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VISUAL COMPARISON OF ROTATED AND REFLECTED RANDOM-DOT PATTERNS AS A FUNCTION OF THEIR POSITIONAL SYMMETRY AND SEPARATION IN THE FIELD*

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Subjects viewed pairs of random-dot patterns which were presented in a number of arrangements varying in the transformations applied to the patterns, in the distance between the patterns, and in the symmetry of the pattern positions with respect to the point of fixation. The task was to judge whether the patterns were "the same" taking into account possible rotations or reflections, or "different". It was found that correct judgements for identical patterns were most affected by the *distance* between the two patterns, whereas correct judgements for patterns where one had been rotated through 180° or reflected were most affected by the *symmetry* of the pattern positions. A scheme modelling the visual recognition of transformed patterns, sufficient to explain the results, is presented.

Introduction

There have been many studies concerned with the effects of spatial transformations on the visual recognition of objects. It is well known that some transformations have little effect; for example, judgements of the shape of a figure are largely independent of the slant, size and position of the figure, hence the notions of "shape constancy" (Epstein and Park, 1963; Hochberg, 1972), "size invariance" and "position invariance" (Hake, 1966; Sutherland, 1968). On the other hand, certain transformations can have marked effects on the visual recognition of objects. For example, rotations through angles close to 90° can greatly reduce recognisability, although this effect is much less for rotations through 180° and for reflections (Dearborn, 1899; Rock, 1973; Foster, 1978).

How the recognisability of transformed patterns is affected by their *positions* in the visual field is not so well quantified. Judgements of the perceptual similarity

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of figures (Attneave, 1950), discrimination of mirror images (Sekuler and Rosenblith, 1964; Sekuler and Pierce, 1973), and identification of parafoveal figure pairs (Banks, Bachrach and Larson, 1977; Banks, Larson and Prinzmetal, 1979; Chastain and Lawson, 1979), have all been shown to depend on the relative positions of the stimuli. Also the time taken to report sameness of mirror pairs has been found to be shorter when the patterns are presented symmetrically about the point of fixation than when they are both presented to one side (Corballis and Roldan, 1974; Bradshaw, Bradley and Patterson, 1976). Similarly, it has been demonstrated that symmetry in a complex random-dot pattern is best perceived when the observer fixates a point on the axis of symmetry (Julesz, 1971; Barlow and Reeves, 1979; see also Bruce and Morgan, 1975).

The way in which the capacity to detect "sameness" of transformed patterns depends on the positions of the patterns in the visual field is important in the verification of current models of visual recognition. These are discussed later in this paper. The two main types of model, either involving symbolic encoding of the pattern or involving internal spatial transformation of the pattern, naturally predict different effects of stimulus arrangement on the capacity to detect "sameness". We show here that the arrangement of patterns in the field has substantial effects on this capacity, and that these effects differ for different transformations.

We performed two experiments in which pairs of patterns were briefly presented in a number of configurations. Both the distance between patterns and the symmetry of their positions with respect to the point of fixation were varied systematically. The subjects' task was to decide whether the two patterns were "same", taking into account possible rotations or reflections, or "different". The main outcome of these experiments is this: when patterns are identical, the proportion of correct decisions depends mainly on the distance between the patterns; when patterns are inverted or reflected, the proportion depends mainly on the symmetry of the arrangement of the patterns. These results are argued to be inconsistent with each of the two main types of model mentioned above, and an alternative scheme is proposed.

Experiment I

Method

Subjects

The subjects were five male students in the Department of Communication and Neuroscience who were aged between 23 and 27 years and had normal or corrected-to-normal vision. All subjects except one (author JIK) were unaware of the purpose of the experiment.

Apparatus

The stimuli were displayed on a Hewlett-Packard 1300A X-Y display oscilloscope (P4 sulfide phosphor) controlled by computer. The screen was viewed at a distance of 1·7 m through a view-tunnel and optical system which produced a uniform white background field of luminance approximately 60 cd m^{-2}. The stimuli were white and appeared superimposed on the background field; their intensity was adjusted by each subject to ten-times increment luminance threshold. Fixation was aided by a permanent display of four small red light-emitting diodes forming a square whose side subtended 4° of visual angle and by four computer-generated white lines, 0·5° long, pointing to the centre of the square. A central fixation spot was also displayed on the screen at the beginning of each trial. The subject controlled the start of each trial and gave his responses on a hand-held push-button box connected to the computer.

