GEOPHYSICS, VOL. 53, NO. 6 (JUNE 1988); P. 769-777, 7 FIGS., 2 TABLES.

Viscoacoustic wave propagation simulation in the earth

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ABSTRACT

Anelasticity of earth materials produces significant changes in the amplitude and phase spectra of seismic waves. The anelastic properties of real materials, particularly of porous rocks, are described using the theory of linear viscoelasticity based on Boltzmann's superposition principle. Wave-propagation simulation with this model requires implementing the convolutional relation in the equation of motion. The choice of a viscoacoustic constitutive relation based on a spectrum of relaxation mechanisms allows a realistic description of the anelastic effects, and the introduction of memory variables obviates storing the entire strain history required by the time convolution. A pseudospectral time-integration technique is used to solve the equation of motion.

Applications of viscoacoustic modeling suggest the need for considering the correct attenuation-dispersion effects for various fundamental seismic problems in anelastic earth models. Comparison of acoustic and viscoacoustic synthetic seismograms shows differences in the amplitudes and arrival times of the wave fields which are enhanced for particular combinations of anelastic and geometrical effects.

INTRODUCTION

Wave propagation in the earth has always been known to be anelastic. Therefore simulations which attempt to accurately reconstruct amplitudes must be able to account for the effects of attenuation and dispersion. It has been shown (Jones, 1986) that these effects are very important in determining the pore fluid content in porous rocks.

Growing evidence suggests a linear attenuation mechanism (with or without constant Q) for seismic strains and upper crustal conditions (Jones, 1986). Linear viscoelasticity provides a general framework for such behavior. The theory of linear viscoelasticity is embodied in Boltzmann's superposition principle, which establishes that the time Fourier transform of the stress is equal to the time Fourier transform of the strain multiplied by the complex bulk modulus. The concept of a spectrum of relaxation mechanisms is used to define the complex bulk modulus.

Liu et al. (1976) showed that a viscoelastic rheology with multiple relaxation mechanisms gives a framework that can explain experimental observations of wave propagation in the earth and in earth-type materials. In particular, Liu and his coworkers showed that, with a suitable choice of material parameters, both a constant Q value and a dispersion relation which qualitatively explains differences in seismic-wave velocities in different frequency ranges can be obtained.

Implementation of the theory of general linear viscoelasticity to frequency-domain methods is straightforward because of the correspondence principle, which states that the solution of a dynamic problem for a viscoelastic material can be obtained from the solution of the corresponding problem for an elastic solid by applying the time Fourier transform to the elastic solution, replacing the elastic constants by the corresponding viscoelastic complex moduli, and finally inverting the transform (Bland, 1960, p. 96). However, for direct methods in the time domain, the convolutional kernel represented by Boltzmann's superposition principle is difficult to implement in the equations of motion. Day and Minster (1984) implemented the equations of motion using an approach based on Padé approximants to transform the convolution integral into a convergent sequence of constant-coefficient differential operators of increasing order, a formulation equivalent to the rheology of multiple relaxation mechanisms mentioned above (Carcione et al., 1987). For each mechanism, a first-order differential equation was obtained that, together with the scalar equation of motion, was solved by a finite-difference scheme.

In this work, the equations of motion are implemented in the convolution integral by introducing memory variables, one for each relaxation mechanism. The algorithm requires more storage than in the purely acoustic case but not signifi-

Manuscript received by the Editor May 20, 1987; revised manuscript received September 28, 1987.
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cantly more computation. The acoustic case requires storage for the dilatation and time derivative of the dilatation for each grid point; the viscoacoustic problem adds storage of the memory variables.

The new theory explains, within the framework of the most general linear relation between stress and strain, the correct changes in the phase and amplitude spectra of the wave field for any type of frequency-dependent complex modulus function. A more complete description of the physical model can be found in Carcione et al. (1987), where the numerical algorithm is verified by comparison with solutions for wave propagation in a homogeneous viscoacoustic medium.

This work is concerned with viscoacoustic wave propagation in two spatial dimensions. The first section presents the constitutive relation and relaxation function for the viscoacoustic medium. Then the quality factor and phase and group velocities are defined as functions of the complex bulk modulus. The following section briefly derives the basic equations for the viscoacoustic wave field obtained after definition of the memory variables.

Three examples of wave propagation through different types of geologic models are studied. Each example compares acoustic with viscoacoustic synthetic time sections for the same geologic structure, the objective being to identify the wave-field changes due to the presence of the anelastic effects.

CONSTITUTIVE RELATION OF THE VISCOACOUSTIC MEDIUM

In perfectly elastic solids, according to Hooke's law, stress is directly proportional to instantaneous strain but independent of the rate of strain; and the mechanical energy is stored without dissipation. On the other hand, for perfectly viscous liquids, in accordance with Newton's law, the stress is directly proportional to the rate of strain and independent of the strain itself; but in this case the energy is completely dissipated.

A realistic representation of the earth may be achieved by combining the mechanical properties of elastic solids and viscous liquids. In the resulting material, the stress depends upon both the strain and the rate of strain, as well as on higher time derivatives of the strain. Such a medium, which combines the characteristics of solids and liquids, is called viscoelastic. These kinds of materials can both store and dissipate mechanical energy. An introduction to the study of the viscoelastic behavior of materials can be found in Christensen (1982).

In this paper, the time convolution of two functions $f(t)$ and $g(t)$ is expressed by

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau) g(t-\tau)\, d\tau$$

and the definition

$$f^c(t) = f(t) H(t)$$

is used, where $H(t)$ denotes the Heaviside function.

