Review Article

Visiting Power Laws in Cyber-Physical Networking Systems

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Cyber-physical networking systems (CPNSs) are made up of various physical systems that are heterogeneous in nature. Therefore, exploring universalities in CPNSs for either data or systems is desired in its fundamental theory. This paper is in the aspect of data, aiming at addressing that power laws may yet be a universality of data in CPNSs. The contributions of this paper are in triple folds. First, we provide a short tutorial about power laws. Then, we address the power laws related to some physical systems. Finally, we discuss that power-law-type data may be governed by stochastically differential equations of fractional order. As a side product, we present the point of view that the upper bound of data flow at large-time scaling and the small one also follows power laws.

1. Introduction

Cyber-physical networking systems (CPNSs) consist of computational and physical elements integrated towards specific tasks [1–3]. Generally, both data and systems in CPNSs are heterogeneous. For instance, teletraffic data are different from transportation traffic, letting along other data in CPNS, such as those in physiology. Therefore, one of the fundamental questions is what possible general laws are to meet CPNS in theory. The answer to that question should be in two folds. One is data. The other is systems that transmit data from sources to destinations within a predetermined restrict period of time according to a given quality of service (QoS).

In general, both data and systems in CPNS are multidimensional. For instance, data from sources to be transmitted may be from a set of sensors distributed in a certain area.

Destinations receiving data may be a set of actuators, for example, a set of cars distributed in a certain area. Systems to transmit data are generally distributed.

Denote by $\mathbb{R}^n$ the n-dimensional Euclidean space. Denote data at sources and destinations, respectively, by $X(t)$ which is supposed to be n-dimensional and $Y(t)$ which is supposed to be m-dimensional. They are given by

$$X(t) = (x_1(t), \ldots, x_n(t)), \tag{1.1}$$

$$Y(t) = \left(y_1(t), \ldots, y_m(t)\right). \tag{1.2}$$

A stochastic equation describing an abstract relationship between $X(t)$ and $Y(t)$ may be expressed by

$$Y^T(t) = S(t) \otimes X^T(t) \oplus B^T(t), \tag{1.3}$$

where T implies the transposition, $S(t)$ is a servie matrix of $n \times m$ order of a system, and $B(t)$, which is a vector with the same dimension as that of $Y(t)$, may represent uncertainty for the operation of $S(t) \otimes X^T(t)$. The operations $\otimes$ and $\oplus$ are to be studied from a view of systems, and they are out of the scope of this paper.

Note that $X(t)$ is usually a random field, see for example the work of Chilés and Delfiner in [4] in geosciences, the work of Uhlig in [5] in telecommunications, the work of Messina et al. in [6] in power systems, the work of Muniandy and Stanslas in [7] in medical images, the work of Mason et al. in [8] in wind engineering, and of simply citing a few. The statistics of $X(t)$ is obviously crucial for the performance analysis of physical systems in CPNS. It is noted that the physical meaning of $X(t)$ is diverse. For example, it may represent a two-dimensional aeromagnetic data (Spector and Grant [9]), a medical image (Fortin et al. [10]), vegetation data (Myrhaug et al. [11]), surface crack in material science (Tanaka et al. [12]), and data in physiology (Werner [13], West [14]), DNA (Cattani [15]), data in stock markets (Rosenow et al. [16]), just mentioning a few. Therefore, seeking for possible universalities of $X(t)$ in CPNS is desired.

