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Vibrations of viscoplastic under impact load

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Abstract

In the present paper theoretical and experimental investigations are made to verify viscoplastic theories applied to thin-walled structures. The vibrations of plates are studied under the assumption of small strains and moderate rotations theoretically and numerically by using constitutive models of Chaboche and Bodner-Partom. The numerical results are compared with experiments.

1 Introduction

In the literature different attempts have been made to study the validity of plastic and viscoplastic models if applied to structural dynamic problems. Structures were subjected to shock waves and detonations and the deformations were compared to calculations, in order to develop theoretical models as simple as possible yielding correlations with experiments. Florence [9] considered rigid-plastic material behaviour applied to impulsively loaded plates. The experiments were performed with sheet explosives; final deformations were measured and compared to numerical simulations. Discorrelations were explained by an insufficient plate theory, not taking membrane forces into account. Wierzbicki et al. [20] modified the constitutive model by using a viscoplastic law for small and a rigid-plastic law for large displacements. Batra et al. [2] applied a theory for impulsively loaded plates including membrane forces and bending moments but neglected shear deformations orthogonal to the midsurface of the plate. Also strain rate sensivity is ignored. Idczak [10] used rigid-viscoplastic laws for the simulation and compared the results with measured deformations of plates caused by shock waves. Argyris [1] presented a shell model with an elasto-plastic hardening law and analysed deformations of shells caused by detonations of sheet explosives. Good correlations to the experimentally observed dynamic response of shells was found. These articles showed, that it is necessary to use viscoplastic laws for rate-sensitive materials. Progress in the field of such constitutive models was made e.g. by Cormeau [7], who studied shells by using solid finite elements and a von-Mises flow rule. Kollmann and Bergmann [14] used hybrid strain finite elements based on a geometrically linear shell theory developed by Kollmann & Mukherjee [13]. Kłosowski et al. [11,12] and Woznica et al. [21,22] applied models of Chaboche and Bodner-Partom and made comparitive studies of structures under various types of impulsive loads. They found that the predicted response of structures depends strongly on the constitutive model. To estimate the range of applicability of each model, the comparison to experiments is necessary. Lindholm et al. [16] compared viscoplastic models of Bodner-Partom and Walker with uniaxial tensile, creep and cyclic tests at different temperatures and obtained special ranges of applicability for the different laws.

The purpose of the present study is to simulate the dynamic response of plates with a nonlinear shell model and constitutive laws for elasto-viscoplasticity with nonlinear hardening and to compare it with experiments. The aim is to determine the ranges of validity of each model by pointing out the correlations to experiments. In chapter 2 we discuss the Chaboche model, its extension for high strain rates, and the Bodner-Partom model, and give also a short description on the shell theory. In the experiments we measure the pressure and the deflection of the plate as a function of time, i.e. we record the entire dynamic structural response and not only the final displacements. The used equipment is described in chapter 3. The comparison in chapter 4 shows the ranges of applicability of the constitutive models.

2 Theoretical model

2.1 Constitutive equations

2.1.1 Chaboche model

The Chaboche model, see e.g. [5], is expressed by the following equations:

$$\dot{\epsilon}^p_{ij} = \frac{3}{2}\dot{p}\frac{\sigma'_{ij} - X'_{ij}}{J_2(\sigma'_{rs} - X'_{rs})}; \; \dot{p} = \left\langle \frac{J_2(\sigma'_{rs} - X'_{rs}) - R - k}{K} \right\rangle^n \tag{1}$$

$$\dot{X}_{ij} = \frac{2}{3}a\dot{\epsilon}^p_{ij} - cX_{ij}\dot{p}; \; \dot{R} = b\,(R_1 - R)\,\dot{p}, \tag{2}$$

where ϵ^p_{ij}, $\dot{p}$, σ_{ij}, X_{ij}, R, k, $(\,)'$, $\dot{(\,)}$, $J_2(\,)$ denote the plastic strain, accumulated plastic strain rate, stress tensor, backstress tensor, isotropic hardening, yield limit, deviatoric part, derivation with respect to the time, second invariant, a, b, c, n, K, R_1 are material parameters. If kinematic hardening is assumed, the following parameters are obtained:

$$E = 198000\frac{N}{mm^2}; a = 2500\frac{N}{mm^2}; b = 20.3; K = 63.12\frac{N}{mm^2}; n = 4.22$$

Here and in the following, the material parameters of the different models have been determined by uniaxial tension tests with steel specimens of 2mm thickness at seven different strain rates, beginning with a quasi-static test with a plastic strain rate of $\dot{\epsilon}_p = 1.71 \cdot 10^{-5}\frac{1}{s}$ until to $\dot{\epsilon}_p = 1.45 \cdot 10^{-1}\frac{1}{s}$.

