

# Viscous damping of perforated planar micromechanical structures

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## Abstract

The paper gives an analytical approximation to the viscous damping coefficient due to the motion of a gas between a pair of closely spaced fluctuating plates in which one of the plates contains a regular system of circular holes. These types of structures are important parts of many microelectromechanical devices realized in MEMS technology as microphones, microaccelerometers, resonators, etc.

The pressure satisfies a Reynolds' type equation with coefficients accounting for all the important effects: compressibility of the gas, inertia and possibly slip of the gas on the plates. An analytical expression for the optimum number of circular holes which assure a minimum value of the total damping coefficient is given. This value realizes an equilibrium between the squeeze-film damping and the viscous resistance of the holes.

The paper also provides analytical design formulas to be used in the case of regular circular perforated plates.

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## 1. Introduction

The study of the motion of a thin fluid layer squeezing between a vibrating plate (or proof mass) and a backplate electrode (or between the two vibrating plates in other applications), referred as a planar microstructure, is important in many microelectromechanical devices such as microphones [1], microaccelerometers [2], micromechanical switches [3], various resonators [4], and tunable microoptical interferometers [5]. The motion of the thin film of air in a planar microstructure generates a *squeeze-film damping* that can adversely affect the dynamic response of the system [6].

The excessive gas damping problem is often solved by “drilling” perforations in one of the plates. In fact, the use of perforated plates in surface-micromachined planar microstructures has a double role: it reduces the squeeze-film damping effect and enhances the etching of underlying sacrificial layers in the microfabrication process. Thus, the perforations are compatible with the thin film processing and add very little to the manufacturing cost.

While the squeeze-film damping is reduced by incorporating holes in the backplate, the motion of the air within the backplate holes gives a new viscous resistance which adds to the squeeze-film damping. Previous work devoted to the viscous damping in planar microstructures has considered, in many cases, only the squeeze-film damping. Thus, Škvor [7] using a simplified hydraulic model succeeded in obtaining an analytical formula for evaluating the squeeze-film damping in some acoustical devices. Škvor's result is applicable to incompressible fluids and neglects any added damping due to the flow through the holes. The viscous hole resistance effect was considered by Rossi [8] in some acoustical applications. Recently, Veijola and Mattila [9] and Bao et al. [10] developed damping models incorporating the holes resistance in the linearized Reynolds equation.

These two viscous effects, namely the squeeze-film damping due to the horizontal motion of the air between the plates and the resistance due to the vertical motion of air through the holes are not independent. In order to decrease the squeeze-film damping we have to “drill” more and more holes but each new hole adds its resistance to the total damping. Thus, we expect the existence of an optimum number of perforations that

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minimizes the viscous damping and corresponds to an equilibrium between the two components of the viscous damping. For the case of incompressible fluids such an analysis was performed in [11]. In the present study, we extend some of these results to account for the effects of compressibility, inertia, and the gas slip on surfaces. A primary aim of this paper is to present practical formulae that are more widely applicable than previous results and, in electrically driven or sensed devices, have the minimal gas damping for an assumed capacitance.

The analysis performed in this work is quite general being applicable to a very large spectrum of frequencies and to various fluids. The departing point is the Navier–Stokes system. As the domain of the fluid motion is very thin, an asymptotic analysis of this partial differential equation system was considered. In order to include the inertia effects, important at large frequencies, the terms containing the time derivatives of velocity were maintained in the system. Also, are considered the first-order slip velocity conditions on the plates, important in the case of slightly rarefied gases [12]. The final equation for the pressure has the same form as Reynolds equation but with different coefficients accounting for all the described effects.

The case of a planar microstructure containing a regular web of circular holes is considered as an application. The cell corresponding to a certain hole (by cell we mean the influence domain of the hole) is a regular polygon. The normal derivative of the pressure along the boundary polygonal line is zero while the pressure has a constant value along the holes’ rim. In solving the resulting boundary value problem for a certain cell the external curve will be approximated by an equivalent circle enclosing the same area. The mixed Dirichlet–Neumann problem for an annulus in the case of Reynolds’ equation can be solved explicitly. Hence, the squeeze-film damping can be expressed analytically. For the case of small frequencies and low compressibility, the resulting formula coincides with that given in [11]. Also, the holes resistance can be determined explicitly by solving a similar Helmholtz-type equation for the Poiseuille flow in a circular pipe.

The total damping coefficient, expressed in terms of number of holes on an unit of area, has a minimum value for a certain number of holes (optimum number of holes on a unit area). Afterwards, designing formulas are given which determine the geometry of the structure such that the resulting micromechanical structure has the smallest damping for an assigned open area.

In order to give an estimation of the error involved by changing the external boundary curve of the cell into a circle, a model problem for an incompressible fluid was considered. In this case, the mixed problem for the exact domain was solved by a numerical boundary element technique. The comparison of the two solutions (one approximate analytical and the other one numerical) shows that for the case where the open area ratio AR is smaller than 0.4 the precision of the analytical formulas is very good, while for AR > 0.5 the precision is decreasing.