Stimuli

Stimuli were random-dot patterns each consisting of ten dots distributed within an imaginary disc of diameter 0·5° visual angle. Each dot subtended about 0·03°. Random-dot patterns were used in these experiments so that the stimuli would be unfamiliar to the subject and would have no meaning, name, and conventional handedness or orientation, which can be ascribed to, for example, letters and geometrical figures.

Pattern positions

In each trial two patterns appeared sequentially. Each pattern was presented in one of three positions:

(a) eccentric, with the pattern centre 0·5° left of the fixation spot;
(b) centred on the fixation spot;
(c) eccentric, with the pattern centre 0·5° right of the fixation spot.

The possible combinations of these pattern positions were classified into four groups:

Es: both patterns eccentric and on the same side (left or right);
Ec: one pattern eccentric (left or right), the other central;
Eo: both patterns eccentric and on opposite sides;
Cc: both patterns central.

Pattern transformations

There were four possible transformations (other than translations) relating the patterns in each "same" pair:

Id: the two patterns were identical;
Ro: one pattern was obtained from the other by planar rotation through 90°;
Pi: one pattern was obtained from the other by point inversion, that is, planar rotation through 180°;
Mi: one pattern was obtained from the other by reflection in a vertical line.

For "different" pairs, the two patterns were generated independently of each other.
A fresh pair of patterns was generated for every trial.

Instructions

At the beginning of the experiment, subjects were informed of the nature of the stimuli and of the types of transformation involved. Subjects were instructed to indicate, by responding "same" or "different" after the presentation of each pair of patterns, whether or not the patterns were related by one of the specified transformations. It was emphasised to the subject that steady fixation should be maintained throughout each presentation and responses should be made as quickly as possible whilst maintaining accuracy.

Presentation sequence

Following the initiation of the trial by the subject, the fixation spot was extinguished, and, after a 1·0-s delay, the first stimulus pattern was presented for 100 ms; after a further 1·0-s delay (which is sufficient to obviate masking effects) the second stimulus pattern appeared for 100 ms. The subject's response was recorded by the computer. (Response times were usually less than 1 s, and there was no evidence of a speed–accuracy trade-off.) After a 1·0-s delay the fixation spot was redisplayed indicating that the next trial could be started.

A run consisted of 32 trials and lasted about 3 min. Each subject performed 48 runs over a period of several days.

Experimental design

In each run every position combination occurred once with each of the four transformation-related pairs ("sames") and four times with "different" pattern pairs, so that a run consisted of 16 "sames" and 16 "differents". The sequence of occurrences of the transformations and of position combinations was chosen randomly but balanced for order and carry-over effects over runs. The Ro transformation occurred the same number of times as a clockwise or anticlockwise rotation in every run.

Results

Figure 1 shows "same-different" pattern discrimination performance, which was measured using the discrimination index d' from signal detection theory (Green and Swets, 1966). The index d' increases monotonically with discriminability, and

