Views for the Development of the Port of Thessaloniki Based on the Cost of the Serving Process<br>Dr. Lida Thomo, Dr. Dimitris Ioannides, Evangelos Bellos and Imir Thomo<br>Department of Economics, University of Macedonia, Thessaloniki, Greece.

## Abstract

In this paper an analysis of the serving process in the Port of Thesaloniki is presented using as instruments the methods of Queuing Theory and Mathematical Statistics. The aim is to estimate the serving cost and find the optimal number of servers in order that the cost would be the minimum. To achieve this goal the port is modelled as a serving mechanism. By processing a large amount of data, the approximate distributions of the arrival flow and serving time are estimated and tested. Also, the idle server cost and the cost of ships waiting to be served are defined. Then we find the model of Queuing Theory, which best describes our serving model. It is shown that the number of servers in the Port of Thessaloniki at the present is sufficient in one part and exceeds the optimal in the other parts. Based also on results from forecasting techniques we conclude that the further development of this port must not continue with increasing the number of the servers.

## 1 Introduction

Two undesired situations could occur in the serving process of a port. The first one is the situation of a long queue of ships waiting for serving, and the second one is the situation of idle servers to be waiting to serve. In the first case the number of servers is smaller than the necessary
number. The port in this case suffers loss indirectly because the quality of the service decreases. Consequently the port will lose customers. In the second case the port loss directly for two reasons. First the port pays a lot of money to the servers that do not work and for maintaining the server system. Second the places are occupied and so we cannot add the necessary and modern means in the best time.

In the first case the natural strategy is to increase the number of servers and means; in the second case the strategy is to decrease that number.

Clearly the two above strategies are counter to each other. The aim of this paper is to define the optimal number of servers in order that the loss of money in the serving process would be the minimum. An accurate figure for this situation would be of crucial importance for the development of the port.

## 2 The Mathematical Model

The Port of Thessaloniki is a very complicated system. A lot of different activities are combined there. The mechanisms that operate there are from the very simple to the very compound. The personnel employed in the port is from the most unskilled to the most specialised. In spite of this it is possible to construct a mathematical model that would give us useful information. The arrival process of ships and the service time for them involve many random phenomena. The times at which requests for service will occur and the lengths of time that these requests will engage facilities cannot be predicted except in a statistical sense. Therefore, the best instrument for analysing the system would be the Queuing Theory.

The serving model, the object of the Queuing Theory, is defined by three characteristics [3]:

1. The customers' arrival process.
2. The serving system.
3. The queue discipline.

The customers are to be regarded as the ships. The random variable «The number of arrived ships per day» is considered in the first part of the paper. We have studied the nature of this random variable and have determined its distribution function. The random variable «The service time of a server» will be studied in the second part. We will be able to apply the formulas of Queuing Theory, after the conclusions of the first and second part, supposing that the discipline of serving is FIFO. For example, we may use the formula of the mean waiting time, the formula
of the mean number of idle servers, the formula of the assurance level of the servers available, etc [4].

The port's working space is categorised into three classes according to the kind of activity done there. The groundworks $9,10,11,12,13,14,15$, $16,18,23$ belong to the first group $\alpha$. The groundworks $17,20,21,22$, 24 , belong to the second group $\beta$ and 26 (containers) to the group $\gamma$. The general goods are served in the group $\alpha$, heavy weights goods are served in the group $\beta$ and the goods in containers are served in group $\gamma$. We will define as a server the place of the groundwork where a ship can be served with all the mechanisms and workers that belong to that place. For example: 15 (fifteen) ships can be served in the first group at the same time. The mechanisms that are in place at every server will be assumed one fifteenth of whole mechanism of first group. We will find in the same manner the number of workers that are employed in one server. A big part $40 \%$ of the activity of the port is made in $\gamma$ group.

## 3 The Arrival Process

Here, the random variable «The number of arrived ships per day» by year, is studied for every group. We have considered all the data of the port that show the arrivals of ships in the years we shall be interested in. We estimate the value of this random variable by the day. The $\chi^{2}$ test is used to decide the appropriate distribution function of probability for the random variable in every case. We have worked with admitted levels of 0.05 or 0.01 . We consider the working year to have 365 days because it is possible to have arrival on any day of the year. After the analysis of the 1741 items of data for 1993, 1884 for the 1994 and 1740 for 1995 and 695 for the 1996 we conclude that the latter random variable follows a Poisson distribution.

