

## Viscoelastic modeling of rocks saturated with heavy oil

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### Summary

While properties of bulk heavy oil can be approximated by an appropriate viscoelastic model, only a few attempts to model properties of rocks saturated with heavy oil have been reported (Eastwood, 1993; Leurer and Dvorkin, 2006). Rock physics for heavy oil is different from rock physics for conventional fluids because its viscoelastic rheology makes Gassmann theory and all its extensions inapplicable in principle. In this paper, we estimate the properties of such rock-heavy oil mixtures by considering (1) a system of layers of solid and a viscoelastic medium and (2) by computing Hashin-Shtrikman bounds for this system. These two methods essentially give approximate bounds for the frequency- and temperature-dependent properties of these rocks. We also propose how to compute a realistic estimate of these properties that would lie between these bounds. This proposed estimate is based on one particular equivalent-medium approach known as coherent potential approximation (CPA) (Berryman, 1980). In a more general form, this approximation can be used for approximate fluid substitution for heavy oil.

### Introduction

Heavy oils are an important hydrocarbon reserve that has been actively exploited recently. Thermal recovery methods proved to be the most efficient in the current industry practice of heavy oil production (Butler, 1994). Depending on properties of heavy-oil reservoirs, recovery should be optimized. As with conventional oil and gas, seismic technology is the primary method that can be used both for reservoir characterization and for production monitoring of heavy oil. A few successful 4D case studies for thermal recovery of heavy-oil reservoirs have been reported recently (Lumley et al., 1995; Zhang et al., 2005). It was shown that the key 4D factor is steam saturation. However, to push the envelope toward more quantitative interpretation with higher resolution, a proper rock physics modeling for heavy-oil rocks is necessary. To this end, rock physics relationships are required to link seismic parameters (velocities and attenuation coefficients) to the properties of oil as a function of frequency and PVT conditions. In recent years, a number of laboratory measurements of elastic properties of heavy oils and of rocks saturated with heavy oil have been reported (Eastwood, 1993; Nur et al., 1984; Schmitt, 1999; Batzle et al., 2006). According to these measurements, heavy oils exhibit viscoelastic properties such that at low frequencies they behave like liquids, but at high frequencies almost like

solids. The characteristic frequencies of this viscoelastic transition exhibit strong temperature dependency, such that the oil that at room temperature behaves as a nearly elastic solid (in a wide range of frequencies), and can exhibit Newtonian fluid behavior at temperatures above 200°C.

This study focuses on the effect of temperature on the frequency-dependent shear modulus of bulk heavy oil and its impact on the viscoelastic properties of the saturated rock.

### Properties of bulk heavy oil

#### Shear-wave dispersion

Batzle et al. (2006) shows measured shear modulus of heavy oil in a range of temperatures and frequencies. One can see that properties of heavy oil show viscoelastic behavior, in that for a given temperature, its shear modulus increases with frequency. The simplest model of a viscoelastic medium is a Maxwell model that results in the following dependency of the complex shear modulus  $\mu$  on frequency  $\omega$ :

$$\mu_f(\omega) = \frac{\mu_\infty}{\frac{1}{-i\omega\tau} + 1}, \quad \tau = \frac{\eta}{\mu_\infty}, \quad (1)$$

where  $\mu_\infty$  is the (real) shear modulus of the medium at high frequencies (or quick deformation), parameter,  $\tau = \eta/\mu_\infty$  is called relaxation time (inverse of relaxation frequency  $\omega_0$ ), and  $\eta$  is the dynamic shear viscosity of the same medium at low frequencies (or slow deformation). Note that at high frequencies  $\omega\tau \gg 1$  and hence the complex shear modulus of the medium approaches the real value  $\mu_\infty$ . Conversely, at low frequencies  $\omega\tau \ll 1$ , and  $\mu_f \approx -i\omega\eta$ . This means that in time domain, the shear stress is proportional to the time derivative (rate of change) of the shear deformation, the behavior characteristic of Newtonian fluids. As discussed by Batzle et al. (2006), the frequency dependency described by the Maxwell model is stronger than can be observed in experiments (Batzle et al., 2006). To model this effect, more gradual variation of elastic properties with frequency introduction of a model with more than one relaxation time is required. Batzle et al. (2006) propose to do this using a model with continuous relaxation time spectrum as proposed by Cole and Cole (1941):

$$\mu_f = \mu_0 + \frac{\mu_\infty - \mu_0}{\frac{1}{(-i\omega\tau)^\beta} + 1}, \quad (2)$$

