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Vibratory Compaction of Coarse-Grained Soils

K. Rainer Massarsch and Bengt H. Fellenius

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Abstract: The variation of the coefficient of earth pressure in normally consolidated and overconsolidated soil and the effect of soil compaction on the change of the horizontal effective stress are discussed based on cone penetration test (CPT) data. A method is outlined for estimating the increase in the effective earth pressure based on sleeve friction measurements. Soil compaction increases not only soil density, but also horizontal effective stress. Since the cone stress is influenced by the vertical and horizontal effective stress, particularly at shallow depths, the cone stress needs to be adjusted for effective mean stress. A relation is presented for determining the soil compressibility from the adjusted cone stress. A case history is presented where a 10 m thick sand fill was compacted using vibratory compaction. Cone penetration tests indicated a significant increase in cone stress and sleeve friction and a decrease in compressibility (increase in modulus number) due to compaction. The friction ratio was unchanged. It was concluded that the earth pressure about doubled corresponding to an increase in the overconsolidation ratio of at least 5. The results of settlement calculations based on the Janbu method demonstrate the importance of considering the preconsolidation effect in the analyses.

Key words: sand, CPTU, vibratory compaction, earth pressure, overconsolidation, modulus number, settlement.

Résumé : La variation du coefficient de pression des terres dans un sol normalement et surconsolidé et l'effet du compactage de sol sur le changement de la contrainte horizontale sont discutés sur la base des données de CPT. On décrit une méthode pour estimer l'augmentation de la pression effective des terres basée sur les mesures du frottement du manchon. Le compactage augmente non seulement la densité du sol, mais aussi la contrainte effective horizontale. Puisque la contrainte sur le cône est influencée par les contraintes effectives verticale et horizontale, particulièrement aux faibles profondeurs, la résistance du cône doit être ajustée en fonction de la contrainte moyenne effective. On présente une relation pour déterminer la compressibilité du sol à partir de la résistance ajustée du cône. Une histoire de cas est présentée dans laquelle un remblai de sable de 10 m d'épaisseur a été soumis à un compactage par vibration. Les essais de pénétration au cône ont montré une augmentation significative de la résistance du cône et du frottement sur le manchon et une diminution de la compressibilité (augmentation du nombre modulaire) dues au compactage. Le rapport de frottement est inchangé. On a conclu que la pression des terres a environ doublé, ce qui correspond à une augmentation du rapport de surconsolidation d'au moins 5. Les résultats des calculs de tassement basés sur la méthode de Janbu démontrent l'importance de prendre en compte l'effet de la préconsolidation dans les analyses.

Mots clés : sable, CPTU, compactage par vibration, pression des terres, surconsolidation, nombre modulaire, tassement.

[Traduit par la Rédaction]

Introduction

Where noncohesive soils present inadequate compressibility or strength, compaction is a viable and economical means of soil improvement applicable to both shallow and deep foundations. Compaction refers to densification by dynamic methods, which, depending on the manner of imparting the energy to the soil, can be divided into two main categories: impact compaction or vibratory compaction. Impact compaction ranges from a surface-compacting heavy roller to a heavy weight (tamper) falling in a grid pattern from large heights (dynamic consolidation). Vibratory compaction includes surface-compacting vibratory rollers and plates as well as deep-acting vibratory probes. The methods and their practical applications are discussed extensively in the geotechnical literature, e.g., Mitchell (1982), Massarsch (1991), Massarsch (1999), and Schlosser (1999).

Soil improvement by means of compaction is used increasingly for the solution of different types of foundation problems in coarse-grained soil deposits, in particular where the foundations will be subjected to dynamic and cyclic loading. A large number of compaction projects have been carried out, and numerous case histories are available, illustrating the complexity of the process. This paper discusses the effects of deep vibratory compaction on strength and stiffness, as well as the resulting change of stress conditions in coarse-grained soils. The use of the cone penetrometer for designing and monitoring soil compaction projects will also be discussed.

Basic considerations

The efficient use of compaction methods requires understanding of the possibilities and limitations particular to each

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method, as an inappropriate application and (or) the execution of a compaction method can have severe technical and economic consequences for a project. The geotechnical engineer must therefore take active part in all phases of a project, such as:

(1) selecting and evaluating the applicable compaction method(s);

(2) designing the required compaction effort, including compaction verification;

(3) choosing the appropriate compaction equipment, as well as appointing competent and experienced personnel to lead the project;

(4) deciding on the optimal compaction process in terms of spacing, sequence, and duration;

(5) preparing contract specifications, which must include acceptance criteria based on the methods of verification testing to be applied to the project; and

(6) supervising the project and verifying that the results of the treatment conform to the design and specifications (including project as-performed documentation and quality control observations per the specified acceptance criteria).

The primary objectives can be summarized, as follows:

(1) reduction of the total settlement by increasing the soil stiffness (modulus);

(2) minimizing differential settlement by making the soil more uniform;

(3) improvement with respect to cyclic loading, e.g., in the case of liquefaction;

(4) modification of the dynamic response of the soil with respect to dynamic soil-structure interaction, e.g., for dynamically loaded foundations or infrastructure projects; and

(5) reduction of hydraulic conductivity (permeability) of soil layers, e.g., in the case of earth dams and waterfront structures.

Several factors have contributed to increased application and improved efficiency:

(1) powerful construction equipment has become available (e.g., adjustable vibrators and large and versatile carriers and rigs);

(2) electronic control systems for hydraulic vibrators now make it possible to optimize equipment performance and to minimize environmental effects, such as construction vibrations;

(3) well-trained site personnel, necessary for operating and maintaining sophisticated construction equipment, are becoming available in most countries;

(4) the quality of geotechnical investigations has generally improved (use of efficient field investigation methods with powerful data storage, transmission, and evaluation systems are of particular importance for large construction projects);

(5) field investigation methods and results interpretation, such as the cone penetrometer test (CPT), and correlation of test data with geotechnical design parameters has become more reliable;

(6) project owners have become more cost conscious, requiring evaluation of different foundation alternatives;

(7) modern structures are more sensitive to differential settlement and more stringent design requirements apply;

(8) a better understanding of the static and dynamic stress–strain behavior of soils and advanced analytical tools have in many cases resulted in more cost-effective and reliable designs, especially with respect to settlement analysis; and

(9) a greater level of understanding of compaction has developed amongst geotechnical engineers with regard to the principles of how a foundation design can benefit from compaction, how compaction is performed, and how compaction work is integrated in the overall construction and performance inspection of an engineering project.

It is unsatisfactory that many times even large projects are designed based on indiscriminately chosen empirical compaction criteria. One reason for this may be that no comprehensive method exists for addressing the effects of dynamic compaction in terms of geotechnical parameters. That is, no method directly applicable to the engineering and construction practice. This paper puts forward a method based on results from cone penetration tests and offers a rational approach based on the cone penetrometer data for assessing the soil profile and the soil compressibility to determine the need and potential for compaction, incorporating the calculation of settlement due to the loads imposed on the ground. It also presents aspects of the design of the compaction effort and testing program for verification of the results. As will be illustrated in a case history, deep vibratory compaction preconsolidates the soil, an aspect usually left out of the settlement analysis of foundations placed on compacted soils. Disregarding the preconsolidation effect in the analysis leads to a significant overestimation of the settlement after compaction. The preconsolidation effect is also of importance for other geotechnical problems, such as soil liquefaction and cyclic loading.

Use of cone penetration tests for soil compaction projects

Geotechnical field investigation methods play an important role for planning, implementation, and verification of performance of soil compaction projects. An integral part of the field investigation is the assessment of the in situ characteristics of the soil before and after compaction,. The cone penetration test (CPT) is an efficient and operator-independent tool for assessing the characteristics of sandy coarse-grained soils. It has become the most widely used field investigation method for compaction projects, gradually replacing the standard penetration test (SPT), which previously was the dominant in situ testing method for this purpose.

The CPT can provide a continuous vertical soil profile, and in the process detect the presence of interspersed soil layers. This is important because the variation of hydraulic conductivity (permeability) of the soil, even of thin layers, can have a significant influence on the compactability and on the time effects. When evaluating the efficiency of compaction, the piezocone is preferred as it also measures pore-water pressure. For special purposes, the CPT can be equipped with additional sensors, such as accelerometers for determining shear wave velocity.

The geotechnical literature contains comprehensive information about the cone penetration test and detailed descriptions of test procedures and data evaluation and (or) interpretation (e.g., Mayne et al. 1995; Lunne et al. 1997; Fellenius and Eslami 2000). The cone stress is the most widely used parameter, while the sleeve friction is mainly used in combination with the cone stress for determining the soil type (profiling) from the CPT measurements. However, in spite of its limited accuracy, the sleeve friction can provide information that is directly useful for dynamic compaction projects, because, as will be discussed in more detail below, the sleeve friction reflects the change of earth pressure in a soil deposit and can, therefore, be used to investigate the effect of soil compaction on the state of stress.