2.1.2 Extended Chaboche model for high strain rates

This model was developed by Chaboche [4]. Its purpose is to account for the strain-dependent behaviour at low strain rates, i.e. the increase of the overstress σ_v, and the strain-independent behaviour at high strain rates, if the overstress is saturated. The evolution equation $\dot{p}$ then reads

$$\dot{p} = \left\langle \frac{J_2(\sigma'_{rs} - X'_{rs}) - R - k}{K} \right\rangle^N e^{\alpha\left(\frac{J_2(\sigma'_{rs} - X'_{rs}) - R - k}{K}\right)^{N+1}} . \tag{3}$$

The viscous exponent N is now a function of the overstress, see eqn (4), because the exponent must increase with the strain rate to keep the overstress constant, controlled by a new parameter α, see Chaboche [4]:

$$N = \frac{\sigma_v}{\dot{p}}\frac{d\dot{p}}{d\sigma_v} = f(\sigma_v) = n + \alpha(n+1)\left(\frac{\sigma_v}{K}\right)^{n+1} \tag{4}$$

For low strain rates N=n and eqn (2) remains unchanged. We verify, if the assumption of a saturation of the overstress is justified by comparing the present material behaviour to tension tests of aluminium, which is almost assumed to be rate-independent. The increase of the overstress of steel at strain rates higher than $\dot{\epsilon}_p = 10^{-2}\frac{1}{s}$ was smaller than observed in the aluminium tests. So the assumption of the saturation is justified and tension tests at higher strain rates have not been performed. N, K are both viscous parameters, but in eqn (4) only N is a function of the overstress. Therefore one has to choose K for a special strain rate interval restricting the validity of the material law to a special range of strain rates. The following parameters are obtained:

$$\alpha = 2912.89 \; for \; K = 703.14\frac{N}{mm^2} \; for \; low \; strain \; rates,$$

$$\alpha = 4.14 \; for \; K = 63.12\frac{N}{mm^2} \; for \; high \; strain \; rates.$$

2.1.3 Bodner-Partom model

The Bodner-Partom law does not refer to a yield surface; in this theory each point in the stress space belongs to a viscoplastic state, see [3]:

$$\dot{\epsilon}^p_{ij} = \frac{3}{2}\dot{p}\frac{\sigma_{ij}}{J_2(\sigma_{rs})}; \; \dot{p} = \frac{2}{3}\sqrt{3}D_0 e^{-\frac{1}{2}\left(\frac{R+D}{J_2(\sigma_{ij})}\right)^{2n}\frac{n+1}{n}} \tag{5}$$

$$D = X_{ij}\frac{\sigma_{ij}}{J_2(\sigma_{rs})}; \; \dot{W}_p = \sigma_{ij}\dot{\epsilon}^p_{ij} \tag{6}$$

$$\dot{R} = m_1(R_1 - R)\dot{W}_p; \; \dot{X}_{ij} = m_2\left(\frac{3}{2}D_1\frac{\sigma_{ij}}{J_2(\sigma_{ij})} - X_{ij}\right)\dot{W}_p. \tag{7}$$

Here m_1, m_2, R_1, D_1, D_0, n are material parameters to be determined by tension tests. Using a method described by Chan et al. [6] and Lindholm et al. [16] we obtain

$$R_0 = 167.88\frac{N}{mm^2}, \; R_1 = 260.25\frac{N}{mm^2}, \; D_1 = 402.70\frac{N}{mm^2},$$

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$$m_1 = 0.0462 \frac{mm^2}{N}, \; m_2 = 0.1214 \frac{mm^2}{N}, \; D_0 = 10^6 \frac{1}{s}, \; n = 2.02.$$