## 4 The Service Time

The nature of the random variable «The service time» is studied in this section. The respective data for this purpose does not exist in the port, so we were obliged to combine the data from two offices, the statistic office and the marked office. We found the service time of every server from the correspondence of names of ships in the first office with the names of ships of the second office. The following result comes from the analysis of these data. The random variable «The service time» follows a negative exponential distribution.

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## 5 Fitting of the Model in Each Part of the Port

From the results of the two above parts we conclude:

1. The traffic intensity which is given by the formula:

$$
\begin{equation*}
\rho=\frac{\lambda}{\mu \cdot n} \tag{1}
\end{equation*}
$$

where :
$\lambda$ the mean number of arrivals in a day, $1 / \mu$ is the mean service time, $n$ the number of servers.
is less that one for every group. So in each group we have a statistical equilibrium. [3]
2. We can construct a model for the serving process which is:

Group a. Group $\beta$.
Year 1993. M/M/15, Year 1993. M/M/10, Year 1994. M/M/15, Year 1994. M/M/10, Year 1995. M/M/15. Year 1995. M/M/10.

## Group $\gamma$.

 Year 1993. M/M/2, Year 1994. M/M/2, Year 1995. M/M/2.3. From the result 2 we are able to use the analytical formulas of Queuing Theory which give the significant characteristics of the queue. These formulas for the models $\mathrm{M} / \mathrm{M} / \mathrm{n}$ under the equilibrium condition are [3]:
$P_{k}$ is the statistical equilibrium probability that in the system with n servers there are k ships: $(\alpha=\lambda / \mu)$

$$
\begin{array}{lr}
P_{k}=\frac{a^{k}}{n!n^{k-n}} P_{0} & k \geq n \\
P_{k}=\frac{a^{k}}{k!} P_{0} & 1 \leq k \leq n \tag{3}
\end{array}
$$

where $P_{0}$ is the probability that all the servers are idle and is given by:

$$
\begin{equation*}
P_{0}=\frac{1}{\sum_{k=0}^{k=n-1} \frac{a^{k}}{k!}+\frac{a^{n}}{(n-1)!(n-a)}} \tag{4}
\end{equation*}
$$

$\bar{n}$ is the mean number of ships in the serving system:

$$
\begin{equation*}
\bar{n}=\sum_{k=0}^{m} k P_{k} \tag{5}
\end{equation*}
$$

$\bar{\rho}$ is the average number of idle servers:

$$
\begin{equation*}
\bar{\rho}=\sum_{k=0}^{n}(n-k) P_{k} \tag{6}
\end{equation*}
$$

$\bar{w}$ is the mean of waiting time in the queue:

$$
\begin{equation*}
\bar{w}=\frac{a^{n} \cdot \frac{1}{\mu}}{(n-a)\left[(n-a)(n-1)!\sum_{k=0}^{k=n-1} \frac{a^{k}}{k!}+a^{n}\right]} \tag{7}
\end{equation*}
$$

$R$ is the level of assurance:

$$
\begin{equation*}
R=e^{-\lambda} \sum_{k=0}^{k=\lambda} \frac{\lambda^{k}}{k!} \tag{8}
\end{equation*}
$$

## 6 The Cost Function

In this section we study the function that evaluate the money loss in the serving process. From the random nature of the serving process it is inevitable that always there would be an amount of money that will be lost. This function is made up by two terms [5]. The first one represents the waiting cost of idle servers, and the second one the waiting cost of the ships. The optimal state of the port activity would be when this function takes the minimum value. Then our objective is to find the conditions which minimise this function. These conditions represent the optimal

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number of servers in the port. Therefore, the economical development must be based on this result.
The form of this function is:

$$
\begin{equation*}
S_{n}=c_{1} \bar{\rho}+c_{2} \bar{w} \lambda \tag{9}
\end{equation*}
$$

where:
$\bar{\rho}$ and $\bar{w}$ are given in the section 5 .
$c_{1}$ the current cost of daily idle time of a server.
$c_{2}$ the current daily waiting cost of a ship.
The values of the cost $c$ where not available in the port. This cost was calculated as the sum of three terms: 1. the maintaining cost, 2. the workers' cost and 3. the administration personnel cost. The cost $c_{2}$ is taken from international catalogues.