As a byproduct of this calculation a correction term for the Škvor’s analytical formula for the cases 0.4 < AR < 0.75 was obtained.

For a finite plate the real damping force depends on the plates’ boundary which is decreasing the pressure. Hence, the damping coefficient resulting by the present analysis is conservative; for a better approximation of this coefficient, corresponding to a particular structure, a more elaborate analysis [9,10] working in the specified case has to be performed.

## 2. Formulation of the problem

In order to study the viscous damping on a planar microstructure we model the air in the gap between the two parallel plates as a compressible Newtonian gas. We refer the motion of the fluid to a Cartesian system of coordinates whose origin *O* is halfway between the plates average position the *xOy*-plane parallel to the two plane surfaces (Fig. 1).

### 2.1. Equations of the fluid motion (horizontal flow)

The isothermal motion of the gas is described by the Navier–Stokes system: the continuity equation,

$$\frac{1}{\rho c^2} \left[ \frac{\partial p}{\partial t} + (\mathbf{v} \cdot \nabla)p \right] + \nabla \cdot \mathbf{v} = 0 \tag{1}$$

and the momentum equations,

$$\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{v} + (\mu + \lambda) \nabla(\nabla \cdot \mathbf{v}) \tag{2}$$

where  $\rho$  is the density,  $c$  the isothermal speed of sound,  $p$  and  $\mathbf{v}$  are perturbations of the pressure and velocity,  $\mu$  and  $\lambda$  are the shear and bulk viscosities and  $\mathbf{g}$  the gravity acceleration.

In the case of simple harmonic oscillations (of frequency  $\omega/2\pi$ ) we have

$$p = p_\omega e^{-i\omega t}, \quad \mathbf{v} = \mathbf{v}_\omega e^{-i\omega t}, \quad \frac{\partial \cdot \mathbf{v}}{\partial t} = -i\omega.$$

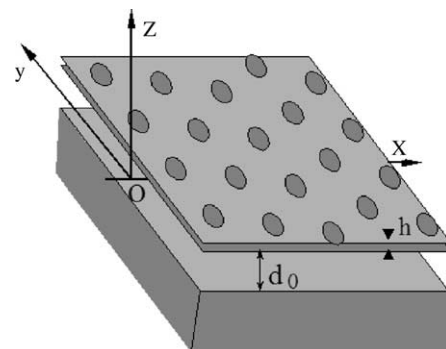


Fig. 1. A perforated planar microstructure.

Eqs. (1) and (2) then become

$$-\frac{i\omega}{\rho c^2} p_\omega + \frac{1}{\rho c^2} (\mathbf{v} \cdot \nabla) p_\omega + \nabla \cdot \mathbf{v}_\omega = 0 \tag{3}$$

$$-i\omega\rho\mathbf{v}_\omega + \rho(\mathbf{v} \cdot \nabla)\mathbf{v}_\omega = \rho\mathbf{g} \exp(i\omega t) - \nabla p_\omega + \mu\nabla^2\mathbf{v}_\omega + (\mu + \lambda)\nabla(\nabla \cdot \mathbf{v}_\omega) \tag{4}$$

It is helpful to recast these equations using dimensionless variables. As the domain in our case is the narrow air gap between the two plates, we will use different scales on the  $x$ ,  $y$ -directions and the  $z$ -direction.

$$\begin{aligned} x &= L_0x', & y &= L_0y', & z &= d_0z', & v_{\omega x} &= V_0v'_x, \\ v_{\omega y} &= V_0v'_y, & v_{\omega z} &= \varepsilon V_0v'_z, & p_\omega - p_a e^{i\omega t} &= P_0p', \\ \rho &= \rho_0\rho', & c &= c_0c' \end{aligned}$$

$d_0$  being the distance between plates at equilibrium,  $L_0$  is a characteristic length connected to the planar domain and  $\varepsilon = d_0/L_0$  is a small parameter. The other reference variables have characteristic values. In the following, we drop the primes and remember that we are now working in dimensionless variables. To the lower order in  $\varepsilon$  we obtain the equations

$$\frac{\partial^2 v_x}{\partial z^2} + iK^2 v_x = \frac{\partial p}{\partial x} \tag{5}$$

$$\frac{\partial^2 v_y}{\partial z^2} + iK^2 v_y = \frac{\partial p}{\partial y} \tag{6}$$

$$\frac{\partial p}{\partial z} = 0 \tag{7}$$

$$\nabla \cdot \mathbf{v} - iK_1 p = 0 \tag{8}$$

where we have denoted

$$P_0 = \frac{\mu V_0 L_0}{d_0^2}, \quad K = d_0 \sqrt{\frac{\rho_0 \omega}{\mu}}, \quad K_1 = \frac{\omega \mu L_0^2}{\rho_0 c_0^2 d_0^2}$$

Most approaches of the thin-film problems are neglecting completely the inertial terms. This is justified when the flow is steady, or slowly oscillating. However, when the oscillation frequency increases the inertia of the gas has a significant influence on the velocity profile. In Eqs. (5)–(8) the terms containing the parameters  $K$  and  $K_1$  include the dependence of the mechanical quantities upon frequency.