The response of the earth is approximated by the viscoacoustic rheology. The most general linear relation between pressure $p(\mathbf{x}, t)$ and dilatation $e(\mathbf{x}, t)$ in an n-dimensional viscoacoustic medium is expressed by

$$p(\mathbf{x}, t) = -\dot{e}(\mathbf{x}, t) * \Psi^c(\mathbf{x}, t), \tag{1}$$

where $\mathbf{x}$ is the n-dimensional spatial vector, t is the time, and the dot above a variable represents a time derivative. The relaxation function of the medium $\Psi(t)$ is given by (Liu et al., 1976)

$$\Psi(t) = M_R\left[1 - \sum_{\ell=1}^{L}\left(1 - \frac{\tau_{\epsilon\ell}}{\tau_{\sigma\ell}}\right)\exp\left(-\frac{t}{\tau_{\sigma\ell}}\right)\right], \tag{2}$$

in which $\tau_{\sigma\ell}(\mathbf{x})$ and $\tau_{\epsilon\ell}(\mathbf{x})$ denote material relaxation times for the ℓth mechanism, L is the number of relaxation mechanisms, and $M_R(\mathbf{x})$ is the acoustic or relaxed modulus of the medium.

Equation (1) is the formulation of Boltzmann's superposition principle, such that the current pressure is the superposition of the responses from previous times. A material of this type is considered to have a memory because the current pressure depends upon the full strain history.

ATTENUATION AND DISPERSION

The relaxation function $\Psi(t)$ completely describes the response of the medium. It is possible to obtain from $\Psi(t)$ the spatial quality factor $Q_e(\omega)$ and the phase and group velocities $c(\omega)$ and $c_g(\omega)$, respectively, where ω is the frequency. These quantities give a measure of the attenuation and dispersion of the wave field.

The constitutive relation (1) is analogous to the purely acoustic relation when viewed in the frequency domain. In that domain, the pressure transform is merely a multiplication of the time Fourier transforms of the dilatation field and the time derivative of the causal relaxation function. The latter is identified as the complex bulk modulus of the medium, given by (Carcione et al., 1987)

$$M_C(\omega) = M_R\left[1 - L + \sum_{\ell=1}^{L}\frac{1 + i\omega\tau_{\epsilon\ell}}{1 + i\omega\tau_{\sigma\ell}}\right]. \tag{3}$$

The spatial quality factor is then

$$Q_e(\omega) = \frac{\mathrm{Re}\,(M_C)}{\mathrm{Im}\,(M_C)}. \tag{4}$$

The phase velocity, defined as the frequency divided by the real part of the complex wavenumber $k_c = [M_C(\omega)/\rho]^{1/2}$ (where ρ is the density of the medium), is

$$c(\omega) = c_a\,\mathrm{Re}^{-1}\left\{\left[\frac{M_R}{M_C(\omega)}\right]^{1/2}\right\}, \tag{5}$$

in which $c_a = (M_R/\rho)^{1/2}$ is the relaxed velocity of the medium. The group velocity of the wave field, obtained as the derivative of the frequency with respect to the real wavenumber $k = \omega/c(\omega)$, is

$$c_g(\omega) = \frac{d\omega}{dk} = \left\{\frac{d}{d\omega}\left[\frac{\omega}{c(\omega)}\right]\right\}^{-1}$$

$$= c_a\,\mathrm{Re}^{-1}\left\{\left[\frac{M_R}{M_C(\omega)}\right]^{1/2}\left[1 - \frac{\omega}{2M_C(\omega)}\frac{dM_C(\omega)}{d\omega}\right]\right\}, \tag{6}$$

where

$$\frac{dM_C(\omega)}{d\omega} = M_R\sum_{\ell=1}^{L}\frac{i(\tau_{\epsilon\ell} - \tau_{\sigma\ell})}{(1 + i\omega\tau_{\sigma\ell})^2}.$$

Real materials behave elastically at both very low and very high frequencies. The relaxation function (2), which is based on the general standard linear solid rheology (Liu et al., 1976), correctly describes this behavior as is clear from the quality factor (4) (in the limits $\omega \rightarrow 0$ and $\omega \rightarrow \infty$, $Q_e \rightarrow \infty$).

In this work we choose the acoustic behavior in the low-frequency limit. For a standard linear solid mechanical model, the acoustic limit is reached when the dashpot is eliminated; elimination implies $\tau_{\epsilon_\ell} \rightarrow 0$ and $\tau_{\sigma_\ell} \rightarrow 0$ (Ben-Menahem and Singh, 1981, p. 856). Eliminating the dashpot is equivalent to $\omega \rightarrow 0$, as can be seen from equation (3); hence, the relaxed and acoustic moduli coincide. In practice, however, we do not need to restrict the representation of real materials to mechanical models; therefore, we can choose the acoustic or "nondispersive" behavior in the high-frequency limit. This choice is the case in Ben-Menahem and Singh (1981, p. 873). In conclusion, when τ_{σ_ℓ} and $\tau_{\epsilon_\ell} \rightarrow 0$ or $\tau_{\sigma_\ell} = \tau_{\epsilon_\ell}$ in equation (3), the complex bulk modulus equals the relaxed bulk modulus; and the acoustic case is obtained. In this limit, the phase and group velocities are constant and equal to the acoustic velocity.

EQUATIONS OF MOTION

The equations of motion are derived first by taking the divergence of the equations of momentum conservation for the n-dimensional viscoacoustic continuous medium. The resulting equation is given by

$$-\mathbf{D}p = \ddot{e} + s, \tag{7}$$

where $s(\mathbf{x}, t)$ is a source term given by the divergence of the body forces divided by the density and $\mathbf{D}$ is a spatial operator defined by

$$\mathbf{D} = \frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial}{\partial x_i}\right), \qquad i = 1, \ldots, n. \tag{8}$$

The convention in which repeated indices imply summation is used throughout this work.

MEMORY VARIABLES

Equations (1) and (7) fully describe the deformation of the viscoacoustic medium, and in principle could be a basis for a numerical solution algorithm. However, the convolution integral in equation (1) poses difficulties because it requires a knowledge of the full strain history, unlike acoustic relations which involve only current values of variables. It was shown in Carcione et al. (1987) that the convolution integral can be avoided by introducing memory variables. In this section, we summarize the main results given in the previous paper.