Depth and stress adjustment

The results of cone and sleeve friction measurements are strongly affected by the effective overburden stress (Jamiolkowski et al. 1988). Therefore, it is necessary to consider this effect when interpreting CPT results. For the depth adjustment of the cone stress, Massarsch (1994) proposed applying a dimensionless adjustment factor, C_M, to the cone stress according to eq. [1], based on the mean effective stress σ'_m.

$$[1] \qquad C_M = \left[\frac{\sigma_r}{\sigma'_m}\right]^{0.5}$$

where C_M is the stress adjustment factor ≤ 2.5; σ_r is a reference stress equal to 100 kPa; and σ'_m is the mean effective stress, determined according to eq. [2].

$$[2] \qquad \sigma'_m = \frac{\sigma'_v(1 + 2K_0)}{3}$$

where σ'_v is the vertical effective stress, and K_0 is the coefficient of horizontal earth pressure (effective stress condition).

Near the ground surface, values per eq. [1] increase disproportionally and it is necessary to limit the adjustment factor to a value of 2.5.

The stress-adjusted cone penetration stress is

$$[3a] \qquad q_{tM} = q_t C_M$$

or

$$[3b] \qquad q_{tM} = q_t\left(\frac{\sigma_r}{\sigma'_m}\right)^{0.5}$$

where q_t is the unadjusted cone stress (as-measured; corrected for pore pressure on the shoulder), and q_{tM} is the stress-adjusted cone stress.

When using the SPT *N*-index for evaluation of compaction results, it is equally important to adjust the values with respect to the effective overburden stress, σ'. The stress adjustment of the cone stress can be compared to the stress adjustment of the SPT *N*-index. Based on settlement observations of footings, Peck et al. (1974) proposed to adjust ("to correct") the measured *N*-index for overburden stress by multiplying it by an adjustment factor, C_N, to obtain a reference value, N_1, corresponding to an effective overburden stress of 1 t/ft^2 (≈100 kPa).

$$[4a] \qquad C_N = 0.77 \log\left(\frac{20}{\sigma'}\right) = 1 - \frac{\log \sigma'}{\log 20}$$

$$[4b] \qquad N_1 = NC_N$$

where C_N is the *N*-index stress adjustment factor; σ' is the effective overburden stress (t/ft^2); N is the SPT *N*-index (blows/ft); and N_1 is the stress-adjusted N.

Seed (1976) proposed a similar factor to adjust the SPT *N*-index when assessing the susceptibility of loose, water-saturated sands to liquefaction. This relationship was developed for earthquake problems and is based on laboratory tests on loose- to medium-dense sands.

$$[5] \qquad C_N = 1 - 1.25 \log\left(\frac{\sigma'}{\sigma_r}\right)$$

Figure 1 presents the stress adjustments for the cone stress according to eq. [1] for two values of the earth pressure coefficient K_0, together with the SPT *N*-index adjustments proposed by Peck et al. (1974) and by Seed (1976) according to eqs. [4] and [5].

As shown in Fig. 1, for an earth pressure coefficient, K_0, equal to 0.5 (typical for loose, normally consolidated sand), the proposed CPT adjustment factor according to eq. [1] is similar to the SPT factors proposed by Seed. However, it should be noted that, in contrast to the CPT adjustment, the SPT adjustment considers only the vertical overburden effective stress and is independent of the horizontal effective stress. For K_0 = 2.0 (typical for overconsolidated sand), the adjustment factors, C_M and C_N are different. As will be shown later, compaction increases horizontal stress significantly, resulting in earth pressure coefficients in the range of 1.5–3. For these cases, the CPT and SPT adjustment factors differ significantly.

Determining the mean stress (eq. [2]) requires knowledge of the earth pressure at rest, K_0. In normally consolidated soils, the magnitude of the horizontal earth pressure is usually assumed to follow eq. [6] (Jaky 1948; Kézdi 1962).

$$[6] \qquad K_0 = 1 - \sin\phi'$$

The effective friction angle for normally consolidated sand and silt ranges between 30 and 36°, which, according to eq. [6], corresponds to a K_0-value ranging from about 0.4 to 0.6.

Compaction results in an increase in the earth pressure coefficient at rest, K_0. However, in overconsolidated soils, i.e., compacted soils, it is more difficult to estimate K_0. Several investigators have proposed empirical relationships between the earth pressure coefficient of normally and overconsolidated sands and the overconsolidation ratio, OCR, as given in eqs. [7*a*] and [7*b*].

$$[7a] \qquad \frac{K_1}{K_0} = \mathrm{OCR}^{\beta}$$

$$[7b] \qquad \mathrm{OCR} = \left[\frac{K_1}{K_0}\right]^{\frac{1}{\beta}}$$

where K_0 is the coefficient of earth pressure at rest for normally consolidated sand; K_1 is the coefficient of earth pressure at rest for overconsolidated sand; and β is an empirically determined exponent.

Based on compression chamber tests, Schmertmann (1975) recommended a value of 0.42 for the β-exponent, and Lunne and Christophersen (1983) suggested 0.45.

Fig. 1. Adjustment factor C_M for CPT with K_0 = 0.5 and 2.0, according to eq. [1] and adjustment factor C_N for SPT indices according to eqs. [4] and [5].

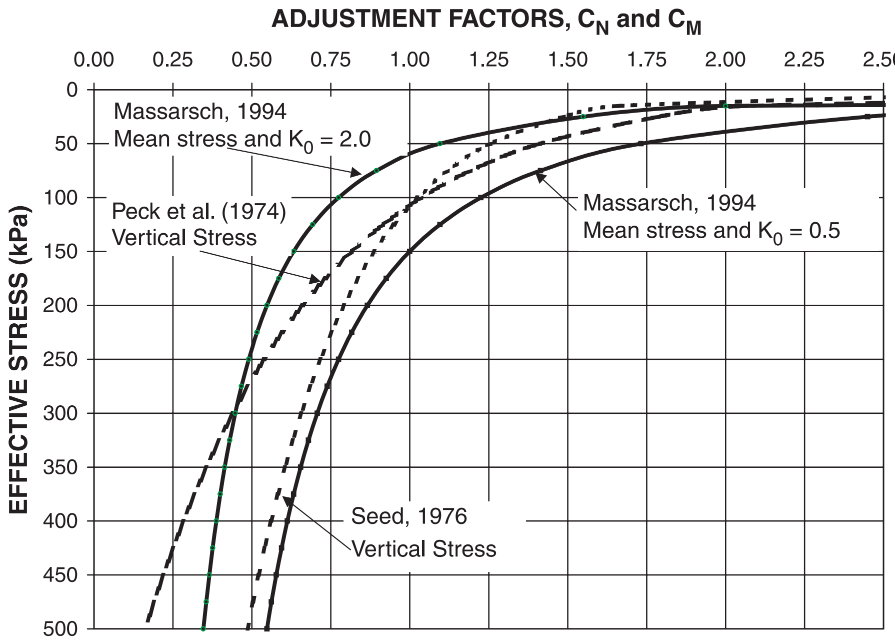
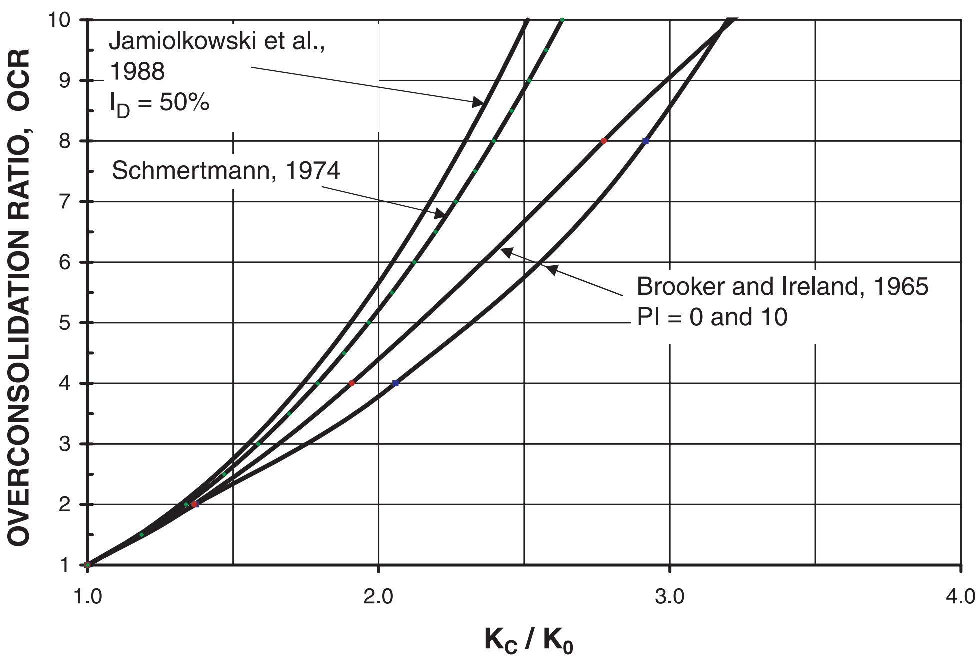


Fig. 2. The relationship between K_0 and OCR for sand (after Brooker and Ireland, 1965).