*2.2. The boundary conditions for the gas velocity and pressure*

For including the case of slightly rarefied compressible gas, we consider first-order slip velocity conditions at solid boundaries instead of the usual nonslip conditions [12]:

$$\begin{aligned} v_x \left( x, y, \pm \frac{1}{2} \right) &= \mp \lambda' \frac{\partial v_x}{\partial z} \left( x, y, \pm \frac{1}{2} \right), & v_y \left( x, y, \pm \frac{1}{2} \right) & \\ &= \mp \lambda' \frac{\partial v_y}{\partial z} \left( x, y, \pm \frac{1}{2} \right) \end{aligned} \tag{9}$$

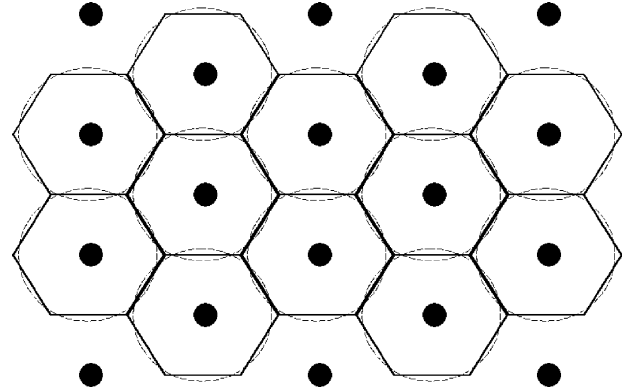


Fig. 2. The regular web of holes on the upper plate.

where  $\lambda'$  is the mean free path of the gas molecules. Also, in the direction normal to the plates we have

$$v_z \left( x, y, -\frac{1}{2} \right) = 0, \quad v_z \left( x, y, \frac{1}{2} \right) = w \tag{10}$$

By  $w$  we denote the  $O_z$ -component of the velocity of the mobile surface, assumed a known quantity. (The classical nonslip condition can still be obtained for  $\lambda' = 0$ .)

At the rim of the holes the pressure is assumed to be equal to the atmospheric pressure. This gives the condition

$$p = 0 \tag{11}$$

on the rim  $\partial D_D$  of the holes. The pressure gradient is zero in a direction that is normal to any line of symmetry of the planar microstructure. On all symmetry lines (denoted by  $\partial D_N$ ), we can then write a new boundary condition as

$$\frac{\partial p}{\partial n} = 0 \tag{12}$$

We suppose that the holes (of circular form of  $r_1$ —radius) are located at the vertices of a regular system of equilateral triangles of  $l$ —side length (Fig. 2). We will take advantage of the repetitive pattern of the perforated plate and define a “cell” as the space occupied by a hole and its surrounding web space (the plane region where the hole is collecting the flow). The *basic domain*  $D$  is defined as the plane region obtained from a cell excluding the hole (Fig. 3). The external boundary of the basic domain is a regular hexagon and the inner boundary is the rim of the hole.

**3. The Reynolds equation for the gas squeezing film**

By integrating Eqs. (5) and (6) and using the boundary conditions (9) we obtain expressions for the horizontal components of velocity

$$\begin{aligned} v_x(x, y, z) &= \left( \frac{\cos(\sqrt{i}Kz)}{\cos(\sqrt{i}K/2) - \lambda' \sqrt{i}K \sin(\sqrt{i}K/2)} - 1 \right) \\ &\times \frac{i}{K^2} \frac{\partial p}{\partial x} \end{aligned} \tag{13}$$

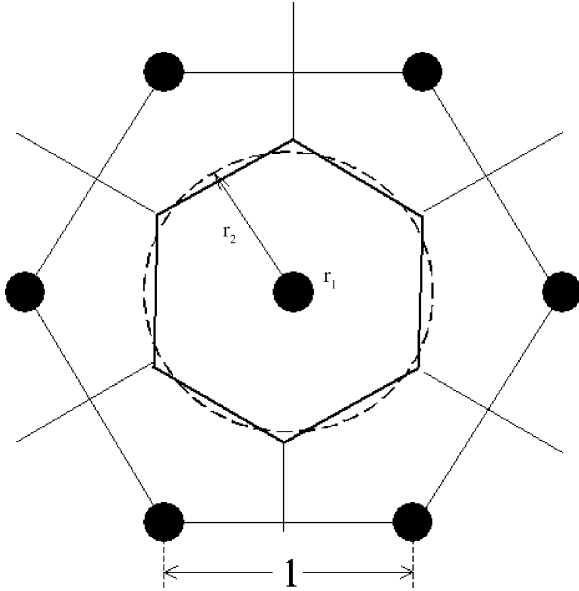


Fig. 3. The influence domain (the cell) of the central hole and its circular approximation.