Equation (1) can also be written as

$$p(t) = -\, e(t) * \dot{\Psi}^c(t). \tag{9}$$

Performing the time derivative and using equation (2) yields

$$p(t) = -\left[e(t)M_u + e(t) * \Phi^c(t)\right], \tag{10}$$

where $M_u \equiv \Psi^c(0)$ is the unrelaxed modulus and

$$\Phi(t) \equiv \dot{\Psi}(t) = \sum_{\ell=1}^{L} \phi_\ell(t) \tag{11}$$

is called the response function of the medium. ϕ_ℓ for each of the L relaxation mechanisms is given by

$$\phi_\ell(t) = \frac{M_R}{\tau_{\sigma_\ell}}\left(1 - \frac{\tau_{\epsilon_\ell}}{\tau_{\sigma_\ell}}\right)\exp\left(-\frac{t}{\tau_{\sigma_\ell}}\right). \tag{12}$$

We now define L memory variables e_{1_ℓ} by

$$e_{1_\ell}(t) = e(t) * \phi_\ell^c(t), \qquad \ell = 1, \ldots, L. \tag{13}$$

Taking derivatives with respect to time and using properties of the Heaviside function, we get

$$\dot{e}_{1_\ell}(t) = e(t) * \dot{\phi}_\ell^c(t) = e(t)\,\phi_\ell^c(0) - \frac{e_{1_\ell}(t)}{\tau_{\sigma_\ell}}, \tag{14}$$

where equation (12) was used.

Substituting equation (10) into equation (7) and using equations (11), (13), and (14), we obtain a coupled first-order ordinary differential equation in time (Carcione et al., 1987):

$$\dot{\mathbf{E}} = \underset{\sim}{\mathbf{M}}\mathbf{E} + \mathbf{S}, \tag{15}$$

where $\underset{\sim}{\mathbf{M}}$ is a spatial operator matrix of dimension $L + 2$ given by

$$\underset{\sim}{\mathbf{M}} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ \mathbf{D}M_u & 0 & \mathbf{D} & \mathbf{D} & \cdots & \mathbf{D} \\ \phi_1^c(0) & 0 & \frac{-1}{\tau_{\sigma_1}} & 0 & \cdots & 0 \\ \phi_2^c(0) & 0 & 0 & \frac{-1}{\tau_{\sigma_2}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_L^c(0) & 0 & 0 & 0 & \cdots & \frac{-1}{\tau_{\sigma_L}} \end{bmatrix} \tag{16}$$

with

$$\mathbf{E}^T = [e, \dot{e}, e_{1_1}, e_{1_2}, \ldots, e_{1_L}] \tag{17}$$

and the source vector expressed by

$$\mathbf{S}^T = [0, s, 0, 0, \ldots, 0]. \tag{18}$$

Equation (15) represents the equation of motion governing the viscoacoustic response of the medium. It correctly describes the anelastic effects observed in wave propagation, namely, attenuation and dispersion, within the framework of the linear response theory. The model can describe wave propagation through any kind of linear viscoacoustic material, for example porous rocks where waves of the first kind can be approximated by the standard linear solid rheology (Geertsma and Smit, 1961), provided the complex bulk modulus of the porous media is given as a function of the frequency. By fitting the observed complex bulk modulus to the viscoacoustic bulk modulus given by equation (3), the corresponding relaxation times and relaxed bulk modulus can be obtained for any frequency range.

Storage requirements increase with the number of memory variables, i.e., with the number of relaxation mechanisms. Depending upon the accuracy required, constant-Q materials in the seismic exploration band (say between 5 and 100 Hz) can be obtained by using two or more sets of relaxation times. It was shown by Geertsma and Smit (1961) that to describe viscoacoustic wave propagation (P waves) in a Biot medium, only one mechanism is necessary. Besides the curve-fitting procedure used in this work, optimal relaxation times can be obtained using the Padé approximant method derived by Day and Minster (1984).

For the spatial derivative operator **D** in equation (15), we use the Fourier pseudospectral method (Kosloff and Baysal, 1982), which consists of a discretization of the space and calculation of spatial derivatives using the fast Fourier transform (FFT).

Propagation in time is performed by a new pseudospectral time-integration technique (Tal-Ezer, 1986; Tal-Ezer et al., 1987; Carcione et al., 1987). Because this approach is very accurate, numerical dispersion is avoided. Avoiding numerical dispersion is very important in viscoacoustic wave propagation, where numerical dispersion could be confused with the real physical dispersion.

ABSORBING BOUNDARIES

Because the Fourier method considers the discretized variables on the grid as periodic functions, absorbing boundaries are implemented to prevent wraparound, the phenomenon in which a pulse exiting the grid on one side reenters it on the opposite side. To eliminate wraparound, we use a method developed by Kosloff and Kosloff (1986) of systematically eliminating the wave amplitude in a strip along the boundary of the numerical mesh. This is achieved by replacing the operator $\underset{\sim}{\mathbf{M}}$ in equation (15) by the operator $(\underset{\sim}{\mathbf{M}} - \alpha\underset{\sim}{\mathbf{I}})$, where $\underset{\sim}{\mathbf{I}}$ is the identity matrix and α, the absorption, is given by

$$\alpha = U_0/\cosh^2(\delta\, m), \tag{19}$$

where U_0 is a constant and δ is a decay factor. The parameter $\alpha(x, z)$ is chosen to differ from zero only in a strip of nodes (m) surrounding the numerical mesh. Equation (19) has the functional form of the complex potentials used in quantum mechanics, where, with an appropriate choice of parameters, it is possible to eliminate reflected or transmitted energy from the absorbing region.