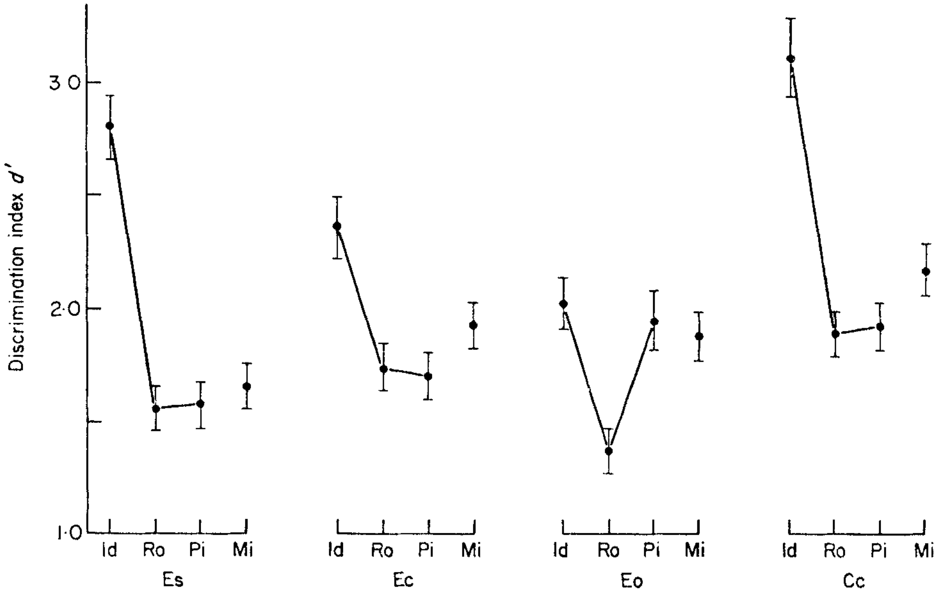

FIGURE 1. "Same" detection performance in Experiment I. Each graph shows the pooled discrimination index d' plotted against pattern transformation for one of the combinations of pattern positions. The position combinations are: Es: both patterns presented 0·5° to one side of fixation spot; Ec: one pattern presented 0·5° to the left or right of fixation spot, the other central; Eo: one pattern presented 0·5° to the left of fixation spot, the other 0·5° to the right; Cc: both patterns presented centrally. The pattern transformations are as follows. Id: the patterns are identical; Ro: the patterns are related by a 90° planar rotation; Pi: the patterns are related by point-inversion; Mi: the patterns are related by reflection in a vertical line.

$d'=0$ corresponds to non-discriminability. The advantages of this index of performance are well established (Swets, 1973); in particular, d' is linear in the sense of additivity (Durlach and Braida, 1969). In each graph the discrimination index, pooled over all subjects (see the Appendix), is plotted against pattern transformation. The various position combinations Es, Ec, Eo, and Cc are indicated.

Effect of distance with symmetry held constant

The separation of the patterns is zero in position combination Cc and 1·0° in combination Eo. In both cases the patterns are positioned symmetrically with respect to the point of fixation. The increase in separation of the patterns causes a large reduction in "same" detection performance for transformations Id and Ro

($P < 0{\cdot}001$ for both, two-tailed tests, see the Appendix). There is no significant change in performance for Pi and a small but not significant change for Mi (respectively, $P > 0{\cdot}5$, $P > 0{\cdot}05$, two-tailed tests).

Effect of symmetry with distance held constant

The patterns are positioned asymmetrically with respect to the point of fixation in the Es position combination and positioned symetrically in the Cc combination. The patterns are not separated in either case. Introduction of symmetry while holding distance constant causes a small but not significant increase in "same" detection performance for transformation Id ($P > 0{\cdot}1$, two-tailed test) and significant increases in performance for transformations Ro, Pi and Mi (respectively, $P < 0{\cdot}05$, $P < 0{\cdot}05$, $P < 0{\cdot}001$, two-tailed tests).

It is made clear below that both symmetry and distance effects cannot be simply ascribed to variations in acuity with retinal eccentricity.

Combined effects of symmetry and distance

The distance between the pattern positions increases linearly from the Es position combination to the Eo combination; the Es combination is positionally asymmetric whereas the Eo combination is positionally symmetric. The Ec position combination is intermediate in both symmetry and distance. From Es to Eo there is a linear decrease in "same" detection performance for transformation Id and a linear increase in detection performance for transformation Pi. (Linear trend in d' for Id is significant, $P < 0{\cdot}001$, two-tailed test, quadratic trend is not significant, $P > 0{\cdot}5$, two-tailed test. Linear trend in d' for Pi is significant, $P < 0{\cdot}05$, two-tailed test, quadratic trend is not significant, $P > 0{\cdot}5$, two-tailed test.) "Same" detection performance for transformation Mi shows no significant increase from Es to Eo. (Linear and quadratic trends in d' are not significant, $P > 0{\cdot}1$ and $P > 0{\cdot}2$ respectively, two-tailed tests.) "Same" detection performance for transformation Ro shows a non-linear trend from Es to Eo (d' has no significant linear trend and a significant quadratic trend, $P > 0{\cdot}1$, $P < 0{\cdot}05$, respectively, two-tailed tests).

Note that the marked qualitative differences between performance in combinations Es and Eo cannot be attributed to retinal-eccentricity and hence acuity effects: eccentricity is identical in the two cases.

To summarise, "same" detection performance for transformation Id is strongly affected by the distance between the patterns. Performance for transformations Pi and Mi is best when the patterns are positioned symmetrically, and the separation of the patterns then has no effect on performance for transformation Pi and a small effect on performance for transformation Mi. Performance for transformation Ro shows no simple dependence on either symmetry or separation. Performance is highest for the Cc position combination, less for the Ec combination and lowest for the Es and Eo combinations. This suggests that it is more appropriate to consider performance for Ro as being determined by the mean distance of the patterns from the fixation point. In fact, d' for Ro shows a highly significant linear dependence on the mean distance ($P < 0{\cdot}001$, two-tailed test).

