Vibration suppression with fractional-order $PI^{\lambda}D^{\mu}$ controller

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Abstract — **Cantilever beams have an important role in day to day life in bridges, towers, buildings and aircraft wings, making active vibration suppression a highly researched field. The purpose of this paper is to detail the design of fractional order PID controllers for smart beams. A novel tuning procedure is proposed based on solving a set of nonlinear complex equations that directly aim at reducing the resonant peak. The control parameters are computed through optimization techniques, making sure that the best ones are chosen. The practical stand was realized using magnet-coil approach and not piezoelectric actuators. The experimentally obtained vibration results prove that fractional order PID controllers can be used in practice to significantly reduce the amplitude and settling time of the vibrating system.**

Keywords-component; PID Controller; Fractional Calculus; Controller Optimization; Vibration Suppresion

I. INTRODUCTION

The most common current fixed aircraft wing design is the cantilever beam, pioneered by Hugo Junkers in 1915. All types of aircraft have the same basic wing construction made of metal. The wing of an airplane has to maintain its shape under extreme stress in order to hold the aerodynamic shape of the plane. Low frequency vibration is normal for all aircrafts, each plane having a unique signature vibration caused by engine operation, speed brakes, landing gear or normal airflow over the surfaces. This is normally perceived as background noise. Unwanted vibrations, with considerably larger magnitudes are caused by turbulences and cause passenger discomfort or even fatigue failure of the aircraft [1]. This justifies the need of advanced controller usage. The closed loop performance of smart structures depends considerably upon the choice of the control algorithm [2]. Previous research on active vibration suppression in smart beams include direct proportional feedback, constant gain velocity feedback and constant amplitude velocity feedback control, optimal control strategies [3], [4], [5], [6]. One of the major issues associated with these control strategies is that they are highly sensitive to modeling errors and require an exact mathematical model of the smart beam [2]. A better choice from this point of view consists in the robust control strategies due to their intrinsic ability of ensuring a robust closed loop behavior, despite modeling uncertainties and estimation errors [7].

Fractional order controllers, more accurately "non-integer-order" controllers [8], are a design alternative for traditional controllers. The derivative and integral orders of a fractional controller are real numbers, usually between 0 and 1. The main advantages are better characterization of the system and increased flexibility. The fractional order $PI^{\lambda}D^{\mu}$ is considered to enhance the closed loop performance, being able to meet more performance specifications, as well as increased robustness, as compared to the classical integer order PID [9], [10]. For implementation purposes, the fractional order controller has to be firstly approximated with an integer transfer function with the same behavior, within a specified frequency range, using approximation methods like continued fraction expansion (CFE), Matsuda, Oustaloup filter and Modified Oustaloup filter [8], [10], to name just a few.

The practical stand used in [11] had a cantilever aluminum beam represented by a 6^{th} order transfer function and 8 piezoelectric patches acting both as a sensor and an actuator independently. In this study, the practical stand uses a spherical magnet - coil approach [12] instead of piezoelectric actuators. The system is characterized in open loop by second order transfer function.

The present paper presents the tuning of a fractional order $PI^{\lambda}D^{\mu}$ controller designed to suppress the vibrations that may occur in a smart beam. The controller is tuned using a novel tuning procedure based on lowering the resonant peak of the system using optimization techniques. The fractional order controller that is obtained is implemented and tested on the experimental unit.

The System Identification, Optimization and FOMCON toolboxes are used for identification, optimization and fractional order transfer functions. The fractional order $PI^{\lambda}D^{\mu}$ controller is approximated to integer order using the Modified Oustaloup Filter [13].The controller is discretized using bilinear transformation with sampling time $T_s=10^{-3}$ seconds.

II. VIBRATION ATTENUATION USING ACTIVE CONTROL

There are a few research papers that deal with active vibration suppression using the theory of fractional calculus, but the tuning procedure adopted is not directly addressing the problem of suppressing the resonance peaks [14], [11]. In fact, in these previous research papers, the authors analyze the performance of fractional order PD controller using different

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orders of Continued Fraction Expansion (CFE) approximations.