To clarify the method, let's consider the one-dimensional (1-D) acoustic limit ($\tau_{\epsilon\ell} = \tau_{\sigma\ell}$, $\ell = 1, \ldots, L$) of equation (15) with constant material properties and zero source term:

$$\frac{d}{dt}\begin{bmatrix} e \\ V \end{bmatrix} = \begin{bmatrix} -\alpha & 1 \\ C_a^2 & -\alpha \end{bmatrix}\begin{bmatrix} e \\ V \end{bmatrix} \tag{20}$$

where c_a is the acoustic wave velocity; in the acoustic limit, $M_u \equiv \Psi^e(0) \rightarrow M_R$ and $\phi_\ell^e(0) \rightarrow 0$, $\ell = 1, \ldots, L$. When $\alpha = 0$, the first equation in (20) expresses the relation $V = de/dt$, whereas the second one is the acoustic wave equation. When α is different from zero, eliminating V makes equation (20) read

$$\ddot{e} = c^2 \frac{d^2 e}{dx^2} - 2\alpha\dot{e} - \alpha^2 e. \tag{21}$$

This equation has a general solution of the form

$$e(x, t) = Af_1(x - c_a t)e^{-(\alpha/c_a)x} + Bf_2(x + c_a t)e^{(\alpha/c_a)x}, \tag{22}$$

with A and B arbitrary constants and f_1 and f_2 arbitrary twice differentiable functions. The solution represents attenuating waves in space, where all frequency components are equally attenuated. This means that the absorbing boundary will gradually attenuate the wave field without changing shape or producing dispersion.

Table 1. Relaxation times (seconds).

ℓ	$\tau_{\epsilon\ell}$	$\tau_{\sigma\ell}$
1	0.3196444	0.3169808
2	0.0850259	0.0842624
3	0.0226023	0.0224139
4	0.0060122	0.0059582
5	0.0016009	0.0015822

EXAMPLES OF WAVE PROPAGATION THROUGH VISCOACOUSTIC MEDIA

Now we examine wave propagation through different types of geologic structures. In the first problem the wave field changes are analyzed in a homogeneous medium. The second example computes a common shot synthetic section for a layered structure with a homogeneous quality factor. In the third case, a zero offset section for a structure with a low Q lens-shaped body is calculated using the exploding reflector concept.

Homogeneous earth

To illustrate the wave-field changes due to attenuation and dispersion, we consider wave propagation in a two-dimensional (2-D) homogeneous medium. The motion is initiated by a point force located at the center of a 132 × 132 grid with spacing DX = DZ = 20 m. The source is a shifted zero-phase Ricker wavelet defined by

$$F(t) = \exp\left[-0.5 f_0^2 (t - t_0)^2\right] \cos \pi f_0 (t - t_0),$$

with $t_0 = 0.06$ s and a high cutoff frequency $f_0 = 50$ Hz. The relaxed bulk modulus of the medium is chosen as $M_R = 8$ GPa and the density is $\rho = 2\text{g/cm}^3$; consequently the acoustic or relaxed velocity is $c_a = 2000$ m/s. The material has five relaxation times (see Table 1), which give an almost constant spatial quality factor $Q_c \approx 100$, a typical value in the exploration seismic band.

Figures 1a and 1b show the quality factor and phase and group velocity dispersion, respectively, as functions of the frequency f. The continuous line in Figure 1b corresponds to the phase velocity.

Figures 2a and 2b compare viscoacoustic and acoustic time histories for identical configurations at stations located 200 m and 800 m from the source, respectively. As the figures show, there is a relatively small difference in amplitude between solutions for the 200 m range; but at a range of 800 m, the difference becomes more apparent. In the latter case, the viscoacoustic pulse arrives earlier than the acoustic one. This early arrival results from the group velocity's (dashed line) being greater than the acoustic velocity, as is clear from Figure 1b. The amplitude reduction is also more pronounced for the viscoacoustic case.

Flat layered structure

A layered geologic structure is used to demonstrate the influence of attenuation on a common shot experiment. The geologic model is shown in Figure 3 with the source located at point S and the recording geophones represented by a set of stars. The velocities indicated are the relaxed velocities of the medium, and the density is considered to be constant with the same value in all of the layers. The same set of relaxation times (Table 1) is used for all the layers. The numerical model uses a 88 × 88 grid with a spacing of DX = DZ = 20 m. The absorbing region surrounding the numerical mesh has a width of 15 grid points with parameters $U_0 = 40\ s^{-1}$ and $\delta = 0.18\ m^{-1}$.

Figures 4a and 4b show the viscoacoustic and purely acoustic time sections, respectively. The four coherent events are the high-amplitude direct wave field and the hyperbolas corresponding to the three flat, not very deep, reflectors. The higher amplitude for the acoustic wave field can be seen mainly in the

