by simulations and experiments if not adequately tuned. Wang et al. [20] derived the theoretical formulation of the ECD in a horizontal TMD and constructed a large-scale horizontal TMD with ECD. They investigated its characteristics experimentally.

The ECD is an effective method for suppressing structural vibrations and it is relatively simple to apply. As previously mentioned it may not be possible to apply the well-known methods like ECD, CLD, and smart materials to the primary structure of the large beam structure because of actuation costs and weight. The objective of the present study is to propose and exploit an effective method to suppress the vibration of a large beam structure, which is large and heavy, such as a gun barrel of a tank without introducing much additional mass.

The key idea of the present study is to apply relatively light-weight TMD to attenuate the vibration of a large beam structure and increase its performance by applying eddy current damping to this TMD. The proposed method is consequently originated from the magnetically tuned mass damper (mTMD) of Bae et al. [19] as shown in Figure 1.



FIGURE 7: Frequency response function of transverse bending displacement.

The design parameters of a TMD are presented based on the parametric study. The vibration analyses of a TMD, a large beam structure, and a beam with a TMD are performed. The results are verified with experiments and the performance of a TMD is discussed to increase the damping performance of a large beam structure. Finally ECD is introduced to the TMD and the damping performance of the proposed method is investigated experimentally.

2. Theoretical Analysis

2.1. Theoretical Modeling of a TMD. The schematic of TMD with damping in both the primary and absorber system is shown in Figure 2. From the previous work [13], the equations of motion are presented as follows:

$$\begin{bmatrix} m_p & 0\\ 0 & m_a \end{bmatrix} \begin{bmatrix} \ddot{x}_p(t)\\ \ddot{x}_a(t) \end{bmatrix} + \begin{bmatrix} c_p + c_a & -c_a\\ -c_a & c_a \end{bmatrix} \begin{bmatrix} \dot{x}_p(t)\\ \dot{x}_a(t) \end{bmatrix} + \begin{bmatrix} k_p + k_a & -k_a\\ -k_a & k_a \end{bmatrix} \begin{bmatrix} x_p(t)\\ x_a(t) \end{bmatrix} = \begin{bmatrix} F_0\\ 0 \end{bmatrix} \sin \omega t.$$
(1)

To solve motion equations of (1), let $F_o \sin \omega t$ be represented in the exponential form by $F_o e^{j\omega t}$ and assume that the steady-state solution can be written as follows:

$$\mathbf{X}(t) = \mathbf{X}e^{j\omega t} = \begin{bmatrix} X_p \\ X_a \end{bmatrix} e^{j\omega t},$$
(2)

where X and X_a are the vibration amplitudes of the primary mass and absorber mass, respectively.

Substituting (2) into (1), the equations of motion can be expressed in

$$\begin{bmatrix} X_p \\ X_a \end{bmatrix} = \frac{1}{\det\left(\mathbf{K} - \omega^2 \mathbf{M} + \omega j \mathbf{C}\right)} \begin{bmatrix} \left(k_a - m_a \omega^2\right) + c_a \omega j & k_a + c_a \omega j \\ k_a + c_a \omega & \left(k_p + k_a - m_a \omega^2\right) + \left(c_p + c_a\right) \omega j \end{bmatrix} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}.$$
 (3)

Assuming that the damping of the primary system c_p can be neglected, (3) can be written in terms of dimensionless ratios as

$$\frac{X_{p}k_{p}}{F_{0}} = \sqrt{\frac{\left(2\zeta r\right)^{2} + \left(r^{2} - \beta^{2}\right)^{2}}{\left(2\zeta r\right)^{2}\left(r^{2} - 1 + \mu r^{2}\right)^{2} + \left[\mu r^{2}\beta^{2} - \left(r^{2} - 1\right)\left(r^{2} - \beta^{2}\right)\right]^{2}},}$$
(4)

where μ is the ratio of the absorber mass to the primary mass $(= m_a/m_p)$, *r* is the ratio of the driving frequency to the primary natural frequency $(= \omega/\omega_p)$, β is the ratio of the decoupled natural frequencies $(= \omega_a/\omega_p)$, and ζ is the ratio of the absorber damping and $2m_a\omega_p$ $(= c_a/2m_a\omega_p)$.

Equation (4) will be used to design the parameters of a TMD and a magnetic TMD. Based on these parameters a TMD and a magnetic TMD will be designed and verified from finite-element method.