Without lose of generality, we rewrite (1.1) by

$$X(\mathbf{t}) = X(t_1, \ldots, t_n), \tag{1.4}$$

where $\mathbf{t} = (t_1, \ldots, t_n)$. The norm of $\mathbf{t}$ is given by

$$\|\mathbf{t}\| = \sqrt{t_1{}^2 + \cdots + t_n{}^2}. \tag{1.5}$$

The autocovariance function (ACF) of $X(\mathbf{t})$ is given, over the hyperrectangle $C = \prod_{i=1}^n [a_i, b_i]$ for $a_i, b_i \in \mathbb{R}$ (Adler [17]), by

$$C(\boldsymbol{\tau}) = \mathrm{E}[X(\mathbf{t})X(\mathbf{t} + \boldsymbol{\tau})], \tag{1.6}$$

where E is the mean operator, $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_n)$, and

$$\|\boldsymbol{\tau}\| = \sqrt{\tau_1^2 + \cdots + \tau_n^2}. \tag{1.7}$$

The ACF $C(\boldsymbol{\tau})$ measures how $X(\mathbf{t})$ correlates to $X(\mathbf{t} + \boldsymbol{\tau})$.

From the point of view of applications of CPNS, we are interested in two asymptotic expressions of $C(\boldsymbol{\tau})$. One is $C(\boldsymbol{\tau})$ for $\|\boldsymbol{\tau}\| \to 0$. The other is $C(\boldsymbol{\tau})$ for $\|\boldsymbol{\tau}\| \to \infty$. The former characterizes the small scaling phenomenon of $X(\mathbf{t})$. The latter measures the large scaling one. It is quite natural for us to investigate two types of scaling phenomena. As a matter of fact, one may be interested in small scaling in some applications, for example, admission control in computer communication or monitoring sudden disaster in geoscience. On the other side, one may be interested in large scaling in applications, for example, long-term performance analysis of systems. Exact expression of $C(\boldsymbol{\tau})$ is certainly useful, but it may usually be application dependent. Consequently, we study possible generalities of $C(\boldsymbol{\tau})$ for $\|\boldsymbol{\tau}\| \to 0$ and $\|\boldsymbol{\tau}\| \to \infty$ instead of its exactly full expressions. The aim of this paper is to explain that both the small scaling described by $C(\boldsymbol{\tau})$ for $\|\boldsymbol{\tau}\| \to 0$ and the large scaling described by $C(\boldsymbol{\tau})$ for $\|\boldsymbol{\tau}\| \to \infty$, in some fields related to CPNS, ranging from geoscience to computer communications, follow power laws.

The rest of paper is organized as follows. Short tutorial about power laws is explained in Section 2. Some cases of power laws relating to computational and physical systems in CPNS are described in Section 3. Stochastically differential equations to govern power-law-type data are discussed in Section 4, which is followed by our conclusions.

2. Brief on Power Laws

Denote by (Ω, T, P) the probability space. Then, $x(t, \varsigma)$ is said to be a stochastic process when the random variable x represents the value of the outcome of an experiment T for every time t, where Ω represents the sample space, T is the event space or sigma algebra, and P is the probability measure.

As usual, $x(t, \varsigma)$ is simplified to be written as $x(t)$. That is, the event space is usually omitted. Denote by $P(x)$ the probability function of x. Then, one can define the general nth order, time varying, joint distribution function $P(x_1, \ldots, x_n; t_1, \ldots, t_n)$ for the random variables $x(t_1), \ldots, x(t_n)$. The joint probability density function (pdf) is written by

$$p(x_1, \ldots, x_n; t_1, \ldots, t_n) = \frac{\partial^n P(x_1, \ldots, x_n; t_1, \ldots, t_n)}{\partial x_1, \ldots, \partial x_n}. \tag{2.1}$$

For simplicity, we write $P(X) = P(x_1, \ldots, x_n; t_1, \ldots, t_n)$ and $p(X) = p(x_1, \ldots, x_n; t_1, \ldots, t_n)$. Then, the probability is given by

$$P(X_2) - P(X_1) = \text{Prob}[X_1 < \xi < X_2] = \int_{X_1}^{X_2} p(\xi) d\xi. \tag{2.2}$$