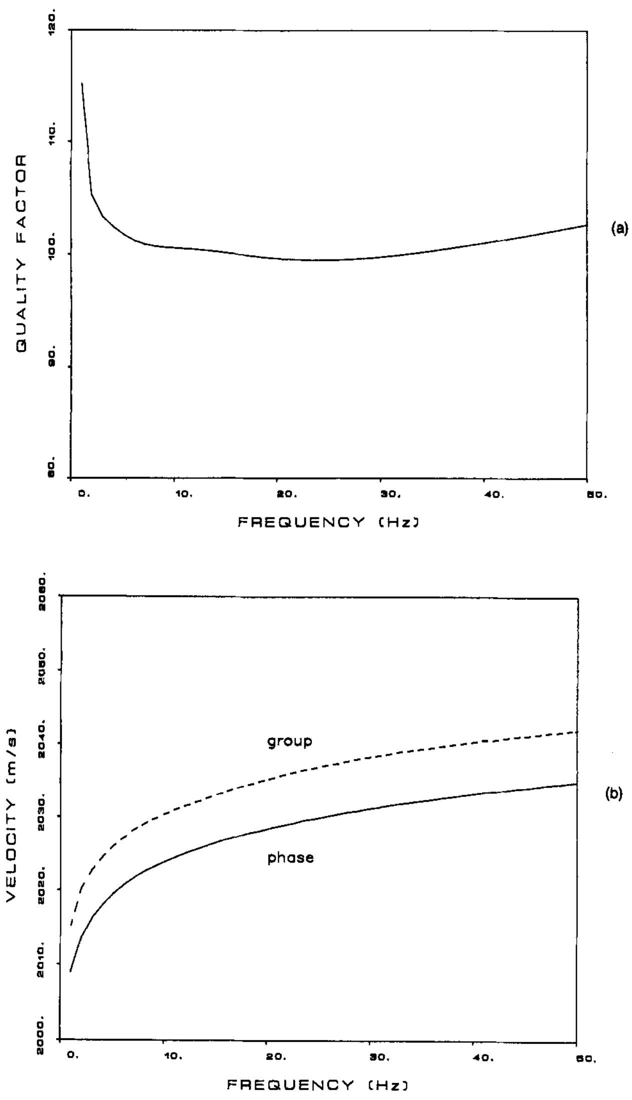


FIG. 1. (a) Spatial quality factor versus frequency. (b) Phase and group velocities versus frequency. The medium is defined by a relaxed modulus $M_R = 8$ GPa, a density $\rho = 2$ g/cm^3, and the five sets of relaxation times given in Table 1.

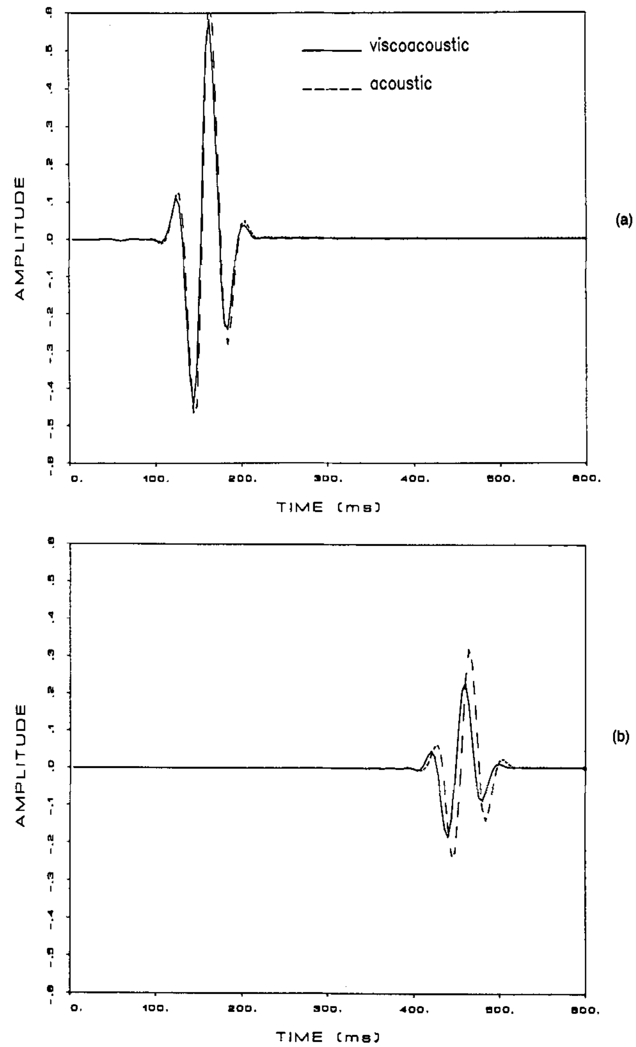


FIG. 2. Time history comparison between the viscoacoustic and acoustic forward modeling algorithms for a homogeneous earth at distances from the source of (a) 200 m and (b) 800 m.

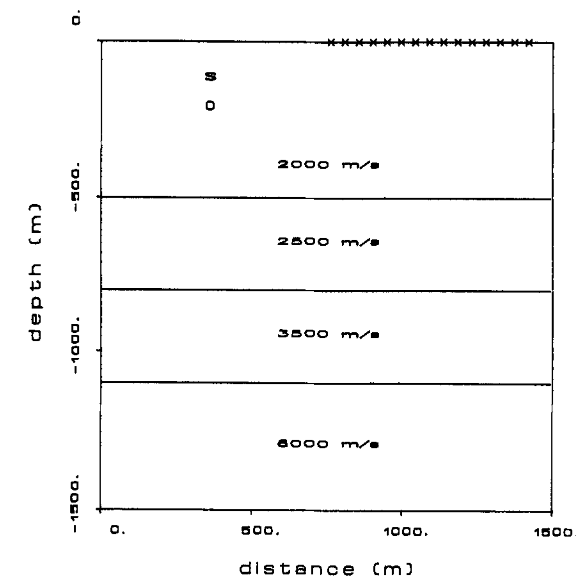


FIG. 3. Geologic model and configuration of a flat layered structure problem. The coordinates of the first receiver relative to the source position are $x_1 = 400$ m and $y_1 = -200$ m. The distance between adjacent receivers is 20 m.

last geophones of the recording line; and the earlier arrival time of the last event for the viscoacoustic wave field is apparent. For deeper reflecting horizons, these effects will be even more pronounced.

Highly anelastic body

The model depicted in Figure 5 shows a highly attenuating body represented by a lens with a low spatial quality factor $Q_e \approx 15$, immersed in a medium having $Q_e \approx 100$ over a flat interface. Table 2 shows the set of relaxation times of the lens' material. Figures 6a and 6b show the quality factor and phase and group velocity dispersion, respectively, as functions of the