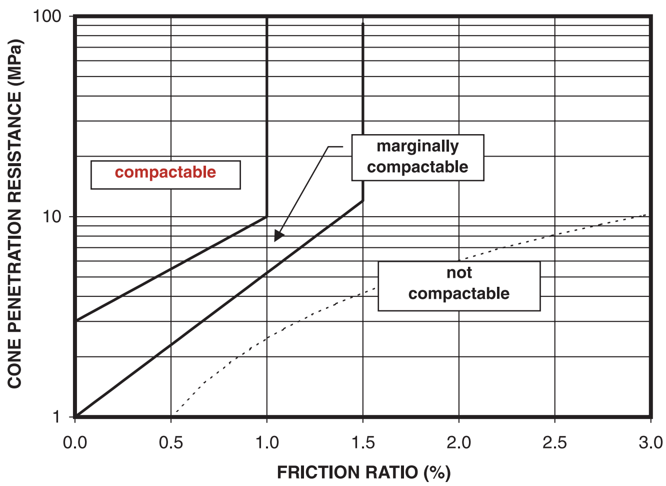
Jamiolkowski et al. (1988) found that the exponent is influenced by the density index ("relative density"), I_D, and suggested a value that ranged from 0.38 through to 0.44 for medium dense sand (I_D = 0.5). Brooker and Ireland (1965) showed that the dependency of the K_0-ratio on the OCR is a function of the plasticity index, PI. Schmertmann (1985) presents an informative discussion on the earth pressure coefficient and influence of horizontal stress (see also Mayne and Kulhawy 1982).

The Brooker and Ireland (1965) data for sand (soil with a low PI value) and the relationships suggested by the above-mentioned authors have been redrawn in Fig. 2, showing OCR as a function of the ratio of earth pressure at rest for overconsolidated and normally consolidated sand, K_1/K_0, respectively. The diagram shows that a relatively small increase in the earth pressure coefficient by a factor of 2 results in an increase in OCR to values ranging from 4 through 7.

Compactability of soils based on the CPT

One of the most important questions to be answered by the geotechnical engineer is whether or not, and to which

Fig. 3. Soil classification for deep compaction based on CPT data (after Massarsch, 1991).



degree, a soil deposit can be improved by dynamic methods (vibratory or impact compaction). Mitchell (1982) identified suitable soil types according to grain size distribution and indicated that most coarse-grained soils with a "fines content" (amount of particles smaller than 0.064 mm) below 10% can be compacted by vibratory and impact methods. However, compaction assessment based on grain-size curves from sieve analysis has the disadvantage that, in order to obtain a realistic picture of the geotechnical conditions, a large number of soil samples and sieve analyses are required — larger than what is usually considered justifiable for a routine foundation project. Going back to a site to obtain additional samples is impractical due to time constraints. Moreover, obtaining representative soil samples may prove to be difficult and costly because the soils at such sites are usually loose and water-saturated. Moreover, soil lenses and layers may not be evident from the inspection of soil samples obtained intermittently. It is therefore preferable to base the assessment of compactability on the results of the CPT, as these measurements present continuous soil profiles reflecting variations in soil strength and compressibility, and, in the case of the piezocone, also variations in the hydraulic conductivity of the soil.

Massarsch (1991) proposed that soils can be classified as "compactable," "marginally compactable," and "not compactable" as a function of where in a CPT chart the cone data plot, as indicated in Fig. 3. (It should be noted that the diagram assumes homogeneous soil conditions. Layers of silt and clay can inhibit the dissipation of excess pore pressures and, therefore, reduce the compaction effectiveness).

Figure 3 presents a conventional soil classification chart with the friction ratio along the abscissa and the cone stress (q_t) along the ordinate. Figure 4 shows the same compaction boundaries in two CPT charts that present the cone stress as a function of the sleeve friction (Eslami and Fellenius 1995; 1997; and Fellenius and Eslami 2000). The left chart uses logarithmic-scale axes. However, as the ranges of cone stress and sleeve friction applicable to the compaction projects are relatively narrow, the usual logarithmic-scale compression can be dispensed with; hence the right chart (Fig. 4) is shown in linear scale.

Compaction criteria

Compaction criteria are frequently expressed in terms of cone stress unadjusted for overburden stress (depth). However, similar to the depth adjustment employed for SPT data, it is preferable to express CPT compaction criteria in terms of a cone stress value adjusted with respect to the mean effective stress. Expressing compaction specifications in terms of the stress-adjusted cone stress will better reflect uniformity of soil density, or lack of uniformity, as opposed to using the unadjusted cone stress. If the cone data are not adjusted according to the stress level (depth), applying a specific value of cone stress as a compaction criterion throughout a soil deposit may lead to the upper layers of the deposit becoming overcompacted while the deeper layers remain loose. When this aspect is not recognized, the result is excessive compaction costs, undesirable loss of ground, and a soil deposit that is not uniformly compacted.

Determination of soil modulus from CPT

A settlement analysis is fundamental to the design of most compaction applications. The analysis requires knowledge of the soil compressibility, that is, of the soil modulus and of the preconsolidation stress. Since the factor of safety against bearing capacity failure is usually high for foundations on coarse-grained soil, the designer is interested in a modulus, E_{25}, for an average applied stress limited to a value equal to about 25% of the estimated ultimate bearing resistance. The modulus can be related to the average cone stress according to the relationship given in eq. [8].

$$[8] \qquad E_{25} = \alpha q_t$$

where E_{25} is the secant modulus for a stress equal to about 25% of the ultimate stress; α is an empirical coefficient; and q_t is the cone stress.

Test data indicate that the coefficient varies considerably and depends on the soil type and stress conditions as well as on the applied load level. According to the Canadian Foundation Engineering Manual (CGS 1992) for plate load tests on sand, the coefficient α varies between 1.5 and 4. Based on a review of the results of cone tests in normally consolidated sand in calibration chambers, Robertson and Campanella (1986) proposed a range for α between 1.3 and 3.0. This range agrees well with the recommendation by Schmertmann (1970) for the use of CPT data to analyze settlement of isolated footings on coarse-grained soils. Dahlberg (1975) performed tests in overconsolidated sand and found that α ranged from 2.4 through to 4, increasing with increasing values of q_t.

The Canadian Foundation Engineering Manual (CGS 1992) states that the ratio between E_{25} and q_t is a function of soil type and compactness, as listed in Table 1.

Table 1. Values of α (= E_{25}/q_t) from static cone penetration tests (CGS 1992).

Soil type	α
Silt and sand	1.5
Compact sand	2.0
Dense sand	3.0
Sand and gravel	4.0

Fig. 4. Soil classification for deep compaction based on the Eslami–Fellenius chart with boundaries from Fig. 3. Numbers in bold on the charts represent the following soil classifications: 1, very soft clays, sensitive and (or) collapsible soils; 2, clay and (or) silt; 3, clayey silt and (or) silty clay; 4a, sandy silt and silt; 4b, fine sand and (or) silty sand; 5, sand to sandy gravel.

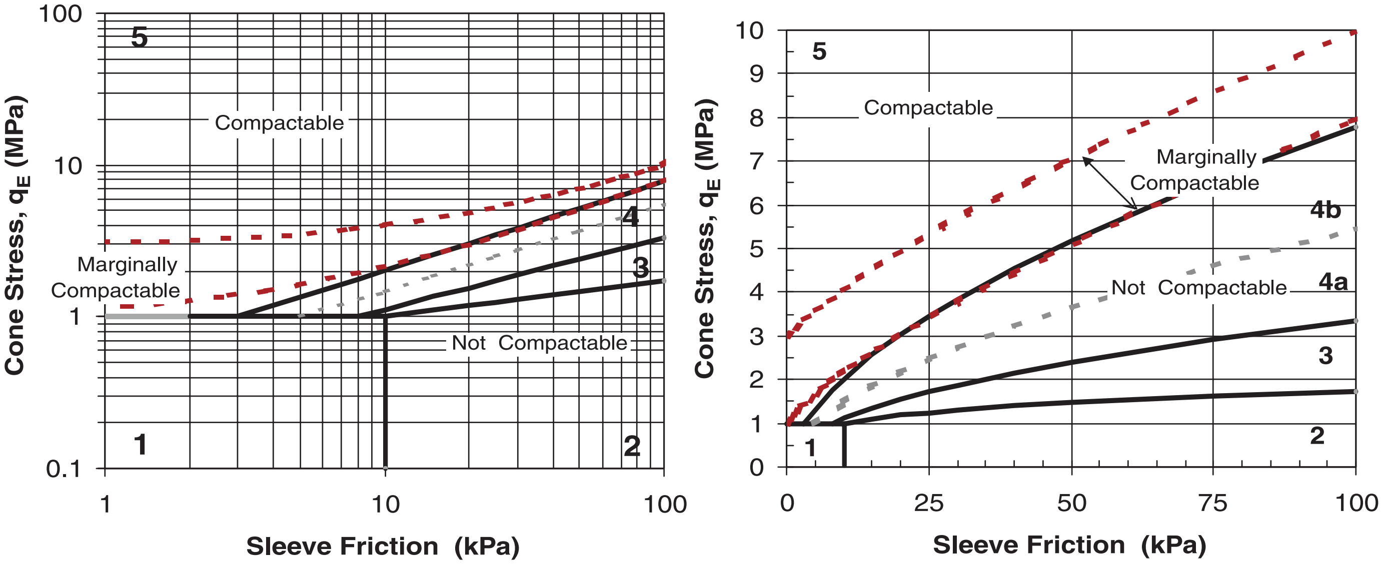


Table 2. Modulus factor, *a*, for different soil types, Massarsch et al. (1999).

Soil type	*a*
Silt, organic soft	7
Silt, loose	12
Silt, compact	15
Silt, dense	20
Sand, silty loose	20
Sand, loose	22
Sand, compact	28
Sand, dense	35
Gravel, loose	35
Gravel, dense	45

The values of the E_{25} modulus shown in Table 1 apply to a settlement analysis in soils that can be assumed to behave as linearly elastic media.