$$v_y(x, y, z) = \left( \frac{\cos(\sqrt{i}Kz)}{\cos(\sqrt{i}K/2) - \lambda'\sqrt{i}K \sin(\sqrt{i}K/2)} - 1 \right) \times \frac{i}{K^2} \frac{\partial p}{\partial y} \quad (14)$$

where  $\sqrt{i} = (1 + i)/\sqrt{2}$ . Eq. (8) then becomes

$$\frac{\partial v_z}{\partial z} = \left( 1 - \frac{\cos(\sqrt{i}Kz)}{\cos(\sqrt{i}K/2) - \lambda'\sqrt{i}K \sin(\sqrt{i}K/2)} \right) \times \frac{i}{K^2} \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) + iK_1 p$$

Integration of this equation with respect to  $z$  yields a formula for the vertical component of velocity and also an equation for the pressure

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \alpha^2 p = 12Mw \quad (15)$$

Here we have used the notations

$$\alpha^2 = 12iMK_1$$

$$M = \frac{i}{12} K^2 \left( \frac{\tan(\sqrt{i}K/2)}{(\sqrt{i}K/2)[1 - \lambda'\sqrt{i}K \tan(\sqrt{i}K/2)]} - 1 \right)^{-1} \quad (16)$$

Eq. (15) is the Reynolds' equation for solving the squeezing film problem in the case of a compressible gas accounting also for the influence of inertia and gas slip on the plates. In

the case we have  $c_0 \gg 1$  (the low compressibility case) Eq. (15) becomes Poisson's equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 12Mw \quad (17)$$

Finally, the physical pressure on the membrane can be expressed by means of the relation

$$p^{\text{phys}}(x, y, t) = -12 \frac{\mu L_0^2}{d_0^3} M \hat{p} \left( \frac{x}{L_0}, \frac{y}{L_0} \right) w^{\text{phys}} \quad (18)$$

where  $w^{\text{phys}} = \varepsilon V_0 w e^{-i\omega t}$  and the function  $\hat{p}$  satisfies the canonical boundary value problem

$$\begin{aligned} \frac{\partial^2 \hat{p}}{\partial x^2} + \frac{\partial^2 \hat{p}}{\partial y^2} + \alpha^2 \hat{p} &= 1, \quad \text{in } D' : \quad \hat{p}|_{\partial D'_D} = 0, \\ \frac{\partial \hat{p}}{\partial n} |_{\partial D'_N} &= 0 \end{aligned} \quad (19)$$

The canonical domain  $D'$  results from the basic domain  $D$  by means of the similarity transformation given by using dimensionless variables.

In the case where the gas is sticking to the wall ( $\lambda' = 0$ ) and for small to moderate values of  $K$ -numbers (slowly oscillating flow), a series expansion enables Eq. (16) to be approximated as

$$M = 1 - \frac{iK^2}{10} + O\left(\frac{K^4}{100}\right)$$

A simple dimensional analysis reveals that for the case of the air and of frequencies between 100 Hz and 50 kHz the value  $M = 1$  is a good approximation.

We will define also the pressure coefficient  $C_p$  of the domain  $D'$  as

$$C_p = - \iint_{D'} \hat{p}(x, y) dx dy \quad (20)$$

Hence, the resulting force due to the squeeze-film damping on a cell of the mobile plate is

$$F^s = 12 \frac{\mu L_0^4}{d_0^3} M C_p w^{\text{phys}} \quad (21)$$

The damping coefficient of a particular structure is obtained by summing the force corresponding to all the cells of the structure and dividing the result by velocity  $w^{\text{phys}}$ .

At small pressures, when the molecular mean free path  $\lambda'$  is not negligible compared with the gap width, the Reynolds equation (15) is still valid if the viscosity coefficient  $\mu$  is substituted by

$$\mu_{\text{eff}} = \frac{\mu}{1 + f(K_n)} \quad (22)$$

$K_n = \lambda'/d$  being the Knudsen number. For a summary of different functions  $f(K_n)$  used in literature to model the gas

flow in a narrow gap and a more elaborate discussion on this topic the reader is directed to the paper [13].

**4. The annular approximation of the basic region: squeeze-film damping**

We consider an approximation of the outer hexagonal boundary of the basic domain by an equivalent circle having the same area (Fig. 3). In this case the domain  $D$  is an annulus of  $r_1 < r_2$  radii. The radius of the outer circle  $r_2$  is connected with the distance  $l$  between the holes by the relationship

$$r_2 = 0.525l$$

resulting by the equality of areas. This approximation works well only in the case of small inner radius  $r_1$  as compared with the linear dimension of the cell. In fact, the comparison of the analytical results obtained by this simplified model with the data provided by the numerical solution, given in the last section, will show the limits of the used approximation. We take as a reference length  $L_0 = r_2$  and denote  $r_0 = r_1/r_2$ . In this case the basic equation (17) becomes in polar coordinates

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \alpha^2 p = 12Mw \quad \text{for } r_0 < r < 1 \quad (23)$$

The boundary conditions for the function  $p(r)$  are

$$p(r_0) = 0, \quad \frac{\partial p}{\partial r}(1) = 0 \quad (24)$$

*4.1. Squeezing viscous damping: the influence of compressibility and inertia*

The solution of Eq. (23) satisfying the conditions (24) can be written as

$$p(r) = \frac{w}{iK_1} \left( 1 - \frac{Y_1(\alpha)J_0(\alpha r) - J_1(\alpha)Y_0(\alpha r)}{Y_1(\alpha)J_0(\alpha r_0) - J_1(\alpha)Y_0(\alpha r_0)} \right)$$

where  $J_0, J_1, Y_0, Y_1$  are the Bessel functions of the first and second kind.