In contrast to previous research regarding fractional order controllers, the present paper proposes a novel tuning procedure for these types of controllers for smart beam vibration suppression. This novel tuning approach is based on solving a set of nonlinear complex equations that directly aim at reducing the resonant peak, rather than the very simple tuning procedure used in [11], which is based on a trial and error method for selecting the optimal fractional order controller parameters. A similar approach to the one proposed in this paper has been used for the tuning of a fractional order PD controller [2]. In this paper, a fractional order PID controller is used instead. Additionally, the proposed fractional order PID controller is also tested experimentally, rather than considering solely Matlab simulations as in [2].

One of the main advantages of the proposed method is that the fractional-order controller can be tuned in a simple manner by analyzing the Bode diagram of the uncompensated system. Certain values for the magnitude of the system are imposed with the purpose of lowering the resonant peak and making the magnitude line smoother in the frequency range of interest.

The frequency domain transfer function for the fractional order $PI^{\lambda}D^{\mu}$ controller is:

$$C_{FO-PID}(j\omega) = k_p \cdot [1 + k_i \cdot \frac{1}{(j\omega)^{\lambda}} + k_d \cdot (j\omega)^{\mu}] \qquad (1)$$

For λ=1 and $\mu = 1$, Equation (1) describes the transfer function of the integer order PID controller, while for $\lambda \in (0,1)$ and $\mu \in (0,1)$ the equation describes the fractional order PID controller. On the Bode magnitude plot, an integrator 1/s introduces a fixed slope of -20 dB/decade while a differentiator s introduces a fixed slope of +20 dB/decade. On the other hand, a fractional order integrator $1/s^{\lambda}$ and a fractional order differentiator s^{μ} having $\lambda, \mu \in (0,1)$ introduces a variable slope between (-20, 0) dB/decade, (0, +20) dB/decade respectively. This means that the fractional order controller has increased flexibility, honoring the imposed constraints more accurately.

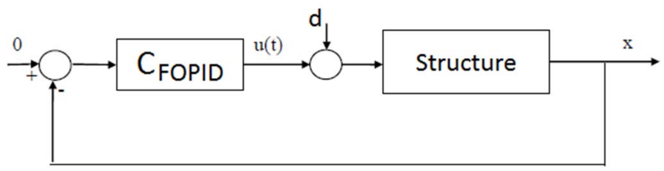

Fig. 1. Schematic diagram of the fractional order PID applied on the structure

The closed loop system with the fractional order PID controller is given in Fig. **1**. The measured structural displacement is given to the controller and it generates the control force for the actuator. Since the purpose of the controller is vibration attenuation, the reference position will be kept at 0. The controller will then treat any excitation as a disturbance and will try to reject it.

Starting from the main purpose of the controller: lower the resonant peak, it is clear that a proportional effect is needed to lower the magnitude. By using a simple proportional controller, the resonant peak will be lowered, but the slope of the magnitude won't be smoother, it will be the same. Also, knowing that the controller has to successfully reject disturbances, it is clear that the integral effect is absolutely necessary. From the Bode diagram point of view, a PI type controller increases the gain at low frequencies, without affecting the gain near and above the crossover frequency. This means that, a PI controller will help honor the constraints imposed for low frequencies, but it will only honor half of the constraints. In order to honor the other half, a derivative effect is needed since the differentiator increases the magnitude at higher frequencies.

The transfer function of the controller $C_{FO\text{-}PID}$ in trigonometric form gives:

$$C_{FO-PID}(j\omega) = k_p \cdot \Big[1 + k_i\omega^{-\lambda}cos\frac{\pi\lambda}{2} - jk_i\omega^{-\lambda}sin\frac{\pi\lambda}{2} + k_d\omega^{\mu}cos\frac{\pi\mu}{2} + jk_d\omega^{\mu}sin\frac{\pi\mu}{2}\Big] \qquad (2)$$

and writing it as a sum of its real and imaginary parts we obtain:

$$C_{FO-PID}(j\omega) = A(\omega) + jB(\omega)$$
$$A(\omega) = k_p + k_i\omega^{-\lambda}cos\frac{\pi\lambda}{2} + k_d\omega^{\mu}cos\frac{\pi\mu}{2} \qquad (3)$$
$$B(\omega) = k_d\omega^{\mu}sin\frac{\pi\mu}{2} - k_i\omega^{-\lambda}sin\frac{\pi\lambda}{2}$$