Janbu (1963; 1965; 1967; 1998) presented a unified method of settlement calculations based on the tangent modulus, M_t, defined by eq. [9].

$$[9] \qquad M_t = \frac{d\sigma}{d\varepsilon} = m\sigma_r\left(\frac{\sigma'_v}{\sigma_r}\right)^{(1-j)}$$

where M_t is the change of stress over change of strain; ε is the strain; m is the Janbu modulus number (dimensionless); σ_r is a reference stress (equal to 100 kPa; originally equal to 1 kg/cm^2); σ'_v is the vertical effective stress; and j is the stress exponent.

The modulus number has a direct mathematical relation to the conventional C_c and e_0 approach in clay soils (where the stress exponent is zero) and in gravel and till (where linear elastic conditions are assumed and the stress exponent is unity). For details, see the Canadian Foundation Engineering Manual (CGS 1992) and Fellenius (1999).

In soils suitable for compaction, i.e., silty and sandy soils, the stress exponent is approximately 0.5, and eq. [9] becomes

$$[10] \qquad \varepsilon = \left(\frac{2}{m}\right)\left[\sqrt{\frac{\sigma'_1}{\sigma_r}} - \sqrt{\frac{\sigma'_0}{\sigma_r}}\right]$$

where σ'_1 is the final vertical effective stress (kPa); and σ'_0 is the initial vertical effective stress (kPa).

Massarsch (1994) proposed a semi-empirical relationship (shown in eq. [11]) between the modulus number and the cone stress adjusted for depth. (This approach is a further development of a concept proposed by Janbu 1974).

$$[11] \qquad m = a\sqrt{\frac{q_{tM}}{\sigma_r}}$$

where a is an empirical modulus modifier, which depends on soil type, and q_{tM} is the stress-adjusted cone stress.

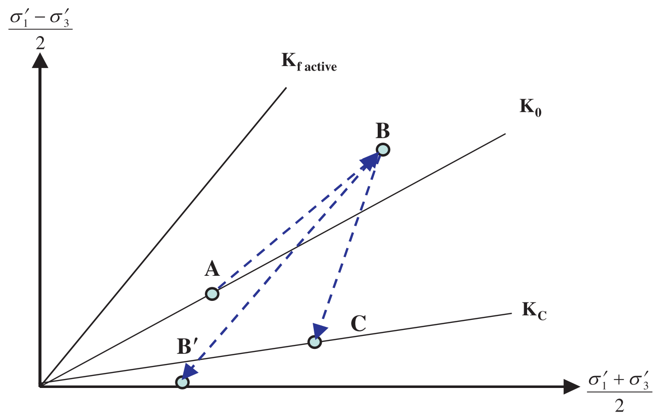
The modulus modifier, *a*, has been determined from the evaluation of extensive field and laboratory data (Massarsch 1994) and shown to vary within a relatively narrow range for each soil type. Massarsch et al. (1997) found the initially proposed values for the modulus modifier to be overly conservative and proposed the revised values listed in Table 2.

An important advantage of determining the modulus number from CPT data and eqs. [10] and [11] is that CPT data are normally available for compaction projects.

Settlement in sands and gravels can usually be analyzed using a constant Young's modulus, such as the E_{25} modulus, according to eq. [8]. Linearly elastic response is characterized by a stress exponent, j, equal to unity, and integration of eq. [9] results in eq. [12].

$$[12] \qquad \varepsilon = \frac{1}{100m}(\sigma'_1 - \sigma'_0) = \frac{1}{100m}\Delta\sigma'$$

Fig. 5. Stress path for a soil element before (A), during (B and B′), and after (C) a single loading and unloading cycle in dry or fully drained coarse-grained soil.



Equation 12 indicates that the E_{25} modulus (eq. [8]) is equal to 100 times the Janbu modulus number determined where a stress exponent equal to unity applies ($\varepsilon = \Delta\sigma'/E_{25}$).

Change of horizontal stress and preconsolidation due to compaction

Compaction of a coarse-grained soil deposit will increase the soil density. With the increase in density comes an increase in stiffness and strength. When the purpose of the compaction is to reduce settlement, the increase in stiffness (i.e., decrease in compressibility) is a highly desirable result. The preceding sections show how CPT measurements, notably the stress-adjusted cone stress, can be used for determining the parameters to use in settlement calculation of conditions before and after compaction. As will be shown below, compaction introduces soil preconsolidation, which is manifested by an increase in horizontal effective stress, i.e., it causes the earth pressure coefficient, K_0, to increase. The K_0 parameter is required for determining the mean effective stress according to eq. [2] and the stress-adjusted cone stress according to eq. [3*b*], which are necessary in order for the modulus number to be established according to eq. [11].

The Jaky relation (eq. [6]) is not valid for overconsolidated soil. However, the CPT sleeve friction measurement can be used for estimating the change in the earth pressure coefficient, as explained in the following paragraph. Moreover, to arrive at representative values, the settlement analysis must also include the beneficial effect of the overconsolidation resulting from the compaction.

As indicated by its name, the sleeve friction is proportional to the soil strength (i.e., the friction between the soil and the steel sleeve). The strength is usually expressed in terms of the friction angle of the soil, ϕ' or, more precisely, as friction, i.e., $\tan\phi'$. Depending on several factors, such as the compaction method, the soil and drainage conditions, and the state of stress prior to compaction, the friction angle increases by about 5–15°. Depending on the actual values, the increase in friction can range from about 20% through to about 40%. However, the sleeve friction value also depends on the horizontal stress acting against the sleeve and the compaction results also in an increase in horizontal stress, that is, an increase in K_0. The increase in K_0, as shown in the following paragraph, results in an even larger contribution to the strength increase.

An hydraulic fill is normally consolidated prior to compaction with an earth pressure coefficient of approximately 0.5. Investigations by Schmertmann (1985), Leonards and Frost (1988), and Massarsch (1991; 1994) have shown that subsequent compaction results in a significant increase in the horizontal stress in the soil. This is demonstrated in Fig. 5, showing a typical change of stress conditions for a soil element before (A), during (B and B′), and after compaction (C). During vibratory compaction, high oscillating centrifugal forces (compression and extension) are generated (up to 4000 kN) that temporarily increase and decrease the vertical and horizontal effective stresses along the compaction probe and at its tip. During vibratory compaction, the soil is subjected to a large number of loading and unloading cycles and the stresses in the soil fluctuate between points B and B′. After compaction and completed dissipation of excess pore-water pressure, the vertical effective stress is again almost equal to the vertical overburden stress prior to compaction, and the stress conditions are represented by point C. Unloading occurs at zero lateral strain and horizontal stresses remain “locked in” at point C. The dynamic compaction has thus caused preconsolidation and increased the horizontal effective stress.

The sleeve friction can be approximated from eq. [13].

$$[13] \qquad f_s = K_0 \sigma_v' \tan\phi'$$

where f_s is the sleeve friction; σ_v' is the effective vertical stress; K_0 is the earth pressure coefficient; and ϕ' is the effective friction angle for the soil–CPT sleeve interface.

The ratio between the sleeve friction after and before compaction, f_{s1}/f_{s0} can be calculated from eq. [14].

$$[14] \qquad \frac{f_{s1}}{f_{s0}} = \frac{K_{01}\sigma_{v1}' \tan(\phi_1')}{K_{00}\sigma_{v0}' \tan(\phi_0')}$$

where f_{s0} is the sleeve friction before compaction; f_{s1} is the sleeve friction after compaction; K_{00} is the coefficient of earth pressure before compaction (effective stress); K_{01} is the coefficient of earth pressure after compaction (effective stress); σ_{v0}' is the vertical effective stress before compaction; σ_{v1}' is the vertical effective stress after compaction; ϕ_0 is the friction angle before compaction; and ϕ_1 is the friction angle after compaction.

If it is assumed that the effective vertical stress, σ_v', is unchanged by the compaction, the ratio of the earth pressure after and before compaction, K_{01}/K_{00} can then be estimated from the relationship according to eq. [15].

$$[15] \qquad \frac{K_{01}}{K_{00}} = \frac{f_{s1}\tan(\phi_0')}{f_{s0}\tan(\phi_1')}$$

Equation [15] shows that the earth pressure coefficient is directly affected by the change of the sleeve friction and of the friction angle of the soil. To illustrate the importance of the relationship, eq. [15] is represented in Fig. 6 for a sand for which the compaction resulted in a friction angle of 36°, improved from values ranging from 21 through to 30° before compaction. The sand is assumed to be normally consolidated before compaction with an earth pressure coefficient,

Fig. 6. Ratio between sleeve friction before and after compaction to OCR for three levels (ranges) of increase in the effective friction angle: 21–36°, 25–36°, and 30–36°.