2.2. Vibration Analysis of a Large Beam Structure. Prior to determining the parameters of a TMD the dynamic characteristics of a primary structure must be investigated. The large beam structure used in the present study is a gun barrel of a tank as shown in Figure 3. The length and weight of the beam are over 6,000 mm and over 1,300 kg, respectively. Table 1 shows the natural frequencies and mode shapes of the first two modes of the beam for the boundary condition of free-free. The fundamental frequency is 21.5 Hz and its shape is first bending mode (1B).

2.3. Parametric Study on TMD. The normalized magnitude equation of the primary structure in (4) is used to determine the design parameters of TMD. Although the ratio of the absorber mass to the primary mass increases the vibration suppression performance of a TMD there is a weight limitation in the present study. The maximum mass ratio must not exceed 0.02. In the case of $\mu = 0.02$ and $\zeta = 0.005$, the normalized magnitude of the primary structure for various β and *r* is presented in Figure 4. As shown in Figure 4, TMD has the good performance of vibration absorption with $\beta = 0.98$ and this value is given by the Den Hartog equations [21]. There in general exists the optimized damping ratio at which



FIGURE 8: Lowest two mode shapes of a beam with TMD.



FIGURE 9: Experimental setup of bungee test.



FIGURE 10: Experimental setup of TMD.

the performance of the vibration suppression becomes best [19].

2.4. Design and Vibration Analysis of TMD. The mass of TMD which is installed at the tip of the beam is determined by 26 kg

and the mass ratio is 0.0195. Figure 5 shows the schematic of TMD installed at the tip of the beam. TMD consists of four aluminum rods, an aluminum fixed holder, and a steel absorber mass. The absorber mass can move freely through rods. The fundamental frequency of TMD can be adjustable by changing the rod length which is the distance between the absorber mass and the fixed holder. From the results of the previous section the frequency ratio is determined by 0.98. When $\beta = 0.98$ the natural frequency, ω_a , of TMD is determined by 21.2 Hz. Figure 6 shows the vibration analysis results of TMD when the rod length is 140 mm. The boundary condition is fixed-free. The lowest natural frequency and mode shape are 19.8 Hz and a torsion mode, respectively. But this mode can be negligible because it does not have any effect on the bending vibration of the beam. The natural frequency of the first bending mode is 21.1 Hz.

Figure 7 shows the frequency response functions of the transverse bending displacement at the tip of the beam with TMD when the rod length is 140 mm. The boundary condition of the beam is free-free. The amplitude of the beam with



TABLE 1: Natural frequencies and mode shapes of free-free beam.

FIGURE 11: Frequency response functions of TMD.



FIGURE 12: TMD installed at tip of beam.

TMD is much smaller than that of the beam without TMD. Figure 8 shows the mode shapes of the lowest two modes of the beam with TMD. From the vibration analysis results it can be concluded that TMD is well designed.

3. Experimental Results of TMD

3.1. Experimental Setup of Beam and Experimental Results. In the present study the bungee test method was used to perform

FIGURE 13: Frequency response functions of beam with and without TMD.

the vibration test of free-free beam as shown in Figure 9. Eight positions on the beam were selected to measure their accelerations and two accelerometers per each position are used to measure both in-plane and out-of-plane bending motions. Table 2 shows the experimental natural frequencies and damping ratios of the free-free beam. The frequency and damping ratio of the 1st out-of-plane mode are 21.48 Hz and 0.009, respectively, and its mode shape is 1st bending mode. The experimental results as shown in Table 2 are in good agreement with the analytical results. The natural frequencies of in-plane modes are almost the same as those of out-of-plane modes.

3.2. Experimental Setup of TMD and Experimental Results. Figure 10 shows experimental setup of TMD and the accelerations at three points are measured by three accelerometers. Figure 11 shows the frequency response functions of TMD at three points and the natural frequencies are the same as 19.44 Hz when the rod length is 140 mm. To investigate the effects of gravity the vibration tests when TMD is placed vertically were performed. The frequency was 19.40 Hz



FIGURE 14: Schematic of mTMD.



FIGURE 15: Experimental setup of mTMD and arrangements of magnets.

and the authors concluded that gravity was negligible. The experimental results show that the natural frequency of TMD obtained from experiment is less than the predicted frequency due to modeling uncertainty of TMD structure. Hence the rod length is determined by 130 mm and the natural frequency and the frequency ratio are 21.3 Hz and 0.99, respectively.