Supposing that the vibration attenuation is done on a process described by a second order transfer function noted H_f:

$$H_f(j\omega) = \frac{k \cdot {\omega_n}^2}{(j\omega)^2 + 2\xi\omega_n s(j\omega) + {\omega_n}^2} \qquad (4)$$

and expressing it in the same manner as the transfer function of the controller we obtain:

$$H_f(j\omega) = ReP(\omega) + jImP(\omega)$$
$$ReP(\omega) = \frac{k \cdot {\omega_n}^2({\omega_n}^2 - \omega^2)}{({\omega_n}^2 - \omega^2)^2 + (2\xi\omega_n \cdot \omega)^2} \qquad (5)$$
$$ImP(\omega) = \frac{2k{\omega_n}^3 \cdot \omega}{({\omega_n}^2 - \omega^2)^2 + (2\xi\omega_n \cdot \omega)^2}$$

The magnitude of the closed loop formula of the compensated system is:

$$|H_o(j\omega)| = \frac{|H_f(j\omega) \cdot C_{FO-PID}(j\omega)|}{|1 + H_f(j\omega) \cdot C_{FO-PID}(j\omega)|} \qquad (6)$$

Rewriting Equation (6) by taking into consideration Equations (3) and (5), the closed loop magnitude is obtained as:

$$|H_o(j\omega)| = \frac{\sqrt{(ReP^2 + ImP^2)(A^2 + B^2)}}{\sqrt{1 + (ReP^2 + ImP^2)(A^2 + B^2) + 2AReP - 2BImP}} \qquad (7)$$

In order to impose the constraints, a frequency range of interest that includes the resonant peak must be chosen. Noting the lower bound of the frequency range with *a*, the resonant

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frequency with b and the upper bound with c the following constraints are obtained:

$$\begin{cases} |H_o(j\omega)| \leq x\, dB, & \omega = a\, rad/s \\ |H_o(j\omega_r)| < y\, dB, & \omega_r = b\, rad/s \\ |H_o(j\omega)| \leq z\, dB, & \omega = c\, rad/s \end{cases} \quad (8)$$

Equation (8) has three imposed constraints, but the fractional order $PI^{\lambda}D^{\mu}$ has five parameters that need to be tuned: k_p, k_i, k_d, λ and μ. As a result, two more constraints will be imposed in the optimization algorithm, both chosen close to the upper and lower bounds.

The optimization is realized replacing Equation (7) in Equation (8) and applying the *fmincon* optimization function in MatLAB. Also, the values of λ and μ should be restricted to be greater than 0 in order to obtain only PID controllers. Note that the results that yield $\lambda, \mu = 1$ represent an integer order PID, while in the case of $\lambda, \mu \in (0,1)$ is obtained a fractional order PID controller.

III. CASE STUDY: SMART BEAM

A. Description of the stand

The stand is built from Plexiglas with the exception of the beam which is made of Polycarbonate, material less breakable than Plexiglas. The dimensions of the beam are 240 × 40 × 3 mm. However, 40 mm from the length of the beam are used to secure it to the stand, leaving 200 mm to vibrate.

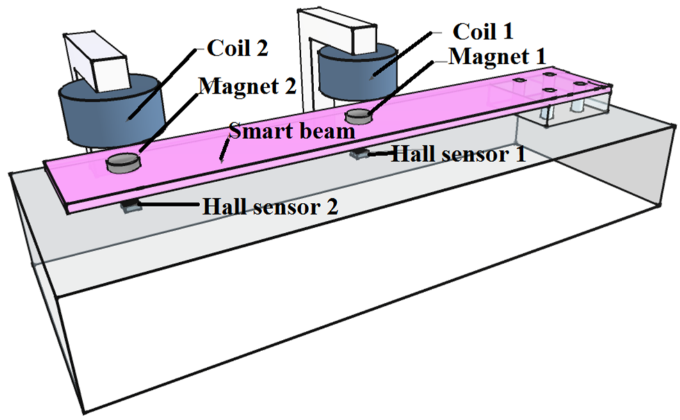

Fig. 2. 3D model of the practical stand

The stand has two magnets and two coils, but only Coil 1 and Magnet 1 are used for vibration suppression. The two coils are identical 24V direct current solenoids with aluminum cores. The aluminum core is preferred in favor of the iron one for a smaller magnetic permeability because the vibration suppression is realized through permanent magnetic disks. An important remark is that the coils can receive between 0 and 10V and only attract the magnet without having the ability to reject them.