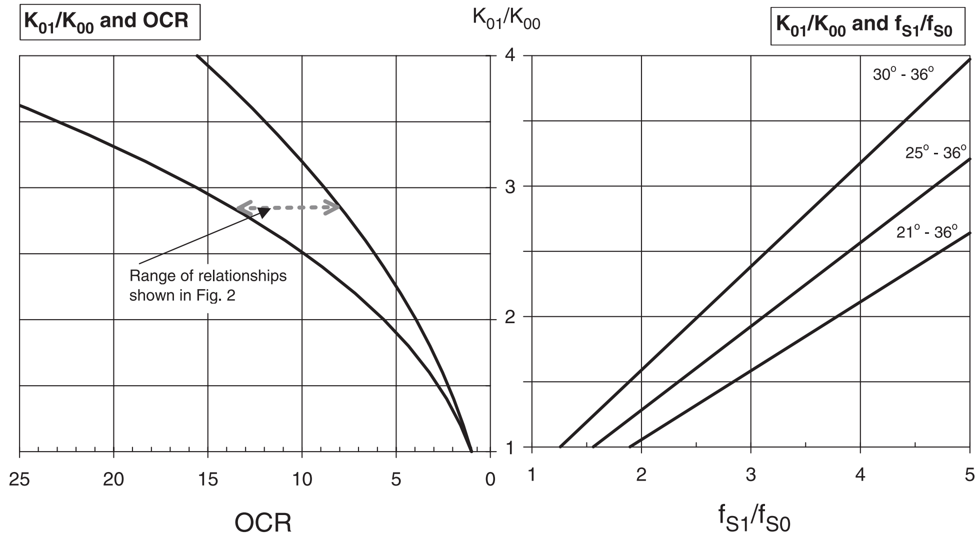
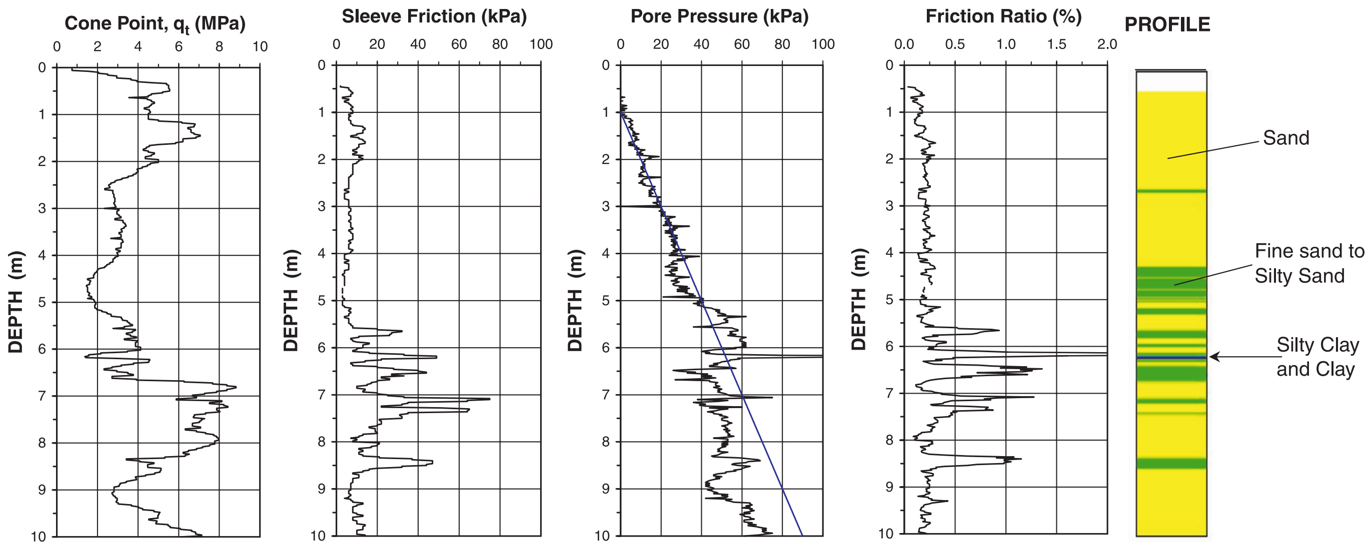


Fig. 7. Results of CPT in trial area prior to compaction. Note the lens of silty clay and clay at 6.1 m depth.



K_{00}, equal to 0.5, according to eq. [6]. The CPT measurements provide the sleeve friction values. As indicated in Fig. 6, the ratio of earth pressure coefficients depends primarily on the ratio of sleeve friction and less on the increase in friction angle.

Figure 6 is supplemented with the diagram showing the relationship between the earth pressure ratio and the overconsolidation ratio, OCR, introduced by the compaction. The two curves are envelopes of the curves shown in Fig. 2. The two diagrams suggest that even a moderate increase in sleeve friction will result in a considerable boost of the OCR value.

In an actual case, reliable values of the friction angles may not be available. In normally consolidated coarse-grained soils, a value for K_{00} = 0.5 can be applied to eq. [15] for determining the value of K_{01}. If the soil is already overconsolidated before compaction, judgment must be used to select a reasonable value of K_{00}. Inserting the value of K_{01} into eqs. [2], and [3] yields the stress-adjusted cone stress, This, in turn, inserted into eq. [11] provides the soil compressibility, that is, the modulus number, to be used in the settlement analysis.

Note that stress applied within the preconsolidation limit only contributes a minor portion of the total settlement. Although, the CPT does not provide the Janbu reloading modulus number, m_r, for a dense to compact coarse-grained soil, the reloading modulus number can usually be adequately assumed as a value about 3–5 times larger than the virgin modulus number.

Increase in soil strength and stiffness with time

Another important factor of soil compaction is the increase in soil strength and stiffness with time after compaction (e.g., Massarsch 1991; Schmertmann 1991; Mitchell 1982). Post-densification SPT and CPT results suggest that natural and man-made deposits of clean sand may gain in

Fig. 8. The CPT data plotted in an Eslami–Fellenius CPT classification chart. The three separate dots near the boundary between zones 2 and 3 are from the clay at depth 6.1 m.

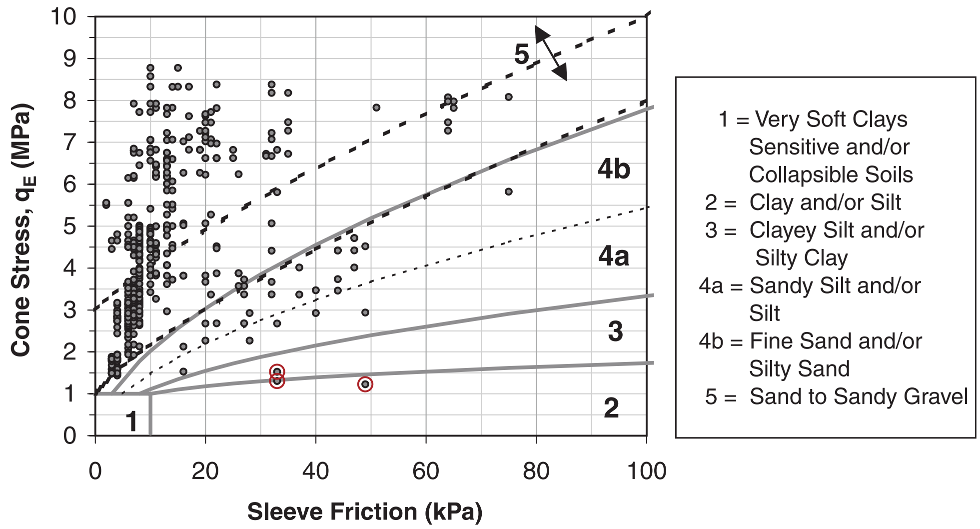
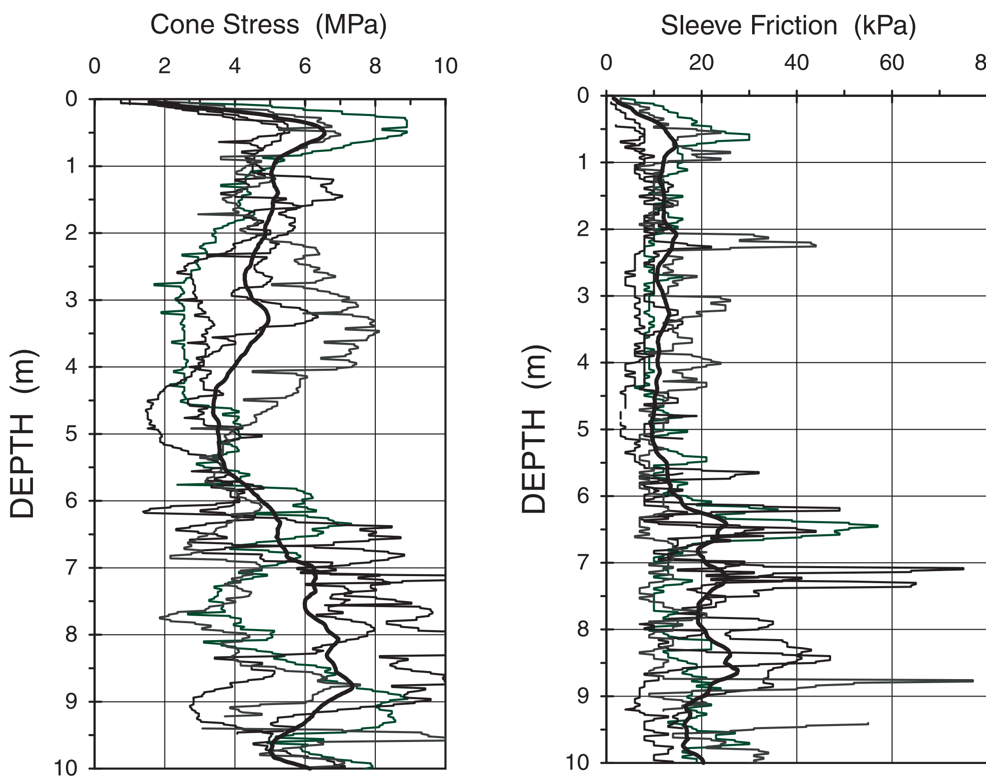


Fig. 9. Cone stress and sleeve friction from four CPTs before compaction. The heavier line shows filtered average values.



strength with time after compaction even after the pore pressures induced during compaction have dissipated. The mechanism of this phenomenon is not yet fully understood.