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where the exponent  $0 < \beta < 1$  is an adjustable parameter, and  $\mu_0$  is constant scalar shear modulus. For appropriate parameters the relationship 2 fits the measurements of Figure 1 reasonably well. However, according to this model, at low frequencies the complex shear modulus is

$$\mu_f \cong \mu_0 + (\mu_\infty - \mu_0)(-i\omega\tau)^\beta \quad (3)$$

Thus, in the low frequency limit, the complex shear modulus approaches a real value  $\mu_0$ , which corresponds to elastic behavior in the low- as well as high-frequency limit. This contradicts the widely assumed notion that, at low frequencies, the oil behaves more like a liquid. If we want to adhere to this notion, we have to set  $\mu_0=0$ . However, even with this assumption, at low frequencies we have:

$$\mu_f \cong \mu_\infty (-i\omega\tau)^\beta = \mu_\infty^{1-\beta} (-i\omega\eta)^\beta \quad (4)$$

Thus, at low frequencies, the absolute value of the complex shear modulus is proportional to  $\eta^\beta$  and not  $\eta$  as in Newtonian fluids. Note that the very notion of viscosity is problematic in this case.

To ensure that in the low-frequency limit, the behavior of oil is Newtonian, we propose the following modification of equation 2:

$$\mu_f = \frac{\mu_\infty}{1 + \frac{1}{(-i\omega\tau)^m} + 1} \quad (5)$$

where  $m$  is an additional dimensionless parameter and  $\tau = \eta/\mu_\infty$  as before. It could be proved that this model satisfies the causality and dissipativity conditions.

Temperature dependency

The temperature dependency is modeled by assuming that the relaxation time is a function of temperature. Known viscosity/temperature relationships are reviewed in Batzle et al. (2006). We approximate these curves by following empirical relationship:

$$\ln(\tau/\tau_0) = A \exp(-T/T_0) \quad (6)$$

where  $T$  is Celsius temperature,  $\tau_0$ ,  $A$  and  $T_0$  adjustable parameters.

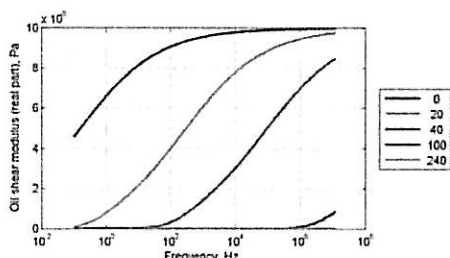


Figure 1: Shear modulus of heavy oil vs. frequency and temperature in bi-logarithmic scale.

Figure 1 shows the dependency of shear modulus of heavy oil as a function of frequency for a number of temperatures as predicted by equations 5 and 6 with the parameters:  $A=32.2$ ,  $\tau_0=10^{12}$ s,  $T_0=75^\circ\text{C}$ ,  $\mu_\infty=1\text{GPa}$ ,  $\beta=1/3$ , and  $m=100$ . We see that, with these parameters, the predicted behavior of the shear modulus is qualitatively similar to that observed in experiments (Batzle et al., 2006).

Modeling properties of rock-oil mixtures

Rock as a system of solid and oil layers

When the pore fluid is Newtonian, low-frequency elastic moduli of a rock saturated with such fluid can be computed from the properties of dry rock and fluid compressibility using Gassmann equation. The corresponding dynamic moduli can be obtained from Biot's equations of poroelasticity. However, neither Gassmann nor Biot's theory is applicable if the pore filling material is viscoelastic. The reason for this is that Gassmann equation is based on the Pascal law, which states that, in the absence of body forces, fluid pressure is the same throughout the pore space. This law does not apply to solids or to any media whose shear modulus has a finite component. Biot's theory is an extension of Gassmann theory to finite frequencies, and is inapplicable to viscoelastic media as well.

To gauge the effect of viscoelasticity of the pore filling material on the overall rock properties, we first model such medium as a simple periodic system of elastic and viscoelastic layers. This simplistic approach to modeling elastic properties of fluid-saturated rocks was previously used by Schoenberg (1984), Schoenberg and Sen (1986), Molotkov and Bakulin (1996), Gurevich (1999, 2002), and Gurevich and Ciz (2006). The advantage of this approach is that, for this simple periodic system, exact dispersion equations are known (Rytov, 1956; Brekhovskikh, 1981). The method is applicable to any rheology and does not have any requirements with respect to size of the pores or the properties of the layers.