In this experiment the patterns under comparison were presented sequentially. It might be suggested that the findings are an artifact of the sequential presentation. For example, it might be argued that, in the Es and Cc position combinations, the first pattern provides an attentional cue for the second. Although this would account for the distance effect for transformation Id, it would not explain the absence of a distance effect for transformations Pi and Mi. More generally, it might be argued that the subject could systematically change his point of fixation between presentations. (There was insufficient time for directed shifts in fixation *during* the presentation of each pattern.) It should be noted, however, that the subject had no *a priori* knowledge of the position in which any pattern would appear, so he could not adopt any advantageous strategy of eye-movements. Experiment II demonstrates that the results are indeed not dependent on the sequential stimulus presentation.

Experiment II

In this second experiment, the two patterns for comparison were presented simultaneously. The duration of the display, 100 msec, was, as noted above, too short for voluntary changes in fixation during presentations (Westheimer, 1954; White and Eason, 1962; Bartz, 1962). Since two patterns could not be presented in the same field position, the replication was limited to the Eo and Ec position combinations.

Method

The subjects and method for this experiment were the same as in Experiment I except for the following:

(i) the distance of the eccentric positions of the patterns from the fixation point was increased to 1·0°, so that in the Ec position combination the patterns would be well separated;

(ii) the Es and Cc combinations were omitted.

Results

Figure 2 shows "same-different" pattern discrimination performance for Experiment II. The pooled discrimination index d' (see the Appendix) is plotted against pattern transformation for the position combinations Ec and Eo. In the Ec combination the pattern positions are asymmetric with respect to the point of fixation and the distance between the patterns is 1·0°. In the Eo position combination the pattern separation is 2·0° and the positions are symmetric with respect to the point of fixation. The increase in distance and symmetry from Ec to Eo reduces "same" detection performance for transformation Id ($P<0{\cdot}001$, one-tailed test) and increases performance for transformation Pi ($P<0{\cdot}01$, one-tailed test). There is no increase in performance for transformation Mi or reduction for transformation Ro ($P>0{\cdot}2$ for both, one-tailed tests).

It thus appears that the results of Experiment I are not a consequence of the difference in the times of presentations of patterns in a trial, or an artifact of eye-movements between the two presentations.

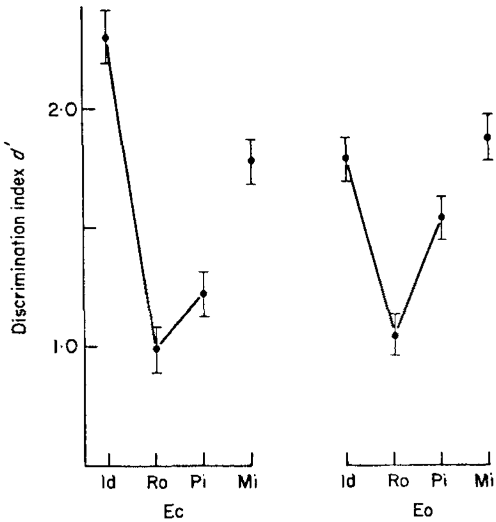

FIGURE 2. "Same" detection performance in Experiment II. The pooled discrimination index d' is plotted against pattern transformation, values Id, Ro, Pi and Mi, for the two combinations of pattern position: Ec: one pattern presented 1° to the left or right of fixation spot, the other central; Eo: one pattern presented 1° to the left of fixation spot, the other 1° to the right..

Discussion

In the experiments we have reported here, subjects made "same-different" judgements about pairs of patterns which were related by certain transformations. The patterns were presented in spatial arrangements which varied in the symmetry of the pattern positions with respect to the point of fixation and in the distance between the two patterns. The results have shown that separation and symmetry both influence "same" detection performance. To summarise:

(i) performance for identical patterns is markedly reduced by separation of the patterns;

(ii) performance for pairs of patterns related by reflection or by point-inversion, that is, rotation through 180° in the plane, is increased by symmetry of pattern position;

(iii) performance for patterns related by 90° planar rotation depends on mean eccentricity, that is, the mean distance of the pattern positions from the point of fixation.