In addition to the complex theories that have been proposed to explain the change of soil parameters with time after compaction, there may be a rather simple explanation. Due to the heterogeneous stress conditions (horizontal stress variation) in a soil deposit after compaction, a rearrangement of soil particles may take place with time in order to adjust to a more homogeneous stress field. This effect depends on several factors, such as geotechnical conditions, type and execution of compaction process, etc., and is difficult to assess quantitatively without in situ testing.

Compaction of hydraulic fill — a case history

Introduction

The geotechnical literature contains only few well-documented case histories of dynamic compaction projects with high-quality CPT measurements. One such case, suitable for analysis in the context of this paper, is the land-reclamation project associated with construction of the new

Fig. 10. Cone stress and sleeve friction from four CPTs 2 days after compaction. The heavier line shows filtered average values.

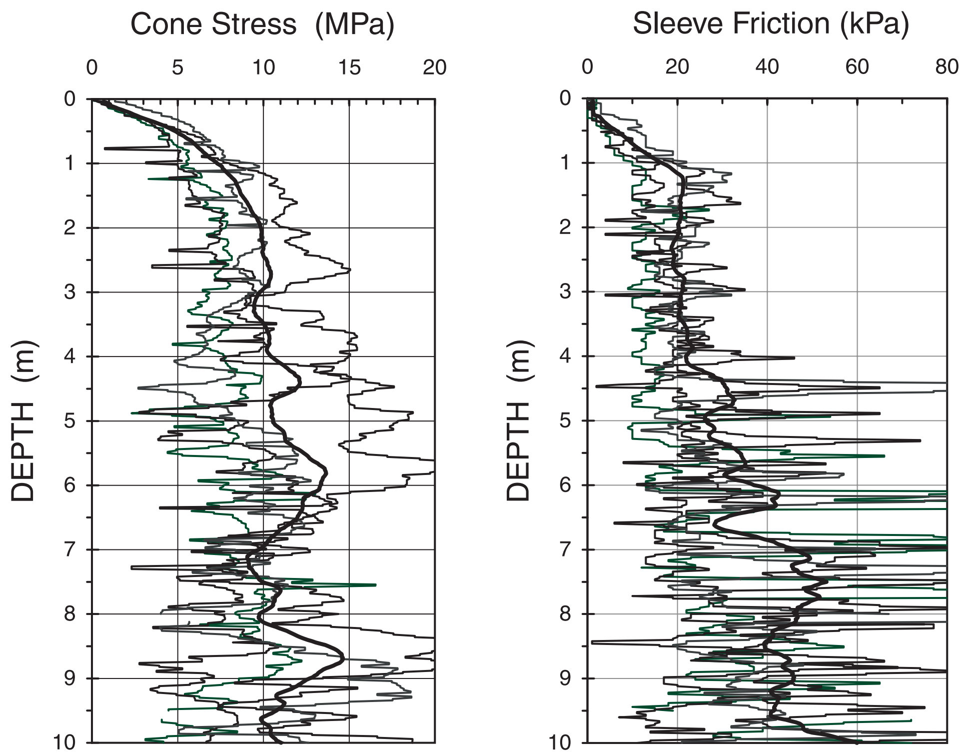
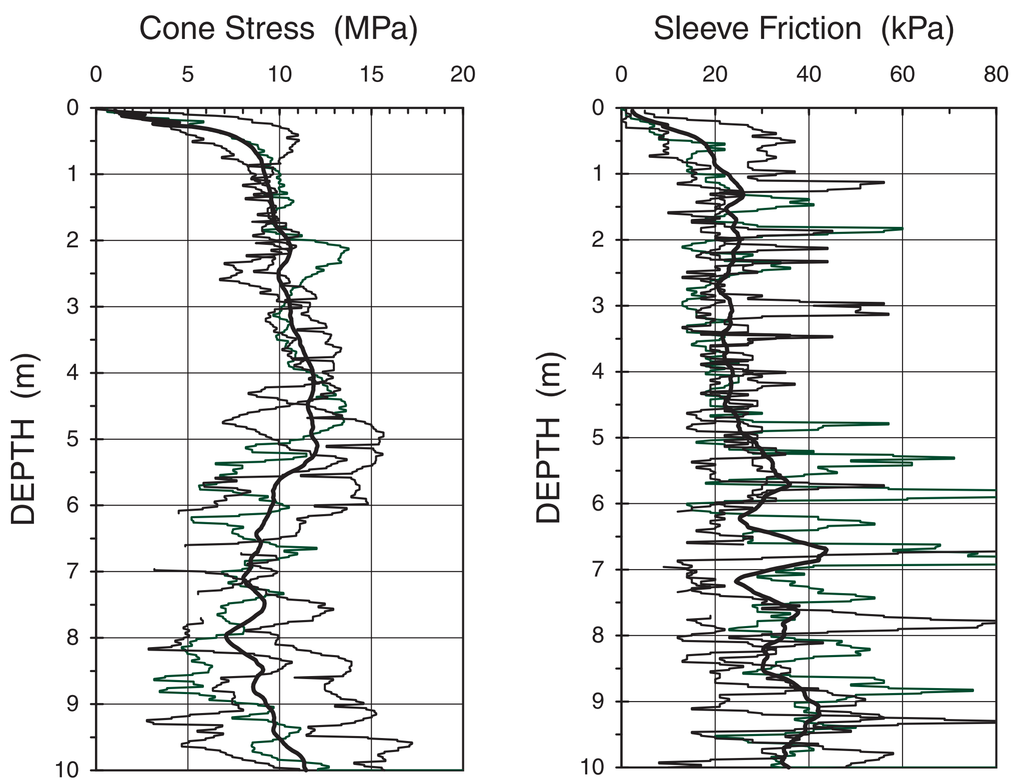


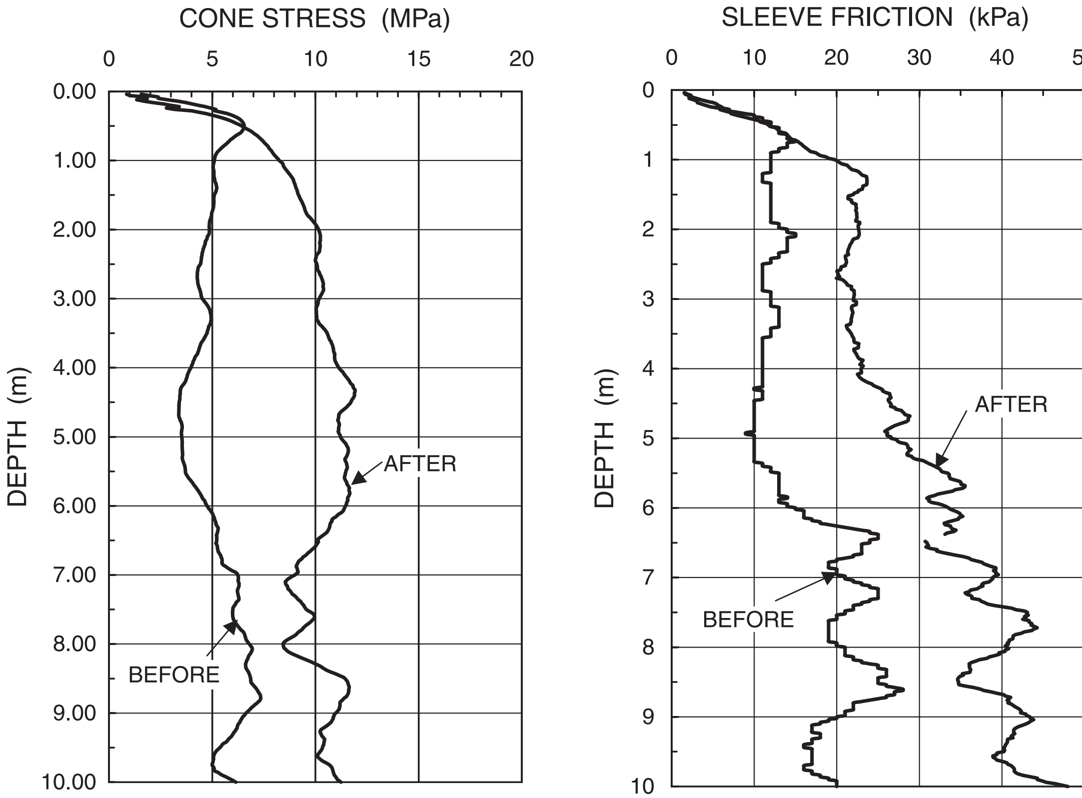
Fig. 11. Cone stress and sleeve friction from three CPTs 7 days after compaction. The heavier line shows filtered average values.



Hong Kong Airport at Chek Lap Kok (Gallon and Nettermann 1996).