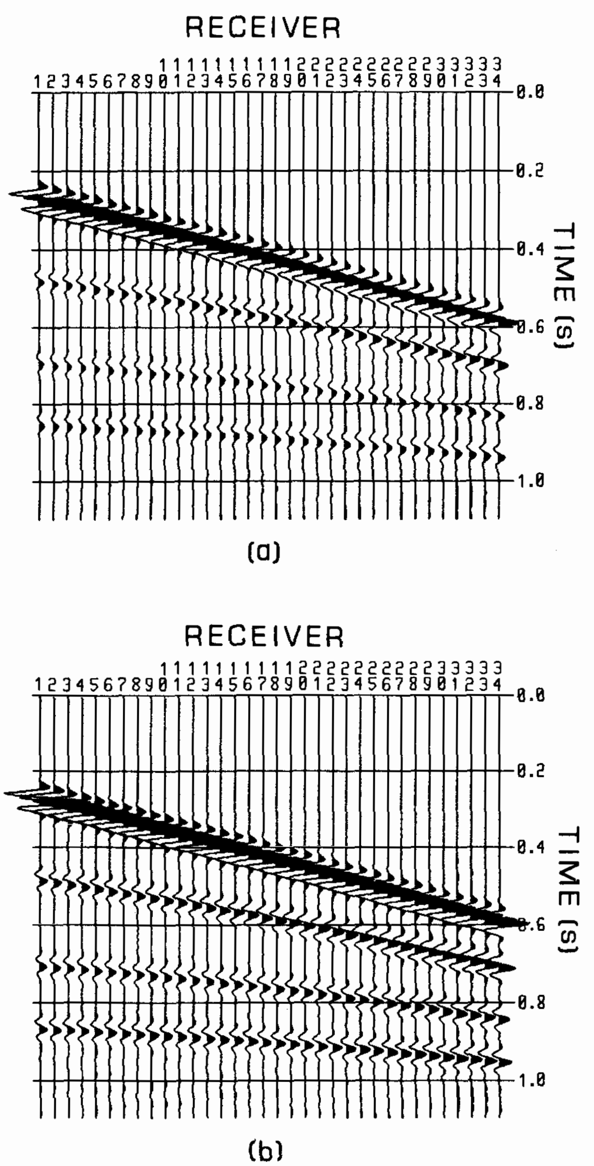


FIG. 4. Synthetic time sections, corresponding to a flat layered structure, for (a) the viscoacoustic model and (b) the purely acoustic model.

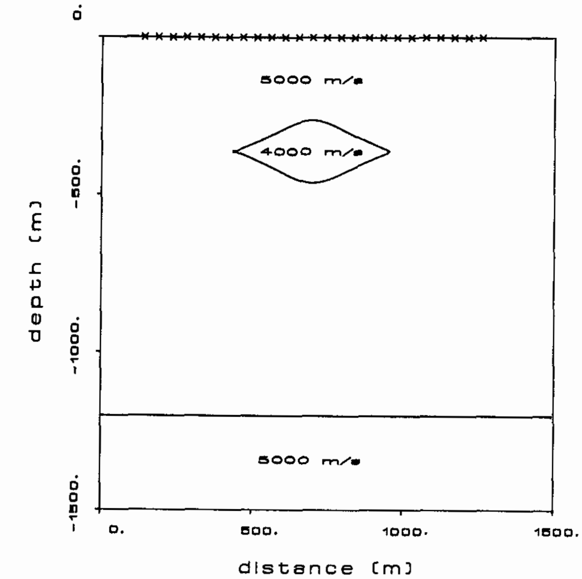


FIG. 5. Geologic model and configuration for the highly anelastic body exploding reflector experiment. The lens limits are constructed by cubic splines interpolation. The coordinates of the first receiver relative to the left extreme of the lens are $x_1 = -300$ m and $y_1 = 360$ m. The distance between adjacent receivers is 20 m.

frequency f. A strong velocity dispersion can be observed. With this attenuation-dispersion pair, although this model is highly idealized, the lens may represent a near-surface unconsolidated material or a strongly anelastic porous rock. The velocities indicated are the acoustic or relaxed velocities. The same source time history as for the previous example is used. The numerical model uses a 99×99 grid with a spacing DX = DZ = 20 m and the absorbing boundaries of the earlier example.

The experiment approximates a zero-offset section by means of the well known exploding reflector model (Loewenthal et al., 1976). To implement the exploding reflector experiment, we assume an earth with a constant impedance, a condition that can be approximately realized by setting $\rho c_a = C$, where C is a constant. A similar procedure for the purely acoustic case was described in Baysal et al. (1984). With this assumption, the operator matrix $\underset{\sim}{\mathbf{M}}$ in equation (15) is expressed by modifying its elements in the following way:

$$\mathbf{D} \rightarrow \frac{\partial}{\partial x_i} c_a \frac{\partial}{\partial x_i} c_a, \qquad i = 1, \ldots, n,$$

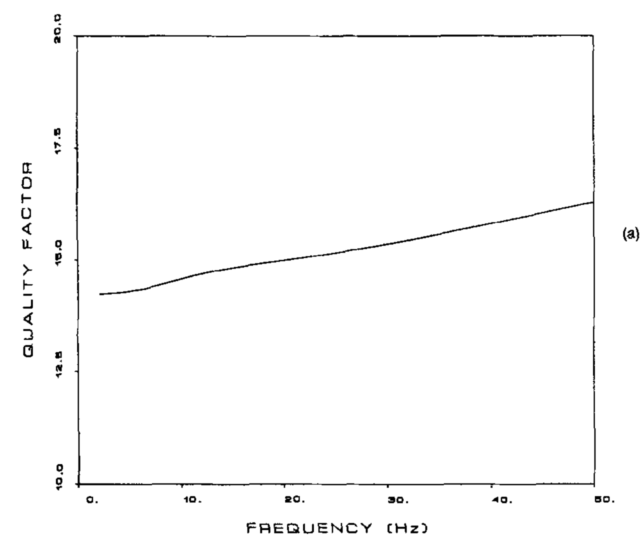
$$M_u \rightarrow \frac{M_u}{M_R},$$

and

$$\phi_\ell^c(0) \rightarrow \frac{\phi_\ell^c(0)}{M_R}.$$

Table 2. Relaxation times (seconds).