The magnets were hot - pressed 2mm into the beam in order to avoid glue which can easily fail under stress. The Neodymium – Iron – Boron permanent magnetic disks have 10mm diameter, 5mm height, 30 grams weight and 1 kg strength. A 3D model of the stand can be seen in Fig. **2**.

By varying the current/voltage that enters the coil, the magnetic field generated by the solenoid is modified, thus the position of the magnetic disk under it also varies. The magnetic disk movement creates beam displacement. A vertical position sensor is used to measure the displacement, namely the Honeywell SS495A Miniature Ratiometric Linear (MRL) Hall Effect sensor.

B. Experimental setup

The experimental setup used to analyze and control the vibration was developed at the Technical University of Cluj-Napoca and is depicted in Figures 3 and 4. The CompactRIO™ 9014 controller manufactured by the National Instruments is used as the main control and acquisition unit. The IO module NI 9215 makes possible reading data from the sensors, while the NI 9263 is used to control the coils. The control is realized using LabVIEW ™.

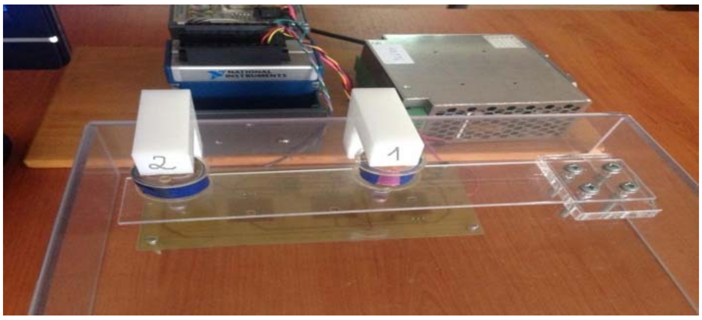

Fig. 3 – Laboratory vibration stand

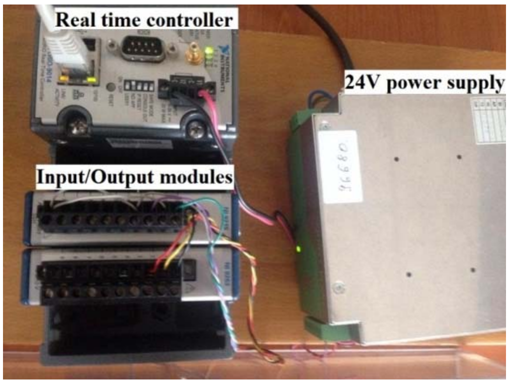

Fig. 4 - Experimental setup used to analyze and control

C. Modeling the system

The system identification of the smart beam was conducted experimentally. Three transfer functions were computed of order two (Fig. **5**), six and ten (Fig. **6**) using the system's impulse response. Due to delays, in order to physically obtain the impulse effect, the coil is given a 10V step input for 35ms.

For the sixth and tenth order transfer functions, MatLAB's System Identification Toolbox is used. In the case of the second order transfer function, the toolbox doesn't find any approximation. The second order system is approximated using the general second order form:

$$H_f(s) = \frac{k \cdot {\omega_n}^2}{s^2 + 2\xi\omega_n s + {\omega_n}^2} \quad (9)$$

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By giving a swept sine input with frequencies between 5 and 60 Hz, the natural frequency was experimentally determined at 15 Hz. Transforming the obtained frequency into rad/s we obtain ω_n =95. The damping factor, ξ, is determined using the logarithmic decrement formulas and the value obtained is 0.01.

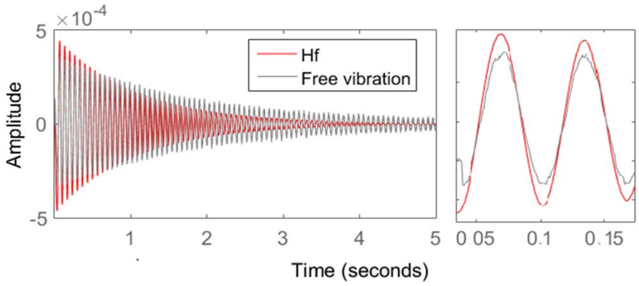

Fig. 5 - Second order transfer function plot compared to experimental data

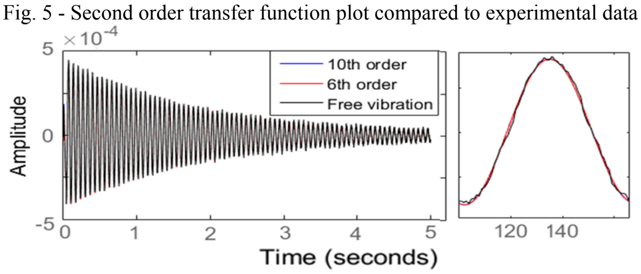

Fig. 6 – 6th and 10th order transfer functions plot compared to experimental data