Let us denote the properties of elastic layers with subscript  $s$  and viscoelastic ("fluid") layers with subscript  $f$ . For shear waves propagating along the layers and polarized also parallel to the layers, the dispersion equation reads (Rytov, 1956; Brekhovskikh, 1981):

$$p \left[ \tan^2 \frac{\beta_s h_s}{2} + \tan^2 \frac{\beta_f h_f}{2} \right] + (1 + p^2) \tan \frac{\beta_s h_s}{2} \tan \frac{\beta_f h_f}{2} = 0 \quad (7)$$

Here  $\beta_s^2 = \omega^2 (1/b_s^2 - 1/b^2)$ ,  $\beta_f^2 = \omega^2 (1/b_f^2 - 1/b^2)$ , where  $b_s = (\mu_s/\rho_s)^{1/2}$ , and  $b_f = (\mu_f/\rho_f)^{1/2}$  are shear velocities in the materials  $s$  and  $f$ , respectively,  $p = \mu_f \beta_f / \mu_s \beta_s$ ,  $\rho_f$  and  $\rho_s$ , are layer densities,  $h_f$  and  $h_s$  are layer thicknesses and  $b$  denotes the unknown complex

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velocity of the SH wave. The real part of the complex velocity yields the phase velocity of the wave, while the ratio of imaginary to real part of the slowness yields the dimensionless attenuation (inverse quality factor):

$$Q^{-1} = 2 \frac{\text{Im } b^{-1}}{\text{Re } b^{-1}} \quad (8)$$

To simulate the properties of rocks, we should associate layer thicknesses with the pore and grain sizes in the rock. Dispersion equation 7 does not have an analytical solution, but can be solved numerically by iteration. The results of this solution for phase velocity and dimensionless attenuation are shown in Figure 2. Also shown in this figure are the predictions of anisotropic variant of Biot's (1956) theory of poroelasticity (with dynamic permeability function adjusted to flat slabs, see Bedford (1986), Gurevich and Ciz (2006)). We see that for low temperatures, the dispersion of the S-wave is consistent with viscoelastic behavior of the bulk oil, while at higher temperatures, it begins to exhibit smaller Biot's dispersion. Viscoelastic attenuation is much higher than Biot's attenuation.

Also shown in Figure 1 are velocities and attenuation factors for the S-wave propagating perpendicular to layers (the corresponding dispersion equation is given in Rytov (1956) and Brekhovskikh (1981)). Clearly, the waves propagating across layers exhibit much higher dispersion.

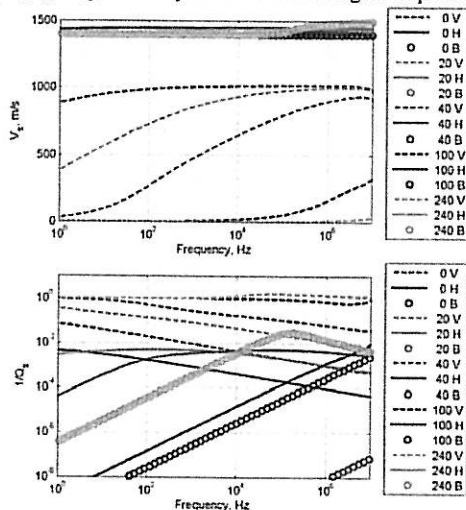


Figure 2: S-wave velocity (upper) and attenuation (lower) in oil/solid mixture vs. frequency and temperature: layered cake.

The dispersion of P-waves (not shown here) along the layers is of the same order of magnitude as that for S-waves propagating in the same direction. However, the dispersion of P-waves propagating perpendicular to layers is weaker

than that for corresponding S-waves. This is caused by the fact that, at low frequencies, the shear modulus of the 'fluid' layers tends to zero, while the P-wave modulus reduces to the bulk modulus of the fluid, which is finite. The obtained characteristics of dispersion and attenuation for waves parallel and perpendicular to layers can serve as bounds on behavior of real rocks.

Hashin-Shtrikman bounds

To obtain alternative estimates of the properties of a rock saturated with heavy oil, we also computed Hashin-Shtrikman bounds for the mixture of elastic and viscoelastic medium. The limitation of using these bounds is that they cannot simulate high-frequency (Biot's) effects, as they are intrinsically designed for static moduli (and the corresponding representative structure corresponds to isolated spherical pores or grains). However, the advantage of these bounds is that they are constrained by an additional requirement of isotropy, and thus correspond to more realistic geometrical structure of the rock.

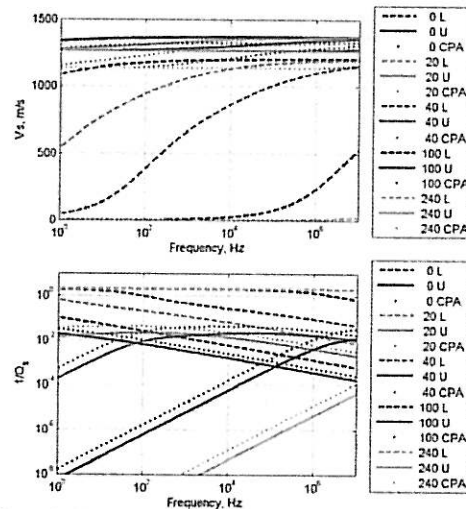


Figure 3: S-wave velocity (upper) and attenuation (lower) in oil/solid mixture vs. frequency and temperature: Hashin-Shtrikman bounds and CPA.