We have also

$$\iint_{D'_1} p(r) \, dx \, dy = -12\pi MC(\alpha, r_0)$$

where

$$C(\alpha, r_0) = \frac{1}{\alpha^2} \left( \frac{2r_0}{\alpha} \frac{J_1(\alpha)Y_1(\alpha r_0) - Y_1(\alpha)J_1(\alpha r_0)}{Y_1(\alpha)J_0(\alpha r_0) - J_1(\alpha)Y_0(\alpha r_0)} - 1 + r_0^2 \right)$$

A power series expansion yields

$$\begin{aligned} & \frac{2r_0}{\alpha} \frac{J_1(\alpha)Y_1(\alpha r_0) - Y_1(\alpha)J_1(\alpha r_0)}{Y_1(\alpha)J_0(\alpha r_0) - J_1(\alpha)Y_0(\alpha r_0)} \\ &= 1 - r_0^2 + c_0\alpha^2 + c_1\alpha^4 + O(\alpha^6) \end{aligned}$$

where

$$c_0(r_0) = \frac{r_0^2}{2} - \frac{3}{8} - \frac{r_0^4}{8} - \frac{1}{2} \ln r_0$$

$$\begin{aligned} c_1(r_0) = & \frac{11}{64} - \frac{5r_0^2}{16} + \frac{11r_0^4}{64} - \frac{r_0^6}{32} + \frac{3}{8} \ln r_0 \\ & + \frac{1}{4} (\ln r_0)^2 - \frac{r_0^2}{4} \ln r_0 \end{aligned}$$

Therefore,

$$C(\alpha, r_0) = c_0(r_0) + c_1(r_0)\alpha^2 + O(\alpha^4) \quad (25)$$

The positive coefficients  $c_0(r_0), c_1(r_0)$  are plotted in Fig. 4. The first term in formula (25) corresponds to the incompressible case and the next term gives the influence of compressibility and inertia. Finally, the total force on the domain  $D$  due to squeeze-film damping can be written as

$$F^s = 12 \frac{\pi \mu r_2^4}{d_0^3} MC(\alpha, r_0) w^{\text{phys}} \quad (26)$$

From Fig. 4 it can be seen that the compressibility is important only for the case of very small  $r_0 = r_1/r_2$  ratio. For an incompressible fluid ( $\alpha = 0$ ), the resulting expression reduces to

$$F^c = \frac{12\pi \mu r_2^4}{d_0^3} M \left( \frac{1}{2} r_0^2 - \frac{1}{8} r_0^4 - \frac{1}{2} \ln r_0 - \frac{3}{8} \right) w^{\text{phys}} \quad (27)$$

given in [11]. In the case of small or moderate frequencies the relationship (27) coincides with Škvor’s formula [7].

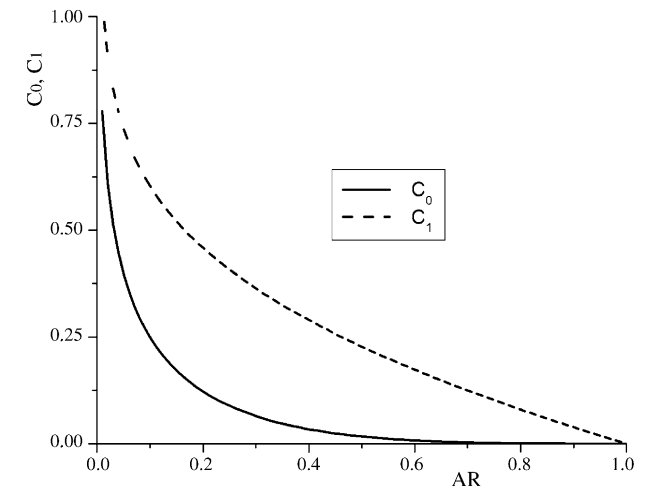


Fig. 4. The functions  $c_0(AR)$  and  $c_1(AR)$ .