3.3. Experimental Results of TMD Performance. Experimental setup of beam with TMD is the same as that of beam only. Figure 12 shows the TMD installed at the tip of the beam and the vibration tests were performed for various rod lengths. Table 3 shows the experimental results for various rod lengths and Figure 13 shows the frequency response functions. When the rod length is between 130 mm and 140 mm the performance of TMD becomes best. The increase of damping due to TMD is about 6 dB while the increase of mass is only 1.95%.

4. Experimental Results of Magnetically TMD

4.1. Experimental Results of Magnetically TMD. Figure 14 shows the schematic of magnetically TMD (mTMD). Different from TMD the tip mass is consisting of steel part and copper part while the total mass is the same. Copper is a conductive material and eddy currents are generated due to the relative motion between copper ring and permanent magnets [13]. Figure 15 shows the experimental setup of mTMD and two different arrangements of magnets. In Case 1 the eddy current damping due to two magnets in horizontal plane is much smaller than that in vertical plane because mTMD moves in vertical plane. In Case 2 the eddy current damping due to four magnets is almost the same. The gap between the copper ring and magnets is about 7 mm. Figure 16 shows the frequency response functions of mTMD. The damping ratios of TMD without ECD, Case 1, and Case 2 are 0.009, 0.025, and 0.032, respectively. Due to the presence of ECD the damping ratio of mTMD increases considerably.

Mode	Frequency (Hz)	Damping ratio	Analysis (Hz)	Mode shape
1	21.48	0.009	21.50	1st bending
2	64.44	0.0038	65.00	2nd bending
3	129.7	0.002	130.5	3rd bending

TABLE 2: Frequencies and damping ratios of free-free beam.

TABLE 3: Frequencies and magnitudes of beam with TMD.

Rod length (mm)	1st mode		2nd mode	
Rou length (mm)	Frequency (Hz)	Magnitude (dB)	Frequency (Hz)	Magnitude (dB)
120	18.2	-28.4	30.5	-32.6
130	17.9	-28.8	28.6	-28.4
140	17.1	-37.9	25.8	-30.0
150	16.6	-42.3	24.9	-24.9

TABLE 4: Comparison of damping ratios.

	Damping ratio
Without TMD	0.009
With TMD	0.025
mTMD	
Case 1	0.033
Case 2	0.038
Case 3	0.055

For vertical movements the magnet arrangement of Case 2 has better damping performance than Case 1.

4.2. Experimental Results of Beam with mTMD. The vibration test of the large beam structure with mTMD was performed for three kinds of magnet arrangements. Figure 17 shows frequency response functions for three different magnet arrangements of mTMD. Case 3 combines magnet arrangements of Case 1 and Case 2 shown in Figure 15. Table 4 shows the damping ratios of the beam with mTMD. These values are determined by the logarithmic decrement method [22]. The damping ratios of the beam with mTMD are greater than those of the beams without and with TMD. Particularly, the damping ratio of Case 3 is 6.1 times of without TMD and 2.2 times of with TMD. It can be concluded that mTMD can be excellent method to attenuate the vibration of a large beam structure.

5. Conclusions

The passive, semipassive, and active methods to suppress structural vibrations are well known. However, these methods have limitations which may render them effective in applications involving large structures. The present study proposed an effective method to suppress the vibrations of a large beam structure and exploit its performance. We apply a light-weight TMD to attenuate the vibration of a large beam structure and increase its performance by applying eddy current damping to this TMD. The parameters of a TMD are designed based on the parametric study of the theoretical model. The vibration analyses of a TMD, a large beam structure, and a beam with a TMD are performed. The analytic results are verified with experimental results. The increase of damping due to TMD is about 6 dB when the increase of mass is only 1.95%.

ECD is introduced to increase the damping performance of TMD. mTMD, whose weight is the same as TMD, is designed, constructed, and tested. The vibrational tests of a large beam structure with mTMD are performed. The damping ratio of the present method is 2.2 times about that of a large beam structure with TMD. And the present damping ratio is 6.1 times about that of a large beam structure without TMD. Hence it can be concluded that the present method is an effective method to suppress the vibration of a large beam structure without much weight increase.

Nomenclature

- C: Damping matrix
- F_o : External force
- *k*: Spring coefficient of system
- K: Stiffness matrix
- *m*: Mass of system
- M: Mass matrix
- r: Standard frequency ratio of system
- *X*: Vibration amplitude of mass
- β : Natural frequency ratio of system
- μ : Mass ratio of system
- ς : Damping ratio of system.

Subscripts

- p: Primary system
- a: Absorber system.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.



FIGURE 16: Frequency response functions of mTMD.



FIGURE 17: Frequency response functions of large beam structure with mTMD.

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