The mean and the ACF of X based on pdf are written by (2.3) and (2.4), respectively,

$$\mu_X = \int_{-\infty}^{\infty} X p(X) dX, \tag{2.3}$$

$$C_{XX}(\tau) = \int_{-\infty}^{\infty} X(t) X(t+\tau) p(X) dX. \tag{2.4}$$

Let V_X be the variance of X. Then,

$$V_X = E\left[(X - \mu_X)^2\right] = \int_{-\infty}^{\infty} (X - \mu_X)^2 p(X) dX. \tag{2.5}$$

The above expressions imply that the integrals in (2.3) and (2.5) are convergent in the domain of ordinary functions if $p(X)$ is light tailed, for example, exponentially decayed (Li et al. [18]). Light-tailed pdfs are not our interests. We are interested in heavy-tailed pdfs. By heavy tail we mean that $p(X)$ decays so slowly that (2.3) and (2.5) may be divergent. In the following subsections, we will describe power laws in probability space, ACF, and power spectrum density (PSD) function, respectively.

2.1. Power Law in pdf

A typical heavy-tailed case is the Pareto distribution. Denote by $p_{\text{Pareto}}(X)$ the pdf of the Pareto distribution. Then,

$$p_{\text{Pareto}}(X) = \frac{ab}{X^{a+1}}, \tag{2.6}$$

where a and b are parameters and $X \geq a$. The mean and variance of X that follows $p_{\text{Pareto}}(X)$ are given by (2.7) and (2.8), respectively,

$$\mu_{\text{Pareto}} = \frac{ab}{a-1}, \tag{2.7}$$

$$V_{\text{Pareto}} = \frac{ab^2}{(a-1)^2(a-2)}. \tag{2.8}$$

It is easily seen that μ_{Pareto} and V_{Pareto} do not exist if $a = 1$. Note that μ_X implies a global property of X while V_X represents a local property of X. Therefore, heavy-tailed pdfs imply that X is in wild randomness due to infinite or very large variance, see the work of Mandelbrot in [19] for the meaning of wild randomness.

Note 1. The Pareto distribution is an instance of power-law-type pdf.

2.2. Power Law in ACF

A consequence of a heavy-tailed random variable in ACF is that $C_{XX}(\tau)$ is slowly decayed. By slowly decayed we mean that $C_{XX}(\tau)$ decays hyperbolically in the power law given by (Adler et al. [20])

$$C_{XX}(\tau) \sim \tau^{-d}, \quad d \in \mathbb{R}. \tag{2.9}$$

The Taqqu theorem describes the relationship between a heavy-tailed pdf and hyperbolically decayed ACF (Abry et al. [21]).

2.3. Power Law in PSD

Denote by $S_{XX}(\omega)$ the PSD of X. Then,

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} C_{XX}(\tau) e^{-j\omega\tau} d\tau, \quad j = \sqrt{-1}. \tag{2.10}$$

According to the theory of generalized functions (Kanwal [22]), one has

$$S_{XX}(\omega) \sim |\omega|^{d-1}. \tag{2.11}$$

Therefore, power law in PSD, which is usually termed $1/f$ noise, see the work of Wornell in [23], the work of Keshner in [24], the work of Ninness in [25], the work of Corsini and Saletti in [26], and the work of Li in [27].

2.4. Power Laws in Describing Scaling Phenomena

We now turn to scaling descriptions. Small scaling phenomenon may be investigated by $C_{XX}(\tau)$ for $\tau \to 0$ and large scaling for $\tau \to \infty$, respectively (Li and Zhao [28]).

On the one side, following Davies and Hall [29], if $C_{XX}(\tau)$ is sufficiently smooth on $(0, \infty)$ and if

$$C_{XX}(0) - C_{XX}(\tau) \sim c_1 |\tau|^\alpha \quad \text{for } |\tau| \longrightarrow 0, \tag{2.12}$$

where c_1 is a constant and α is the fractal index of X, then the fractal dimension, denoted by D, of X is expressed by

$$D = 2 - \frac{\alpha}{2}. \tag{2.13}$$

Note 2. Fractal dimension is a parameter to characterize small scaling phenomenon (Mandelbrot [30], Gneiting and Schlather [31], Li [32]).