Original Hashin-Shtrikman (H-S) bounds are rigorous and exact bounds for static bulk and shear moduli of an isotropic composite consisting of two isotropic elastic constituents (Christensen, 1979). It is also valid if one of the constituents is fluid, in which case its shear modulus is simply set to zero. We use H-S bounds to compute frequency-dependent moduli of a mixture of an elastic solid and a viscoelastic fluid, by first computing shear modulus of the 'fluid' using equation 5, and then using it as if it were static shear modulus. As a result, we obtained bulk and shear moduli for both lower and upper H-S bounds.

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However, the shear moduli so obtained are no longer rigorous bounds, because they are now complex, and the notion of a value lying between two bounds is undefined for complex numbers. However, similarly to the elastic case, the bounds for bulk modulus are still realizable (in the sense that there exists a particular geometrical configuration of the two constituents such that its equivalent modulus is exactly equal to the value of the bound).

The results for S-waves are shown in Figure 3. We see very similar behavior to layered cake for S-waves: weak dispersion for upper bound and much stronger for the lower bound. Similar behavior is observed for P-waves (not shown here).

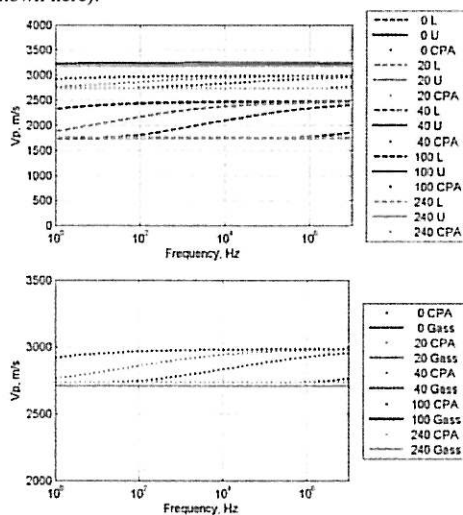


Figure 4: P-wave velocity in oil/solid mixture vs. frequency and temperature: (upper) Hashin-Shtrikman bounds and CPA; (lower) CPA versus Gassmann equation.

#### Coherent potential approximation

The results for layered cake and H-S bounds give upper and lower estimates of the rock properties. To obtain a more realistic 'middle' estimate, we propose to use one of the popular mixing laws of the theory of composite, namely coherent potential approximation, or CPA. CPA, which was originally proposed in quantum field theory, is essentially a self-consistent version of the average T-matrix approximation of Küster and Töksöz (1974); see Berryman (1980). We choose CPA because it has the property that a more abundant constituent is the load-bearing one. Thus, for example, a solid-fluid mixture is modeled as a solid with spheroidal fluid inclusions when fluid concentration is small, and as a suspension of solid particles in the fluid when the solid concentration is small. This is an attractive property, as it is consistent with the concepts of percolation

and critical porosity, and allows one to model both sandstones and unconsolidated sand.

Figure 4 (upper) shows the CPA results for spheres, which as expected, lie between lower and upper H-S bounds. Figure 4 (lower) shows the comparison of CPA velocities against those computed using Gassmann equation (with dry bulk and shear moduli computed by CPA with empty pores). We see that the CPA results are Gassmann-consistent when fluid is Newtonian (i.e., for low frequencies or high temperatures). We note that the CPA for spheres is known to overestimate typical sandstone bulk and shear moduli, simply because typical pores in sands and sandstones are not spherical. More realistic estimates can be obtained by using CPA with the aspect ratio of the pores/grains set to 0.3 or 0.2.

#### Discussions and conclusions

We have shown how velocity and attenuation of a rock saturated with heavy oil can be modeled using simple theoretical constructs adopted from the theory of elastic composite. The key element there was introducing an appropriate model for complex frequency- and temperature-dependent shear modulus of bulk oil. Qualitatively, the results of dispersion trends and their changes with temperature are consistent with the lab observations of Batzle et al. (2006). However, the predicted P-wave dispersion was weaker in our calculations than observed in the experiments. One possible cause of this effect may be bulk viscosity, which is ignored in our study. Another possible cause of this discrepancy is squirt flow of heavy oil in and out of grain contacts (Lerner and Dvorkin, 2006).

This paper has focused on the effect of temperature on attenuation and dispersion and completely ignored other important effects such as pressure, steam saturation, and consolidation. In particular, for loosely consolidated sediments cemented with the heavy oil, the microscopic stress distribution could be very nontrivial. Furthermore, thermal recovery processes may cause significant geomechanical changes in the reservoir and overburden. All these effects need to be analyzed to build a quantitative realistic rock physics model for rocks filled with heavy oil. Description of in situ heavy-oil rheology and monitoring of the thermal recovery is a big challenge, but is very important for static and dynamic modeling and production optimization.

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## EDITED REFERENCES

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