As it can be seen in Fig. **6**, the 6th and 10th order transfer functions are almost a perfect match over the experimental data. Fig. **65** shows that the second order fit is less accurate than the 6th and 10th order ones, but still satisfactory. Furthermore, the second order transfer function is chosen to represent the plant because of its much simpler form and satisfactory fit:

$$Hf(s) = \frac{-0.2121}{s^2 + 1.9s + 9025} \tag{10}$$

The resonant peak has a magnitude of -56.8 dB at frequency 95 rad/s, as can be seen on the Bode magnitude plot of the second order transfer function given in Fig. **7**.

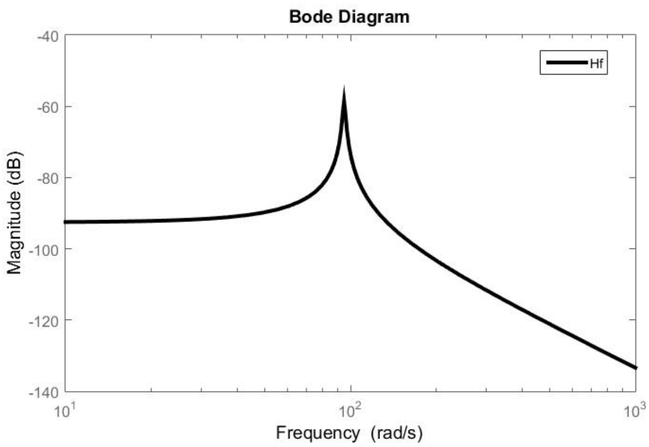

Fig. 7 – Bode magnitude plot of H_f

IV. Tuning and Experimental Results

The controllers were computed based on the frequency response magnitude of the transfer function. The resonant peak has a magnitude of -56.8 dB at frequency 95 rad/s. The controller is tuned with the purpose of lowering the resonant peak and making the magnitude line smoother between 10 and 10^3 rad/s which is the frequency range of interest.

The equations needed for optimization are deduced by imposing the closed loop (noted H_o) frequency response to have the value of the magnitude at frequency ω_n =95 less than -56.8 dB, while keeping the actual or lowering the values at frequencies 10 and 10^3. The system is:

$$\begin{cases} |H_o(j\omega)| \le -92\,dB, & \omega = 10\,rad.s \\ |H_o(j\omega)| \le -95\,dB, & \omega = 34\,rad/s \\ |H_o(j\omega)| < -80\,dB, & \omega_r = 95\,rad/s \\ |H_o(j\omega)| \le -115\,dB, & \omega = 294\,rad/s \\ |H_o(j\omega)| \le -133\,dB, & \omega = 10^3\,rad/s \end{cases} \tag{11}$$

In order to reduce computation time and obtain better results it is strongly recommended to reduce the value of the resonant peak in the inequality constraint. A new resonant peak close to the actual one (-56.8 dB) is far from ideal since it wouldn't bring a significant improvement. This is the reason the value of the resonant peak has been chosen at -80 dB. The ideal result consists in a value of the magnitude peak at the resonant frequency below the imposed value, which will result in less vibrations of the smart beam under active control.

Using the constraints from Equation (11), the equation of the closed loop magnitude from Equation (7) and applying the *fmincon* function in MatLAB, the following fractional order PID controller parameters were obtained: k_p=0.0288; k_i=1.66; k_d=0.0039; λ=0.64; μ=0.59. The controller was then implemented on the experimental stand. The Bode magnitude diagram of the closed loop system with the fractional order PI controller can be seen in Fig. **8**.