## 5. Viscous resistance due to the microstructure holes (vertical flow)

In this section, we extend the previous results to account for the supplementary pressure due to the resistance of the flow through holes. As in the above formulation, the effects of compressibility and the inertia effects will be accounted for. In order to determine the “holes resistance” we assume a pressure  $p_1$  along the upper edge of a perforation and model a plate hole as a pipe of  $2r_1$  diameter and of length  $h$  equal to the plate thickness. In this case the only nonvanishing component of velocity in the hole is  $v_z$  (Poiseuille flow) [14] and we can write the equation

$$\Delta v_z + \frac{i\omega}{\nu} v_z = \frac{1}{\mu} \frac{\partial p}{\partial z}$$

$\nu = \mu/\rho$  being the dynamic viscosity. In polar coordinates there results

$$\frac{1}{r} \left( r \frac{\partial v_z}{\partial r} \right) + \frac{i\omega}{\nu} v_z = -\frac{p_1}{\mu h}$$

an equation similar to (23). Its solution, finite in the domain  $r < r_1$  and vanishing on the pipe wall  $r = r_1$  is

$$v_z(r) = -\frac{p_1}{i\omega\rho h} \left\{ 1 - \frac{J_0(\beta r)}{J_0(\beta r_1)} \right\}$$

where  $\beta^2 = i\omega/\nu$ . The total volume rate of flow results by integration in the form

$$Q^h = -\frac{\pi p_1}{i\omega\rho h} \left\{ r_1^2 - \frac{2r_1}{\beta} \frac{J_1(\beta r_1)}{J_0(\beta r_1)} \right\}$$

In the case  $\beta r_1 < 1$  we can write

$$Q^h = \frac{\pi p_1 r_1^4}{8\mu h} \left\{ 1 + \frac{i\omega r_1^2}{4\nu} + O\left(\frac{\omega^2 r_1^4}{\nu^2}\right) \right\}$$

In the incompressible case the pressure  $p_1$  at the rim of the hole can be obtained by balancing this volume rate of flow with the volume rate of flow  $Q = \mathcal{A}w$  entering (or leaving) the space between the microstructure plates (by  $\mathcal{A}$  we denoted the cell's area). Hence there results

$$p_1 = \frac{8\mu h \mathcal{A}}{\pi r_1^4} \left( 1 - i \frac{\omega r_1^2}{4\nu} \right) w \quad (28)$$

This rim pressure gives a supplementary force on the cell, which may be written as

$$F^h = \frac{8\mu h \mathcal{A}^2}{\pi r_1^4} \left( 1 - i \frac{\omega r_1^2}{4\nu} \right) w \quad (29)$$

The first term in Eq. (28), which is in phase with  $w$ , causes the pressure in Eq. (29) to correspond to static Poiseuille flow

and has been given by Rossi in [8]. The second term in Eq. (28), accounts for the effect of the oscillation frequency.

The formula (29) shows that the resistance due to the flow through the holes is important in the case of small diameter holes (i.e. small  $r_1$ ) and thick plates (i.e. large  $h$ ). It is quite common in devices such as microaccelerometers that the proof mass thickness is as much as 10 times larger than the gap dimension so that the hole resistance is an important component of the viscous damping.

The difficult fluid dynamic problem of the motion of the gas in the perforated micromechanical system has been decomposed in two simpler flows: a horizontal (squeezing film) flow and a vertical (Poiseuille) flow. In the case where the thickness of the plate  $h$  and the radius  $r_1$  are of comparable dimensions a correction has to be made for the effect of the holes' end. Sharipov and Seleznev [15] have shown that this effect can be included in formula (29) by replacing the holes' length  $h$  with

$$h_{\text{eff}} = h + \frac{3\pi r_1}{8}$$

On the other side, if the holes are very thin (Knudsen number is large) the flow resistance can be determined again by the same formula if the effective viscosity

$$\mu_{\text{eff}} = \frac{\sqrt{\pi} \mu r_1}{8\lambda G_{tb}}$$

stands for viscosity. Values of the function  $G_{tb}$  can also be found in [15].

## 6. Optimal number of circular holes and designing relationships

By adding the two terms modelling the viscous damping: the squeezing mechanical damping given in Eq. (26) or (27) and the plate holes resistance in Eq. (29) we obtain the total force on a microstructure cell as

$$\begin{aligned} F^T &\equiv F^s + F^h \\ &= \left[ \frac{12\pi\mu' r_2^4}{d_0^3} MC(\alpha, r_0) + \frac{8\pi\mu'' h}{r_0^4} \left( 1 - i \frac{\omega r_1^2}{4\nu} \right) \right] w \end{aligned} \quad (30)$$

where different viscosities  $\mu'$ ,  $\mu''$  have been introduced accounting for possible different effective viscosities on the horizontal and/or vertical flow and, again,  $r_0 = r_1/r_2$ . We introduce as new variables  $N$  the number of holes on a unit area  $u^2$  and AR the area ratio (area fraction of holes)

$$N = \frac{u^2}{\pi r_2^2}, \quad \text{AR} = \frac{\pi r_1^2}{\pi r_2^2} = r_0^2$$



For a given plate thickness  $h$ , air gap thickness  $d_0$ , and area ratio AR, it is often desirable to determine the number of holes,  $N$ , and their dimensions in order to minimize the damping pressure. The total damping coefficient  $\mathcal{B}$  on a unit area of the diaphragm is

$$\mathcal{B} \equiv \frac{NF^T}{w} = \frac{12\mu'C(\alpha, r_0)}{\pi d_0^3 N} Mu^4 + \frac{8\pi\mu''hN}{(AR)^2} \left(1 - i\frac{\omega r_1^2}{4\nu}\right)$$