In the light of these results, we now consider two current schemes for the modelling of visual recognition of patterns: one scheme involves a process of internal pattern transformation, and the other involves the encoding of patterns as structural descriptions. After making some plausible assumptions about the detection performance which might be expected for each of these schemes we show that neither is *naturally* compatible with the results. We then propose a scheme which combines elements of both the above schemes and which *is* compatible with the results.

In transformation schemes it is supposed that a stimulus is internally represented as an approximate point-for-point "image". These "images" can be modified, for the purposes of comparison, by internally effecting certain continuous families of transformations (Pitts and McCulloch, 1947; Shepard and Metzler, 1971; Marko, 1973; Foster and Mason, 1979). The "sameness" of two stimuli is assumed to be detected if it is possible to find a family of shape-preserving transformations which brings the two images into coincidence. It seems plausible that the efficiency of this matching operation should depend on the magnitude of the modification which has to be effected; if this is so, the probability of a correct "same" detection should decay with increasing size of objective transformation for which the system has to compensate. For translated patterns (labelled Id in this experiment) we have found such a decay with the separation of the patterns. Although the results for identical patterns are consistent with a transformation scheme, the "same" detection performance found here for reflected or point-inverted patterns is not naturally implied by such a scheme. If these transformations were compensated for by an appropriate family of rotations in three- or two-dimensions respectively, then one might expect the following:

(a) detection performance would fall off with separation of the patterns, since translations then have to be combined with rotations to bring the representations into coincidence;

(b) "sameness" of 180° rotated patterns would not be easier to detect than that of 90° rotated patterns under any arrangement of the pattern positions.

In the present experiments neither of these predictions has been fulfilled.

Next, consider structural-description schemes. Here the internal representation of a stimulus is thought to specify certain local pattern features in the stimulus and the spatial relations obtaining between these features (Sutherland, 1968, 1973; Barlow, Narasimhan and Rosenfeld, 1972). A local feature might consist of an edge or a corner, and a spatial relation might indicate that one feature is, for example, "left of" another, or "joined to" or "above". The "sameness" of two patterns is assumed to be evaluated by the extent to which their structural descriptions concur. By virtue of their discrete nature, such schemes are potentially capable of describing the observed elevation in detection performance for reflected or point-inverted patterns (Foster and Mason, 1979). This could be achieved, for example, by a simple relabelling of relations "left of" as "right of" and vice versa, in the case of reflected patterns. However, structural-description schemes fail to predict the marked effects on performance of positional symmetry and separation.

Thus, neither transformation schemes nor structural-description schemes as outlined above adequately explain the data, although clearly they each characterise the form of the results in some experimental conditions. In order to develop an adequate scheme, one must decide how position information might be encoded in the internal representation. The form of this encoding is suggested by the "same" detection performance for reflected or point-inverted patterns. Although performance for these patterns does not depend on their separation, it does depend on positional symmetry. Since symmetry must be defined with respect to a particular point in the visual field, it is reasonable to hypothesise that, within the internal representation, the position of the stimulus is expressed with respect to the point of fixation.

We propose the following recognition scheme which is neither exclusively structural nor transformational. Patterns are assumed to give rise to internal representations consisting of collections of elements specifying local pattern features, the spatial relations between these features, and the position of the pattern with respect to the point of fixation. Attneave (1968) has suggested a similar framework in which there are separate local and global cartesian axes in which the stimulus is represented. We suppose that there are two distinct types of operation which can be performed on the representation. First, any element of the representation can be modified, but this modification can be effected only in a progressive continuous fashion. Second, all the elements of a given kind can be relabelled in a single step. The renaming nature of the second operation means that it may be applied only to the representation as a whole, not to single elements of the representation. Each of these operations may be used in the comparison of two different internal representations, but the efficiency of the matching process depends on the extent of the modification required and on the number of different operations needed to bring the representations into coincidence.

Within this scheme, the results of the present experiment may be interpreted in the following way. (Note that the scheme includes no specific ability to respond to patterns which have been rotated through 90°.)* Pairs of "identical" patterns differing only in position are detected as "same" by continuous modification of the position component in the internal representations of the patterns, until the representations coincide. Increased pattern separation requires more modification of the position component before the match can be achieved, and so reduces the "same" detectability of the patterns.