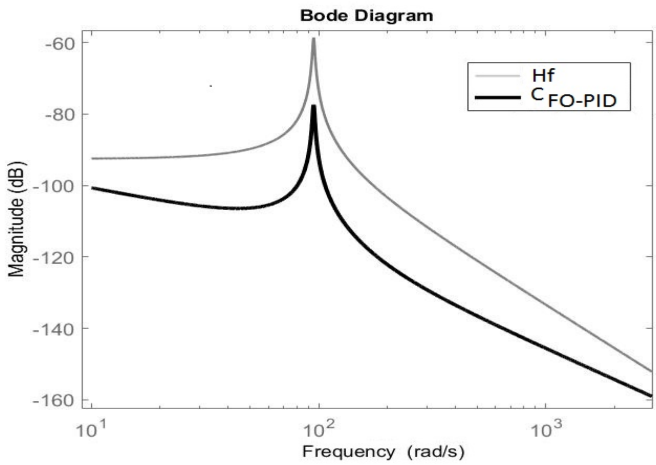

Fig. 8 – Bode diagram of the smart beam (H_f) and of the closed loop system with the designed fractional order PID controller ($C_{FO\text{-}PID}$)

All tests were performed on impulse perturbations with maximum allowed amplitude by the construction of the stand. Because the input signal is not filtered, a simple filter is used on the command signal consisting of a moving average of the last 30 values. This introduces a subtle delay in the command signal, but filters the command efficiently.

In order to practically implement the fractional order controller, the fractional order integrator is approximated to an integer order transfer function using the Modified Oustaloup Filter and discretized using the bilinear transformation with the sampling time $T_s=10^{-3}$ s.

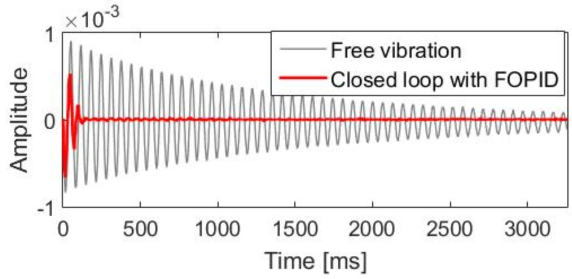

Fig. 9 – Experimental results of the impulse responses of the smart beam: free vibration and active fractional order PID control

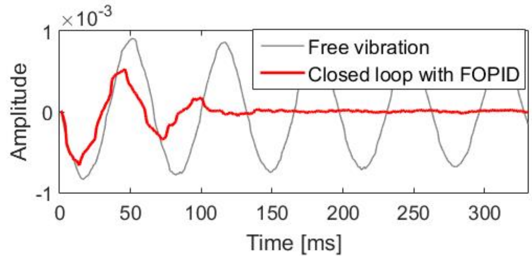

Fig. 10 – Zoomed impulse response of the smart beam: free vibration and active fractional order PID control

As can be seen in Figures 9 and 10 the closed loop exhibits a settling time between 100 and 150ms (approx. 125ms). In terms of statistics, the controller stabilizes the system in 3% of the free vibration's settling time (4120ms). In conclusion, the settling time is improved by 97%.

In the experiment presented above, the amplitude of the impulse disturbance was the maximum amplitude allowed by the construction of the stand. Multiple tests were performed and the improvement remained the same.

V. CONCLUSIONS

The results obtained during the study prove experimentally the superiority of fractional order controllers and that optimization techniques can be efficiently used to compute controller parameters.

The use of the second order transfer function to describe the plant significantly simplified calculus. If the 6^{th} order transfer function, or even the 10^{th} order one, had been chosen to describe the smart beam dynamics, the calculus volume would have increased exponentially.

The study also shows that good results can be used in active vibration suppression without expensive equipment. Also, the cantilever beam can be accurately described by a second order transfer function. In addition, compared to the study realized by [11], [14] briefly presented in the introduction, the results obtained through second order transfer function approximation of the plant and one coil – magnet actuator are as good as results obtained by 6^{th} order model of the plant and eight piezoelectric actuators.

The obtained controllers can be successfully implemented in real-life applications where they could make a visible difference in increasing safety for the passengers of an airplane, construction workers on a bridge, spectators on a stadium, swimmers in a suspended pool or residents of a cantilever home during seismic activity and extreme windy conditions.

As future research, the study can be easily improved by using piezoelectric sensors and actuators on the cantilever beam and comparing the results obtained on exactly the same stand, material and beam length.

A control algorithm can be implemented on both coils simultaneously, making them work together to suppress vibration. Another future plan is to characterize the plant by fractional order transfer functions and to perform robustness tests by adding weight to the free end of the beam.

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