The first term of this function is a monotonically increasing function over the number of servers while the second is a monotonically decreasing function of this number.

The graphical representation of each term and of the function is given below:


Figure 1: The graphical representation of the cost function. (the above curve, and its two components below)

Obviously the minimum of the function will be at the point $n_{0}$.
The advantage of the application of Queuing Theory results is that we have analytically expressed by $\lambda, \mu, n$ the magnitudes making up the
function of money loss. So, we can see the variation of this function when the $\lambda, \mu, n$ parameters variate, namely when situations change. With this we mean the change in the arrival flux and the change in the serving power.
(We begin with the $\gamma$ group for the reason of the planed extension taking soon for this group.)

### 6.1 Group $\gamma$

We are in the case of the model $\mathrm{M} / \mathrm{M} / 2$. In this case the formula $S_{n}=c \bar{\rho}+c_{2} \bar{w} \lambda$ takes the form:

$$
\begin{equation*}
S_{n}=c_{1} \sum_{k=0}^{n}(n-k) P_{k}+c_{2}\left(\frac{a^{n} \cdot \frac{1}{\mu}}{(n-a)\left[(n-a)(n-1)!\sum_{k=0}^{k=n-1} \frac{a^{k}}{k!}+a^{n}\right]}+\frac{1}{\mu}\right) \lambda \tag{10}
\end{equation*}
$$

where:

$$
\begin{equation*}
P_{k}=\frac{a^{k}}{k!} P_{0} \quad 1 \leq k \leq n \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
P_{0}=\frac{1}{\sum_{k=0}^{k=n-1} \frac{a^{k}}{k!}+\frac{a^{n}}{(n-1)!(n-a)}} \tag{12}
\end{equation*}
$$

After calculations the function of money loss takes the form:

$$
\begin{equation*}
S_{n}=c_{1}(n-a)+c_{2}\left(\frac{a^{n} \cdot \frac{1}{\mu}}{(n-a)\left[(n-a)(n-1)!\sum_{k=0}^{k=n-1} \frac{a^{k}}{k!}+a^{n}\right]}+\frac{1}{\mu}\right) \lambda \tag{13}
\end{equation*}
$$

where:
$\lambda=2.02295$
$c_{1}=301.085+5025+4325$ (the first term concerns the mechanisms, the second term concerns the workers and the third concerns the management personnel)
$c_{2}=5000$.
$1 / \mu=0.314859$
After the evaluation of the above function for different values of $n$ we took the following results. We recall that the money loss is composed by two parts. The first part is the money loss of the port and the second the money loss of the ships. Next of the total loss is given the port money loss $S_{n}^{\prime}$ and the customers' money loss $S_{n}^{\prime \prime}$.

| $\mathrm{n}=1$ | $S_{1}=12275.86$ | $S_{1}^{\prime}=3503.88$ | $S_{1}^{\prime \prime}=8771.98$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{n}=2$ | $S_{2}=16699.15$ | $S_{2}^{\prime}=13154.96$ | $S_{2}^{\prime \prime}=3544.19$ |
| $\mathrm{n}=3$ | $S_{3}=26029.67$ | $S_{3}^{\prime}=22806.05$ | $S_{3}^{\prime \prime}=3223.62$ |
| $\mathrm{n}=4$ | $S_{4}=35645.94$ | $S_{4}^{\prime}=32457.13$ | $S_{4}^{\prime \prime}=3188.81$ |
| $\mathrm{n}=5$ | $S_{5}=45293.33$ | $S_{5}^{\prime}=42108.22$ | $S_{5}^{\prime \prime}=3185.11$ |