2.2 Shell model

The shell model used in this paper is described in detail by Schmidt et al. [18,19], Palmerio et al. [17]. It assumes small strains and moderate rotations and is based on the hypothesis of first-order shear deformation theory neglecting thickness change, i.e. the components of the displacement vector in the shell space are described by

$$v_\alpha = \overset{0}{v}_\alpha + \theta \overset{1}{v}_\alpha; \; v_3 = \overset{0}{v}_3, \; \alpha = 1, 2, \tag{8}$$

where $\overset{0}{v}_\alpha, \overset{0}{v}_3$ denote the displacement components, $\overset{1}{v}_\alpha$ are the rotations at the midsurface, and θ is the normal coordinate. By dividing the shell into layers one can trace the evolution of the material properties in each layer, which is necessary when plastic deformations are considered. The simulation is performed using a finite-element program developed by Palmerio et al. [17], Kreja et al. [15] and extended for viscoplastic dynamic analysis by Kłosowski et al. [11,12]. The plate is meshed with isoparametric nine-node elements.

3 The shock tube

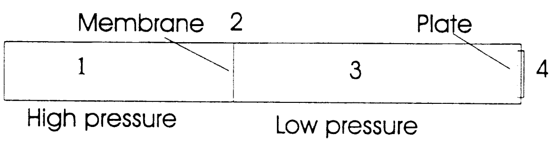

Figure 1: Principle of the shock tube.

A conventional shock tube is used for the experiments consisting of a high pressure part (1) and a low pressure part (3). They are seperated from each other by a flexible hostaphan-membrane (2). At the end of the tube a steel plate (Steel St37, 2mm thick) is clamped between two ring flanges (4). The plate has a diameter of 138 mm. Because of the clamping device the shock wave acts on an area with 108 mm diameter. Part (1) is filled with gas until the membrane (2) is destroyed causing a shock wave running into part (3) and striking

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the plate (4) at the end of the tube. The pressure acting on the plate is measured with piezoelectric elements suitable for fast pressure changes.
The middle point deflection is recorded with a capacitor. One plate of the capacitor is the specimen and the other is the device.

4 Comparison between simulation and experiment

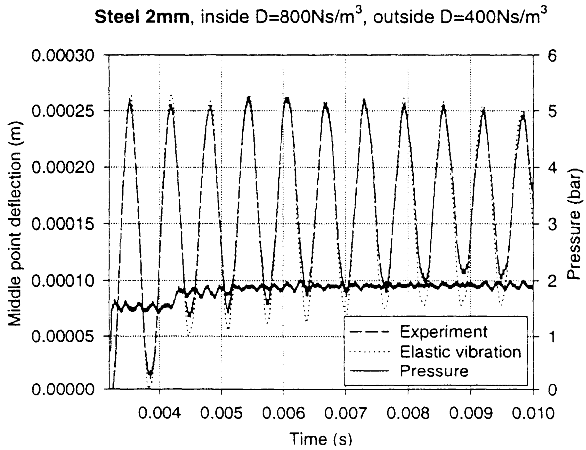

Figure 2: Measured and simulated elastic vibration.

To obtain the boundary conditions and damping D, elastic vibrations are performed in the shock tube and compared with simulated vibrations, see fig. 2. The frequency of the vibration depends on the stiffness of the clamping, so the correct boundary condition is found by adjusting the frequency in the simulation to the frequency observed in the experiment. This way, one finds as equivalent stiffness of the outer ring a Young's modulus of $E = 450000\frac{N}{mm^2}$. For the damping is considered that the densities inside and outside of the tube are different. Outside normal air density is present and the density inside is calculated by a computer program developed at the Institute of General Mechanics by Fieweger [8]. The outer damping coefficient is once determined by comparing measured and simulated elastic vibration, see fig. 2, and the inner damping coefficient is calculated for each experiment. Because of the varying pressure in the