The modulus of  $\mathcal{B}$  can be written as

$$|\mathcal{B}|^2 = \left( \frac{12\mu'|C(\alpha, r_0)||M|}{\pi d_0^3} \frac{u^4}{N} - \frac{8\pi\mu''h}{(AR)^2} \left|1 + i\frac{\omega r_1^2}{4\nu}\right| N \right)^2 + \frac{192\mu'\mu''h}{d_0^3 AR^2} u^4 (|Z| + \text{Re}(Z))$$

where  $Z = C(\alpha, \sqrt{AR})M(1 + i\omega r_1^2/4\nu)$ . For  $N = N_{\text{opt}}$

$$N_{\text{opt}} = \sqrt{\frac{3\mu'|C(\alpha, \sqrt{AR})||M|}{2hd_0^3\mu''} \frac{AR}{\pi} u^2} \quad (31)$$

the modulus of the damping coefficient attains its minimum value

$$|\mathcal{B}|_{\text{min}} = \frac{8\sqrt{3}\mu'\mu''h}{(AR)d_0^{3/2}} \sqrt{|Z| + \text{Re}(Z)} u^2 \quad (32)$$

In the case of an incompressible gas and moderate frequencies these formulas yield

$$N_{\text{opt}}^0 = \sqrt{\frac{3}{2hd_0^3} \left( \frac{AR}{2} - \frac{(AR)^2}{8} - \frac{\ln(AR)}{4} - \frac{3}{8} \right) \frac{AR}{\pi} u^2} \quad (33)$$

$$|\mathcal{B}|_{\text{min}}^0 = \frac{8\sqrt{6}\mu}{(AR)} \sqrt{\frac{h}{d_0^3} \left( \frac{AR}{2} - \frac{(AR)^2}{8} - \frac{\ln(AR)}{4} - \frac{3}{8} \right) u^2} \quad (34)$$

Denoting by  $l$  the distance (in  $u$ —units) between the centers of two neighboring circular holes (see Fig. 3), we have the designing relationships

$$r_{2\text{opt}} = \frac{u}{\sqrt{\pi N_{\text{opt}}}}, \quad l_{\text{opt}} = 1.905r_{2\text{opt}},$$

$$r_{1\text{opt}} = AR \cdot r_{2\text{opt}} \quad (35)$$

For example, in the case of a microstructure with  $AR = 0.2$ ,  $d_0 = 0.005$  mm,  $h = 0.004$  mm and neglecting the air compressibility ( $\alpha = 0$ ) there results

$$N_{\text{opt}} = 1.220 \text{ holes/mm}^2, \quad l_{\text{opt}} = 0.031 \text{ mm},$$

$$r_{1\text{opt}} = 0.0032 \text{ mm}, \quad \mathcal{B}_{\text{min}} = 0.11 \times 10^{-3} \text{ N s/m}$$

**Remark 1.** It is possible that in some cases the value  $r_{1\text{opt}}$  will be too small to be realized technologically. In this case  $r_{1\text{opt}} = r_{1\text{min}}$  ( $r_{1\text{min}}$  being the radius of the minimum circle which can be “drilled”) and the formulas (100) will be used for determining the designing variables  $r_{2\text{opt}}$ ,  $N_{\text{opt}}$  and  $l_{\text{opt}}$ . Correspondingly, the squeeze-film damping will be the dominating part in the total viscous damping.

## 7. Optimal number of holes: a numerical estimation of the accuracy

The results presented above are based on a circular approximation of the real polygonal external boundary of the cell (Fig. 3). To determine the error involved in this approximation a model problem for the case of an incompressible gas was simulated numerically by using a boundary element method. Thus, the mixed boundary value problem for Eq. (17) and the real basic domain  $D$  (delimited by the inner circle of radius  $r_1$  and the external polygonal line) was integrated numerically by using a complex variable boundary element algorithm [16] and yielding finally the pressure coefficient  $C_p$  of the canonical domain. Now the force on a cell due to squeeze-film damping can be written as

$$F^s = 12 \frac{\mu l^2}{d_0^3} M C_p w^{\text{phys}}$$

and, also, the viscous resistance of the hole (29), gives the force

$$F^h = \frac{6}{\pi} \frac{\mu h l^4}{r_1^4} \left(1 - i\frac{\omega r_1^2}{4\nu}\right)$$

The geometrical parameters AR and  $N$  (the number of holes on a unit of area  $u^2$ ) are now

$$AR = \frac{2\pi}{\sqrt{3}} \frac{r_1^2}{l^2}, \quad N = \frac{2}{\sqrt{3}} \frac{u^2}{l^2}$$

An analysis similar to that in the previous section gives the optimal number of holes

$$N_{\text{opt}}^{\text{num}} = \sqrt{\frac{2C_p}{\pi h d_0^3}} (AR) u^2$$

and the minimum value of the principal part of the damping coefficient

$$\mathcal{B}_{\text{min}}^{\text{num}} = 16\sqrt{2\pi}\mu \frac{\sqrt{hC_p}}{AR\sqrt{d_0^3}} u^2 \quad (36)$$