Pairs of symmetrically positioned patterns which are related by point-inversion are detected as "same" by relabelling with an appropriate opposite term all those elements that specify spatial direction or sense in one of the internal representations. This brings the two representations into coincidence. Under this relabelling, the feature relation "above" becomes "below" and the component "1° to the left of the fixation point" becomes "1° to the right of the fixation point". If the two point-inverted patterns are not symmetrically positioned with respect to the point of fixation, this operation is not sufficient to bring the two representations into coincidence. In this case the "sameness" of the stimuli is less detectable, since further modification of the positional component of the representation is necessary to achieve a match.

*Our finding that "same" detection performance for 90° rotated patterns is on average lower than that for the other transformations is consistent with the results of Dearborn (1899), Rock (1973) and Foster (1978). The proposed scheme thus contains no specific capacity to detect the "sameness" of such stimuli. Sutherland (1973) has indicated that recognition of 90° rotated stimuli may not occur unless it is facilitated by special features in the stimulus. Such stimulus features may have occurred in these experiments. Subjects reported that it was easy to detect "sameness" in pairs of patterns which were elongated or which had a distinctive feature such as a spur or a cluster. These patterns were easily detected as "same" in any position combination or after any of the pattern transformations. This suggests that when such special features are present, detection might be achieved by a more direct non-structural feature-matching process, which would account for significant non-zero d' values for 90° rotated patterns and would imply a minimum level of correct response for all transformations.

Pairs of symmetrically positioned patterns which are related by reflection in a vertical line are detected as "same" by relabelling with an appropriate opposite term all those elements which specify horizontal direction or sense in one of the representations. This again brings the two representations into coincidence. Under this relabelling the feature relation "left of" becomes "right of" and the component "1° to the left of the fixation point" becomes "1° to the right of the fixation point". Relations such as "above" and "below" are unaffected. If the two reflected patterns are not positioned symmetrically with respect to the point of fixation this operation is not sufficient to bring the two representations into coincidence, which means that, as for point-inverted patterns, the "sameness" of these stimuli is less detectable than that of symmetrically positioned stimuli.

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Appendix

The scores for each subject were converted into the discrimination index d' using the false-alarm rate (that is, the proportion of incorrect "same" responses) from each of the position combinations to set the level for all the transformations presented in that combination. Variances were estimated using the method described by Gourevitch and Galanter (1967). The values of d' were then pooled across subjects.

The following statistical tests were performed on the data of Experiment I to investigate differences between subjects.

(i) *Chi-squared test for differences between subjects.* The discrimination indices d'_{ijk} and variances v_{ijk} were calculated where $i=1, \ldots, 5$ specifies the subject, $j=1, \ldots, 4$ specifies the pattern transformation, and $k=1, \ldots, 4$ specifies the position combination. Under the hypothesis that there are no differences between subjects' performances, the quantity

$$\chi^2 = \Sigma_{ijk} \, (d'_{ijk} - \bar{d}'_{.jk})^2 / v_{ijk},$$

where $\bar{d}'_{.jk} = \frac{1}{5} \Sigma_i \, d'_{ijk}$,
should be distributed as chi-squared with 64 degrees of freedom.

(ii) *Chi-squared test for differences between subjects allowing for each subject's overall performance level.* The mean performance level for each subject $\bar{d}'_{i..}=\frac{1}{16}\Sigma_{jk}\, d'_{ijk}$ was subtracted from his d' scores to give $e_{ijk}=d'_{ijk}-\bar{d}_{i..}$. Under the hypothesis that there are no differences between subjects' performances when each of these is expressed relative to the subject's mean performance level, the quantity

$$\chi^2=\Sigma_{ijk}\,(e_{ijk}-\bar{e}_{.jk})^2/v_{ijk}$$

should be distributed as chi-squared with 59 degrees of freedom.

For the data from Experiment I, test (ii) yielded no significant differences between subjects ($P>0{\cdot}5$) whereas test (i) yielded highly significant differences between subjects ($P<0{\cdot}001$). Similar tests on the data from Experiment II also yielded no significant differences in the type (ii) test ($P>0{\cdot}2$) and significant differences in the type (i) test ($P<0{\cdot}001$).

It follows that although subjects' overall abilities to detect the "sameness" of transformed patterns are not identical, the form of the effects of the experimental treatments is similar for all subjects.

The tests quoted in the text were trend analyses and, where appropriate, contrast tests as described by Lindman (1974). These were performed using the standard normal variable

$$z_{jk}=\bar{d}'_{.jk}/(\tfrac{1}{5}\Sigma_i\, v_{ijk})^{\frac{1}{2}}$$