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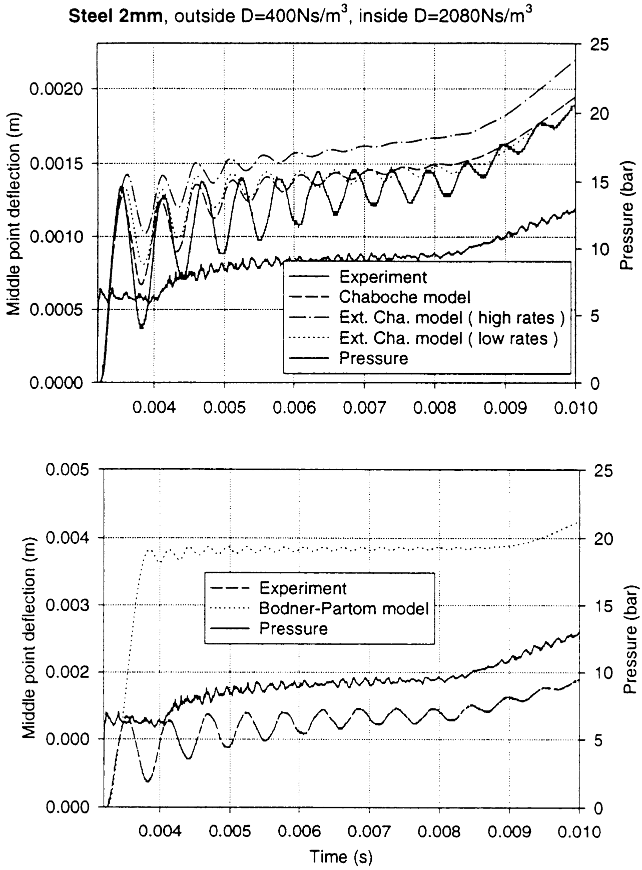

Figure 3: Comparison between measured and simulated viscoplastic vibrations, a) Chaboche, b) Bodner-Partom.

tube the damping coefficient is not constant during one experiment. This may explain differences between the calculated and measured vibrations at the lower peak of the vibration.

Now let us regard the viscoplastic vibrations in fig.3. The obtained damping is indicated in the figures. As expected, the classical model of Chaboche does not simulate the vibrations correctly because it does not take into account the saturation process. As has been shown before, the extended version of the Chaboche model needs a restriction to special ranges of strain rates. However in the present study the vibrating deformations cover the whole interval of strain rates , beginning with $\dot{\epsilon}_p = 0$ at the peak until to the maximum velocity in the turning point.

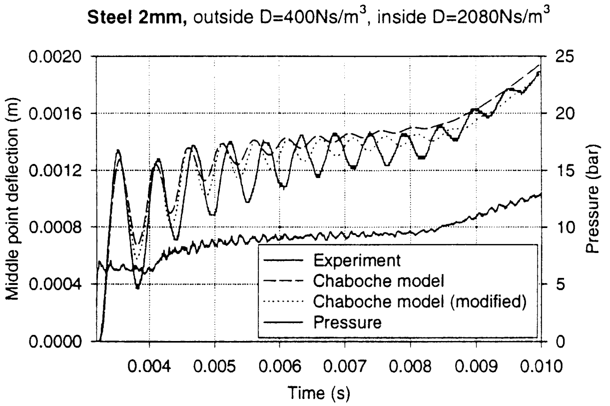

Figure 4: Comparison between measured and simulated viscoplastic vibrations with different boundary conditions.

The Bodner-Partom law yields too large displacements with too small amplitudes. The differences between the models of Bodner-Partom and Chaboche were observed in comparative simulations of shell structures before, e.g. Kłosowski et al. [11,12], but in these studies no comparison to a measured vibration was considered.
For the future, the authors suggest to calculate the error margins of the simulated and measured vibrations, generated by errors in tension tests and measurements of the vibrations. Here, it is found that the errors of the measured vibrations are neglectible in comparison to those of the simulated vibrations generated by error propagation from the parameter identification. In the simulations the boundary conditions have a great influence on the results, e.g. if we vary the yield limit of the clamping device we obtain the vibration in fig. 4 'Chaboche modified'.

5 Conclusions

Using the Chaboche models the simulated plate vibrations are in qualitatively good correlation to the experiments. It has been shown that a better correlation to the measured values is possible by taking into account the error margins, e.g. in the material identification, simulation of the boundary conditions, etc.. Furthermore we considered the advantage of the extended Chaboche model taking the saturation into account, but till now it was not possible to incorpo-

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rate this model into the analysis of the dynamic plate response. So a combination of both Chaboche models could lead to better results. This gives rise to another strategy in simulating the dynamic response of stuctures: to develope a finite-element code which uses different constitutive law depending on the strain rate, or in other words, which combines different laws for an optimal description of material behaviour. In the present case this would mean to use a viscoplastic law like the classical Chaboche model in the slow parts of the vibrations and to change to a plastic time-independent law in the case of high velocities. In the opinion of the authors this step is necessary, because, as shown in this study, the considered constitutive laws have their limits of applicability. Furthermore in comparative studies error margins of simulations and measurements should be determined and the development of the errors from their generation until to the results should be taken into account to estimate how exactly simulations can be performed.

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