In Fig. 5 we present a comparison of the optimum number of holes computed by using the analytical formula (33) (small circles) and the value resulting from numerical integration

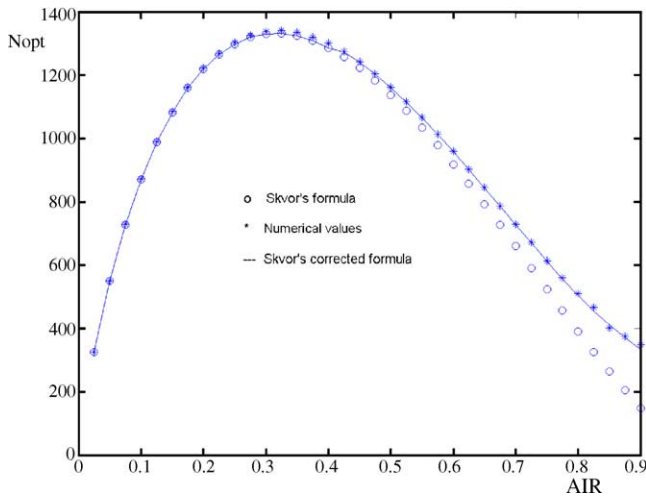


Fig. 5. A comparison of the optimal number of holes given by analytical (Škvor's formula (circles), numerical simulation (asterisks) and modified Škvor's formula (continuous line).

(asterisks) in this section. It is evident that for  $AR < 0.4$  the two values are very close.

By comparing the formulas (34) and (36) there results

$$C_p = \frac{3}{4\pi} C(0, \sqrt{AR}) \quad (37)$$

In Fig. 6 we plot the value of the coefficient  $C_p$  given by formula (37) (small circles) and the value obtained by numerical computation (asterisks). By using the obtained numerical values, we have determined also a corrected value  $C^*(0, \sqrt{AR})$  (the improved Škvor's formula) to the coefficient  $C(0, \sqrt{AR})$  in the form

$$C^*(0, \sqrt{AR}) = C(0, \sqrt{AR}) + 10^{-4} \times (8.7 - 10AR + 26AR^2 - 23AR^3) \quad (38)$$

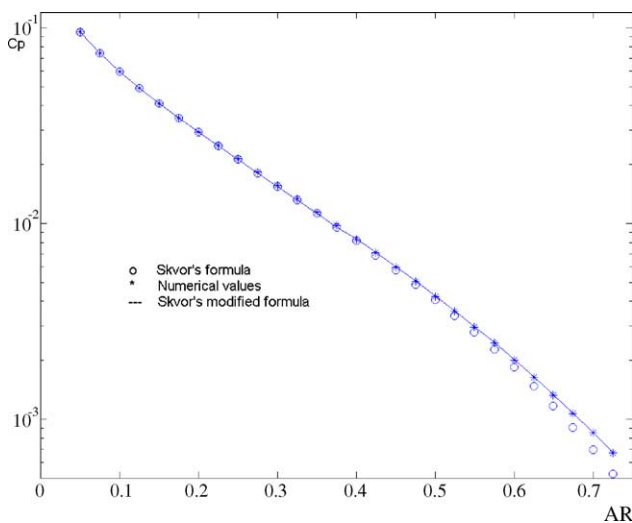


Fig. 6. A comparison of the minimum total damping coefficient given by analytical formula (circles) by numerical simulation (asterisks) and modified analytical formula (continuous line).

valid for  $0.4 < AR < 0.75$ , which is also plotted in Fig. 6 as a continuous line. The optimum number of holes resulting by using the function  $C^*(0, \sqrt{AR})$  in formula (33) is also plotted in Fig. 5 as a continuous line. It is evident that the analytical designing formulas presented in Section 6 can be used for all the area ratio values of practical interest if we consider the coefficient  $C^*(0, \sqrt{AR})$  instead of  $C(0, \sqrt{AR})$  in the case the area ratio  $AR$  is larger than 0.4.

### 8. Conclusions

The paper provides damping coefficients for the case of regularly perforated plates valid for all frequencies and including the effect of compressibility, inertia, and gas slip on solid surfaces. For a regular web of circular holes, the paper gives designing analytical formulas for determining the optimal number of holes (on a unit of area) which give the smallest total damping coefficient for an assumed open area.

The analysis in the last section (based on an incompressible model) has shown that the obtained approximate analytical formulas given in Section 6 can be used as long as  $AR < 0.4$ . For the incompressible fluid and  $0.4 < AR < 0.75$  a correction term to Škvor's formula is obtained.

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## Biographies

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**R.N. Miles** received a BSEE from the University of California at Berkeley in 1976, and his MSE and Ph.D. in mechanical engineering from the University of Washington. Beginning in 1977 he worked on structural acoustics and noise control at Boeing for eight years. He was an assistant research engineer and lecturer in the Department of Mechanical Engineering at UC Berkeley and has been with the Department of Mechanical Engineering at the State University of New York at Binghamton since 1989 where he is now Professor. His primary research is on the development of biologically-inspired acoustic sensors.