ℓ	$\tau_{\epsilon \ell}$	$\tau_{\sigma \ell}$
1	0.3290970	0.3078763
2	0.0876762	0.0817153
3	0.0232707	0.0217701
4	0.0061996	0.0057781
5	0.0016604	0.0015255



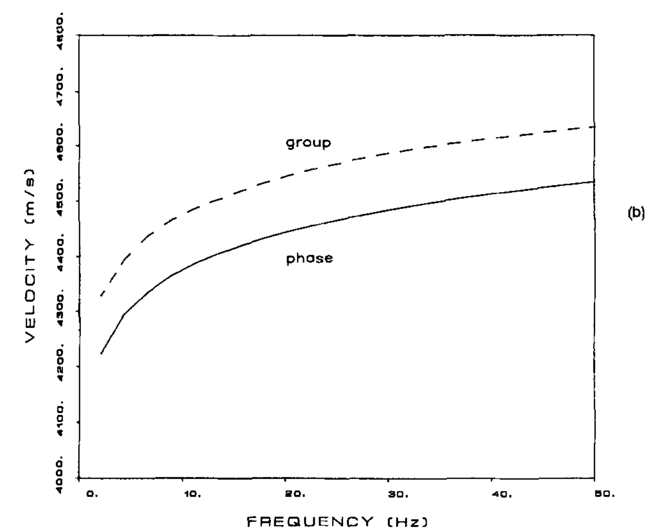


FIG. 6. (a) Spatial quality factor versus frequency. (b) Phase and group velocities versus frequency. The medium (the lens in Figure 5) is defined by a relaxed velocity $c_o = 4000$ m/s and the five sets of relaxation times given in Table 2.

In order to obtain the correct time events and anelastic effects, the relaxed velocity is halved.

Viscoacoustic and acoustic synthetic time sections are shown in Figures 7a and 7b, respectively. Three coherent events can be identified in the synthetic seismograms. They are the time responses of the top and bottom interfaces that define the lens, and after 0.4 s, the time response of the flat interface. The strong "bright spot" in the acoustic time section is a geometrical effect caused by focusing of the flat interface due to the presence of the lens. This focusing effect has completely disappeared in the viscoacoustic seismogram, where the amplitude of the wave field was affected by the strong absorption in the lens. Also, the pull-down effect that is appreciable at the onset times of the acoustic time section has disappeared on the viscoacoustic section, a consequence of the strong velocity dispersion inside the lens.

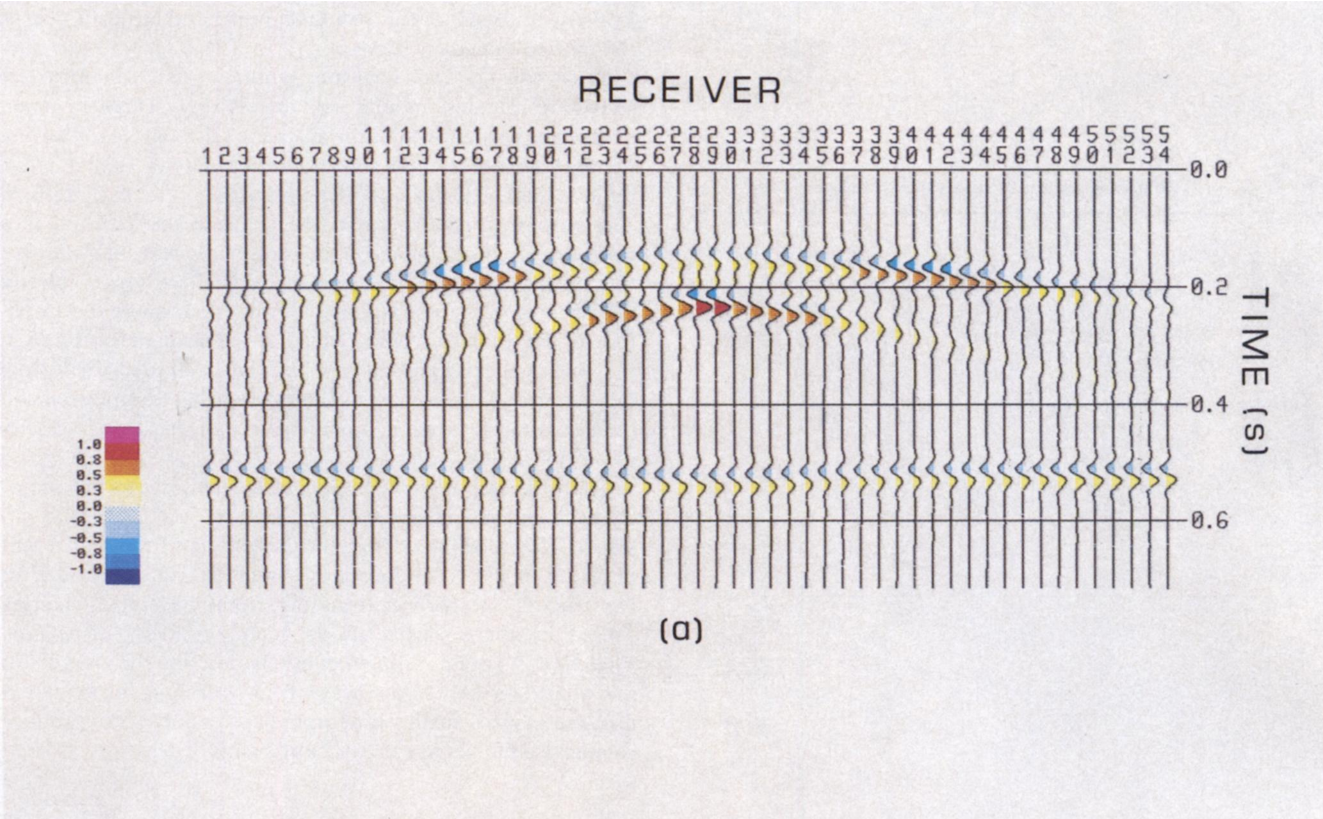
The time response of the top of the lens shows no major changes, but the response of the bottom is considerably affected. The strong amplitude in the middle of the later event observed in the viscoacoustic seismogram is probably due to constructive interference resulting from wave-field changes caused by the anelastic effects, mainly velocity dispersion. Therefore, it is important to correctly describe the changes in the phase spectra of the wave field. For long propagation distances, even a small variation in phase velocity can produce significant changes in waveforms and amplitudes.