The Chek Lap Kok reclaimed land was located along the seashore, where, prior to the placement of dredged sand fill, existing soft clay was dredged from the sloping seabed. The sand fill consisted partly of calcareous material (fragments of shells and clams), and was placed by bottom dumping, where the water depth exceeded 4 m, and by spraying where

Fig. 12. Filtered average values of cone stress and sleeve friction from before and after compaction (2 and 7 days combined).



the water depth was shallower. The final thickness of the sand fill prior to compaction was on average about 10 m, but ranged from about 6 m through to about 20 m. The groundwater level before compaction was located about 1 m below the fill surface. The sand fill was specified to contain less than 10% of silt and clay. However, this was difficult to achieve by the dredging contractor and, while it was achieved for most of the fill volume, the as-placed fill contained occasional pockets of clay and silt. Due to the sloping seabed, the thickness of the hydraulic fill varied significantly. The construction time schedule was very short, which prevented a laboratory study to determine conventional geotechnical parameters of the sand.

Cone penetration tests were obtained with a piezocone within a relatively small trial area (12 m × 12 m). Figure 7 presents the results of one of four CPT soundings through the as-placed fill before compaction, illustrating that the fill consisted mainly of loose sand with frequent zones of silty sand and an occasional lens of silty clay and even clay. The homogeneity of the fill is demonstrated in the profiling chart shown in Fig. 8. The chart includes all CPT records (readings were taken every 20 mm) from one CPT sounding. The silty clay and clay lens indicated in Fig. 7 at about 6 m depth is 60 mm thick and the profiling chart (Fig. 8) shows it to be made up of three closely located values, one value indicating clay and two values indicating silty clay.

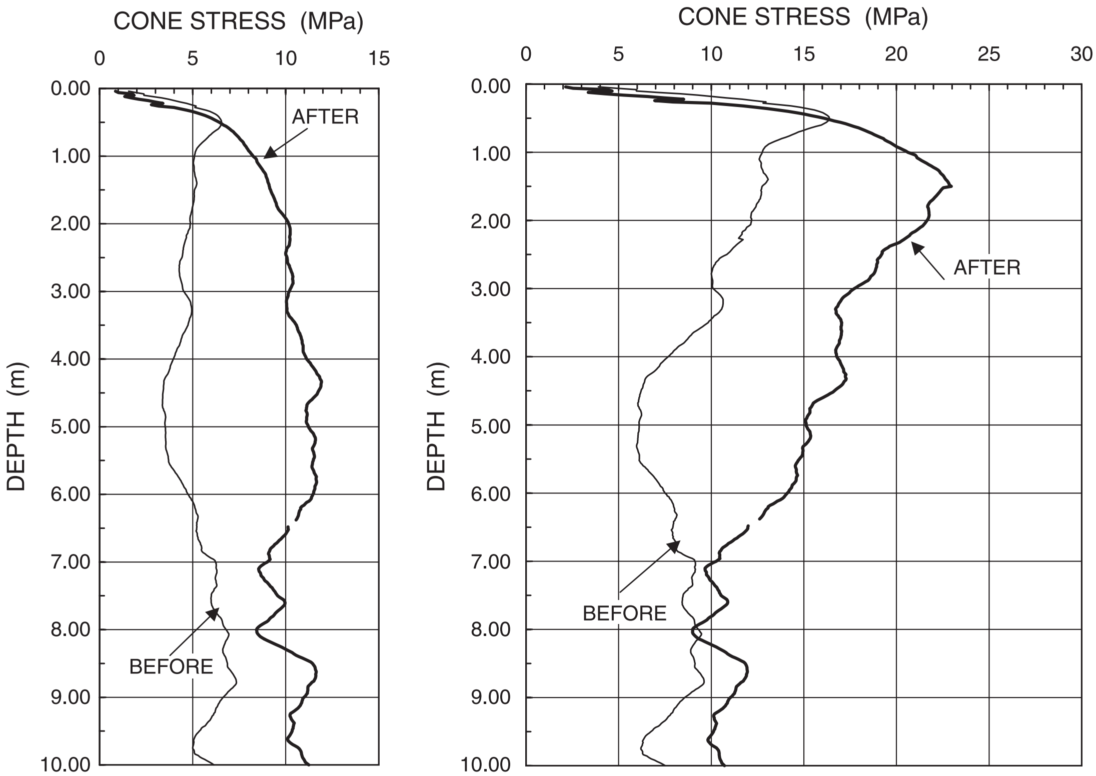
Figure 9 presents a compilation of the cone stress and sleeve friction values from the four CPT soundings made before the compaction work. Note that in spite of the relatively consistent placement method of the fill and the short distance between test points, the cone stresses varied significantly between the points; by more than a factor of 2. The heavier line in Fig. 9 shows the arithmetic average of the four sets of values, filtered to smooth out the peaks and troughs of the records. The filtering is made by a running geometric average over a 0.5 m record length.

The sleeve friction was low prior to compaction and ranged from 10 through to 15 kPa, with locally higher values, indicating the presence of layers of silt and clay also reflected in the friction ratio values, ranging from 0.2 through to 0.5%. The low values are typical for loose, calcareous sand.

The contractor for the compaction work elected to use the Müller resonance compaction (MRC) method, which incorporates a powerful vibrator that is clamped to the upper end of a flexible compaction probe (Gallon and Nettermann 1996, Massarsch and Westerberg 1995). By changing the vibration frequency, the system makes use of the vibration amplification that occurs when the soil deposit is excited at the resonance frequency. Different vibration frequencies are used during the particular phases of the compaction process to achieve optimal probe penetration and soil densification, as well as to facilitate probe extraction and to avoid undoing the compaction ("uncompacting" the soil). The compaction process is monitored and documented using an electronic process control system, which records various parameters of importance for the soil densification process. The mode of probe insertion and extraction in the soil layer to be compacted plays a significant role, as does the sequence in which compaction is performed.

The spacing between compaction points and the duration of compaction were determined by first compacting a 12 m × 12 m trial area. During the trial compaction, the densification

Fig. 13. Filtered average values of cone stress from before and after compaction; unadjusted in left diagram and adjusted in right diagram.



effect was monitored by recording ground surface settlement and performing cone penetration tests, as well as ground vibration velocity and frequency measurements. The ground surface was also compacted due to the movement of heavy construction equipment and tidal variations.

Compaction and testing programme

Extensive field tests were carried out to optimize the compaction procedure and to evaluate the change of soil characteristics after densification (Gallon and Nettermann 1996). Another objective was to investigate whether the soil stiffness would increase after compaction. However, because of the pressing time schedule, investigation of this effect had to be limited to the course of one week.

To verify the suitability of vibratory compaction and to determine compaction criteria, a trial area was selected. The compaction was carried out using a Müller MS100 vibrator with variable operating frequency and a maximum centrifugal force of 2000 kN. The vibration frequency could be varied between 8 and 30 Hz using an electronic process control system and vibration sensors placed on the ground surface. The vibrator was guided by leads mounted on an 150-ton crawler crane.

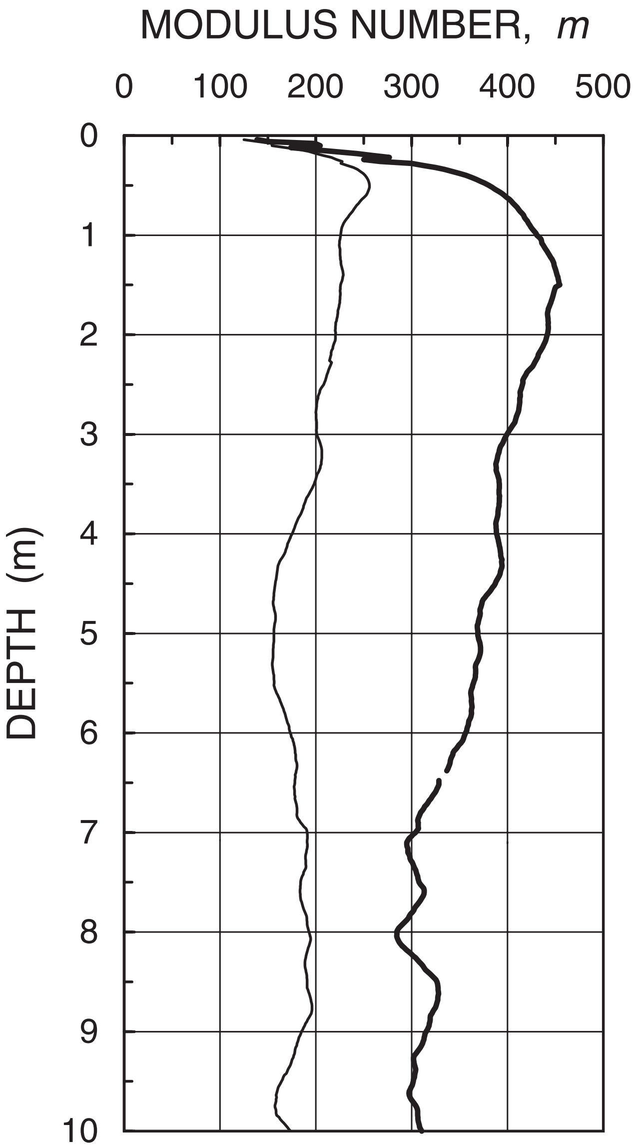
The compaction in the trial area was performed in two passes. During the first pass, the soil was compacted in 13 points with a grid spacing of 4.2 m × 3.6 m. At seven of the points, the compaction duration was 5 min and at the remaining six points, the duration was 10 min. All of the 13 points were compacted during one afternoon. The vibrator frequency during probe penetration and extraction was 25 Hz, and the optimal compaction frequency (resonance frequency) was 14 Hz at the start of compaction and 16 Hz at the end of compaction. During the second pass, the intermediate points at the center of the initial grid were compacted.