Minimum of $S_{n}=12275.86 \$$ and this is achieved for $n=1$
As we can see the optimal number of servers is 1 . But because the second part of the loss (that concerns the money loss of customers) is relatively big, thus provoking loss of customers, it is best to have two servers as the number of the servers in this group. The corresponding total money loss is $S_{2}=16699.15 \$$ and the part concerning the port is $S_{2}^{\prime}$ $=13154.96 \$$. If another server (the third) were added the money loss would be $S_{3}=26029.67 \$$ and the part of this concerning the port is $S_{3}^{\prime}=$ 22806.05\$. So, the money the port would lose is $S_{3}^{\prime}-S_{2}^{\prime}=9651.09 \$$ per day that is $9651.09 \$ * 365=3,522,648 \$$ in one year. In the same manner the adding of a fourth server will increase the money loss concerning the port to the amount of $7,045,000 \$$ in one year.

We discuss this issue because the port authorities had planned to extend this group by adding two further servers. If we consider the decrease of the money loss incurred by the customer we will see that this is $S_{2}^{\prime \prime}-S_{4}^{\prime \prime}=3544.19-3188.81=355.38 \$$ per day or, in other words, the customers will lose $10 \%$ less while the port will lose $147 \%$ more. Let us do the same calculation for the case when we go from three to four servers. The loss to the customers will decrease only by $35 \$$ per day or
$1 \%(12,775 \$$ in one year = cost of 2 waiting days of a ship $)$ while the loss of the port will be $9651 \$$ per day or $42.3 \%(3,522,000 \$$ in one year) more. It is then obviously that the adding of the fourth server in the $\gamma$ group would be pointless.
Note.
a. The optimal number of the servers will be 4 (four) in the case when the mean number of arrivals is 7.6 .

If we are interested for the state when the flux of the arrived ships is twice more of the daily rate as present, then the optimal number would be 3 (three). However it is recognised that is difficult to increase the daily mean number of customers' arrivals. It is fact, [1] that in the other part of the port, where the serving capacity is approximately twice to what is necessary; here for a long time, the mean number of customers' arrivals has not increased.
b. From the trend analysis, using a quadratic function for regression, we can predict that there will be the need for adding a fourth server after many years.

### 6.2 Group $\boldsymbol{a}$

We are in the case of $\mathrm{M} / \mathrm{M} / 15$ model. In this case we can use the same formulas as in the above group $\gamma$ where:
$n=15$
$\lambda=1.926$
$c_{1}=560.95 / 15+1350+445.7$ (the first term concerns the mechanisms, the second term concerns the workers and the third concerns the management personnel)

$$
\begin{aligned}
c_{2} & =5000 \\
\tau & =2.6087
\end{aligned}
$$

The result is:

| $\mathrm{n}=6$ | $S_{6}=42264.08$ | $S_{6}^{\prime}=1796.79$ | $S_{6}^{\prime \prime}=40467.29$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{n}=7$ | $S_{7}=32953.97$ | $S_{7}^{\prime}=3637.77$ | $S_{7}^{\prime \prime}=29316.2$ |
| $\mathrm{n}=8$ | $S_{8}=32042.44$ | $S_{8}^{\prime}=5478.75$ | $S_{8}^{\prime \prime}=26563.69$ |
| $\mathrm{n}=9$ | $S_{9}=32962.88$ | $S_{9}^{\prime}=7319.72$ | $S_{9}^{\prime \prime}=25643.16$ |
| $\mathrm{n}=10$ | $S_{10}=34470.04$ | $S_{10}^{\prime}=9160.70$ | $S_{10}^{\prime \prime}=25309.34$ |