CONCLUSIONS

We have presented a model for viscoacoustic wave propagation simulation that is completely general within the class of linear constitutive relations. The theory, based on Boltzmann's superposition principle, includes as special cases any linear model such as those which describe acoustic wave propagation in sedimentary rocks, for instance, the theory of Murphy et al. (1986). Extension of the theory to the viscoelastic rheology allows us to describe elastic wave propagation through porous media (Biot, 1956a, b; Burridge and Keller, 1981; de la Cruz and Spanos, 1986). In addition to a viscoelastic representation of theoretical complex moduli, experimental data can be used when quality factors versus frequency and velocity dispersion are available.

Applications of viscoacoustic modeling suggest a strong need to consider the correct attenuation-dispersion effect for fundamental seismic problems in anelastic earth models. Comparisons between viscoacoustic and acoustic seismic responses of the two geologic models presented here show differences in the amplitudes and arrival times of the wave field. The first example shows that the viscoacoustic wave pulse arrives earlier than the acoustic one. This is a consequence of the dispersion effect, which becomes more important with increasing distance. The example of the highly attenuating body indicates that the combination of geometrical and anelastic effects can considerably alter the recorded wave field.

Existing algorithms that simulate the process of wave propagation are mainly based on the acoustic wave equation. The present theory improves the acoustic assumption and therefore represents a more suitable tool for solution of geophysical problems and interpretation of seismograms. In the future, we plan to expand this effort by incorporating a truly viscoelastic



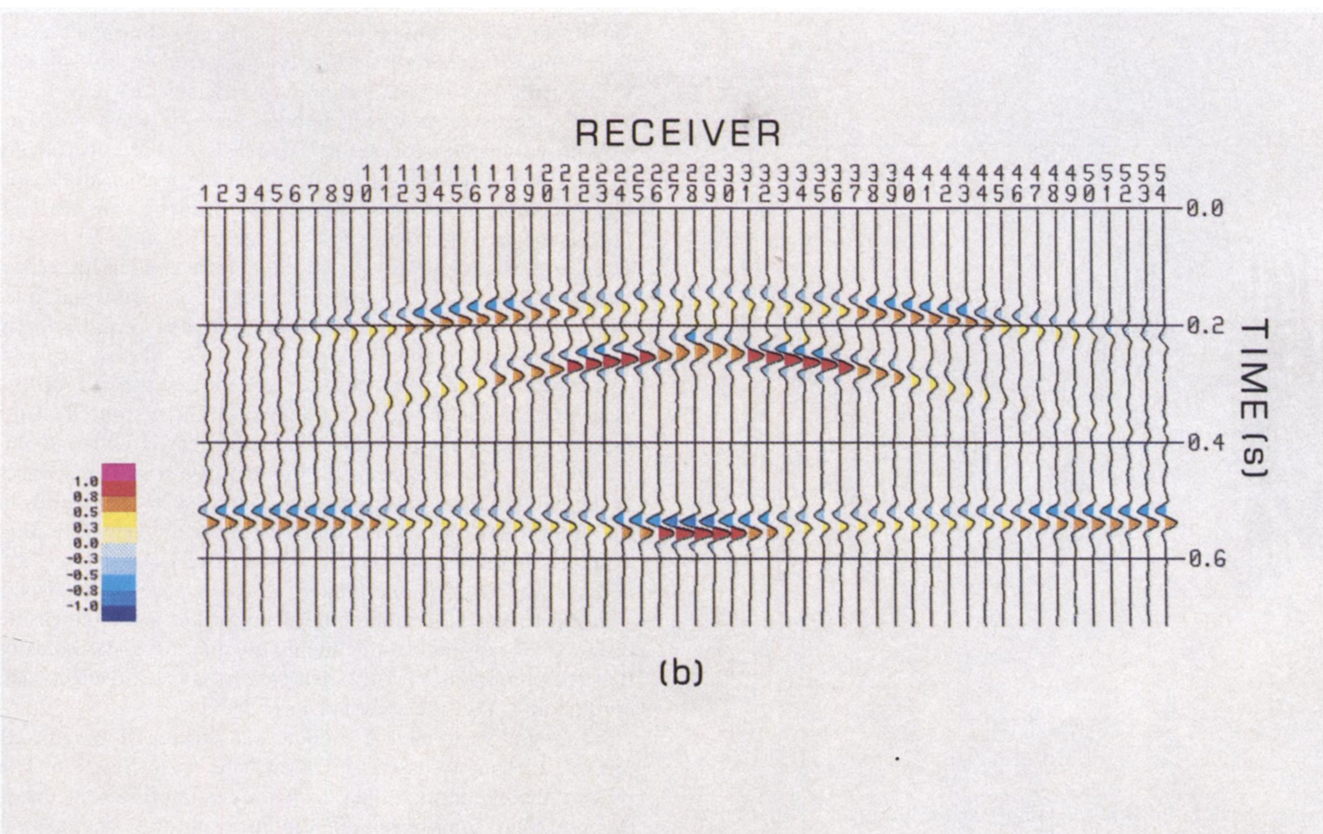


FIG. 7. Synthetic time sections corresponding to the highly attenuating body, for (a) the viscoacoustic model and (b) the purely acoustic model.

rheology to address the problem of wave propagation in a porous medium.

ACKNOWLEDGMENTS

This work was supported by grants from the Israel Academy of Sciences and Humanities and from the Geophysical Institute of Hamburg University under project 03E-6424-a of the BMFT, West Germany and project EN3C-0008-D of the Commission of the European Communities.

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