The acceptance criterion for the sand fill required a cone stress, q_t, after compaction, of at least 10 MPa. This criterion did not consider adjustment for depth (overburden stress). To meet the requirement, the contractor therefore devoted considerably larger effort and time in compacting the upper portion of the soil deposit as opposed to the deeper portion. The compaction contractor had long experience using the MRC method and based the required compaction effort (duration of compaction in each compaction point) during the second compaction pass on ground vibration measurements, on measurement of the probe penetration speed, and on the gradual increase in the resonance frequency.

Compaction results

CPT soundings were carried out two days (four tests) and seven days (three tests) after completion of the first compaction pass. The CPT soundings were placed midway between compaction points. The distance from each CPT sounding to the closest compaction point was 2.0 m. Figures 10 and 11 show the cone stress and sleeve friction measurements for the two sets of soundings. The figures show measured values of cone stress and sleeve friction with a heavier line representing the filtered average value. Before the start of the compaction trials, a tendency of time-dependent increased soil strength and (or) stiffness had been expected following the compaction. However, it is difficult to distinguish any distinct difference between the 2- and the 7-day tests, and the 5-day interval may not have been long enough for a noticeable time-dependent improvement to develop.

Fig. 14. Janbu modulus number, *m*, before and after compaction.



It had been expected that the pore-water pressure should show decreased values reflecting an increased soil dilatancy due to the denser soil, but this was not observed. As the increase in cone stress and sleeve friction are of the same magnitude, the sleeve friction ratio remained almost unchanged after compaction.

As shown in Figs. 9–11, in spite of the relatively uniform compaction procedure and the close proximity between the test points (closer than 8 m horizontal distance), significant variations in cone stress and sleeve friction can be noted. The difference between the individual measurements means that despite the consistent soil compaction, significant variations of soil strength, density, and earth pressure occurred. Figure 12 shows a comparison of the results of the CPT soundings in terms of the filtered average cone and sleeve friction before and after compaction and illustrates the compaction effect.

The cone stress and the sleeve friction have increased in the sand deposit as a result of the vibratory compaction. On average, the increase in cone stress is a doubling or higher, indicating a definite densification of the sand fill. The specifications requirement of a cone stress of at least 10 MPa was satisfied. The effect of dynamic soil compaction on the stress conditions is also evidenced by the increase in sleeve friction, on average about 2.5 times, which is about the same increase ratio as that of the cone stress.

The cone stress data shown in Figs. 9–11 are unadjusted for overburden stress. For determining the C_M value to apply to the adjusted q_t values, a K_0 of 0.5 for the "before" conditions was applied, representing a friction angle of 30°. The friction angle after compaction was not determined, but it is assumed that it is about 36°, which results in a sleeve friction ratio of 0.8. Inserting this ratio and the ratio of sleeve friction of 2.5 into eq. [15] gives a ratio of earth pressure coefficient of 2.0. Because the earth pressure coefficient before compaction, K_{00}, was assumed to be 0.5, the earth pressure coefficient after compaction, K_{01}, is 1.0, which allows for the adjusted cone stress after compaction to be determined according to eqs. [3*a*] and [3*b*].

Figure 13 shows the filtered average unadjusted and adjusted cone stress data before and after compaction. The figure clearly demonstrates the results of the extra compaction effort in the upper portion of the fill deposit. Such excessive compaction results in an upper soil layer that is stiffer than intended, as well as a loss of soil volume that will have to be compensated for by import and placement of additional fill. This can be an important cost aspect for larger compaction projects.

The adjusted values of cone stress were inserted in eq. [11] to determine the Janbu modulus numbers before and after compaction, as plotted in Fig. 14. Before compaction, the modulus number was about 250 near the surface and decreased slightly and approximately linearly to a value of 180 at 10 m depth. After compaction, the modulus number was 450 near the surface, decreasing approximately linearly to 300 at 10 m.

The compaction also left the soil in a preconsolidated stress state. According to Fig. 6, a ratio of earth pressure coefficient of 2 indicates that the OCR value is about 5.

Settlement calculations

Foundations at the site can typically be represented by a 10 m wide square slab placed at about 0.5 m depth and exerting a foundation stress of 300 kPa. The modulus numbers determined from the CPT data ranged from 250 through to 180 before compaction. According to CGS (1992), this range corresponds to a loose-to-medium (water-saturated) sand with a porosity of about 45%. After compaction, the modulus numbers increased to between 450 and 300, corresponding to a compact-to-dense sand. The average reduction of porosity is about 5–7%. This reduction of the soil volume agreed well with the observed settlements of the about 10 m thick sand deposit, where the measured surface settlements were 0.55–0.75 m. The modulus numbers were used as input to eq. [10] and the resulting strain values were integrated over the sand thickness to calculate the settlement for the slab. The calculations were performed with the UniSettle program (Fellenius and Goudreault 2000), applying the Janbu method for silty and sandy soils (applying a stress exponent, j, equal to 0.5) and a Boussinesq distribution of the applied stress below the characteristic point. (At the characteristic point, the stress along a vertical line is the same for an infinitely flexible slab and an infinitely stiff slab. Settlement calculations using this stress distribution are considered representative for the settlement beneath a reasonably stiff slab; CGS 1992; Fellenius 1999).

The calculations for the conditions prior to the compaction show a total slab settlement of 60 mm. For the conditions after compaction, assuming normally consolidated soil, the calculated slab settlement is 35 mm, that is, 58% of the settlement for the uncompacted soil. This settlement reduction would be considered not sufficient in most cases, and, probably, not believed, as compacted soils usually show minimal settlement. In many cases, the calculated settlements would therefore be "adjusted based on engineering judgment." However, if taking into account that the compacted soil is overconsolidated with an OCR value of at least 5, then, below the depth of about 2.5 m, the imposed stress is smaller than the preconsolidation stress. Assuming that the reloading modulus numbers are three times the virgin numbers, the calculated settlement is smaller than 20 mm (30% of the settlement in the uncompacted soil), well below the usually accepted limit and in good agreement with "practical experience."

Conclusions

Soil compaction can offer effective solutions for many foundation problems, and it is especially useful for reducing total and differential settlements in sands. However, efficient use of soil compaction methods requires that the geotechnical engineer understands all of the factors that influence deep soil compaction and plans, designs, and monitors the compaction process carefully. The most useful tool for deciding which soils can be compacted by dynamic methods is the cone penetration test, notably the piezocone. The authors' simple classification charts based on cone stress and sleeve friction measurements can be used to judge the efficiency of compaction work.

Many soil compaction projects require that settlements can be estimated before and after compaction. This paper presents how compressibility parameters (modulus numbers) are estimated from CPT data. The variation of the coefficient of earth pressure in normally consolidated and overconsolidated soil is discussed, showing that even a relatively small increase in the earth pressure coefficient increases the overconsolidation ratio significantly.

An important aspect is the effect of dynamic soil compaction on the horizontal effective stress. As a result of repeated vibration cycles, the horizontal effective stress increases significantly. This effect is shown to be important for settlement calculations. It can also be significant for cyclic and dynamic foundation problems (liquefaction), but this aspect is not elaborated on.

The effect of soil compaction on the change of the horizontal effective stress is discussed. A method is outlined that makes it possible to estimate the increase in the lateral effective earth pressure based on sleeve friction measurements.

Since the cone stress is influenced by the vertical and horizontal effective stress, the measured cone stress needs to be adjusted for effective mean stress, rather than the effective overburden pressure. It is recommended that compaction criteria be based on adjusted cone stress values to avoid unnecessary compaction in the upper soil layers.

Soil compaction increases not only soil density but also horizontal effective stress. This is evidenced by the fact that the sleeve friction can double or triple as a result of compaction, resulting in a substantial increase in the overconsolidation ratio. This aspect is currently often neglected in settlement calculations and leads to an overestimation of the calculated settlement.

A case history is presented where an about 10 m thick sand fill was compacted using vibratory compaction. Soil conditions were determined using cone penetration tests before and at 2 and 7 days after compaction. The compaction charts provided realistic information concerning the compactability of the soil deposit to be compacted. The results show a significant increase in cone stress, which demonstrates a decrease in compressibility (increase in modulus number). The sleeve friction increased proportionally, and the friction ratio was unchanged. The pore-water pressure did not show different response after as opposed to before compaction. From the increase in sleeve friction, it was concluded that the earth pressure against the friction sleeve about doubled corresponding to an increase in the overconsolidation ratio of at least 5. The observed ground surface settlement due to compaction was 0.55–0.75 m, which corresponds to a volume decrease of approximately 5–7%.

The results of settlement calculations based on the Janbu method demonstrate the importance of taking the preconsolidation effect into account in the analyses. If the settlement analysis had been based only on the densification effect (increase in modulus number), the settlement reduction would have been only about 40% of that of the uncompacted fill. When including the preconsolidation effect, the reduction was 70%.

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