| $\mathrm{n}=11$ | $S_{11}=36189.00$ | $S_{11}^{\prime}=11001.68$ | $S_{11}^{\prime \prime}=25187.32$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{n}=12$ | $S_{12}=37986.43$ | $S_{12}^{\prime}=12842.66$ | $S_{12}^{\prime \prime}=25143.77$ |
| $\mathrm{n}=13$ | $S_{13}=39812.46$ | $S_{7}^{\prime \prime}=14683.64$ | $S_{3}^{\prime \prime}=25128.82$ |
| $\mathrm{n}=14$ | $S_{14}=41648.54$ | $S_{4}^{\prime}=16524.61$ | $S_{4}^{\prime \prime}=25123.93$ |
| $\mathrm{n}=15$ | $S_{15}=43488.00$ | $S_{5}^{\prime}=18365.59$ | $S_{5}^{\prime \prime}=25123$ |
| $\mathrm{n}=16$ | $S_{16}=45328.52$ | $S_{6}^{\prime}=20206.57$ | $S_{6}^{\prime \prime}=25121.95$ |
| $\mathrm{n}=17$ | $S_{17}=47169.37$ | $S_{7}^{\prime}=22047.55$ | $S_{7}^{\prime \prime}=25121.82$ |

Minimum of $S_{n}=32042.44 \$$ and is achieved for $n=8$
As we can see the optimal number is $n=8$. The port in this part has 15 (fifteen) servers. So the daily loss is $11445 \$$, and in a year $4,177,629.4 \$$. If we inspect the above sequence of $S_{n}^{\prime \prime}$ we note that the improvement in the customers' loss from $n=11$ to $n=15$ is negligible (64\$) while the loss of the port is $7364 \$$ in a day and in a year $2,687,860 \$$.

### 6.3 Group $\beta$

We are now in the case of $\mathrm{M} / \mathrm{M} / 10$ model. In this case we can use the same formulas as in the above groups $\alpha$ and $\gamma$ where:
$n=10$
$\lambda=1.370115$
$c_{1}=452.78 / 10+1350+445.7$ (the first term concerns the mechanisms, the second term concerns the workers and the third concerns the management personnel)
$c_{2}=5000$
$\tau=1.9927$
The result is:

| $\mathrm{n}=3$ | $S_{3}=56355.34$ | $S_{3}^{\prime}=497.06$ | $S_{3}^{\prime \prime}=55858.28$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{n}=4$ | $S_{4}=20309.3$ | $S_{4}^{\prime}=2338.04$ | $S_{4}^{\prime \prime}=17971.26$ |
| $\mathrm{n}=5$ | $S_{5}=18879.91$ | $S_{5}^{\prime}=4179.02$ | $S_{5}^{\prime \prime}=14700.89$ |
| $\mathrm{n}=6$ | $S_{6}=19955.63$ | $S_{6}^{\prime}=6019.99$ | $S_{6}^{\prime \prime}=13935.64$ |
| $\mathrm{n}=7$ | $S_{7}=21588.58$ | $S_{7}^{\prime}=7860.97$ | $S_{7}^{\prime \prime}=13727.61$ |

$$
\begin{array}{llll}
\mathrm{n}=8 & S_{8}=23372.7 & S_{8}^{\prime}=9701.95 & S_{8}^{\prime \prime}=13670.75 \\
\mathrm{n}=9 & S_{9}=25198.8 & S_{9}^{\prime}=11542.93 & S_{9}^{\prime \prime}=13655.87 \\
\mathrm{n}=10 & S_{10}=27036.11 & S_{10}^{\prime}=13383.91 & S_{10}^{\prime \prime}=13652.2 \\
\mathrm{n}=11 & S_{11}=28876.25 & S_{11}^{\prime}=15224.88 & S_{11}^{\prime \prime}=13651.37 \\
\mathrm{n}=12 & S_{12}=30717.05 & S_{12}^{\prime}=17065.86 & S_{12}^{\prime \prime}=13651.19
\end{array}
$$

Minimum of $S_{n}=18879.91$ and is achieved for $\mathrm{n}=5$
As we can see the optimal number is $\mathrm{n}=5$. The port in this part has 10 (ten) servers. So the daily loss is $8156 \$$, and in a year $2,977,013 \$$. If we inspect the above sequence of $S_{n}^{\prime \prime}$ we note that the improvement in the customers' loss from $n=7$ to $n=10$ is negligible (75\$) while the loss of the port is $5523 \$$ in a day and in a year $2,015,873 \$$.

## 7 Conclusions

From the analysis of the serving cost, it follows that the service power of the Port of Thessaloniki is greater than the optimal. As a consequence, the port suffers loss of money. In this way, the further development of this port should not be in the extension but in the quality of the old parts.

## References

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