On the other side, if

$$C_{XX}(\tau) \sim |\tau|^{-\beta} \quad (\tau \longrightarrow \infty), \tag{2.14}$$

then the parameter β is used to measure the statistical dependence of X. If $\beta > 1$, $C_{XX}(\tau)$ is integrable, and accordingly X is short-range dependent (SRD). If $0 < \beta < 1$, $C_{XX}(\tau)$ is nonintegrable and X is long-range dependent (LRD), see the work of Beran in [33]. Representing β by the Hurst parameter $H \in (0, 1)$ yields

$$H = 1 - \frac{\beta}{2}. \tag{2.15}$$

Note 3. Statistical dependence, either SRD or LRD, is a property for large scaling phenomenon.

3. Cases of Power Laws in CPNS

We address some application cases of power laws in CPNS in this section.

3.1. Power Laws in the Internet

Let $x(t)$ be the teletraffic time series. It may represent the packet size of teletraffic at time t. Denote the ACF of $x(t)$ by

$$R(\tau) = \mathrm{E}[X(t + \tau)X(t)]. \tag{3.1}$$

Then, we have (Li and Lim [34])

$$R(\tau) \sim |\tau|^{\alpha}, \quad \tau \longrightarrow 0,$$

$$R(\tau) \sim |\tau|^{-\beta}, \quad \tau \longrightarrow \infty. \tag{3.2}$$

From (3.2), the fractal dimension D and the Hurst parameter H of teletraffic are, respectively, given by

$$D = 2 - \frac{\alpha}{2},$$

$$H = 1 - \frac{\beta}{2}. \tag{3.3}$$

The above exhibits that both the small scaling and the large one follow power laws.

It is worth noting that the upper bounds of teletraffic also follow power laws. In fact, the amount of teletraffic accumulated in the interval $[0, t]$ is upper bounded by

$$\int_0^t x(u)du \le \sigma + \rho t, \tag{3.4}$$

where σ and ρ are constants and $t > 0$ (Cruz [35]). Following Li and Zhao [28], we have the bounds of both the small-time scaling and the large one, respectively expressed by

$$\int_0^t x(u)du \le r^{2D-5} \, \sigma \text{ for small } t,$$

$$\int_0^t x(u)du \le a^{-H} \, \rho \text{ for large } t, \tag{3.5}$$

where $r > 0$ is a small-scale factor and $a > 0$ is a large-scale factor. Therefore, we have the following theorem.

Theorem 3.1. *Both the small-scale factor and the large one of teletraffic obey power law, that is, r^{2D-5} and a^{-H}.*

Proof. Two scaling factors follow r^{2D-5} and a^{-H}, respectively. Thus, they obey power laws. This completes the proof. $\square$

In addition to teletraffic, others with respect to the Internet also follow power laws. Some are listed below.

Note 4. Barabasi and Albert [36] studied several large databases in the World Wide Web (WWW), where they defined vertices by HyperText Markup Language (HTML) documents. They inferred that the probability $P(k)$ that a vertex in the network interacts with k other vertices decays hyperbolically as $P(k) \sim k^{-\gamma}$ for $\gamma > 0$, hence, power law.

Note 5. Let $P_{\text{out}}(k)$ and $P_{\text{in}}(k)$ be the probabilities of a document to have k outgoing and incoming links, respectively. Then, $P_{\text{out}}(k)$ and $P_{\text{in}}(k)$ obey power laws (Albert [37]).

Note 6. The probability of web pages among sites is of power law (Huberman and Adamic [38]).

3.2. Power Laws in Geosciences

Let $(x, y, z) \in \mathbb{R}^3$ be a spatial point. The physical meaning of a random function $U(x, y, z)$ may be diverse in the field. For instance, it may represent prospected gold amount at (x, y, z) in a gold mine, or a value of pollution index for pollution alert at (x, y, z) in a city.

For simplicity, denote a vector by $l = (x, y, z)$. Let

$$\rho = |l| = \sqrt{x^2 + y^2 + z^2}. \tag{3.6}$$

Then, one may be interested in the covariance function of $U(\rho)$. Denote by $C(\tau)$ the covariance function of $U(\rho)$. Then,

$$C(\tau) = \mathrm{E}\big\{ [U(\rho) - \mathrm{E}U(\rho)] [U(\rho + \tau) - \mathrm{E}U(\rho)] \big\}. \tag{3.7}$$

One of the commonly used models of covariance functions in geosciences is given by

$$C(\tau) = \frac{1}{1 + |\tau|^2}. \tag{3.8}$$

The above constant power is the case of the standard Cauchy process (Webster and Oliver [39]). It fits with some cases in geosciences, see, for example, the work of Wackernagel in [40]. We list some in the following notes.

Note 7. Let $C(s)$ be the covariance function between yield densities at any two points in a region, where s represents the distance difference between two points. Then,

$$C(s) \sim |s|^{-\nu} (\nu > 0) \text{ for large } \nu, \tag{3.9}$$

see the work of Whittle in [41].

Note 8. Sea-level fluctuations, river flow, and flood height follow power laws (Li et al. [42], Lefebvre [43], Lawrance and Kottegoda [44]).

Note 9. Urban growth obeys power laws (Makse et al. [45]).

3.3. Power Laws in Wind Engineering

Wind engineering is an important field relating to wind power generation and disaster preventions from a view of CPNS. In this field, studying fluctuations of wind speed is essential.

The PSD introduced by von Kármán [46], known as the von Kármán spectra (VKS), is widely used in the diverse fields, ranging from turbulence to acoustic wave propagation in random media, see for example, the work of Goedecke et al. in [47] and the work of Hui et al. in [48]. For the VKS expressed in (3.10), we use the term VKSW for short,

$$S_{\mathrm{von}}(f) = \frac{4u_f^2 b_v w}{f(1 + 70.8w^2)^{5/6}}, \qquad w = \frac{fL_u^x}{U}, \tag{3.10}$$

where f is frequency (Hz), L_u^x is turbulence integral scale, U is mean speed, u_f is friction velocity (ms^{-1}), and b_v is friction velocity coefficient such that the variance of wind speed $\sigma_u^2 = b_v u_f^2$. Equation (3.10) implies that VKSW obeys power law for $f \in (0, \infty)$.

Another famous PSD in wind engineering is the one introduced by Davenport [49], which is expressed by

$$\frac{fS_{\text{Dav}}(f)}{u_f^2} = 4\frac{u^2}{(1+u^2)^{4/3}}, \qquad u = \frac{1200n}{z}, \tag{3.11}$$

where n is the normalized frequency (fz/U (10 m)), U (10 m) is the mean wind speed (ms^{-1}) measured at height 10 m, $U(z)$ is the mean wind speed (ms^{-1}) measured at height z. Davenport's PSD exhibits a power law of wind speed. Other forms of the PSDs of wind speed, such as those discussed by Kaimal [50], Antoniou et al. [51], and Hiriart et al. [52], all follow power laws, referring [50–52] for details.

4. Possible Equations for Power-Law-Type Data

The cases of power laws mentioned in the previous section are a few that people may be interested in from a view of CPNS. There are others that are essential in the field of CPNS, such as power laws in earthquake, see for example, the work of Pisarenko and Rodkin in [53]. Now, we turn to the discussions about the generality about the equations that may govern data of power law type.

Conventionally, a stationary random function $y(t)$ may be taken as a solution of a differential equation of integer order, which is driven by white noise $w(t)$. This equation may be written by

$$\sum_{i=0}^{p} a_i \frac{d^{p-i}y(t)}{dt^{p-i}} = w(t), \tag{4.1}$$

where p and i are integers.

Let $v > 0$ and $f(t)$ be piecewise continuous on $(0, \infty)$ and integrable on any finite subinterval of $[0, \infty)$. For $t > 0$, denote by $_0D_t^{-v}$ the Riemann-Liouville integral operator of order v [54–57]. Then,

$$_0D_t^{-v} f(t) = \frac{1}{\Gamma(v)} \int_0^t (t-u)^{v-1} f(u) du, \tag{4.2}$$

where Γ is the Gamma function. For simplicity, we write $_0D_t^{-v}$ by D^{-v} below.

Let $v_p, v_{p-1}, \ldots, v_0$ be a strictly decreasing sequence of nonnegative numbers. Then, for the constants a_i, we have

$$\sum_{i=0}^{p} a_{p-i} D^{v_i} y(t) = w(t). \tag{4.3}$$

The above is a stochastically fractional differential equation with constant coefficients of order v_p. This class of equations yield random functions with power laws (Li [27]). In the case of

random fields, (4.3) is extended to be a partial differential equation of fractional order given by

$$\sum_{i=0}^{p} a_{p-i} D^{v_i} y = w, \tag{4.4}$$

where both w and y are multidimensional and D is an operator of partial differentiation.

Another class of stochastically differential equations of fractional order is given by (Lim and Muniandy [58])

$$\left(\sum_{i=0}^{p} a_i \frac{d^{p-i} y(t)}{dt^{p-i}} \right)^{\beta_1} = w(t) \quad (\beta_1 > 0). \tag{4.5}$$

Note that (4.3), (4.4), or (4.5) should not be taken as a simple extension of conventional equation (4.1) from integer order to fractional one. As a matter of fact, there are challenging issues with respect to differential equations of fractional order. Since data of power-law-type may be with infinite variance (Samorodnitsky and Taqqu [59]), variance analysis which is a powerful tool in the analysis of conventional random functions fails to describe random data with infinite variance. Power-law type data may be LRD, which makes the stationarity test of data a tough issue, see for example, the work of Mandelbrot in [60], the work of Abry and Veitch in [61], the work of Li et al. in [62]. Owing to power laws, stability of systems that produce such a type of data becomes a critical issue in theory, see the work of Li et al. in [63] and the references therein. In addition, the prediction of data with power laws considerably differs from that of conventional data (M. Li and J. Y. Li [64], Hall and Yao [65]). Topics in power laws are paid attention to, see for example, the work of Kamoun in [66], the work of Ng et al. in [67], the work of Song et al. in [68], the work of Cattani et al. in [69–71], and in [72–79].

5. Conclusions

We have discussed the elements of power laws from both a mathematical point of view and with respect to applications to a number of fields in CPNS. The purpose of this paper is to exhibit that power laws may yet serve as a universality of data in CPNS. We believe that this point of view may be useful for data modeling and analysis in CPNS.
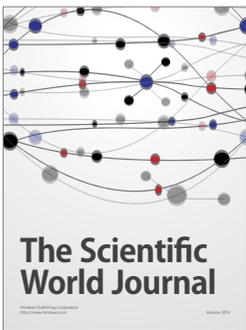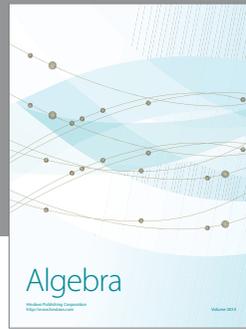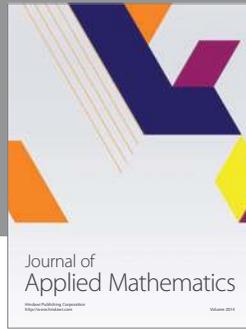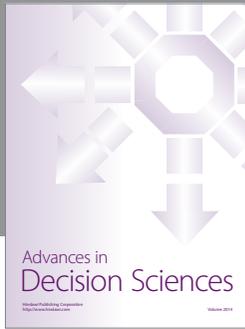
Acknowledgments

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